Quark Mixing and CP Violation

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The Standard Model

Gauge group: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_\mathrm{Y} \quad \xrightarrow{\textbf{SSB}} \quad \mathrm{SU}(3)_c \times \mathrm{U}(1)_\mathrm{em}$

$$\mathcal{L}_\mathrm{SM} = \underbrace{\mathcal{L}_{\substack{\text{kinetic}\\ \text{\& gauge}}} + \mathcal{L}_\mathrm{Yukawa} - \underbrace{V(h)}_{\text{Higgs potential}}}_{\textbf{dimension-four part}} + \underbrace{\mathcal{O}\left(\frac{1}{\Lambda_\mathrm{Huge}}\right)}_{\substack{\text{neutrino masses,}\\ \text{gravity, ...}}} \qquad \langle 0|h|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix}$$

Fermion fields: $\begin{pmatrix} u_L^j \\ d_L^j \end{pmatrix}, \; u_R^j, \; d_R^j, \; \begin{pmatrix} \nu_L^j \\ e_L^j \end{pmatrix}, \; e_R^j \qquad j = 1, 2, 3.$

$$\mathcal{L}_\mathrm{Yukawa} \quad \xrightarrow{\textbf{SSB}} \quad \mathcal{L}_{\substack{\text{fermion}\\ \text{mass}}} + \mathcal{L}_{\substack{\text{fermion}-\text{higgs}\\ \text{interactions}}}$$

$$\mathcal{L}_{\substack{\text{fermion}\\ \text{mass}}} = -\frac{v}{\sqrt{2}}\left(\bar{u}_R^i Y_u^{ij} u_L^i + \bar{d}_R^i Y_d^{ij} d_L^j + \bar{e}_R^i Y_e^{ij} e_L^j + \text{h.c.} \right)$$

Y_u, Y_d, Y_e : **complex** 3×3 **Yukawa coupling matrices. They are the only source of CP-violation in the dimension-four part of the SM Lagrangian.**

The CP transformation maps the SM fields onto themselves:

$$\psi(t,\vec{x}) \xrightarrow{\text{CP}} \gamma_0 C \bar{\psi}^T(t,-\vec{x}), \qquad C^\dagger \gamma_\mu C = -\gamma_\mu^T$$

$$h(t,\vec{x}) \xrightarrow{\text{CP}} h^\star(t,-\vec{x}),$$

$$A_0^a(t,\vec{x}) \xrightarrow{\text{CP}} \mp A_0^a(t,-\vec{x}), \qquad (T^a)^\star = \pm T^a$$

$$\vec{A}^a(t,\vec{x}) \xrightarrow{\text{CP}} \pm \vec{A}^a(t,-\vec{x}).$$

When applied to free fields, it is identified as mirror reflection combined with simultaneous interchange of particles and antiparticles.

The dimension-four part of the SM action would be CP-invariant if (and only if) the Yukawa couplings contained no physical phases.

$$\mathcal{L}_{\substack{\text{fermion}\\ \text{mass}}} = -\frac{v}{\sqrt{2}}\left(\, \bar{u}_R^i Y_u^{ij} u_L^i \;+\; \bar{d}_R^i Y_d^{ij} d_L^j \;+\; \bar{e}_R^i Y_e^{ij} e_L^j \;+\; \text{h.c.}\,\right).$$

The fermion mass terms are diagonalized via chiral rotations, e.g. $u_L^{\text{new}} = S_{uL} u_L^{\text{old}}$**.**
The resulting 3×3 **diagonal fermion mass matrices read:**

$$M_u = S_{uR} Y_u S_{uL}^\dagger, \qquad M_d = S_{dR} Y_d S_{dL}^\dagger, \qquad M_e = S_{eR} Y_e S_{eL}^\dagger.$$

$$\mathcal{L}_{\substack{\text{fermion}\\ \text{mass}}} = -\left(\, \bar{u}_R^i M_u^{ij} u_L^i \;+\; \bar{d}_R^i M_d^{ij} d_L^j \;+\; \bar{e}_R^i M_e^{ij} e_L^j \;+\; \text{h.c.}\,\right).$$

At the same time, the W-boson couplings to quarks become flavour off-diagonal:

$$W^- \bar{u}_L^i d_L^i \to W^- \bar{u}_L^i V_{ij} d_L^j \qquad V = S_{uL} S_{dL}^\dagger$$ – **unitary Cabibbo-Kobayashi-Maskawa matrix**

Standard parametrization of the CKM matrix V.

