

# Co nowego w modelowaniu oddziaływań neutrin w zakresie energii kilku GeV?



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(współpraca: C. Juszczak, T. Golan, S. Lee [JINR Dubna])

# Plan

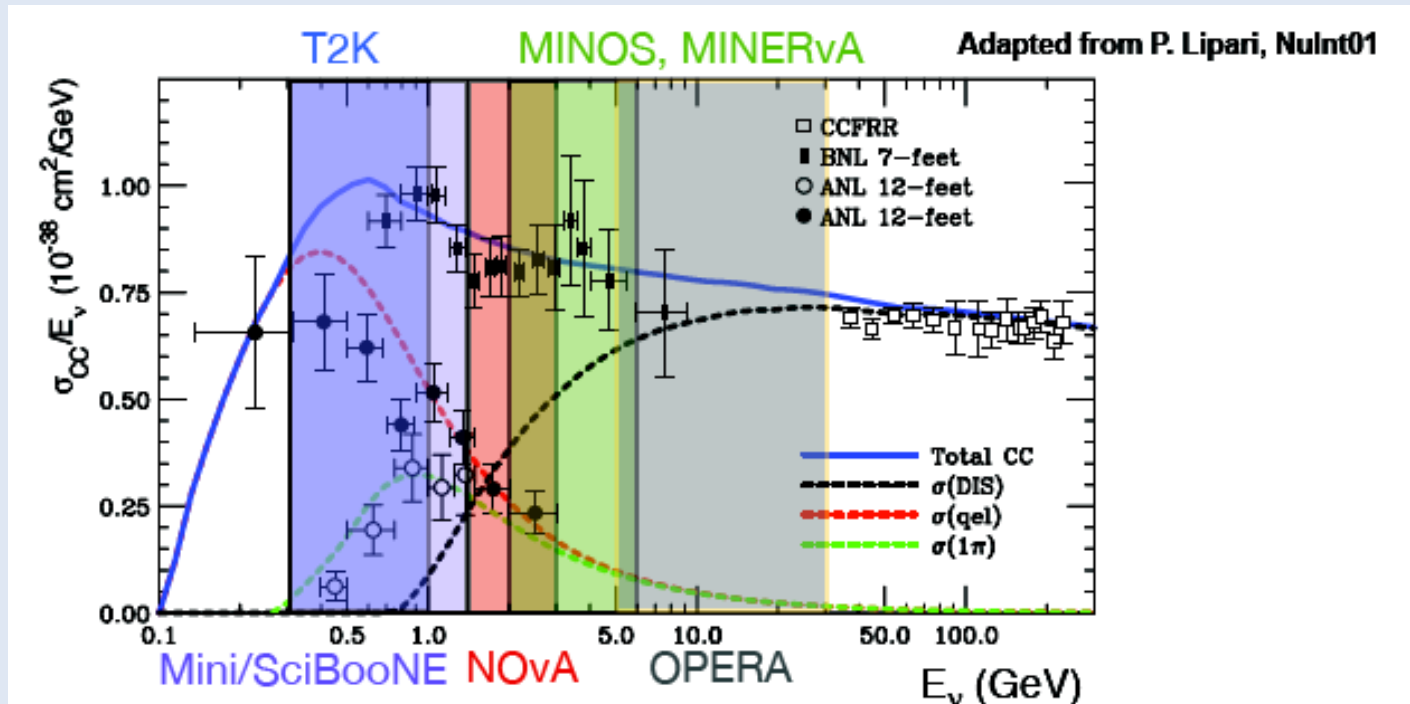


- motywacja
- zagadka quasi-elastycznej masy aksjalnej
- koherentna produkcja pionów
- produkcja  $\text{Pi}^0$  przez prąd neutralny
- inne pomiary
- generatory MC, znaczenie funkcji spektralnej
- wnioski

# Motivation



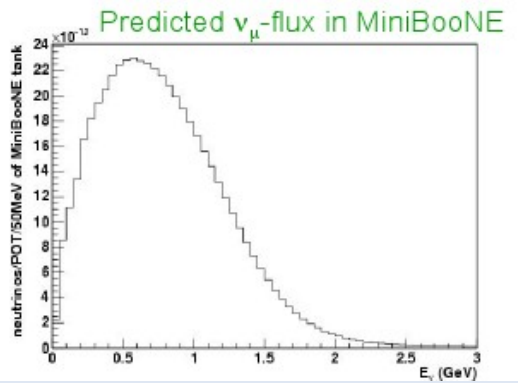
1 GeV is the typical energy region of all long baseline neutrino oscillation experiments.



(from Hiroshi Tanaka)



# Motivation



Why do we need cross sections?

