Neutrino Masses and Supersymmetry

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1. Introduction: neutrino mass models
   - common approach - heavy right neutrinos and see-saw mechanism
   - supersymmetric modifications/alternatives: with and without superheavy particles
2. Supersymmetrized heavy sector: right sneutrinos and their effects
3. Alternative to heavy right sector: R-parity and Lepton Flavour Violation (LFV)
4. Conclusions

1. Introduction

No FCNC processes with charged leptons observed, but numerous experiments measuring neutrino oscillation has been made or are currently collecting data. Brief summary based on Particle Data Group review:

- Gallium ($E_\nu \geq 0.2$ MeV), chlorine ($E_\nu \geq 0.8$ MeV) and Cherenkov (water) detectors ($E_\nu \geq 5$ MeV) provide a more than 5σ evidence of oscillation of solar produced electron neutrinos $\nu_e \to \nu_{\mu,\tau}$. A global analysis including reactor experiments (KAMLAND) gives neutrino mass splitting $\Delta(m^2) \approx 6 - 9 \cdot 10^{-5}\ eV^2$ and large mixing angle.
- Underground detectors observing neutrino produced by cosmic rays in the atmosphere measure ν_μ/ν_e ratio much less than expected. This can be explained by $\nu_\mu \to \nu_\tau$ oscillations with $\Delta(m^2) \approx 1-3 \cdot 10^{-3}\ eV^2$ and almost maximal mixing angle of ν_μ and ν_τ. The effect has been confirmed by K2K experiment with accelerator neutrinos.

Major experimental breakthrough - massive neutrinos require going beyond the minimal version of the SM!

Neutrino masses by many orders of magnitude lighter then any other fermions. Why? Theoretical explanation(s) necessary.

Common approach: “see-saw mechanism”.

Assume existence of the right handed neutrinos. Then, one can add neutrino Yukawa coupling to the Lagrangian. After electroweak symmetry breaking and replacing Higgs field by the VEV one gets the neutrino mass term (“Dirac” mass):

$$Y_\nu^{IJ} H_i^\star l_i^I \nu_R^J \quad \rightarrow \quad \frac{v}{2} Y_\nu^{IJ} \nu_L^I \nu_R^J \quad \equiv \quad m_D^{IJ} \nu_L^I \nu_R^J$$

Not enough to explain mass hierarchy. But, ν_R are singlets of all SM gauge groups- one can add to the Lagrangian gauge invariant explicit mass term (forbidden for all other non-singlet particles):

$$m_D^{IJ} \nu_L^I \nu_R^J \quad - \quad M_R^{IJ} \nu_R^I \nu_R^J$$

"See-saw" mass matrix:

$$(\nu_L, \nu_R) \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \end{pmatrix}$$

Correct mass hierarchy: suppose further that SM is a low energy effective theory for some more general Grand Unified Theory with bigger gauge symmetry, valid at the high energy scale $E_{GUT} \sim \mathcal{O}(10^{14} - 10^{16})$ GeV. In many such GUTs, right neutrinos would be also a singlet of a "big" gauge group. Mass term is thus still permitted, and has natural value $M_R \sim \mathcal{O}(E_{GUT})$.

In such case neutrino mass matrix can be "block-diagonalized" ("Takagi diagonalization", Takagi 1925!)

Define 6×6 matrix U:

$$U = \begin{pmatrix} i & m_D^* M_R^{-1} \\ -iM_R^{-1} m_D^T & 1 \end{pmatrix}$$

and rotate light and heavy neutrino states with the use of U matrix. New effective mass matrix:

$$U^T \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} U \approx \begin{pmatrix} m_D^T M_R^{-1} m_D & 0 \\ 0 & M_R \end{pmatrix} + \text{small corrections}$$

Effective 3×3 mass matrix for light neutrinos:

$$M_\nu \approx m_D^T M_R^{-1} m_D$$

Finally, diagonalize m_ν:

$$\nu_l^I = (U_{MNS}^*)^{IJ} (\nu_l^J)^{phys}$$

$$U_{MNS}^\dagger M_\nu U_{MNS}^* = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$$

Assuming "Dirac mass" to be $m_D \sim \mathcal{O}(0.1 - 100)$ GeV, like typical masses of other SM fermions, one has neutrino masses in the correct range: $m_\nu \sim \mathcal{O}(10^{-1} - 10^{-5})$ eV!

See-saw: very elegant solution, but require presence of heavy right neutrinos. We assume existence of something very difficult to confirm experimentally - very heavy and gauge singlet... Indirect clues - cosmology, leptogenesis?

See-saw mechanism can be introduced within the extended SM. What changes in SUSY models? Two questions:

1. Is the see-saw neutrino mass structure modified comparing to SM?

2. Or, can SUSY offer an alternative model of neutrino masses, not based on heavy unobservable beings?

Answer to both questions is positive!

2. Supersymmetric heavy sector: right sneutrinos

Work in progress - not published yet.