Three angles θ_{ij} and one phase δ. Notation: $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$.

$$V \equiv \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Introducing the Wolfenstein parameters:

$\lambda = s_{12} \simeq 0.22$, $\quad A = \frac{s_{23}}{s_{12}^2}$, $\quad \rho = \frac{s_{13}}{s_{23}s_{12}}\cos\delta$, $\quad \eta = \frac{s_{13}}{s_{23}s_{12}}\sin\delta$.

Then $c_{12} = \sqrt{1-\lambda^2}$, $\quad c_{23} = \sqrt{1-A^2\lambda^4}$, $\quad c_{13} = \sqrt{1-A^2\lambda^6(\rho^2+\eta^2)}$,

$$V = \begin{pmatrix} c_{12}c_{13} & \lambda c_{13} & A\lambda^3(\rho - i\eta) \\ -\lambda c_{23} - A^2\lambda^5(\rho+i\eta)c_{12} & c_{12}c_{23} - A^2\lambda^6(\rho+i\eta) & A\lambda^2 c_{13} \\ A\lambda^3[1 - c_{12}c_{23}(\rho+i\eta)] & -A\lambda^2 c_{12} - A\lambda^4(\rho+i\eta)c_{23} & c_{23}c_{13} \end{pmatrix}$$

$$\simeq \begin{pmatrix} 1-\frac{\lambda^2}{2}+\mathcal{O}(\lambda^4) & \lambda+\mathcal{O}(\lambda^7) & A\lambda^3(\rho-i\eta) \\ -\lambda+\mathcal{O}(\lambda^5) & 1-\frac{\lambda^2}{2}+\mathcal{O}(\lambda^4) & A\lambda^2+\mathcal{O}(\lambda^8) \\ A\lambda^3(1-\bar\rho-i\bar\eta)+\mathcal{O}(\lambda^7) & -A\lambda^2+\mathcal{O}(\lambda^4) & 1+\mathcal{O}(\lambda^4) \end{pmatrix},$$

where $\bar\rho = \left(1-\frac{\lambda^2}{2}\right)\rho$ **and** $\bar\eta = \left(1-\frac{\lambda^2}{2}\right)\eta$.

$V^\dagger V = 1 \;\Rightarrow$ **6 unitarity triangles, e.g.** $V_{ub}^\star V_{ud} + V_{cb}^\star V_{cd} + V_{tb}^\star V_{td} = 0$.

$$A\lambda^3(\bar\rho+i\bar\eta) - A\lambda^3 + A\lambda^3(1-\bar\rho-i\bar\eta) = \mathcal{O}(\lambda^7)$$

