

Formula $E = mc^2$ in the Year of Physics

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Year of Physics

2005 is the first year of physics in the history of humankind. It celebrates the papers on relativity, quanta, and atoms, published in 1905 by Einstein. The whole building of modern physics has these papers at its basis.

Rest Energy E_0

1905: Einstein introduced the notion of rest energy E_0

$$\Delta mc^2 = \Delta E_0$$

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According to Special Relativity

$$(mc^2)^2 = E^2 - \mathbf{p}^2 c^2$$

$(E, \mathbf{p}c)$ – Lorentz 4-vector

m – Lorentz scalar

$E^2 - (\mathbf{p}c)^2 = (mc^2)^2$, E – energy, \mathbf{p} – momentum, m – mass.

$$E = mc^2\gamma, \quad \mathbf{p} = m\mathbf{v}\gamma, \quad \gamma = 1/\sqrt{1 - \mathbf{v}^2/c^2},$$

$$\begin{aligned} E^2 - \mathbf{p}^2c^2 &= m^2c^4\gamma^2 - m^2c^2\mathbf{v}^2\gamma^2 = \\ &= m^2c^4(1 - \mathbf{v}^2/c^2)^{-1}(1 - \mathbf{v}^2/c^2) = m^2c^4 \end{aligned}$$

For a free body at rest $\mathbf{p} = 0$, $\mathbf{v} = 0$, $E_0 = mc^2$, where E_0 – rest energy.

“The famous formula” $E = mc^2$ was wrong in 1905 and even more so in 2005!

$E = Mc^2$ is misleading. M is redundant,
 $M = m\gamma$, M is sometimes called “relativistic mass”.
Einstein never used this term.

$$E_0 = mc^2$$

Mass m – relativistically invariant!

Origin of $E = mc^2$

1900: Poincaré “proved” that mass of a pulse of the light is proportional to its energy.

Poincaré misused non-relativistic Newton’s relation

$$p = mv$$

at $v = c$. And from Poynting relation

$$p = E/c$$

derived $E = mc^2$.

Longitudinal, transverse masses

1899: Lorentz introduced two masses of electron, which depend on its velocity

$$\mathbf{F} \perp \mathbf{v} : \quad \mathbf{F} = m_t \mathbf{a}$$

$$\mathbf{F} \parallel \mathbf{v} : \quad \mathbf{F} = m_l \mathbf{a}$$

$$m_t = m\gamma$$

$$m_l = m\gamma^3$$

$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

Childhood of $E = mc^2$

Influential papers:

Gilbert Lewis and Richard Tolman (1909)

R. Tolman (1912)

Influential monographs:

Vladimir Fock (1955, 1961, 1967, in Russian)

Max Born (1920, 1922, 1964 in German; 1962, 1965 in English)

As for Einstein, he “oscillated” between $E_0 = mc^2$ and $E = mc^2$ till 1921.

Evolution of concept of matter

At present matter includes massless and extremely light particles: photon and neutrinos.

The masses of relativistic particles are not additive, while their energies are.

Mass of the system of two photons: $m^2 = (p_1 + p_2)^2$, where $p = (E, \mathbf{p})$.

$$m = 0, \quad \text{if } \mathbf{p}_1 = \mathbf{p}_2 ;$$

$$m = 2|\mathbf{p}_1|, \quad \text{if } \mathbf{p}_1 = -\mathbf{p}_2 .$$

Example: Photosynthesis

The light from the sun is absorbed by vegetation on the earth to produce carbohydrates via reaction of photosynthesis:



The total energy of photons required to produce one molecule of $\text{C}_6\text{H}_{12}\text{O}_6$ is about 4.9 eV.

The kinetic energy of photons is transformed into the rest energy of carbohydrates.

$E = mc^2$ in the Year of Physics

Revival of $E = mc^2$ and of “relativistic mass”:
 $m_r = m_0 / \sqrt{1 - v^2/c^2}$, where m_0 is “rest mass”.

“Scientific American” September 2004 full of these notions.

Many other magazines, journals and books in 2005, e.g. “Physics World”, January 2005.

Max Jammer: “Concepts of Mass in Contemporary Physics and Philosophy”. Princeton, 2000.

Roger Penrose. The Road to Reality. A complete guide to the laws of the universe. A.A. Knopf, New York, 2005.

Who is guilty?

Of course, light-minded journalists.

But first of all, renowned professors of physics, who promote $E = mc^2$ and relativistic mass as authors, lecturers, and members of editorial boards.

They try to conform the prevailing opinions of ignorant readers, instead of educating them.

Niels Bohr on truth and clarity

Niels Bohr once said that the antonym to truth is clarity. A true statement cannot be clear, and a clear one cannot be true. This maxim might be valid for the deepest truths at the front line of science, but it should be applied with some reservation in the fields, such as Special Relativity, behind the front line, where everything is firmly established.

Still many authors consider that to be clear is “politically incorrect”.

It seems that this belief is the basis of a longstanding confusion which surrounds the relation between energy and mass.

What to do?

To reach a consensus in the community of experts in Relativity Theory on the concept of unique relativistically invariant mass, m .

Experts should discard from their writings the terms “rest mass” and “relativistic mass” and the famous but wrong formula

$$E = mc^2 .$$

The rest energy should be promoted:

$$E_0 = mc^2 .$$

Those who are indifferent

All true believers break their eggs at the convenient end.

J. Swift,
Gulliver's Travels,
big-endians,
small-endians,
lilliput.