Superpotential (N^I - heavy neutral superfields, neutrinos + sneutrinos):

$$W = \epsilon_{ij}(\mu \widehat{H}_i^1 \widehat{H}_j^2 + Y_\ell^I \widehat{H}_i^1 \widehat{L}_j^I \widehat{R}^I + Y_\nu^{IJ} \widehat{H}_i^2 \widehat{L}_j^I \widehat{N}^J) + \frac{1}{2} M^I N^I N^I \,,$$

Soft terms (Higgs and sleptons):

$$\begin{aligned} V_S &= m_{H_1}^2 H_i^{1*} H_i^1 + m_{H_2}^2 H_i^{2*} H_i^2 \\ &+ (m_L^2)^{IJ} \tilde{L}_i^{I*} \tilde{L}_i^J + (m_R^2)^{IJ} \tilde{R}^{I*} \tilde{R}^J + (m_N^2)^{IJ} \widetilde{N}^{I*} \widetilde{N}^J \\ &- \left[(m_B^2)^{IJ} \widetilde{N}^I \widetilde{N}^J + \epsilon_{ij} \left(m_{12}^2 H_i^1 H_j^2 + A_\ell^{IJ} H_i^1 \tilde{L}_j^I \tilde{R}^J + A_\nu^{IJ} H_i^2 \tilde{L}_j^I \widetilde{N}^J \right) + \text{H.c.} \right] \end{aligned}$$

In general, Y_ν, A_ν are complex 3×3 matrices, M is a real diagonal, m_N^2 is hermitian matrices, m_B^2 is complex symmetric matrix.

m_B^2 leads to mass splitting of real and imaginary part of sneutrino fields: one gets $6 + 6$ real instead of $3 + 3$ complex fields.

Mass scale assumptions:

1. Neutrino sector: $\|Y_\nu\| \lesssim \mathcal{O}(1)$ (or equivalently $\|m_D\| \lesssim m_t$), $\|M\| \sim M_{GUT} \gg v$.

2. Charged slepton sector $\|m_L^2\| \sim \|m_R^2\| \sim v^2$.

3. Sneutrino sector $\|A_\nu\| \lesssim v$ (constrained by charged slepton mass naturalness), $\|m_B^2\| \lesssim v\|M\|$ (SUGRA), $\|m_N^2\| \sim v^2$ or $\|m_N^2\| \sim \|M^2\|$, depending on details of SUSY breaking mechanism.

Further constraints *aposteriori*, after comparing to experimental data.

Consequences of new mass terms? Left slepton mass matrix:

$$M_L^2 = m_L^2 + m_l^2 + D - term$$

"Obvious" generalization:

$$M_{\tilde{\nu}_L}^2 = m_L^2 + m_D^* m_D^T + D - term$$

Not true, $m_D^* m_D$ term is supressed! Errors in many papers.

Correct approach - see-saw mechanism for sneutrinos. More complicated - one needs to start from 12×12 mass matrix and get 6×6 effective mass matrix for 6 light real sneutrino fields. It reads as:

$$\mathcal{M}^2_{\tilde{\nu}_\ell} \equiv \begin{pmatrix} M^2_{LC} & (M^2_{LV})^* \\ M^2_{LV} & (M^2_{LC})^* \end{pmatrix}$$

Denoting $A_v = a_v Y_v$ and $X_v = a_v + \mu^* \cot\beta$, one gets:

$$\begin{aligned} M^2_{LC} &\approx m^2_L + m^*_D \frac{1}{M} m^2_N \frac{1}{M} m^T_D + \frac{1}{2} M^2_Z \cos 2\beta \\ M^2_{LV} &\approx M_\nu X^T_\nu + X_\nu M_\nu - 2 m_D \frac{1}{M} m^2_B \frac{1}{M} m^T_D = \mathcal{O}(m_\nu M_{SUSY}) \end{aligned}$$

Immediate consequence - sneutrino mass splitting:

$$\begin{aligned} m^2_{S_i} &= (M^2_{LC})_{ii} - \mathrm{Re}(M^2_{LV})_{ii} \\ m^2_{S_{i+3}} &= (M^2_{LC})_{ii} + \mathrm{Re}(M^2_{LV})_{ii} \qquad i = 1,2,3 \end{aligned}$$

Mass splitting $\Delta m^2_{S_{i,i+3}} = 2\mathrm{Re}(M^2_{LV})_{ii} \sim \mathcal{O}(m_\nu M_{SUSY})$. Sneutrino oscillations possible? Discussed later...

Sneutrino mass structure is already subject of experimental constraints.

1. Diagonal M^2_{LC} entries, or the typical sneutrino mass scale - constrained by $g_\mu - 2$.

2. Off diagonal M^2_{LC} entries constrained by $\mu \to e\gamma$ and $\tau \to e\gamma$, $\tau \to \mu\gamma$ decays:

$$BR(\mu \to e\gamma) = \frac{48\pi^2 e^2}{m_\mu^2 G_F^2} |C^{12}|^2$$

where

$$C^{12} \approx \frac{e^2 m_\mu}{2(4\pi)^2 s_W^2} \left(M^2_{LC}\right)^{12} \left(\frac{\sqrt{2}}{\cos\beta} \frac{m_{\chi_i^+}}{M_W} Z_+^{1i*} Z_-^{2i*} D_{11} - |Z_+^{1i}|^2 D_{23} \right)$$

and similarly $BR(\tau \to e\gamma) \sim |\left(M^2_{LC}\right)^{13}|^2$, $BR(\tau \to \mu\gamma) \sim |\left(M^2_{LC}\right)^{23}|^2$

Numerical estimate:

$$M^2_{LC} = \begin{pmatrix} \gtrsim 200^2 & \lesssim 3^2 & \lesssim 32^2 \\ \ldots & \gtrsim 200^2 & \lesssim 28^2 \\ \ldots & \ldots & \gtrsim 200^2 \end{pmatrix}.$$

Knowledge of M^2_{LC} can give insight into Y_ν flavour structure independent from the one derived from neutrino oscillations, particularly if m^2_L is flavour-diagonal (SUGRA type models)

Further conclusion: $\frac{||m^2_N||}{||M^2||} \leq 0.01$.

