Nobel Prize 2008

- broken symmetries -

Bohdan GRZADKOWSKI
University of Warsaw

- Nobel Prize Laureates in Physics 2008
- Yoichiro Nambu and spontaneous chiral symmetry breaking
- The Goldstone theorem
- Makoto Kobayashi and Toshihide Maskawa and explicit CP violation
- Comments:
  - Giovanni Jona-Lasinio - "the Nambu-Jona-Lasinio model"
  - Jeffrey Goldstone - "the Nambu-Goldstone bosons"
  - Robert Brout, Francois Englert, Peter Higgs, . . . - the Higgs mechanism
  - Nicola Cabibbo - the Cabibbo angle
- Cosmological illustrations e.g. baryogenesis
- We are not yet done: the strong CP problem
- Hints for beyond the Standard Model physics
- Conclusions

Nobel Prize Laureates in Physics 2008

- "for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics":
  - **Yoichiro Nambu**, Enrico Fermi Institute, University of Chicago Chicago, IL, USA, $1/2$ of the prize, born 1921,
    "A "Superconductor" Model of Elementary Particles and its Consequences" given at a conference at Perdue (1960) reprinted in "Broken Symmetries, Selected Papers by Y.Nambu" ed. T. Eguchi and K. Nishijima, World Scientific 1995

- "for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature":
  - **Makoto Kobayashi**, High Energy Accelerator Research Organization (KEK) Tsukuba, Japan, $1/4$ of the prize, born 1944
  - **Toshihide Maskawa**, Kyoto Sangyo University; Yukawa Institute for Theoretical Physics (YITP), Kyoto University Kyoto, Japan, $1/4$ of the prize, born 1940
    M. Kobayashi and T. Maskawa, "CP Violation in the Renormalizable Theory of Weak Interaction", Prog. Theor. Phys. **49**, 652 (1973), 5503 citations

Yoichiro Nambu and spontaneous chiral symmetry breaking

The state of the art in strong interactions in 1960:

- Isospin symmetry, $SU(2)$ (introduced by Werner Heisenberg in 1932 to explain properties of the then newly discovered neutron):
  - $m_p = 938.3$ MeV and $m_n = 939.6$ MeV.
  - The strength of the strong interaction between any pair of nucleons is the same, independent of whether they are interacting as protons or as neutrons.

- The baryon number conservation $\leftrightarrow U(1)_B$ symmetry

- 3 pions: $\pi^{\pm}$, π^0, with $m_{\pi^{\pm}} = 139.6$ MeV and $m_{\pi^0} = 135.0$ MeV, so

$$m_{\pi^{\pm}} \simeq m_{\pi^0} \ll m_p \simeq m_n$$

$$\Downarrow$$

The hypothesis of spontaneous chiral symmetry breaking
Nambu 1960, Goldstone 1961

- symmetry: $SU(2)_A \times SU(2)_V$ with Q_5^a and Q^a $(a = 1, 2, 3)$ are generators of $SU(2)_{A,V}$

- vacuum $|0\rangle$: $\langle 0|H|0\rangle = \min\langle H\rangle$

The hypothesis of spontaneous chiral symmetry breaking: $Q_5^a|0\rangle \neq 0$ and $Q^a|0\rangle = 0$

Nambu 1960, Goldstone 1961

$SU(2)_A$ is *spontaneously* broken

$$\Downarrow$$

The theory contains massless bosons corresponding to each generator that does not annihilate the vacuum (since $[Q_5^a, H] = 0$ therefore $Q_5^a|0\rangle \neq 0$ is degenerate with $|0\rangle$ applying successively Q_5^a one can generate infinite number of degenerate states - massless particle).