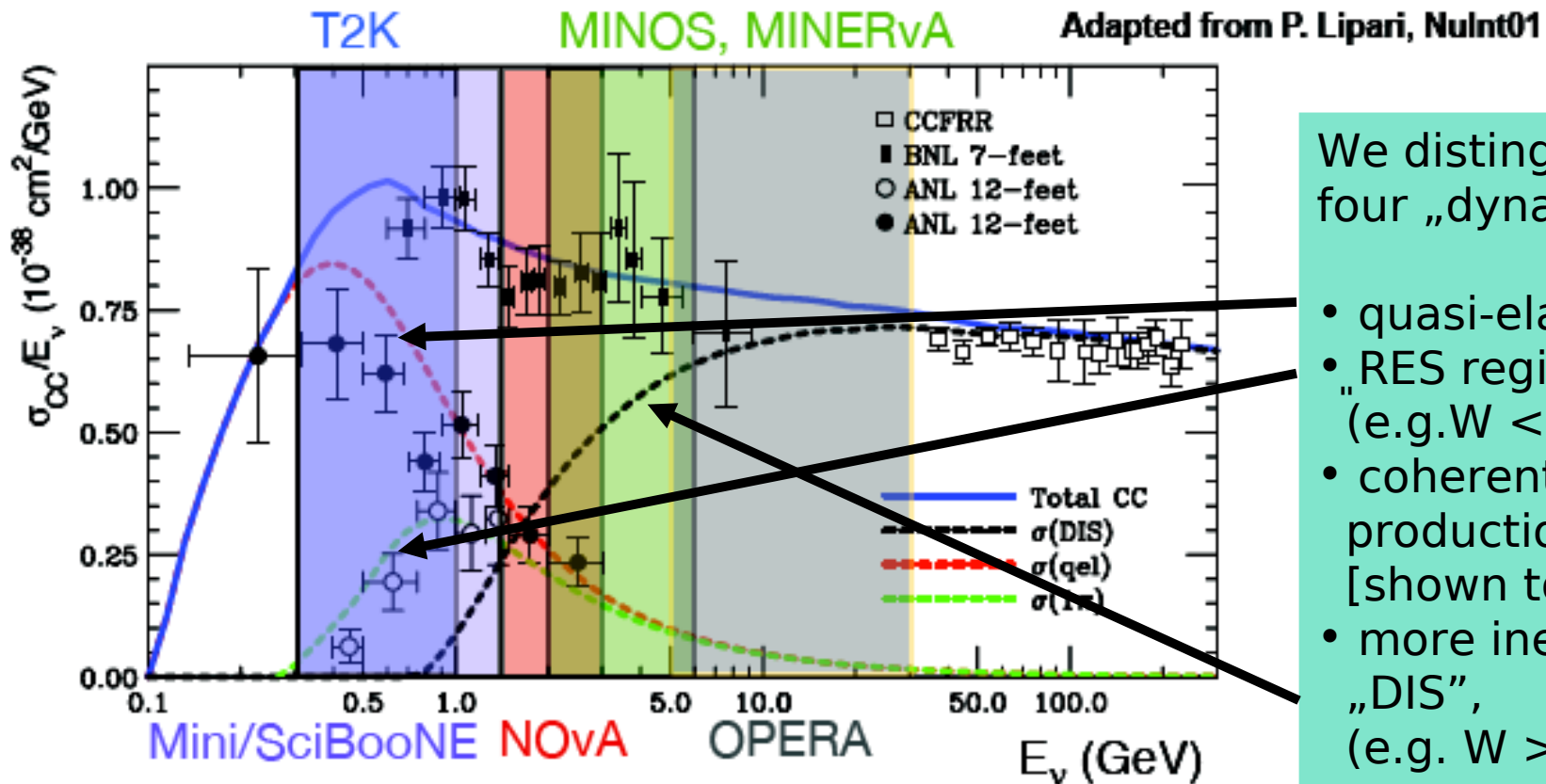
- We do not know neutrino energy, we only see final states.
- Oscillations are energy dependent !
- In order to investigate oscillations we must reconstruct neutrino energy or to investigate observed distribution of muons - in both cases we should understand cross sections.

In particular nuclear effects are important for targets like: carbon, oxygen, argon, iron.

# Motivation



On the theoretical side, several dynamical mechanisms must be considered together.



We distinguish four „dynamics“:

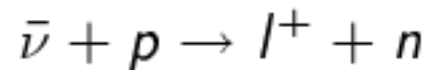
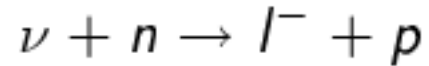
- quasi-elastic
- „RES region“, (e.g.  $W < 1.6 \text{ GeV}$ )
- coherent pion production [shown together]
- more inelastic „DIS“, (e.g.  $W > 1.6 \text{ GeV}$ )

# Quasi-elastic axial mass puzzle

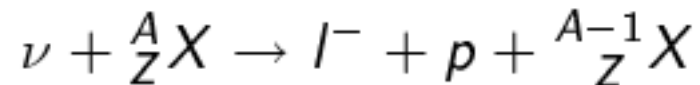


How do we define „quasi-elastic” reaction?