In standard MSSM, left slepton and sneutrino mass difference is fixed:

$$m^2_{\tilde{e}} - m^2_{\tilde{\nu}_e} = -M^2_W \cos 2\beta$$

Right sneutrino effects alter the relation:

$$m^2_{\tilde{e}} - m^2_{\tilde{\nu}_e} = -M^2_W \cos 2\beta - (m^*_D \frac{1}{M} m^2_N \frac{1}{M} m^T_D)_{11} \approx -M^2_W \cos 2\beta + \mathcal{O}(x m^2_D)$$

With sufficient accuracy, such effect can be measured?

Off-diagonal block M^2_{LV} affects directly the neutrino mass matrix via loop corrections:

$$M_\nu = M_\nu^{tree} + M_\nu^{1-loop} = M_\nu^{tree} - \frac{m_{\chi_i^0}}{32\pi^2} \text{Re}\left[(g_2 Z_N^{2i} - g_1 Z_N^{1i})^2 \left(M^2_{LV}\right)\right] C_0$$

Assumption that the loop corrections are at most of the order of the tree-level mass gives constraint on M^2_{LV}:

$$M^2_{LV} = \begin{pmatrix} \lesssim 2\times 10^{-9} & ... & ... \\ ... & \lesssim 2\times 10^{-6} & ... \\ ... & ... & \lesssim 10^{-5} \end{pmatrix},$$

Alternatively, one can ask how far some particular form of M^2_{LV} may change the the neutrino masses and mixing.

Example: assume diagonal Y_ν, i.e. no mixing at the tree level: $U_{MNS} = 1$! Denote also:

$$\alpha = -\frac{1}{32\pi^2} m_{\chi_i^0} (g_1 Z_N^{1i} - g_2 Z_N^{2i})^2 C_0$$

Then

$$M_\nu = m_D \frac{1}{M} m_D - \alpha \mathrm{Re}(M^2_{LV})$$

Using

$$M^2_{LV} = m_D \frac{1}{M} m_D X^T_\nu + X_\nu m_D \frac{1}{M} m_D - 2 m_D \frac{1}{M} m^2_B \frac{1}{M} m^T_D$$

and assuming X_ν to be real, one gets

$$\begin{aligned} M_\nu &= (1+\alpha X_\nu)\, m_D M^{-1} m_D\, (1+\alpha X^T_\nu) \\ &- \alpha^2 X_\nu m_D M^{-1} m_D X^T_\nu - 2\alpha\, m_D \frac{1}{M} \mathrm{Re}\, m^2_B \frac{1}{M} m_D \end{aligned}$$

Choose now m^2_B such that the last two terms cancels - fine tuning, but can be always done. Then we require first term to restore physical neutrino masses and mixings:

$$(1+\alpha X_\nu)\, m_D M^{-1} m_D\, (1+\alpha X^T_\nu) = U^{\mathrm{phys}}_{\mathrm{MNS}}\, m^{phys}_{\nu_\ell}\, (U^{\mathrm{phys}}_{\mathrm{MNS}})^T .$$

This can be solved as:

$$X_\nu = \frac{1}{\alpha}\left[U^{\rm phys}_{\rm MNS}\,{\rm diag}(\beta_1,\beta_2,\beta_3) - 1\right]\,, \qquad \text{with} \qquad \beta_I \equiv \frac{\sqrt{2\,M_I\,m^{\rm phys}_{\nu^I}}}{v_2 Y^I_\nu}\,.$$

Some algebra, but conclusion is clear - starting from diagonal Y_ν (m_D), one can find soft parameters in right sneutrino sector restoring <u>any</u> set of neutrino masses and mixings.

This is possible even if tree level neutrino masses were degenerated, i.e. no tree level mass splitting and oscillations!

The explicit analytical solution given above is rather ugly and strongly fine-tuned, but numerically one can find more realistic ones.

In general - corrections from right sneutrino sector potentially very important!

One more interesting effects: sneutrino oscillations?

$$P_{\tilde{\nu}_I \to \tilde{\nu}_I(\tilde{\nu}_I^*)} \approx \frac{1}{2} e^{-\frac{t}{\tau_I}} \left[1 \pm \cos \Delta m_{\tilde{\nu}_I} t\right]$$

$P_{\tilde{\nu} \to \tilde{\nu}(\tilde{\nu}^*)}$ is proportional to the number $N_{l^-}(N_{l^+})$ of $l^-(l^+)$'s final state in the reaction $\tilde{\nu} \to l^- + \chi^+ (\tilde{\nu}^* \to l^+ + \chi^-)$. Asymmetry

$$A_l = \frac{N_{l^-} - N_{l^+}}{N_{l^-} + N_{l^+}} \sim \cos(\Delta m\, t)$$

Can be measured if

$$\frac{\Delta m}{\Gamma_{\tilde{\nu}}} \simeq O(1)$$

Not very likely - unfortunately for most parameter choices $\Delta m \ll \Gamma_{\tilde{\nu}}$.

Many interesting effects, nice piece of physics. But, is the presence of right heavy sector really unavoidable?