The sigma model of the strong interactions
The theory of pions as Nambu-Goldston bosons
-Gell-Mann & Lévy-

The symmetry: $SU(2)_L \times SU(2)_R \quad \Leftrightarrow \quad SU(2)_A \times SU(2)_V$

The elementary fields:

$$N = \begin{pmatrix} p \\ n \end{pmatrix} \quad \text{and} \quad \vec{\pi}\ (0^-),\ \ \sigma'\ (0^+)$$

The Lagrangian:

$$\mathcal{L} = \bar{N} i \not{\partial} N - g \bar{N} \left(\sigma' + i \vec{\sigma} \cdot \vec{\pi} \gamma_5\right) N + \frac{1}{2}\left[(\partial_\mu \vec{\pi})^2 + (\partial_\mu \sigma')^2\right] - \frac{1}{2}\mu^2({\sigma'}^2 + \vec{\pi}^2) - \frac{1}{4}\lambda({\sigma'}^2 + \vec{\pi}^2)^2$$

where σ' is needed to make $\mathcal{L}$ chirally symmetric, $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ and

$$\begin{aligned} \bar{N} i \not{\partial} N &= \bar{N}_R i \not{\partial} N_R + \bar{N}_L i \not{\partial} N_L \\ \bar{N}\left(\sigma' + i\vec{\sigma}\cdot\vec{\pi}\gamma_5\right) N &= \bar{N}_L \left(\sigma' + i\vec{\sigma}\cdot\vec{\pi}\right) N_R + \bar{N}_R \left(\sigma' - i\vec{\sigma}\cdot\vec{\pi}\right) N_L \end{aligned}$$

The transformation properties of $\sigma' \pm i\vec{\sigma}\cdot\vec{\pi}$ are determined by the requirement of the invariance of $\mathcal{L}$ under $SU(2)_L \times SU(2)_R$:

- $SU(2)_L$: $\sigma' \to \sigma' + \frac{1}{2}\vec{\pi}\cdot\delta\vec{\alpha}_L$ and $\vec{\pi} \to \vec{\pi} - \frac{1}{2}\vec{\pi}\times\delta\vec{\alpha}_L - \frac{1}{2}\sigma'\delta\vec{\alpha}_L$
- $SU(2)_R$: $\sigma' \to \sigma' - \frac{1}{2}\vec{\pi}\cdot\delta\vec{\alpha}_R$ and $\vec{\pi} \to \vec{\pi} - \frac{1}{2}\vec{\pi}\times\delta\vec{\alpha}_R + \frac{1}{2}\sigma'\delta\vec{\alpha}_R$

The ground state is (at the tree level) the minimum of the potential:

$$V(\sigma', \vec{\pi}) = \frac{1}{4}\lambda\left(\sigma'^2 + \vec{\pi}^2 + \frac{\mu^2}{\lambda}\right)^2$$

So, either

- $\sigma' = \vec{\pi} = 0$ for $\mu^2/\lambda > 0$, or
- $\sigma'^2 + \vec{\pi}^2 = \left|\frac{\mu^2}{\lambda}\right|$ for $\mu^2/\lambda < 0$

To satisfy $Q_5^a|0\rangle \neq 0$ and $Q^a|0\rangle = 0$ we must choose

$$\langle 0|\vec{\pi}|0\rangle = 0 \quad \text{and} \quad \langle 0|\sigma'|0\rangle = \left(\left|\frac{\mu^2}{\lambda}\right|\right)^{1/2} \equiv v$$

Expanding around v, $\sigma = \sigma' - v$ one obtains:

$$\begin{aligned}\mathcal{L} &= \bar{N}(i\,\not{\partial} - gv)N - g\bar{N}\left(\sigma + i\vec{\sigma}\cdot\vec{\pi}\gamma_5\right)N + \\ &+\frac{1}{2}\left[(\partial_\mu\vec{\pi})^2 + (\partial_\mu\sigma)^2\right] - |\mu^2|\sigma^2 - \frac{1}{4}\lambda(\sigma^2+\vec{\pi}^2)^2 - \lambda v(\sigma^3 + \sigma\vec{\pi}^2) + \text{const.}\end{aligned}$$

- So, we have massive nucleons $m_N = gv$, a sigma meson of mass $m_\sigma^2 = 2|\mu^2|$ and the isospin triplet of massless pions: (π^+, π^0, π^-).