The name refers to the free target CC processes:



But typically, the reaction occurs on nucleus targets:

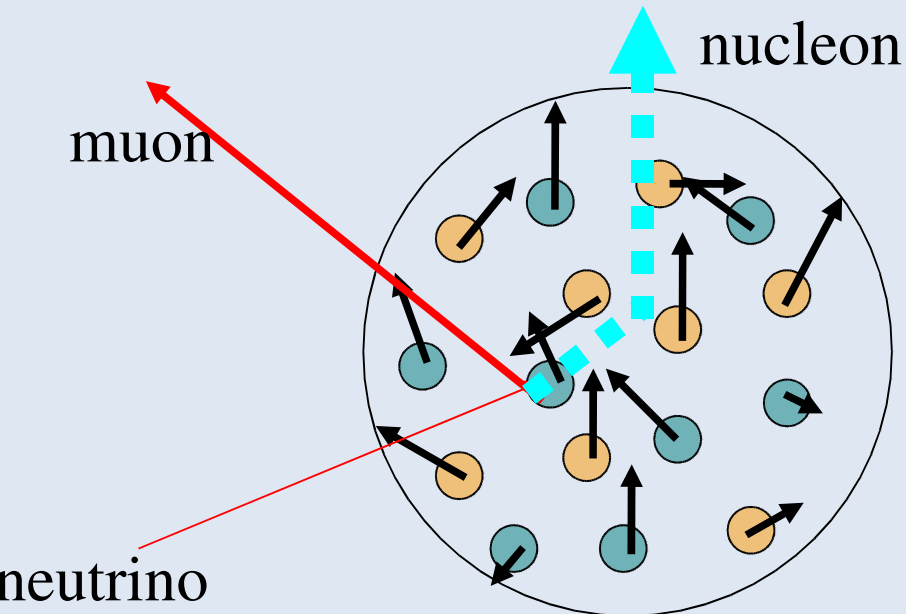


# Quasi-elastic axial mass puzzle



We assume *factorization*, every interaction is a two-step process:

- a *primary interaction* on a quasi-free nucleon
- *final state interactions* affecting only hadrons, here viewed as a unitary transformation in the space of final hadronic states



**We define „quasi-elastic” events as coming from the quasi-elastic primary interaction.**

But keep in mind that experimentalists observe only final states!

# Quasi-elastic axial mass puzzle



$$\nu + n \rightarrow l^- + p$$

$$\bar{\nu} + p \rightarrow l^+ + n$$

$$\Gamma_\mu = \gamma_\mu F_1(Q^2) + i\sigma_{\mu\nu} q^\nu \frac{F_2(Q^2)}{2M} + \gamma_\mu \gamma_5 F_A(Q^2) + \gamma_5 q_\mu \frac{F_P(Q^2)}{M}$$

F1 and F2 are determined by isospin symmetry, electromagnetic data is used

For the axial part the PCAC hypothesis is used to fix Fp

$$F_P(Q^2) = \frac{2M^2 F_A(Q^2)}{m_\pi^2 + Q^2}$$

We still need FA: the dipole form is assumed

$$F_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

$g_A = 1.26$  from the beta decay,  
 $M_A$  a free parameter (the only one!)



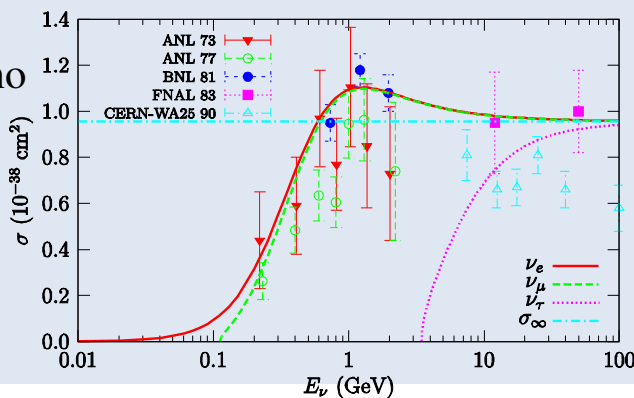
# Quasi-elastic axial mass puzzle



$$F_A(Q^2) = \frac{g_A}{\left(1 + \frac{Q^2}{M_A^2}\right)^2}$$

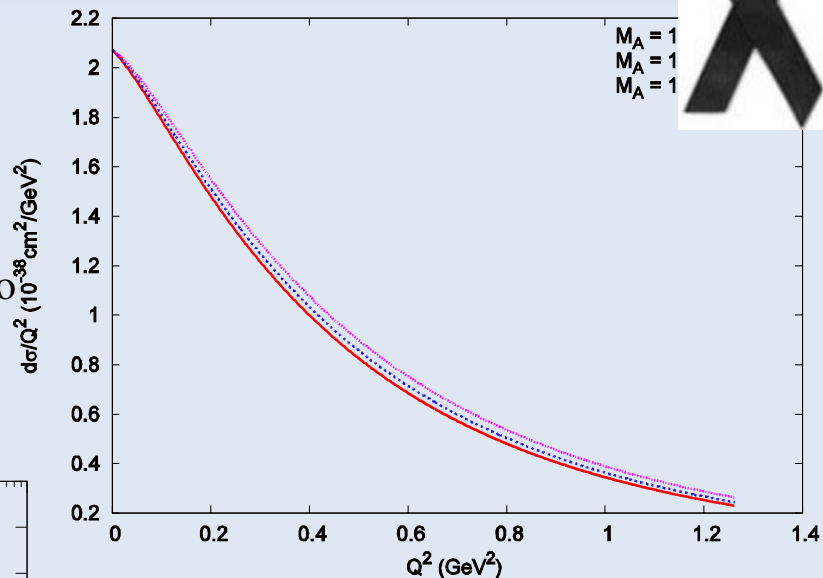
Axial mass determines **the shape** of differential cross section in  $Q^2$  and also **the total cross section**.