3. Alternative to see-saw mechanism: SUSY with broken R-parity.

Standard recipe for MSSM construction:

- add supersymmetric partners to the SM particles; add additional Higgs doublet
- extend SM couplings to incorporate new particles.
- add "soft" supersymmetry breaking terms - they do not involve matter fermions

Complete? Not really!

SM: lepton number conservation is not due to an imposed symmetry, just reflects the fact that all such combinations of SM fields are ruled out by gauge invariance and renormalisability! [Weinberg].

MSSM: lepton and baryon number violating terms can appear naturally.

Standard Yukawa couplings (indices suppressed):

$$L_Y = Y_l H_1 L_L E_R + Y_d H_1 Q_L D_R + Y_u H_2 Q_L U_R + \mu_0 H_1 H_2$$

In SM Yukawa couplings describe interactions of scalar Higgs fields to leptons or quarks. In MSSM all combinations exist - Higgs-fermions or Higgsino-fermion-sfermion.

Additional couplings (λ's, like Y's, are matrices in flavour space):

$$L_{R-par} = \lambda L_L L_L E_R + \lambda' L_L Q_L D_R + \lambda'' D_R D_R U_R + \mu L H_2$$

$+$ analogous terms in soft breaking sector...

New couplings break lepton and/or baryon number.

To distinguish between various couplings, let's introduce discrete symmetry - "R-parity".

$$R = \begin{cases} 1 & \text{SM particles and Higgs fields} \\ -1 & \text{superpartners} \end{cases}$$

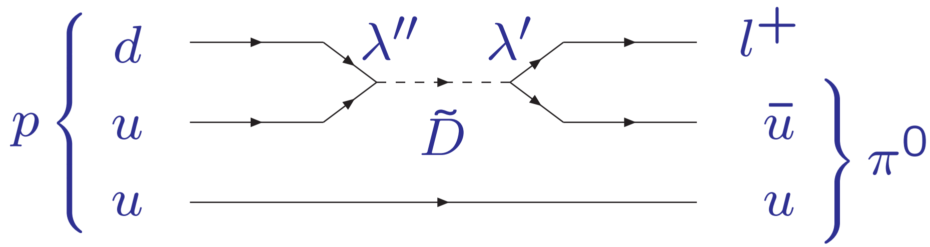
Formally $R = (-1)^{L+3B+2S} \equiv (-1)^{2S+(B-L)}$

SM-like Yukawa interactions preserve R-parity. The new ones break it. Breaking of R-parity drastically alters phenomenology of SUSY models!

Most importance consequence: SUSY particles do not need to be produced in pairs - lightest SUSY particle not stable. No CDM candidate, but non-vanishing neutrino masses instead!

Is R-parity breaking phenomenologically feasible? With limitations, yes.

Simultaneous presence of both B and L violating terms excluded - fast proton decay mediated by squarks:



Most realistic models assume only lepton flavour violation, i.e. $\lambda''_{IJK} = 0$.

Without λ'', new terms break R-parity (Z_2), but preserve some other Z_3 global symmetry - Lepton Number Violating, or LFV MSSM.

Bounds on λ, λ' mostly from various rare processes. General conclusion - R-parity breaking constrained in magnitude, but certainly possible in the view of current data.

Consequences for neutrino masses

R-parity breaking leads to mixing of standard and supersymmetric fermions.

Neutralinos take the role of right neutrinos - mixing of neutralinos and neutrinos gives neutrino masses. Simple idea but precise calculation quite sophisticated due to the complexity of R-parity violating MSSM - several papers published on the subject, but based on major simplifications and approximations.

Complete analysis: - complicated and interesting problem. This seminar based on:

Dedes, Rimmer, JR, Schmidt-Sommerfeld, *Phys. Lett. B 2005*
Dedes, Rimmer, JR - *JHEP 2006*.

How to attack the problem? Very numerous input parameters, complicated vaccum (VEV) structure, large size mass matrices etc. Kind of nightmare...

Do it step by step:

1. Scalar sector. Neutral scalar complicated and quite tricky, charged sleptons and squarks not different from the RPC MSSM.

2. Neutral and charged fermion sector - simplified with extensive use of see-saw mechanism.

3. Parameter initialisation - another tricky point, careful approach required.

I. Neutral scalar sector.

5 complex neutral fields: 2 Higgs bosons, 3 sneutrinos. LFV: H_d and lepton superfields can mix! Define: $\mathcal{L} = (\mathcal{L}_0, \mathcal{L}_1, \mathcal{L}_2, \mathcal{L}_3) \equiv (H_d, L_1, L_2, L_3)$.

Neutral scalar potential, including soft terms:

$$\begin{aligned} V_{\text{neutral}} &= \left(\mathcal{M}^2_{\tilde{\mathcal{L}}}\right)_{\alpha\beta} \tilde{\nu}^*_{L\alpha} \tilde{\nu}_{L\beta} + m_2^2\, h_2^{0*}\, h_2^0 - (b_\alpha \tilde{\nu}_{L\alpha} h_2^0 + \text{H.c}) \\ &+ \frac{1}{8}(g^2 + g_2^2)[h_2^{0*} h_2^0 - \tilde{\nu}^*_{L\alpha}\tilde{\nu}_{L\alpha}]^2 \,. \end{aligned}$$

First step - gauge symmetry breaking. Calculations of VEV require finding minimum of V_{neutral} - quartic polynomial of 10 real variables! Previously sold only approximately or numerically.