- Using the current implied by $SU(2)_A$ one finds $\langle 0|J^a_{A\ \mu}(x)|\pi^a(q)\rangle = -\langle 0|\sigma'|0\rangle i q_\mu \delta^{ab} e^{-i$ so

$$f_\pi = -\langle 0|\sigma'|0\rangle$$

- since in reality $m_\pi \neq 0$, one should break the chiral symmetry explicitly preserving isospin $SU(2)$: $\delta\mathcal{L} = -m^3\sigma'$. Then

$$m_\pi^2 = \frac{m^3}{f_\pi} \neq 0$$

- The axial current is not conserved and one obtains PCAC (Partial Conservation of Axial Current)

$$\partial^\mu J^a_{A\ \mu}(x) = m_\pi^2 f_\pi \pi^a(x)$$

The Goldstone theorem

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_i \partial^\mu \phi_i - V(\phi_i)$$

If $\mathcal{L}$ is symmetric with respect to $\phi_i \to \phi_i' \simeq \phi_i - i\Theta^a T^a_{ij}\phi_j$ then

$$0 = \delta V(\phi_i) = \frac{\partial V}{\partial \phi_i}\delta\phi_i = -i\frac{\partial V}{\partial \phi_i}\Theta^a T^a_{ij}\phi_j \tag{1}$$

Expanding $V(\phi_i)$ around its minimum $\phi_i = v_i$ one gets ($\phi_i' = \phi_i - v_i$)

$$V(\phi_i) = \text{const.} + \frac{1}{2}M^2_{ij}\phi_i'\phi_j' + \cdots$$

where

$$M^2_{ij} = \left(\frac{\partial^2 V}{\partial\phi_i \partial\phi_j}\right)_{\phi_i = v_i}$$

Differentiating (1) at the minimum one gets

$$M^2_{ij}\, T^a_{ij} v_j = 0 \tag{2}$$

So, for all broken generators ($T^a_{ij}v_j \neq 0$), (2) implies the existence of a massless Nambu-Goldstone boson.

Makoto Kobayashi and Toshihide Maskawa and explicit CP violation

The state of the art in quark weak interactions in 1973 :

- James Cronin, Val Fitch, 1964 $\Longrightarrow$ CP violation in $K \to \pi\pi$
- Steven Weinberg, "A Model of Leptons", *Phys.Rev.Lett.* 19, 1264 (1967)
- Charm quark predicted in 1970 by S. Glashow, J. Iliopoulos, and L. Maiani, *Phys. Rev.* **D**2, 1285 (1970)
- Quarks: u,d,s known while charm (c) still unobserved till its discovery in 1974

Kobayashi-Maskawa: $N \geq 3$ implies CP violation in the Weinberg model

- Parity:

$$
\begin{aligned}
(t,\vec{x}) &\xrightarrow{P} (t,-\vec{x}) \\
S(t,\vec{x}) &\xrightarrow{P} +S(t,-\vec{x}) \quad \text{scalar} \\
P(t,\vec{x}) &\xrightarrow{P} -P(t,-\vec{x}) \quad \text{pseudoscalar} \\
\bar{\psi}_a(t,\vec{x})\gamma_{\mu}\psi_b(t,\vec{x}) &\xrightarrow{P} +\bar{\psi}_a(t,-\vec{x})\gamma^{\mu}\psi_b(t,-\vec{x}) \quad \text{vector current} \\
\bar{\psi}_a(t,\vec{x})\gamma_{\mu}\gamma_5\psi_b(t,\vec{x}) &\xrightarrow{P} -\bar{\psi}_a(t,-\vec{x})\gamma^{\mu}\gamma_5\psi_b(t,-\vec{x}) \quad \text{axial current} \\
V_{\mu}(t,\vec{x}) &\xrightarrow{P} +V^{\mu}(t,-\vec{x}) \quad \text{vector} \\
A_{\mu}(t,\vec{x}) &\xrightarrow{P} -A^{\mu}(t,-\vec{x}) \quad \text{axial}
\end{aligned}
$$

- Charge conjugation:

$$
\begin{aligned}
\phi &\xrightarrow{C} \phi^{\dagger} \\
\bar{\psi}_a\gamma_{\mu}\psi_b &\xrightarrow{C} -\bar{\psi}_b\gamma_{\mu}\psi_a \quad \text{vector current} \\
\bar{\psi}_a\gamma_{\mu}\gamma_5\psi_b &\xrightarrow{C} +\bar{\psi}_b\gamma_{\mu}\gamma_5\psi_a \quad \text{vector current} \\
V_{\mu} &\xrightarrow{C} -V_{\mu}^{\star} \quad \text{vector} \\
A_{\mu} &\xrightarrow{C} +A_{\mu}^{\star} \quad \text{axial}
\end{aligned}
$$

where $C^{-1}\gamma_{\mu}C = -\gamma_{\mu}^{T}$ and e.g. $C = i\gamma^2\gamma^0$.