The limiting value of the cross section at large neutrino energy under assumption of dipole vector form factors:

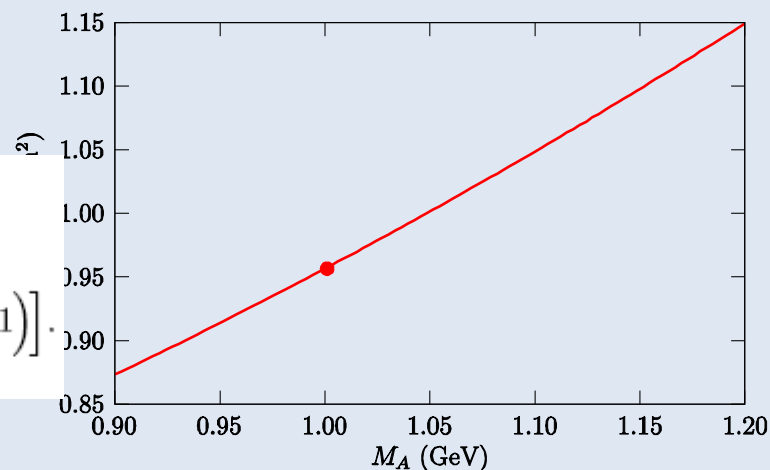


$$\sigma_\infty = \frac{G_F^2 \cos^2 \theta_C}{6\pi} \left[ M_V^2 + g_A^2 M_A^2 + \frac{2\xi(\xi + 2)M_V^4}{(4M^2 - M_V^2)^2} (M^2 - M_V^2) + \frac{3\xi(\xi + 2)M_V^8}{(4M^2 - M_V^2)^3} \left( \frac{4M^2}{4M^2 - M_V^2} \ln \frac{4M^2}{M_V^2} - 1 \right) \right]$$

(A. Ankowski, Acta Phys. Pol. B37 (2005) 377)



