# Conformal Symmetry and the Standard Model

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# Hierarchy Problem

- **Fact: Standard Model (= SM) of elementary particle physics is conformally invariant except for tree level mass term $-m^2\Phi^\dagger\Phi$ $\rightarrow$ masses for vector bosons, quarks and leptons (via *Brout-Englert-Higgs* mechanism).**

- **Quantum corrections $m^2 \sim \Lambda^2 \Rightarrow$ why $m_H \ll M_P$? (with UV cutoff $\Lambda$ = scale of 'new physics')**
  - ***explanation of hierarchy?***
  - ***stabilization of hierarchy?***

- **Most popular proposal: SM $\longrightarrow$ MSSM or NMSSM: use supersymmetry to control quantum corrections via cancellation of quadratic divergences $\Rightarrow$**

$$m^2 \sim \Lambda_{SUSY}^2 \ln(\Lambda^2/\Lambda_{SUSY}^2)$$

# Coleman-Weinberg Mechanism (1973)

- **Idea: spontaneous breaking of conformal symmetry by quantum corrections $\Longrightarrow$ small mass scales arise via *conformal anomaly* and *effective potential***

$$\lambda\varphi^4 \to V_{\rm eff}(\varphi) = \lambda\varphi^4 + \frac{\lambda^2\varphi^4}{64\pi^2}\left(\ln\left(\frac{\lambda\varphi^2}{v^2}\right) - \frac{1}{2}\right)$$

- **In its original form this proposal does not work:**
  – **Higgs mass too small ($\sim \mathcal{O}(10\,\mathrm{GeV})$), or**
  – **Scalar self-couplings too large $\to$ Landau poles**

[See e.g.: Sher, Phys.Rep.179(1989)273; Ford,Jones,Stephenson,Einhorn, Nucl.Phys.B395(1993)17]

- **Also, must accommodate:**
  – $m_H > 115\,\mathrm{GeV}$ **and** $m_{top} = 174\,\mathrm{GeV}$
  – $m_\nu < 1\,\mathrm{eV}$ $\quad\to$ ***large intermediate scale*?**

# Our Proposal [= hep-th/0612165]

- Classical conformal symmetry (i.e. no tree level mass terms) in SM
- plus right-chiral neutrinos
- plus enlarged scalar sector: $\Phi$ and $\varphi$
- All mass scales from effective (CW) potential
- all coupling constants small and positive up to $M_{Pl}$
- No large intermediate scales $\Longrightarrow$
  - no grand unification (GUTs)
  - no new scales required to explain $m_\nu < 1$ eV
- No low energy SUSY

# The Model

- **Start from conformally invariant (and therefore renormalizable) Lagrangian $\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}'$ with:**

$$\mathcal{L}' := \Big(\bar{L}^i \Phi Y_{ij}^E E^j + \bar{Q}^i \epsilon \Phi^* Y_{ij}^D D^j + \bar{Q}^i \epsilon \Phi^* Y_{ij}^U U^j + \\ + \bar{L}^i \epsilon \Phi^* Y_{ij}^\nu \nu_R^j + \varphi \nu_R^{iT} \mathcal{C} Y_{ij}^M \nu_R^j + \text{h.c.}\Big) - \\ - \frac{\lambda_1}{4} (\Phi^\dagger \Phi)^2 - \frac{\lambda_2}{2} \varphi^2 (\Phi^\dagger \Phi) - \frac{\lambda_3}{4} \varphi^4$$

- **Besides usual $SU(2)$ doublet $\Phi$: new scalar field $\varphi$ (here taken to be *real*, but could be complex and in non-trivial representation of family symmetry)**
- **No mass terms, all coupling constants dimensionless**
- **$Y_{ij}^U$, $Y_{ij}^E$, $Y_{ij}^M$ real and diagonal**
  **$Y_{ij}^D$, $Y_{ij}^\nu$ complex $\rightarrow$ parametrize family mixing**