Our paper in PLB267: strict analytical analysis. Convenient field basis choice, effective procedure of finding VEV's, proof of CP-conservation, stability analysis etc.

II. Fermion sector

General form of superpotential:

$$\mathcal{W} = \epsilon_{ab}\left[\frac{1}{2}\lambda_{\alpha\beta k}\,\mathcal{L}^a_\alpha\,\mathcal{L}^b_\beta\,\bar{E}_k + \lambda'_{\alpha jk}\,\mathcal{L}^a_\alpha\,Q^b_j\,\bar{D}_k - \mu_\alpha\,\mathcal{L}^a_\alpha\,H^b_2 + (Y_u)_{ij}\,Q^a_i\,H^b_2\,\bar{U}_{ij}\right]$$

Quark superfield rotation can diagonalize $(Y_u)_{ij}$ and $(Y_d)_{ij} = \lambda'_{0ij}$, but not lepton Yukawa $(Y_l)_{ij} = \lambda_{0ij}$ - here rotation freedom fixed already in simplifying the scalar sector. At best, Y_l can be chosen to be hermitian.

Problems with parameter initialization - only lepton masses, i.e. Y_l eigenvalues, known!

Start from block diagonalization of neutrino-neutralino and charged lepton-chargino mass matrices.

Neutral fermions

Neutrinos and neutralinos mix via bi-linear R-parity violating terms - 7×7 mass matrix:

$$\mathcal{M}_N = \left(\begin{array}{cccc|ccc} M_1 & 0 & \frac{gv_u}{2} & -\frac{gv_d}{2} & 0 & 0 & 0 \\ 0 & M_2 & -\frac{g_2v_u}{2} & \frac{g_2v_d}{2} & 0 & 0 & 0 \\ \frac{gv_u}{2} & -\frac{g_2v_u}{2} & 0 & -\mu_0 & -\mu_1 & -\mu_2 & -\mu_3 \\ -\frac{gv_d}{2} & \frac{g_2v_d}{2} & -\mu_0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & -\mu_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 \end{array}\right) \equiv \left(\begin{array}{cc} M_{N\,4\times 4} & d_{N\,4\times 3} \\ d^T_{N\,3\times 4} & 0_{3\times 3} \end{array}\right)$$

Diagonalization matrix

$$Z_N = \left(\begin{array}{cc} 1 & -M_N^{-1} d_N \\ d_N^\dagger M_N^{\dagger -1} & 1 \end{array}\right) \left(\begin{array}{cc} \mathcal{Z}_N & 0 \\ 0 & \mathcal{Z}_\nu \end{array}\right)$$

$\mathcal{Z}_N, \mathcal{Z}_\nu$ diagonalize effective "neutralino" and "neutrino" mass matrices:

$$\mathcal{Z}_\nu^T m_\nu^{eff} \mathcal{Z}_\nu = m_\nu^{eff-diag} \qquad \mathcal{Z}_N^T M_N \mathcal{Z}_N = M_N^{diag}$$

Notation: physical "neutralinos" - 4 heavy states
physical "neutrinos" - 3 light states

See-saw corrections to "neutralino" mass matrix negligible, standard analyses hold.

Effective neutrino mass matrix:

$$m_\nu^{eff} = -d_N^T M_N^{-1} d_N = \frac{v_d^2 (M_1 g_2^2 + M_2 g^2)}{4\mathrm{Det}[M_N]} \begin{pmatrix} \mu_1^2 & \mu_1\mu_2 & \mu_1\mu_3 \\ \mu_1\mu_2 & \mu_2^2 & \mu_2\mu_3 \\ \mu_1\mu_3 & \mu_2\mu_3 & \mu_3^2 \end{pmatrix}$$

Charged fermions

Charged leptons, gauginos and Higgsinos mix - 5×5 mass matrix

$$\mathcal{M}_C = \left(\begin{array}{cc|ccc} M_2 & \frac{g_2 v_u}{\sqrt{2}} & 0 & 0 & 0 \\ \frac{g_2 v_d}{\sqrt{2}} & \mu_0 & 0 & 0 & 0 \\ \hline 0 & \mu_1 & & & \\ 0 & \mu_2 & & \frac{v_d}{\sqrt{2}} Y_l & \\ 0 & \mu_3 & & & \end{array}\right) \equiv \begin{pmatrix} M_{C\,2\times2} & 0 \\ d_{C\,3\times2} & m_{C\,3\times3} \end{pmatrix}$$

Diagonalized by:

$$Z_- \approx \begin{pmatrix} 1 & -M_C^{\dagger\,-1} d_C^\dagger \\ d_C M_C^{-1} & 1 \end{pmatrix} \begin{pmatrix} \mathcal{Z}_- & 0 \\ 0 & \mathcal{Z}_{l-} \end{pmatrix}$$

$$Z_+ \approx \begin{pmatrix} \mathcal{Z}_+ & 0 \\ 0 & \mathcal{Z}_{l+} \end{pmatrix}$$

$$\hat{\mathcal{M}}_C = Z_-^\dagger \mathcal{M}_C Z_+ \approx \begin{pmatrix} \mathcal{Z}_-^\dagger M_C \mathcal{Z}_+ & 0 \\ 0 & \mathcal{Z}_{l-}^\dagger m_C \mathcal{Z}_{l+} \end{pmatrix}$$

m_C hermitian (lepton Yukawa), so $\mathcal{Z}_{l+} = \mathcal{Z}_{l-} \equiv \mathcal{Z}_l$.