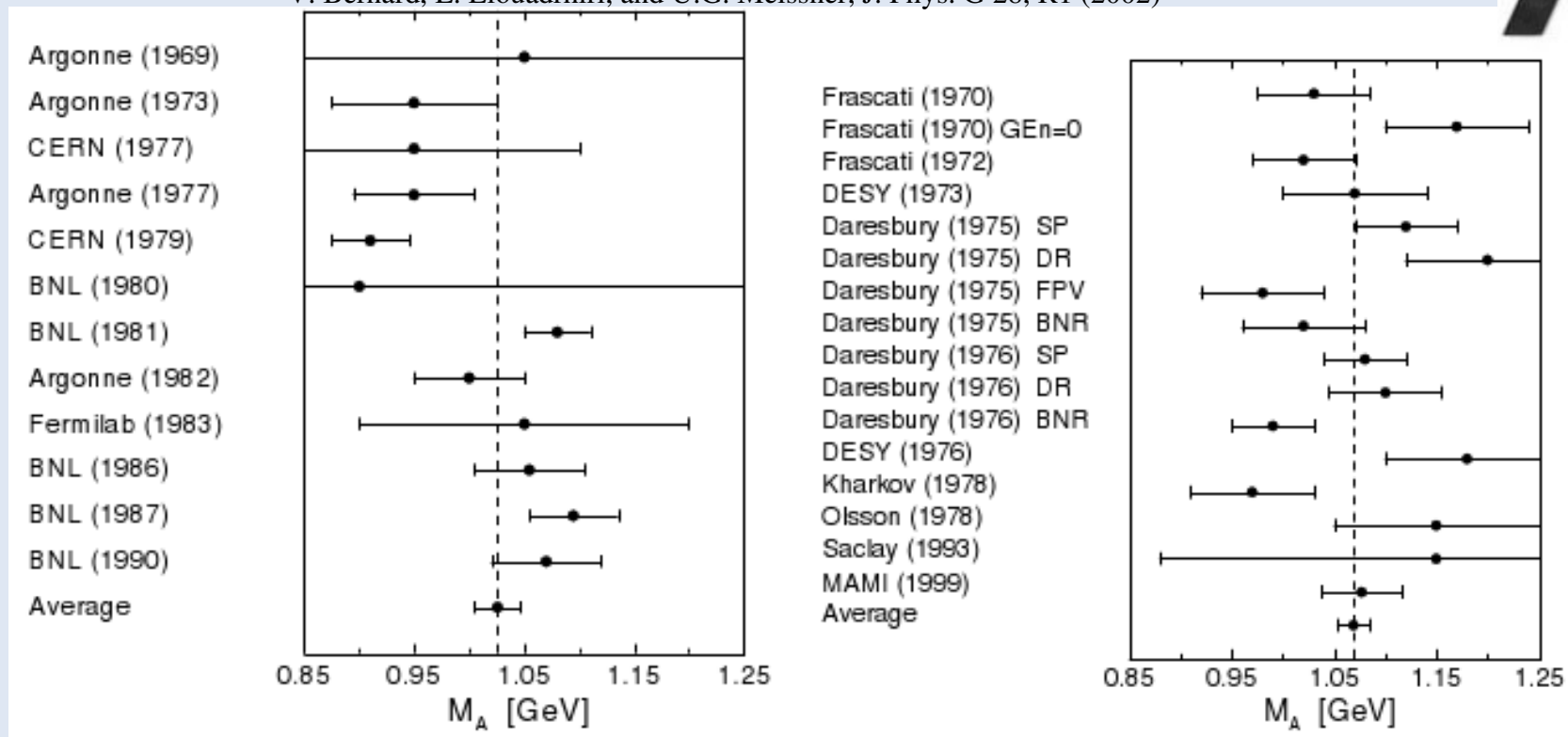
1e-38cm2



# Quasi-elastic axial mass puzzle



V. Bernard, L. Elouadrhiri, and U.G. Meissner, J. Phys. G 28, R1 (2002)



Neutrino experiments:  
 $M_A = 1.026 \pm 0.021$

Charged pion electroproduction (via PCAC!):  
 $M_A = 1.069 \pm 0.016$  GeV  
 but ... there are corrections to be calculated  
 within chiral perturbation theory.

# Quasi-elastic axial mass puzzle



Most recent neutrino data:

**TABLE I.** Modern determinations of  $M_A$  determined from shape fits to neutrino QE data assuming the FG model. Note: the K2K and MiniBooNE data were collected at lower neutrino energies than the MINOS and NOMAD samples.

experiment	$M_A$ (GeV)	target	fit range
K2K	$1.20 \pm 0.12$ [4]	$^{16}\text{O}$	$Q^2 > 0.2 \text{ GeV}^2$
K2K	$1.14 \pm 0.11$ [5]	$^{12}\text{C}$	$Q^2 > 0.2 \text{ GeV}^2$
MiniBooNE (2009)	$1.27 \pm 0.14$ [6]	$^{12}\text{C}$	$Q^2 > 0.25 \text{ GeV}^2$
MiniBooNE (2009)	$1.35 \pm 0.17, \kappa = 1.007 \pm 0.007$ [6]	$^{12}\text{C}$	$Q^2 > 0 \text{ GeV}^2$
MINOS	$1.26 \pm 0.17$ [7]	$^{56}\text{Fe}$	$Q^2 > 0.3 \text{ GeV}^2$
MINOS	$1.19 \pm 0.17, p_F \text{ scale} = 1.28$ [7]	$^{56}\text{Fe}$	$Q^2 > 0 \text{ GeV}^2$
NOMAD	$1.07 \pm 0.07$ [8, 9]	$^{12}\text{C}$	$Q^2 > 0 \text{ GeV}^2$

SciBooNE  $\rightarrow$  „consistent with  $M_A=1.21$ ” (within Neut MC)

**If the value of axial mass is increased from 1.03 to 1.23, the expected number of QE events is raised by  $\sim 20\%$  !**

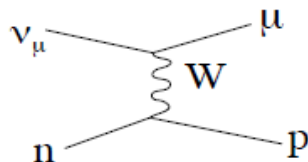
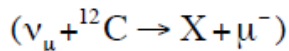
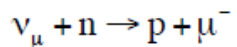
# Quasi-elastic axial mass puzzle



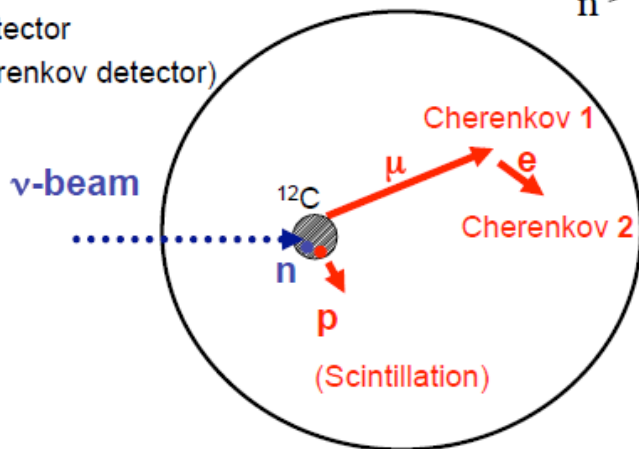
Possible explanations:

- statistical fluctuations (after all the discrepancy is on the  $2\sigma$  level)
- MiniBooNE overestimates the beam (the claim is that it is know with uncertainty of 10.7%; but all the cross sections reported by MB are very large – see later)
- something goes wrong in the data analysis...