# Effective Potential

$$\begin{aligned} V_{\text{eff}}(H,\varphi) \;=\; & \frac{\lambda_1 H^4}{4} + \frac{\lambda_2 H^2 \varphi^2}{2} + \frac{\lambda_3 \varphi^4}{4} \\ & + \frac{3}{256\pi^2}(\lambda_1 H^2 + \lambda_2 \varphi^2)^2 \ln\left[\frac{\lambda_1 H^2 + \lambda_2 \varphi^2}{v^2}\right] \\ & + \frac{1}{64\pi^2} F_+^2 \ln\left[\frac{F_+}{v^2}\right] + \frac{1}{64\pi^2} F_-^2 \ln\left[\frac{F_-}{v^2}\right] \\ & - \frac{6}{32\pi^2} g_t^4 (H^2)^2 \ln\left[\frac{H^2}{v^2}\right] - \frac{1}{32\pi^2} g_M^4 \varphi^4 \ln\left[\frac{\varphi^2}{v^2}\right] \end{aligned}$$

$g_M^4 \equiv \mathrm{Tr}\, Y_M^4$, $v$ **dimensionful scale (conformal anomaly)**
**Not included: contributions from** $SU(2)_w \times U(1)_Y$ **gauge fields (because respective gauge couplings are small), nor from** $SU(3)_c$ **(because it is a two-loop effect)**

**Search for minima: must perform *numerical analysis*!**

# Numerics

**Choice of parameters is *strongly constrained* by experimental data and RGE analysis $\rightarrow$ 'trial and error' method leads to following choice of parameters**

$$\lambda_1 = 3.4\,, \quad \lambda_2 = 2.6\,, \quad \lambda_3 = 3.3\,, \quad g_t = 1\,, \quad g_M^2 = 0.4$$

**Minimum lies at**

$$\langle H \rangle = 0.415 \cdot 10^{-5}\, v\ , \quad \langle \varphi \rangle = 2.506 \cdot 10^{-5}\, v$$

**Normalize this by setting** $\langle H \rangle = 174$ **GeV** $\quad \Rightarrow$

$$H' = H\cos\beta + \varphi\sin\beta\ , \qquad \varphi' = -H\sin\beta + \varphi\cos\beta$$

$$m_{H'} = 217 \text{ GeV}, \qquad m_{\varphi'} = 439 \text{ GeV}; \qquad \sin\beta = 0.119$$

**'Higgs mixing': only the components along $H$ of the mass eigenstates couple to the usual SM particles.**

**NB: Not (yet?) a definitive prediction.**

# Renormalization Group Equations

**Effective couplings (≡ 4th derivatives at minimum):**

$$\lambda_1^{\rm eff} = 1.463, \quad \lambda_2^{\rm eff} = 0.348, \quad \lambda_3^{\rm eff} = 0.626$$