- CP:

$$
\begin{array}{rcll}
(t, \vec{x}) & \stackrel{CP}{\to} & (t, -\vec{x}) & \\
\phi & \stackrel{CP}{\to} & \phi^\dagger & \\
\bar{\psi}_a \gamma_\mu \psi_b & \stackrel{CP}{\to} & -\bar{\psi}_b \gamma^\mu \psi_a & \text{vector current} \\
\bar{\psi}_a \gamma_\mu \gamma_5 \psi_b & \stackrel{CP}{\to} & -\bar{\psi}_b \gamma^\mu \gamma_5 \psi_a & \text{vector current} \\
V_\mu & \stackrel{CP}{\to} & -V^{\mu\,\star} & \text{vector} \\
A_\mu & \stackrel{CP}{\to} & -A^{\mu\,\star} & \text{axial}
\end{array}
$$

The Standard Model in the interaction basis:

- gauge boson self-interactions symmetric under CP,
- gauge boson $\leftrightarrow$ fermion interactions symmetric under CP,
- gauge boson $\leftrightarrow$ Higgs boson interactions symmetric under CP,
- Higgs boson self-interactions symmetric under CP,
- Higgs boson $\leftrightarrow$ fermion (Yukawa) interactions ??? under CP.

Hadronic Yukawa interactions in the SM

$$\mathcal{L}_Y = -\sum_{j,k=1}^{N} \left\{ \left[\Gamma^u_{jk} (\bar{u}', \bar{d}')_{j\,L} \hat{H} u'_{k\,R} + \Gamma^{u\,\star}_{jk} \bar{u}'_{k\,R} \hat{H}^\dagger \begin{pmatrix} u' \\ d' \end{pmatrix}_{j\,L} \right] \right.$$

$$\left. \left[\Gamma^d_{jk} (\bar{u}', \bar{d}')_{j\,L} H d'_{k\,R} + \Gamma^{d\,\star}_{jk} \bar{d}'_{k\,R} H^\dagger \begin{pmatrix} u' \\ d' \end{pmatrix}_{j\,L} \right] \right\}$$

where

$$H = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad \text{and} \quad \hat{H} \equiv i\sigma_2 H^\star = \begin{pmatrix} \phi^{0\,\star} \\ -\phi^- \end{pmatrix}$$

$$(\bar{u}', \bar{d}')_{j\,L} \hat{H} u'_{k\,R} \quad \stackrel{CP}{\rightarrow} \quad \bar{u}'_{k\,R} \hat{H}^\dagger \begin{pmatrix} u' \\ d' \end{pmatrix}_{j\,L}$$

$$(\bar{u}', \bar{d}')_{j\,L} H d'_{k\,R} \quad \stackrel{CP}{\rightarrow} \quad \bar{d}'_{k\,R} H^\dagger \begin{pmatrix} u' \\ d' \end{pmatrix}_{j\,L}$$

If $\Gamma^u_{jk} \neq \Gamma^{u\,\star}_{jk}$ and/or $\Gamma^d_{jk} \neq \Gamma^{d\,\star}_{jk}$ then CP invariance seems to be broken explicitly by the Yukawa interactions !