# Quasi-elastic axial mass puzzle



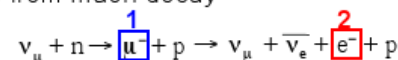
MiniBooNE detector  
(spherical Cherenkov detector)



muon like Cherenkov light and subsequent decayed electron (Michel electron) like Cherenkov light are the signal of CCQE event

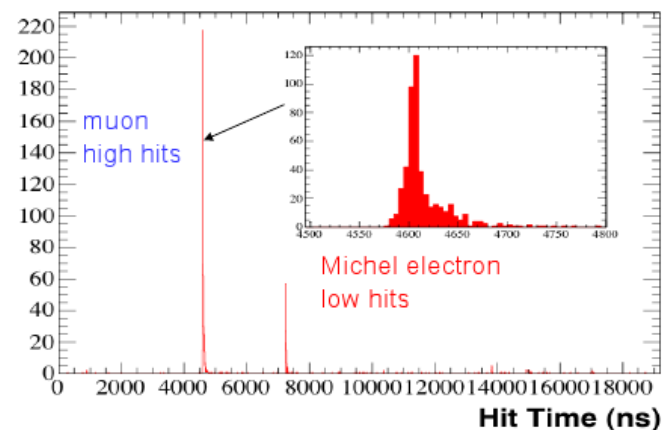
MiniBooNE collaboration tried to make the analysis independent on the models implemented in the Monte Carlo generator (Nuance).

$\nu_\mu$  CCQE interactions ( $\nu + n \rightarrow \mu + p$ ) has characteristic two "subevent" structure from muon decay



(from Teppei Katori)

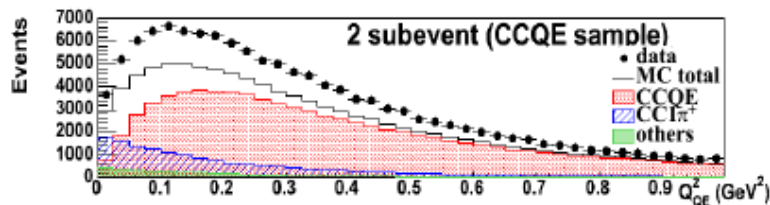
27% efficiency  
77% purity  
146,070 events  
with 5.58E20POT



# Quasi-elastic axial mass puzzle



The background is dominated with  $CC1\pi$  without pion ( $CCQE$ -like). We need a background prediction with an absolute scale.



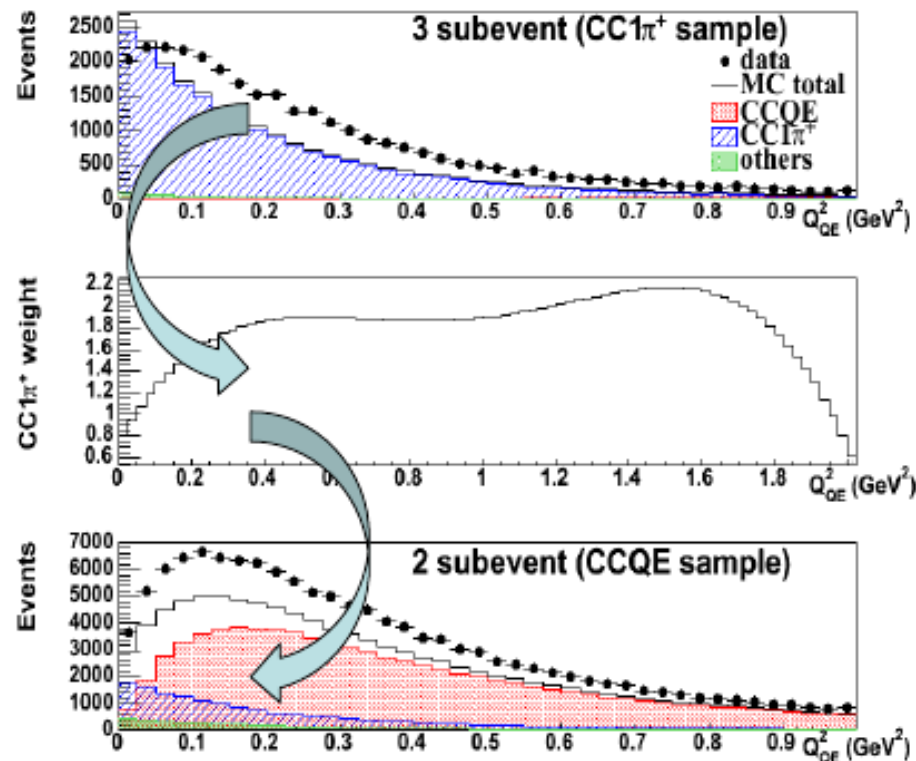
## Solution

Use data-MC  $Q^2$  ratio in  $CC1\pi$  sample to correct all  $CC1\pi$  events in MC.