Finally, U_{MNS} matrix:

$$U_{\mathsf{MNS}} = \mathcal{Z}_\nu^\dagger \mathcal{Z}_l^* + \mathcal{O}\left(\frac{d_c d_N}{M_C M_N}\right) ,$$

Unitarity violation in U_{MNS}: $\mathcal{O}\left(\frac{d_c d_N}{M_C M_N}\right) \sim \frac{m_\nu^{\mathsf{tree}} M_{\mathsf{SUSY}}}{M_Z^2} \tan^2\beta \sim 10^{-12} \tan^2\beta$, well below sensitivity of current (or planned) experiments.

Technical problem: neutrino mixing matrix, $\mathcal{Z}_\nu$ defined at tree level up to a $U(2)$ rotation - complete definition of $\mathcal{Z}_\nu$ and thus also $\mathcal{Z}_l$ requires a one-loop corrected neutrino mixing matrix. Then, lepton Yukawa matrix Y_l can be found iteratively, such that physical (i.e. loop corrected) $\mathcal{Z}_\nu$ and $\mathcal{Z}_l$ produce the correct experimentally measured U_{MNS} matrix.

Neutrino mass generation and possible hierarchies

Many papers on the subject, starting from Hempfling 1996. Many of them misleading or even wrong - problems with correct parameter initialization and with incorrect expansions/approximations used.

Tree level masses simple:

$$\begin{aligned} m_{\nu_1} &= m_{\nu_2} = 0 \\ m_{\nu_3} &\approx \left| \frac{v_d^2 (M_1 g_2^2 + M_2 g^2)}{4\mathrm{Det}[M_{N_{4\times4}}]} \right| (|\mu_1|^2 + |\mu_2|^2 + |\mu_3|^2) \end{aligned}$$

Massless neutrino states degenerate at tree level, mixing matrix $\mathcal{Z}_\nu$ not well defined - loop corrections necessary.

Dominant contributions: neutralino-neutral Higgs, chargino-charged Higgs and/or down quark/down squark in loop; other vanishing or small.

Higgs-neutralino diagram - proportional to bilinear soft LFV breaking parameters B_i. Other diagrams proportional to trilinear LFV breaking parameters:

Chargino-charged Higgs diagram - driven by λ^2
Down quark-down squark diagram - driven by $(\lambda')^2$

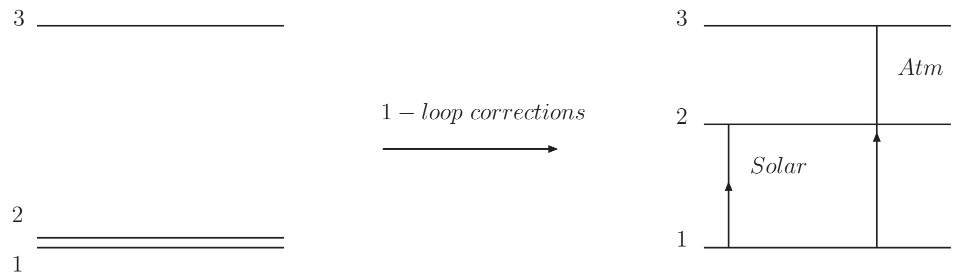
At loop level, the correct neutrino mass hierarchy can be always generated by the proper choice of just two of the LFV parameters – one sets the scale of the atmospheric mass^2 difference, the second the solar mass^2 difference.

General choices of RPV parameters lead to non-diagonal lepton Yukawa, problems with leptonic FCNC ($\mu \to e\gamma$ etc.)

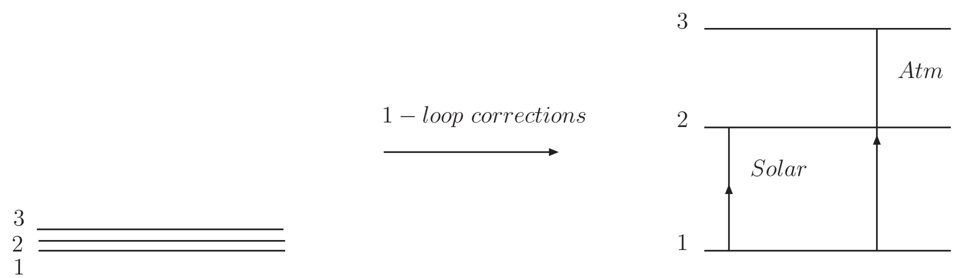
Can one keep diagonal Y_l and still obtain correct pattern of neutrino masses and mixings? Yes, assuming special scenario: hierarchy of LFV parameters chosen to set the neutrino mass scale must match the hierarchy of two different U_{MNS} rows!

Two scenarios:

- Tree level dominance: the atmospheric mass^2 difference originates from tree level contributions to neutrino masses.



- Loop level dominance: The atmospheric mass^2 difference originates from one-loop contributions to neutrino masses.



In either case, the solar mass^2 difference originates from loop effects.