Spontaneous symmetry breaking $\Longrightarrow$ $H \to \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\Downarrow$$

$$\mathcal{L}_Y \to \mathcal{L}_m = -\sum_{j,k=1}^{N} \left\{ \bar{u}'_{j\,L} \underbrace{\left[\frac{v}{\sqrt{2}}\Gamma^u_{jk}\right]}_{\mathcal{M}^u_{jk}} u'_{k\,R} + \bar{d}'_{j\,L} \underbrace{\left[\frac{v}{\sqrt{2}}\Gamma^d_{jk}\right]}_{\mathcal{M}^d_{jk}} d'_{k\,R} + \text{H.c.} \right\}$$

Mass matrix diagonalization (optional):

$$u_L = U_L u'_L \quad d_L = D_L d'_L \quad \text{for} \quad M^u = U_L \mathcal{M}^u U_R^\dagger \quad \text{diagonal}$$
$$u_R = U_R u'_R \quad d_R = D_R d'_R \quad \text{for} \quad M^d = D_L \mathcal{M}^d D_R^\dagger \quad \text{diagonal}$$

$$\mathcal{L}_Y = -\sum_{j=1}^{N} \left[\bar{u}_L M^u u_R + \bar{d}_L M^d d_R + \text{H.c.}\right]\left(1+\frac{h}{v}\right) = -\sum_{j=1}^{N}\left[\bar{u} M^u u + \bar{d} M^d d\right]\left(1+\frac{h}{v}\right.$$

$$W^+_\mu \bar{u}'_L \gamma^\mu d'_L + \text{H.c.} = W^+_\mu \bar{u}_L \gamma^\mu \underbrace{U_L D_L}_{V_{CKM}} d_L + \text{H.c.} = W^+_\mu \bar{u}_L \gamma^\mu V_{CKM} d_L + W^{-\,\mu} \bar{d}_L \gamma_\mu V^\dagger_{CKM}$$

$$W^+_\mu \overset{CP}{\leftrightarrow} -W^{-\,\mu} \quad \text{and} \quad \bar{u}_{j\,L}\gamma^\mu d_{k\,L} \overset{CP}{\leftrightarrow} -\bar{d}_{k\,L}\gamma_\mu u_{j\,L}$$

$V_{CKM} = V^\star_{CKM}$ $\Longrightarrow$ CP conservation

Rephasing for $N = 3$:

$$\begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \to \begin{pmatrix} e^{i\alpha_u} & & \\ & e^{i\alpha_c} & \\ & & e^{i\alpha_t} \end{pmatrix} \begin{pmatrix} u_L \\ c_L \\ t_L \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} \to \begin{pmatrix} e^{i\alpha_d} & & \\ & e^{i\alpha_s} & \\ & & e^{i\alpha_b} \end{pmatrix}$$

Then

$$V_{CKM} \to \begin{pmatrix} e^{-i\alpha_u} & & \\ & e^{-i\alpha_c} & \\ & & e^{-i\alpha_t} \end{pmatrix} V_{CKM} \begin{pmatrix} e^{i\alpha_d} & & \\ & e^{i\alpha_s} & \\ & & e^{i\alpha_b} \end{pmatrix}$$

so

$$V_{CKM}^{ij} \to e^{-i(\alpha_j - \alpha_j)} V_{CKM}^{ij}$$

$2N - 1$ phases could be removed (as unphysical), so eventually the number of relevant phases

$$N^2 - \frac{N(N-1)}{2} - (2N-1) = \frac{(N-1)(N-2)}{2}$$

CP violation appears for $N \geq 3$

Comments

- Spontaneous symmetry breaking:
  - 1960: Yoichiro Nambu was 39 years old, Giovanni Jona-Lasinio, who presented Nambu's work at Perdue was 28 years old.
  - Jeffrey Goldstone - "the Nambu-Goldstone bosons":
    Jeffrey Goldstone, "Field Theories With Superconductor Solutions",
    Nuovo Cim. **19**, 154 (1961).
  - Robert Brout, Francois Englert and Peter Higgs - the Higgs mechanism:
    Awarded the High Energy and Particle Prize of the European Physical Society in 1997 and the Wolf Prize in Physics in 2004 for the "Higgs mechanism" (the mechanism that generates mass for gauge vector bosons).
  - Gerald Guralnik, Thomas Kibble, Carl Hagen - the Higgs mechanism, but no prize.

- CP violation:
  - Nicola Cabibbo - the Cabibbo angle, no CP violation.

Cosmological illustrations e.g. baryogenesis

Baryogenesis is where CP violation and spontaneous symmetry breaking merge.