(from Teppei Katori)

Then, this "new" MC is used to predict  $CC1\pi$  background in  $CCQE$  sample

This correction gives both  $CC1\pi$  background normalization and shape in  $CCQE$  sample



Background subtraction is essentially MC independent !

# Quasi-elastic axial mass puzzle



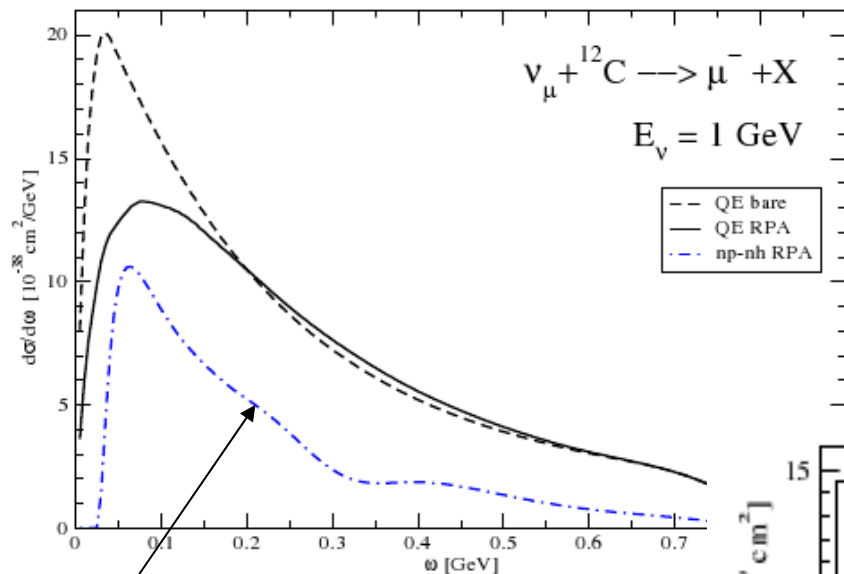
## Possible explanations:

- statistical fluctuations (after all the discrepancy is on the  $2\sigma$  level)
- MiniBooNE overestimates the beam (the claim is that it is know with uncertainty of 10.7%; but all the cross sections reported by MB are very large)
- something is wrong in the data analysis...
- large 2p-2h contribution ?!

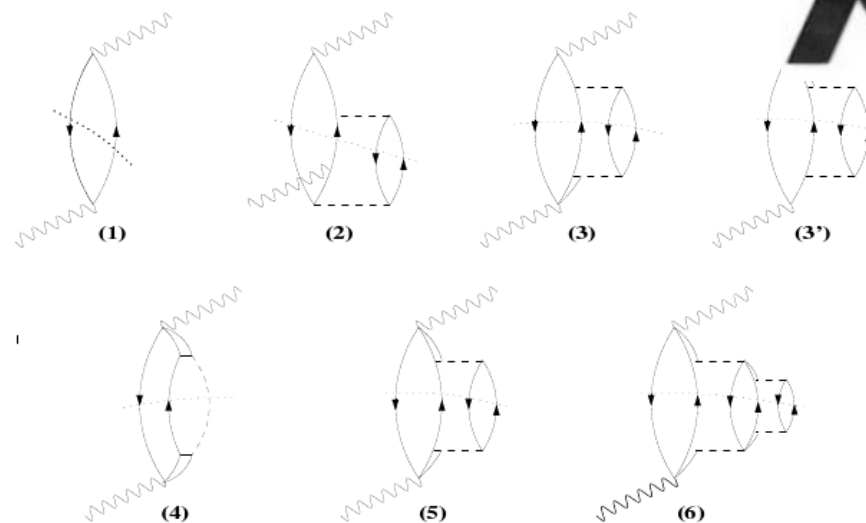
# Quasi-elastic axial mass puzzle



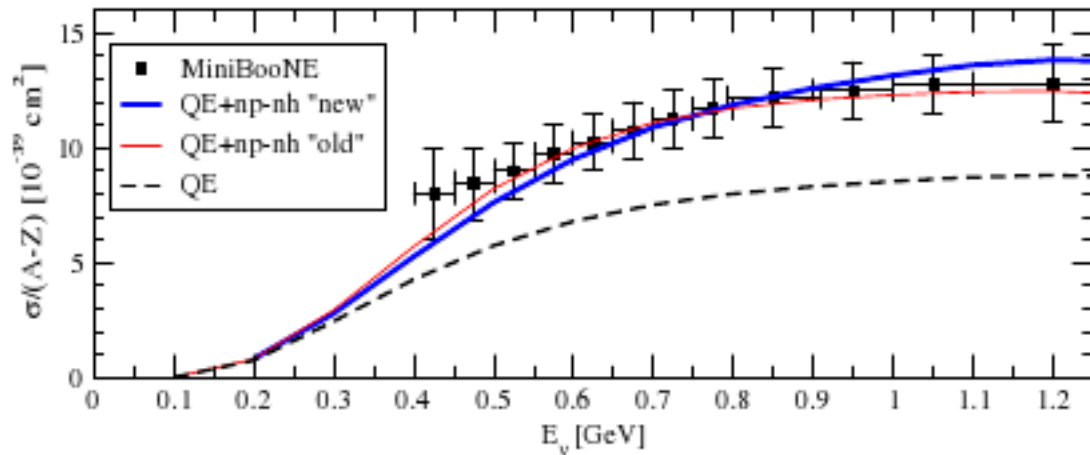
Martini-Marteau model  
(many body RPA computations)



new contribution claimed  
to be disregarded in „standard”  
computations



(M.Martini, M.Ericson, G.Chanfray, J. Marteau,  
arXiv: 0910.2622 [nucl-th])