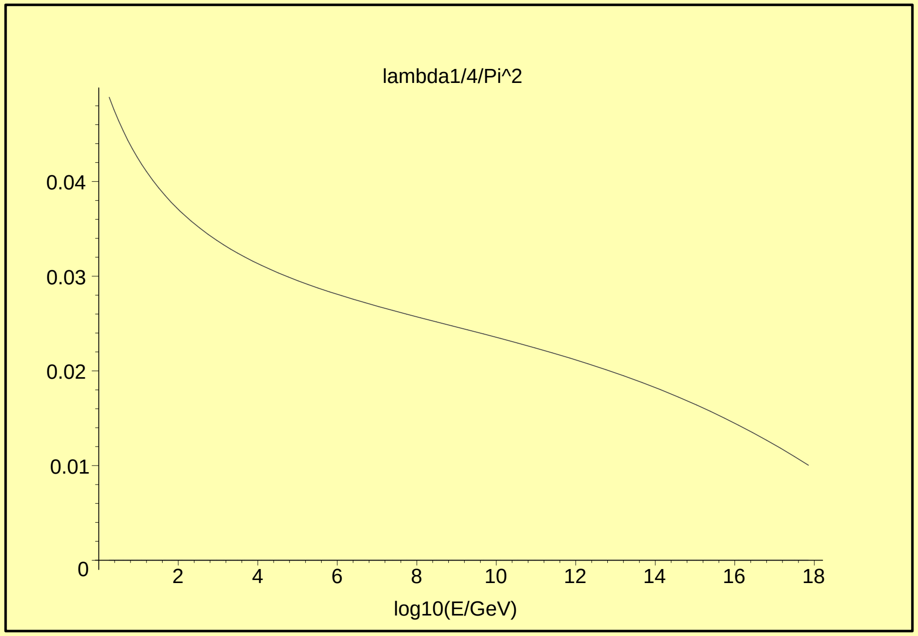
**With**

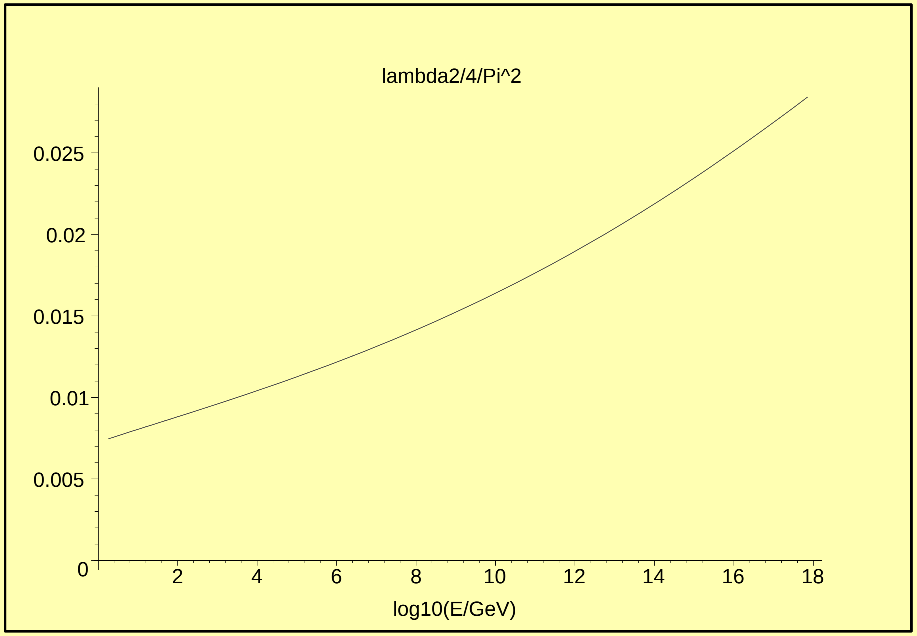
$$y_1 = \frac{\lambda_1^{\rm eff}}{4\pi^2},\ y_2 = \frac{\lambda_2^{\rm eff}}{4\pi^2},\ y_3 = \frac{\lambda_3^{\rm eff}}{4\pi^2},\ x = \frac{g_t^2}{4\pi^2},\ u = \frac{g_M^2}{4\pi^2},\ z = \frac{\alpha_s}{\pi}$$

**we get**

$$\begin{aligned}
\mu\frac{\mathrm{d}y_1}{\mathrm{d}\mu} &= \frac{3}{2}y_1^2 + \frac{1}{8}y_2^2 - 6x^2,\\
\mu\frac{\mathrm{d}y_2}{\mathrm{d}\mu} &= \frac{3}{8}y_2(2y_1 + y_3 + \frac{4}{3}y_2),\\
\mu\frac{\mathrm{d}y_3}{\mathrm{d}\mu} &= \frac{9}{8}y_3^2 + \frac{1}{2}y_2^2 - u^2, \quad \mu\frac{\mathrm{d}u}{\mathrm{d}\mu} = \frac{3}{4}u^2\\
\mu\frac{\mathrm{d}x}{\mathrm{d}\mu} &= \frac{9}{4}x^2 - 4xz, \qquad \mu\frac{\mathrm{d}z}{\mathrm{d}\mu} = -\frac{7}{2}z^2.
\end{aligned}$$