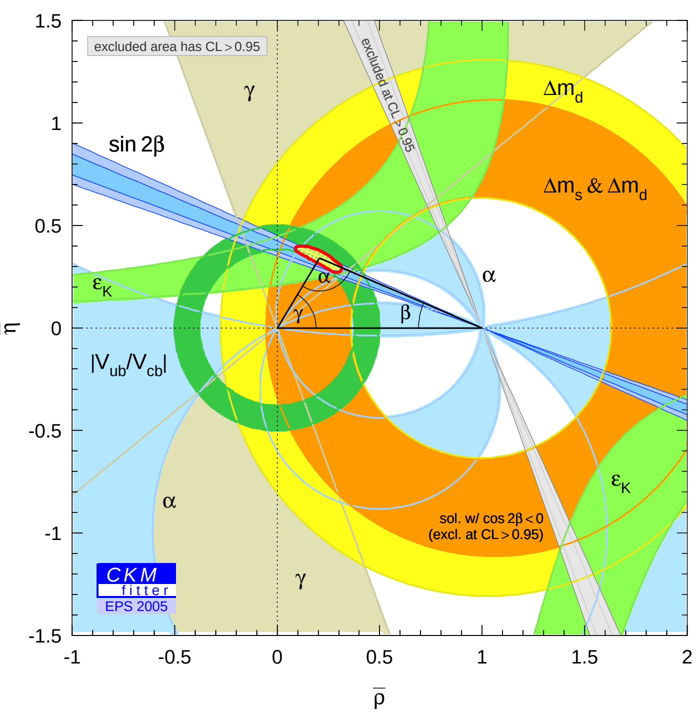
Current results for the CKM parameters

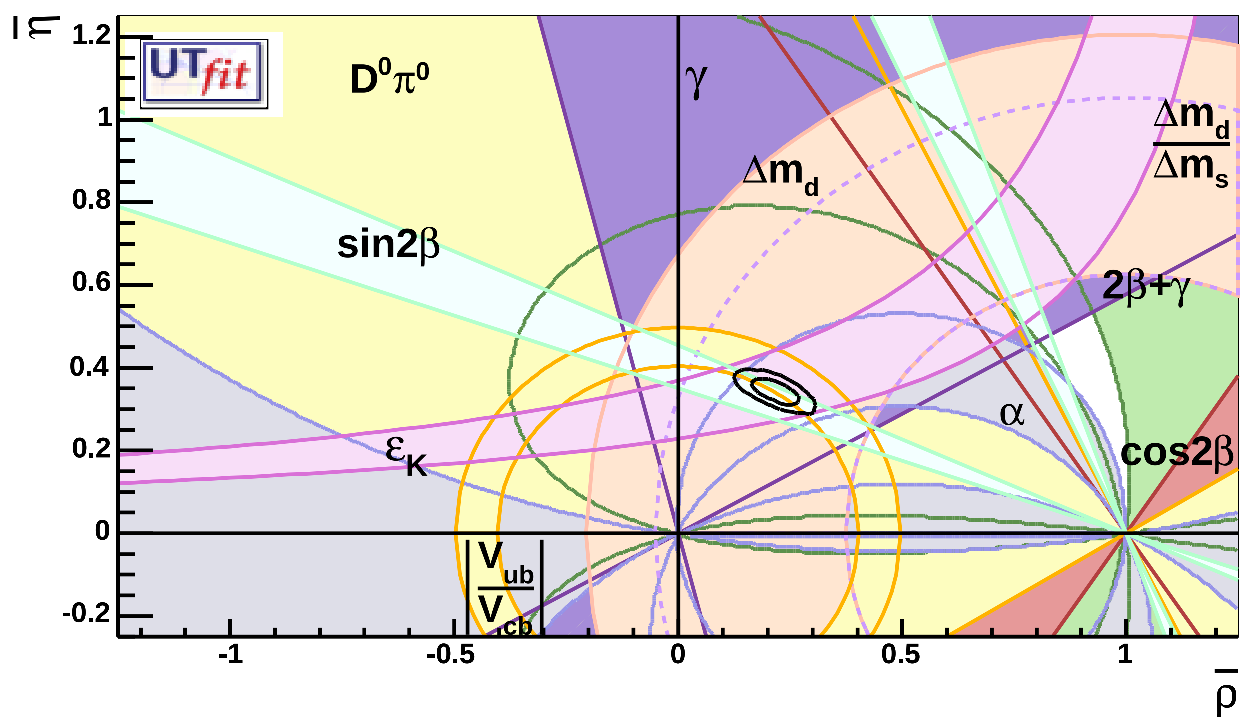
	λ	A	$\bar{\rho}$	$\bar{\eta}$
CKM Fitter (ferquentist)	0.2262 ± 0.0010	$0.825^{+0.011}_{-0.019}$	$0.207^{+0.036}_{-0.043}$	0.340 ± 0.023
UTfit (bayesian)	0.2258 ± 0.0014	0.816 ± 0.017	0.216 ± 0.036	0.342 ± 0.022

from K and τ decays ($|V_{us}|$)

from $b \to c$ decays ($|V_{cb}|$)

$b \to u$ decays ($|V_{ub}|$)
CP-violation in $K^0\bar{K}^0$ mixing (ε_K)
Mixing-induced CP-violation in B_d decays ($\sin 2\beta$)
$B^0 - \bar{B}^0$ mass differences (Δm_d, Δm_s)
CP-violation in exclusive B_d decays (α, β, γ)
Rare $b \to s, d$ decays and $K \to \pi\nu\bar{\nu}$
Direct CP-violation in K decays (ε'/ε)