In further numerical examples we fix neutrino mixing angles to reproduce the "tri-bimaximal mixing"

$$\sin^2\theta_{12} = \frac{1}{3}, \qquad \sin^2\theta_{23} = \frac{1}{2}, \qquad \sin^2\theta_{13} = 0$$

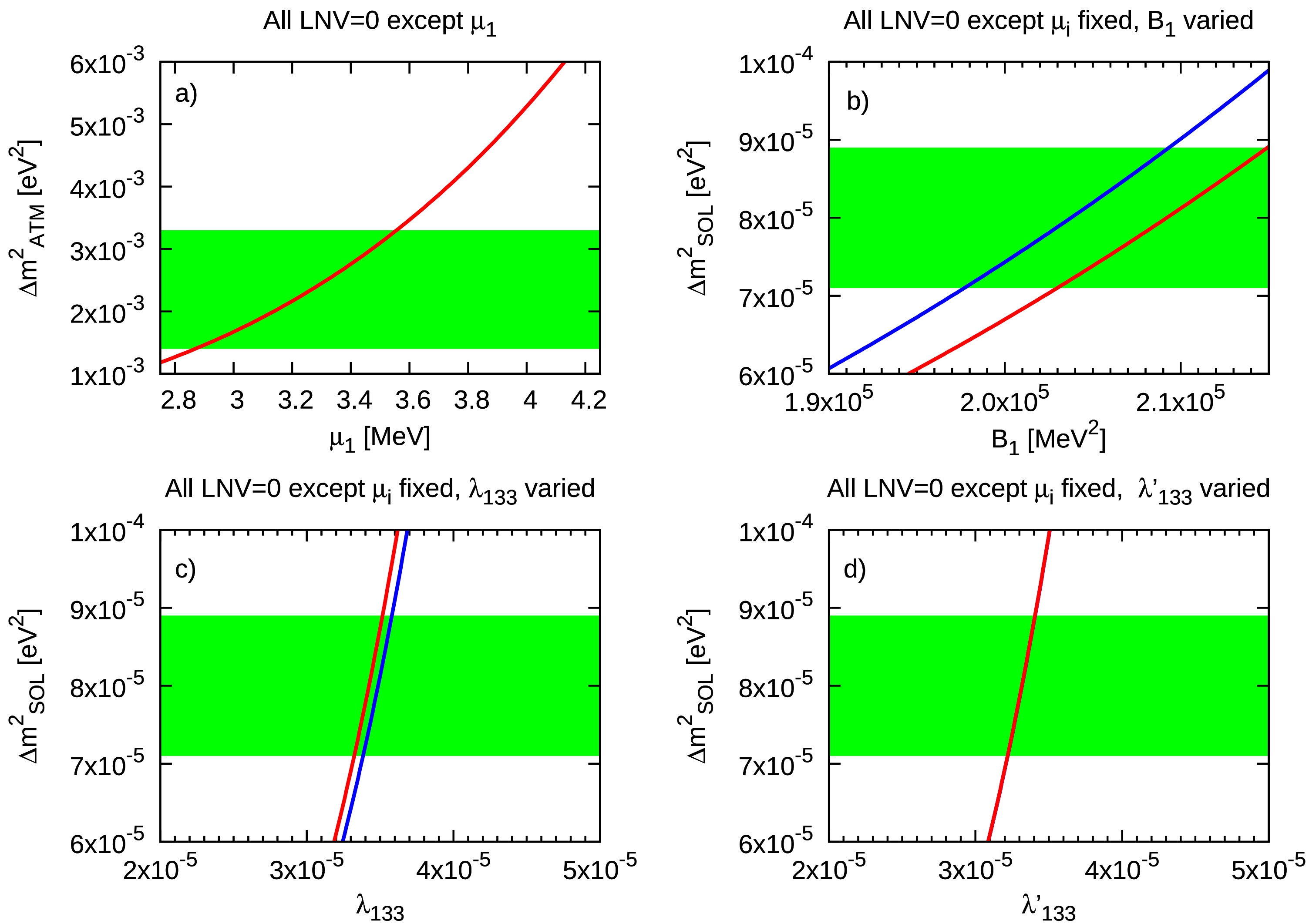
Tree level dominance.

Fix atmospheric mass difference by setting $\mu_1 = 1.47$ MeV $= \frac{\mu_2}{\sqrt{2}} = \frac{\mu_3}{\sqrt{3}}$.

Single other LFV parameter initialized - loop effects approximately proportional to tree level masses - U_{MNS} structure preserved after rediagonalization (so also lepton Yukawa remains diagonal).

Correct mass hierarchies e.g. for

$$\begin{aligned} B_1 &\sim 0.21\,\mathrm{GeV}^2 \sim \left[300\,\mu_1\right]^2, \\ \lambda_{133} &\sim 3.4\times10^{-5} \sim Y_e\,, \\ \lambda'_{133} &\sim 3.2\times10^{-5} \sim 0.1 Y_d\,, \end{aligned}$$



Red curve - full result. a) Only μ_1 is varied. Other figures, μ_i fixed as explained and b) B_1, or c) λ_{133}, or d) λ'_{133}, is varied respectively.

Loop level dominance: $\mu_i \approx 0$, mass scales set by λ's.

Diagrams dominated by trilinear couplings - the flavour of the external legs of the loop can be "swapped independently" of the flavour of the particles in the loop, just changing the appropriate indices of the λ, λ' matrices in the loop vertices.