# Evolution of Coupling Constants



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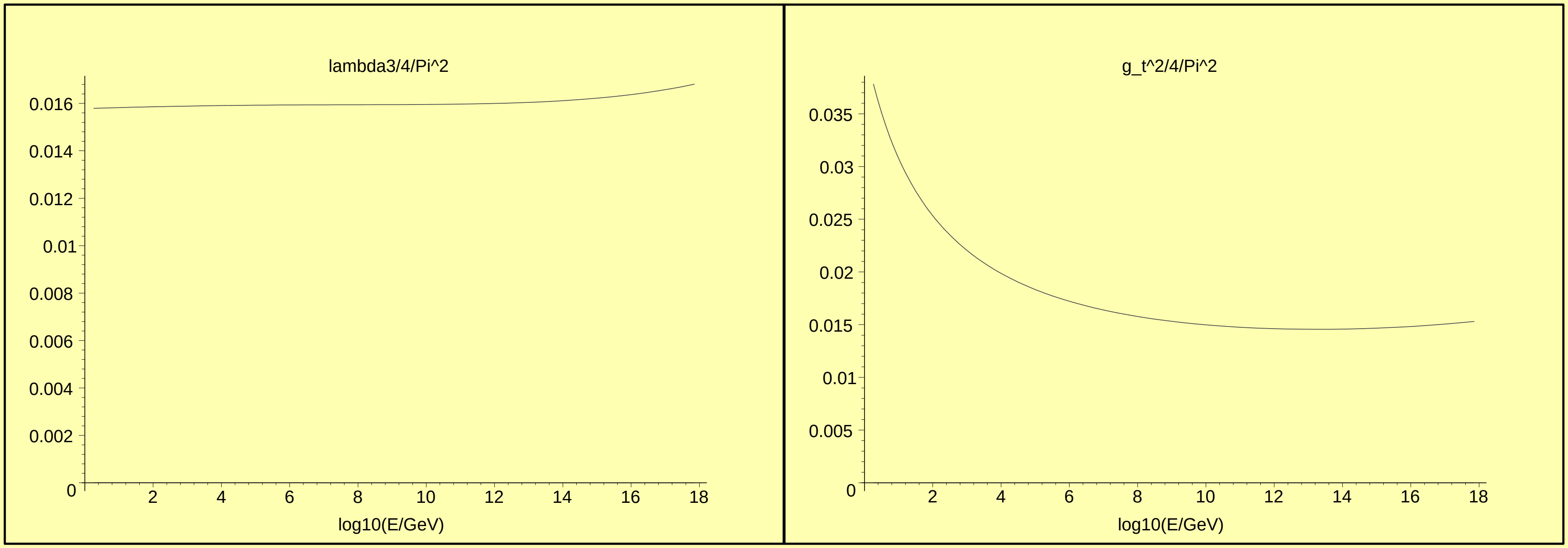