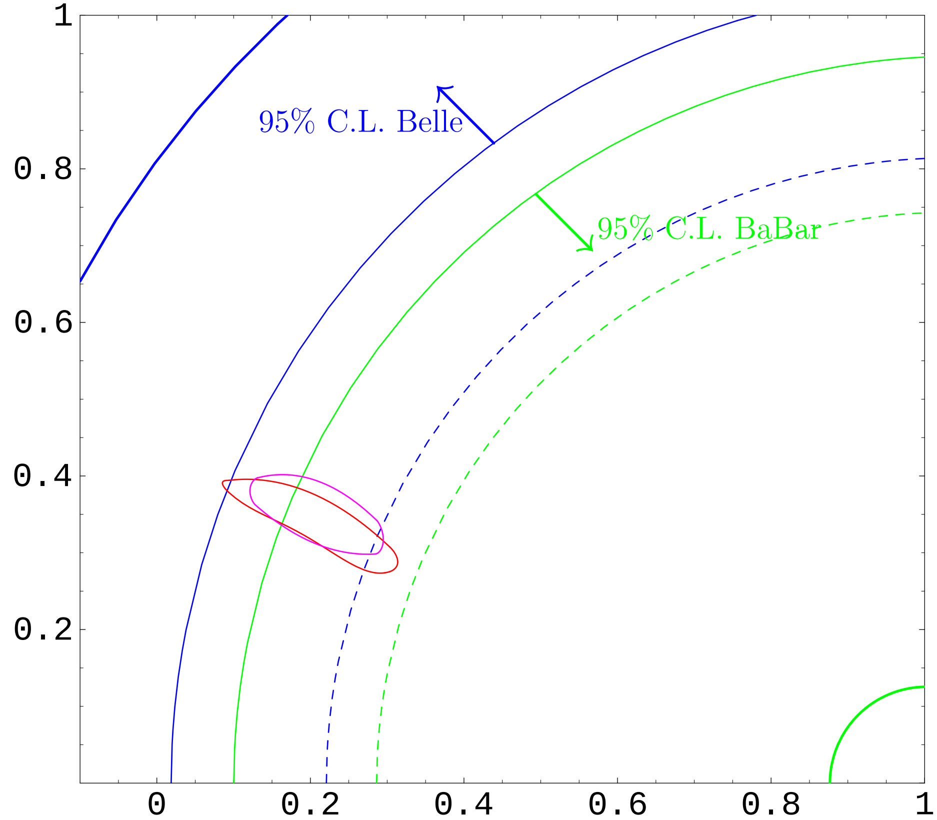




Constraints in the $\bar{\rho} - \bar{\eta}$ plane from $\Gamma(B_d^0 \to \rho^0\gamma)/\Gamma(B_d^0 \to K^\star\gamma)$