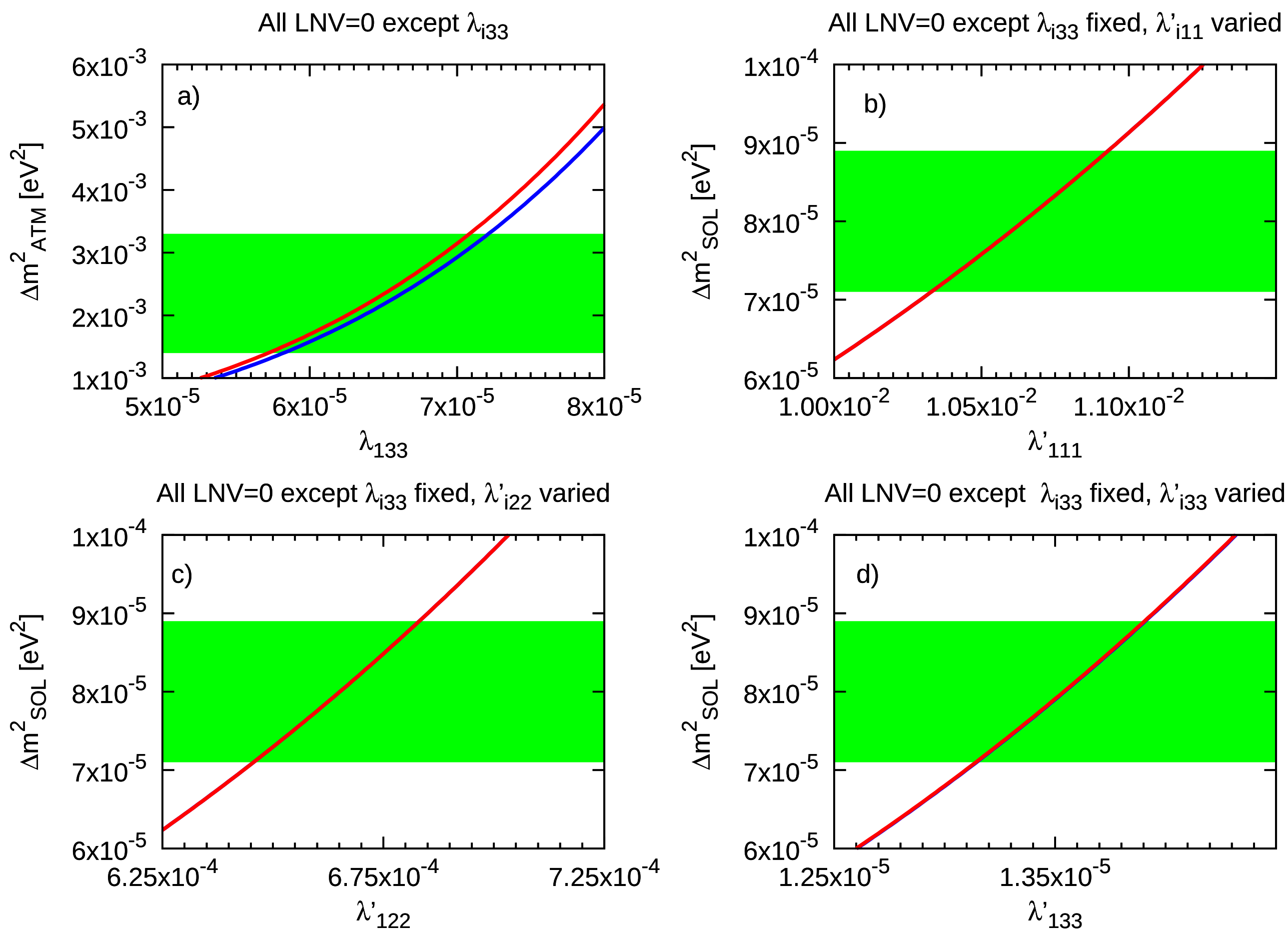
Setting the λ and λ' entries which control the couplings of the external legs in certain hierarchies, one can ensure that also the ratios of the various entries in the one loop corrected neutrino mass matrix are such that they give rise to the correct U_{MNS} rotation matrix.

Three hierarchies allowed:

$$\text{Hierarchy (B)} \quad : \quad \lambda'_{1jj} = \frac{\lambda'_{2jj}}{\sqrt{2}} = \frac{\lambda'_{3jj}}{\sqrt{3}}$$

$$\text{Hierarchy (C)} \quad : \quad \lambda'_{1jj} = \frac{\lambda'_{2jj}}{\sqrt{2}} = -\frac{\lambda'_{3jj}}{\sqrt{3}}$$

$$\text{Hierarchy (D)} \quad : \quad \lambda_{1jj} = -\sqrt{2}\lambda_{2jj} \quad , \quad \lambda_{3jj} = 0 \,.$$



a) λ_{i33} varied in hierarchy (D). For b,c,d), λ_{i33} fixed to a value consistent with the atmospheric mass^2 difference and b) only λ'_{i11} is varied in hierarchy (B) or c) only λ'_{i22} in hierarchy (B) or d) only λ'_{i33} in hierarchy (B) in order to accommodate the solar mass^2 difference.

4. Conclusions

- SUSY has important influence on neutrino masses and mixings

- heavy right neutral scalar sector can significantly modify neutrino masses via radiative corrections. Interesting effects also in the light sneutrino sector - mass splitting between CP-even and CP-odd states, possible sneutrino oscillations, modifications to selectron-sneutrino mass relation.

- SUSY can explain neutrino masses even in the absence of super-heavy right sector. Masses can be entirely set by LFV parameters, keeping simultaneously correct U_{MNS} structure and diagonal lepton Yukawa couplings. Special hierarchies of LFV parameters required.