**Important: top Yukawa stays bounded up to $M_{Pl}$ due to large $\alpha_s$!**

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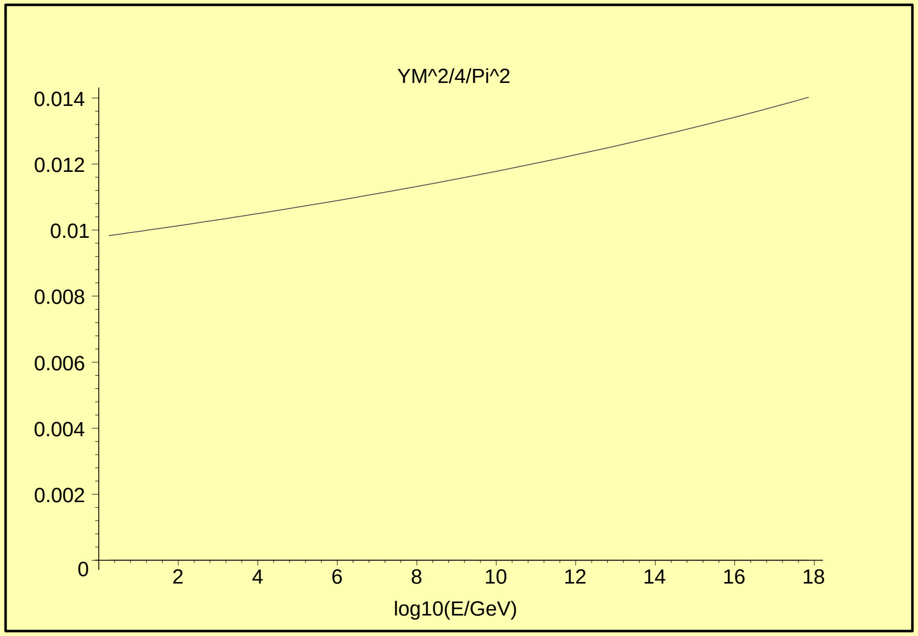
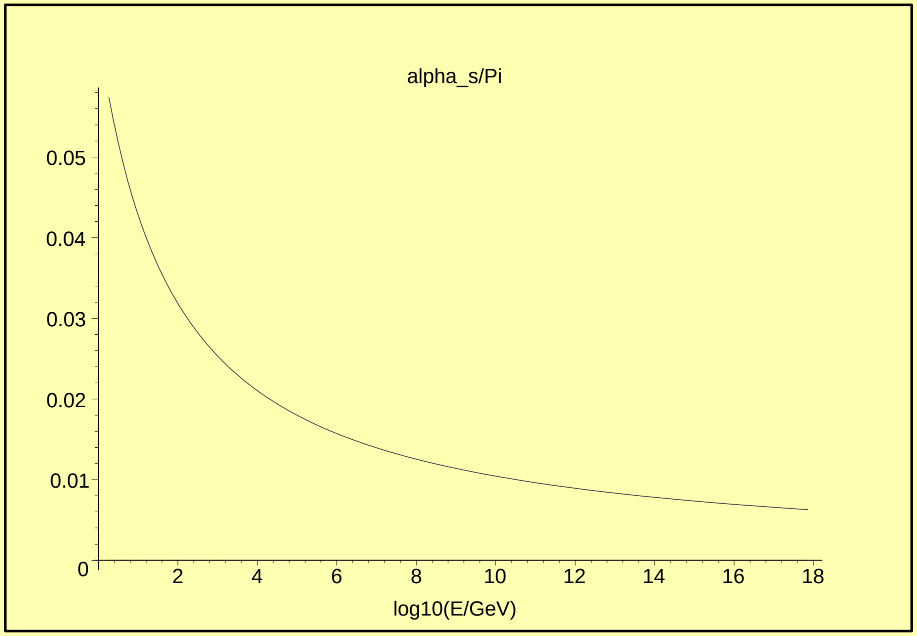
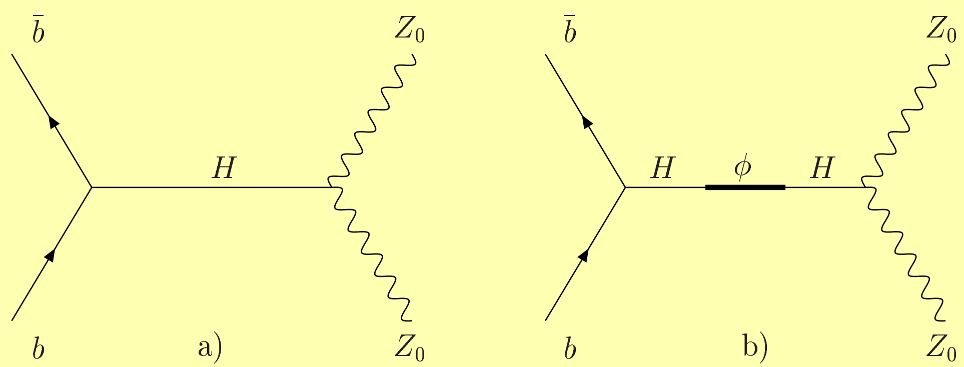


# Phenomenology

- **New scalar is a 'heavier brother' of the SM Higgs (the same BRs except for the mass difference and lower cross sections)** **unique signature**



- **if $m_\phi > 2m_Z$ a clear signal in LHC**
- **no other new particles at LHC except standard Higgs and a new scalar (scalars?)**
- **if $|\sin(\beta)| < 0.1$ then no contradiction with LEP precision data**

# Discussion and outlook

- **conformal symmetry is a different (much simpler than SUSY...) explanation of hierarchy of scales**
- **SM with massive neutrinos and the new scalar can be viable up to the Planck scale so LHC may see just the SM Higgs and (several?) new scalar particles...**
- **LHC will tell...**