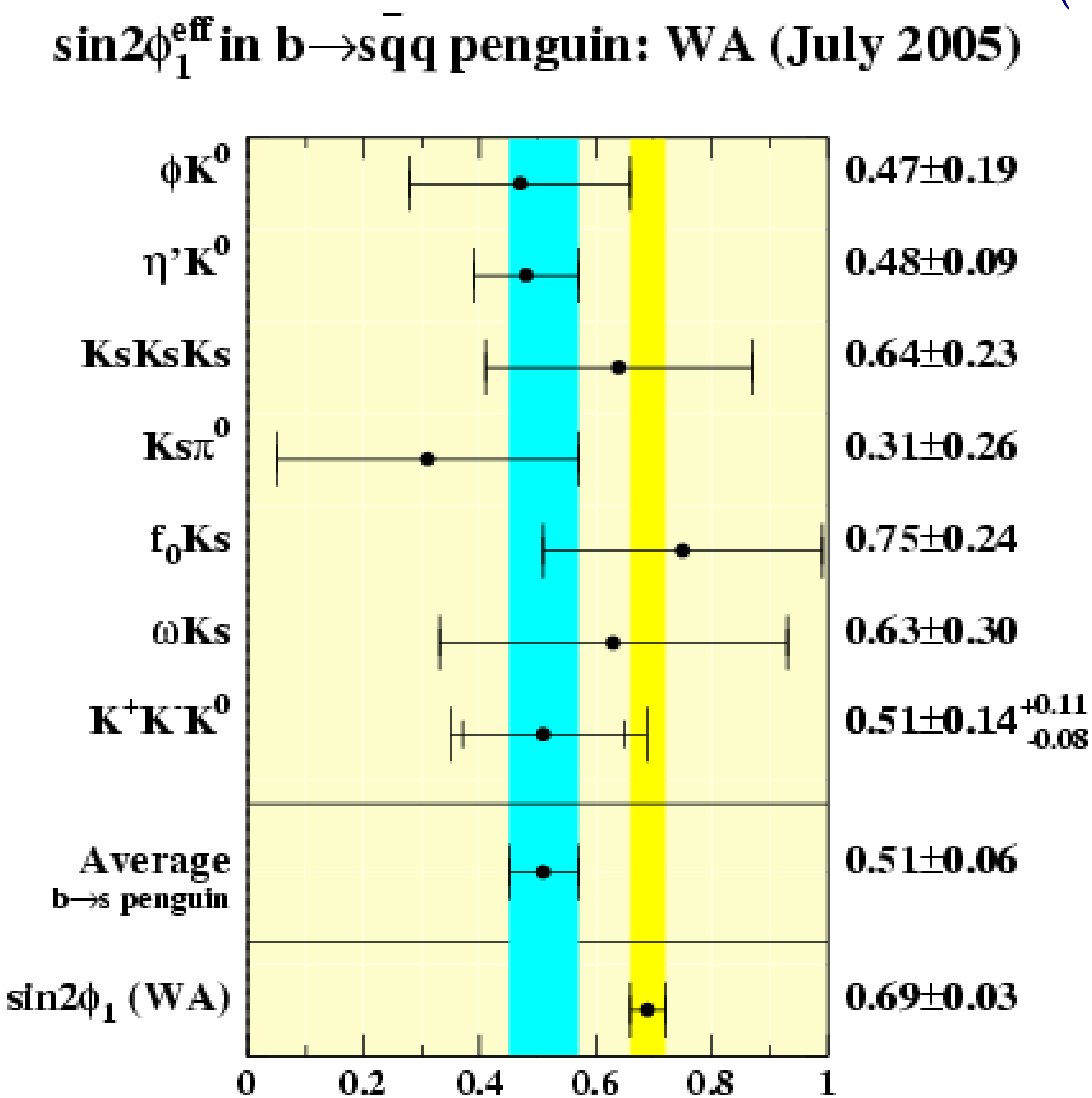
Mode	Belle	BaBar	Combined
$B^0 \to \rho^0\gamma$	$1.17\ ^{+0.35}_{-0.31}\ ^{+0.09}_{-0.08}$ $<??$	$0.01^{+0.22}_{-0.16} \pm 0.05$ < 0.36	$0.30^{+0.19}_{-0.15}$ < 0.61
$B^\pm \to \rho^\pm\gamma$	$0.55\ ^{+0.43}_{-0.37}\ ^{+0.12}_{-0.11}$ $<??$	$0.92^{+0.55}_{-0.50} \pm 0.13$ < 1.76	$0.69^{+0.35}_{-0.31}$ < 1.26
$B^0 \to \omega\gamma$	$0.58\ ^{+0.35}_{-0.27}\ ^{+0.07}_{-0.08}$ $<??$	$0.46^{+0.31}_{-0.25} \pm 0.10$ < 0.97	$0.52^{+0.24}_{-0.19}$ < 0.91
$B \to (\rho,\omega)\gamma$ averaged	$1.34\ ^{+0.34}_{-0.31}\ ^{+0.14}_{-0.10}$ $<??$	$0.64^{+0.32}_{-0.28} \pm 0.10$ < 1.16	$0.96^{+0.25}_{-0.22}$ < 1.36

Belle and BaBar results for the branching ratios of $B \to (\rho,\omega)\gamma$ decays in units 10^{-6}. Averaging charge conjugate modes is implicitly assumed. The corresponding 90% CL upper bounds are indicated below each result. In their combination, Gaussian errors have been assumed.



$''\sin 2\phi_1''$ from *hadronic $b \to s$ penguins*

(BaBar + Belle)



Deviation between $\sin 2\phi_1(b \to c\bar{c}s)$ and $\sin 2\phi_1(b \to s\bar{q}q)$ is 2.6σ using a naive average…

Summary

(i) All the experimental results up-to-date are in agreement with the SM description of quark mixing and CP-violation. In particular, the single CKM phase δ is sufficient to describe all the measured CP-violating observables and, in addition, survives constraints from CP-conserving quantities.

(ii) Between 1964 and 1988, the CP-violation was observed only in $K^0\bar{K}^0$ mixing. The direct CP-violation in K-decays ($\varepsilon'/\varepsilon \neq 0$) was observed in 1988 (CERN) and 1999 (FNAL). CP-violation in $B_d^0\bar{B}_d^0$ mixing was established in 2000. In 2004/2005, direct CP-violation in B decays was established through measurements of γ and α in various exclusive modes.

(iii) Former ideas that the bulk of CP-violaton may come from beyond the SM are now out-of-date.