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} \simeq \frac{n_b}{n_\gamma} \simeq (1-6) \times 10^{-10}$$

The three necessary "Sakharov conditions" are:

1. Baryon number B violation.

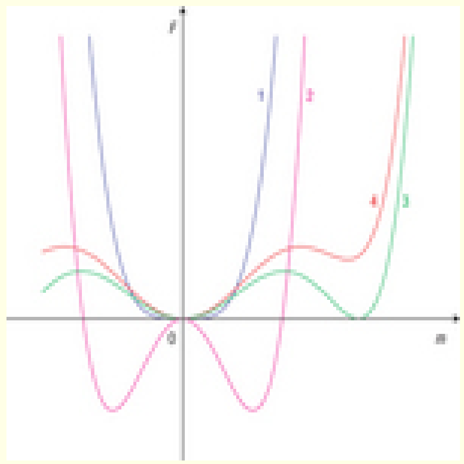
2. C-symmetry and CP-symmetry violation.

3. Interactions out of thermal equilibrium.

Electroweak baryogenesis:

- "CP-symmetry violation: Jarlskog invariant, $\mathbf{Im}(V_{ud}V_{cb}V^{\star}_{ub}V^{\star}_{cd})$ from the Kobayashi-Maskawa phase,

- "out of thermal equilibrium": first order electroweak phase transition:



We are not yet done: the strong CP problem

$$\mathcal{L}_{QCD\,CPV} = \theta \frac{\alpha_s}{16\pi} \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

Neutron EDM:

$$\theta \lesssim 10^{-9}$$

while there is no symmetry which prohibits the θ term, so we expect $\theta \sim 1$.

Why is θ so small?

Peccei-Quin axion?

$$\mathcal{L}_{QCD\,CPV} = \frac{a(x)}{f_a} \frac{\alpha_s}{16\pi} \mathrm{Tr}(F_{\mu\nu}\tilde{F}^{\mu\nu})$$

Hints for beyond the Standard Model physics

- Scalar fields provide a realistic (although possibly effective) description of existing particles.

- In the presence of many fermions and gauge (vector) bosons just one scalar (Higgs boson) is unlikely. Motivation for scalar extensions of the SM:
  - multi-singlet models (DM, inflation, DE)
  - multi-doublet models (little hierarchy problem, DM)

- Higgs boson as a $t\bar{t}$ bound state (top "colour" dynamics required), $h \sim (\bar{t}t)$, $m_t \sim 230$ GeV, W. Bardeen, Ch. Hill, and M. Lindner (stimulated by Nambu).

- Higgs boson as a pseudo-Nambu-Goldstone boson (motivated by the little hierarchy problem), "little Higgs models", H.Georgi and A. Pais.

- Axion physics $\propto aF^a_{\mu\nu}F^{a\,\mu\nu}$ (CP and scalar fields).

Conclusions

- Yoichiro Nambu received the prize for the hypothesis of spontaneous chiral symmetry breaking in strong interactions, with $(\pi^{\pm}, \pi^0)$, as massless Nambu-Goldston bosons.

- Makoto Kobayashi and Toshihide Maskawa received the prize for discovering the origin of CP violation (within the Glashow-Salam-Weinberg model) which has defended itself for 35 years as the proper source of CP violation in the quark sector and for predicting 3 generations of quarks.

$$Q_5^a|0\rangle \neq 0 \quad \text{and} \quad Q^a|0\rangle = 0$$

$$\Downarrow$$

- The Goldstone's theorem:

$$\langle 0|\partial^\mu J^a_{A\ \mu}(x)|\pi^b(q)\rangle = e^{-iqx} f_\pi m_\pi^2 \delta^{ab}$$

Since $f_\pi \neq 0$ ($Q_5^a|0\rangle \neq 0$) it follows (for $\partial^\mu J^a_{A\ \mu}(x) = 0$) that $m_\pi = 0$.

- The Goldberger-Treiman relation (satisfied within 7%):

$$m_n g_A(0) = g_{\pi n} f_\pi$$

where $f_{\pi n}$ pion-nucleon coupling constant and

$$\langle n(k')|J^-_{A\ \mu}(x)|p(k)\rangle = e^{iqx}\bar{u}(k')\left[\gamma_\mu\gamma_5 g_A(q^2) + q_\mu\gamma_5 h(q^2) + \cdots\right]u(k)$$

with $q = k' - k$.