# Quasi-elastic axial mass puzzle



Martini-Marteau model  
(many body RPA computations)

**Test: antineutrinos!**  
**The effect is expected to be smaller.**

$$R_\tau = \sum_n \langle n | \sum_{j=1}^A \tau(j) e^{i\mathbf{q}\cdot\mathbf{x}_j} | 0 \rangle \times \langle n | \sum_{k=1}^A \tau(k) e^{i\mathbf{q}\cdot\mathbf{x}_k} | 0 \rangle^* \delta(\omega - E_n + E_0).$$

$$\tau_j^\pm, (\boldsymbol{\sigma}_j \cdot \hat{\mathbf{q}}) \tau_j^\pm, (\boldsymbol{\sigma}_j \times \hat{\mathbf{q}})^i \tau_j^\pm,$$

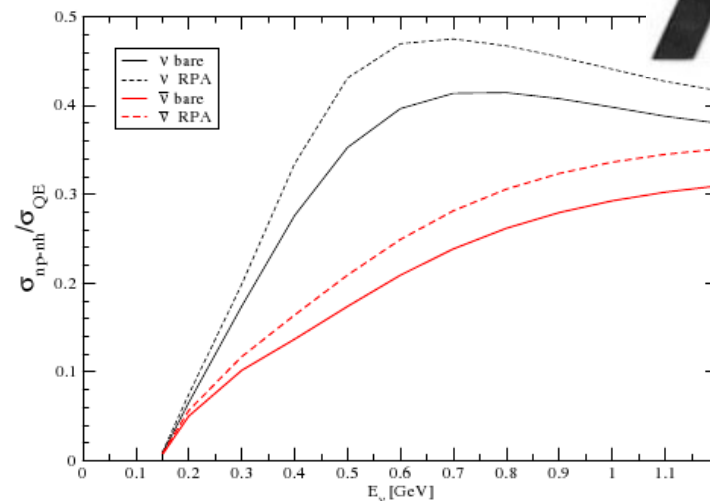


FIG. 6: Ratio of multinucleon component of “quasielastic” cross section on  $^{12}\text{C}$  to the single nucleon one for  $\nu_\mu$  and  $\bar{\nu}_\mu$  as a function of neutrino energy.

(M.Martini, M.Ericson, G.Chanfray, J. Marteau, arXiv: 1002.4538 [hep-ph])

	$\nu$		$\bar{\nu}$	
	np-nh	QE+np-nh	np-nh	QE+np-nh
ba	2.77	2.73	1.52	1.47
RPA	0.40	2.73	1.00	0.47

TABLE I: MiniBooNE flux-integrated CC  $\nu_\mu$ - $^{12}\text{C}$  and  $\bar{\nu}_\mu$ - $^{12}\text{C}$  total cross sections per neutron and per proton respectively in unit of  $10^{-39} \text{ cm}^2$ . The experimental CCQE  $\nu_\mu$ - $^{12}\text{C}$  value measured by MiniBooNE is  $9.429 \times 10^{-39} \text{ cm}^2$  with a total normalization error of 10.7 % [10].

# Quasi-elastic axial mass puzzle



MiniBooNE provided double differential cross section data which is very useful in more detail discussions

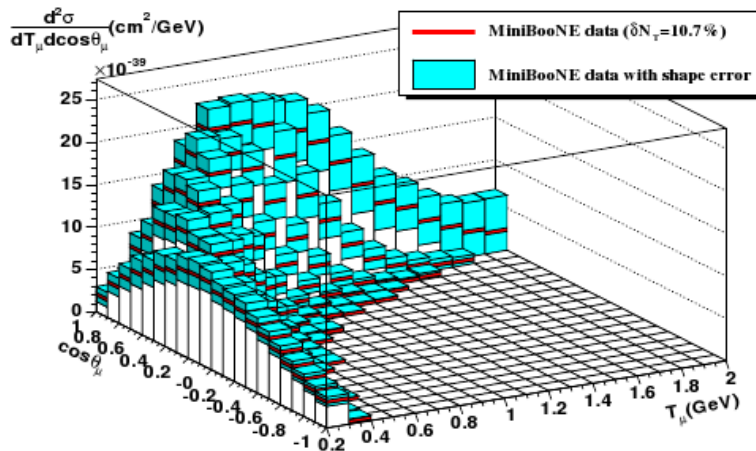
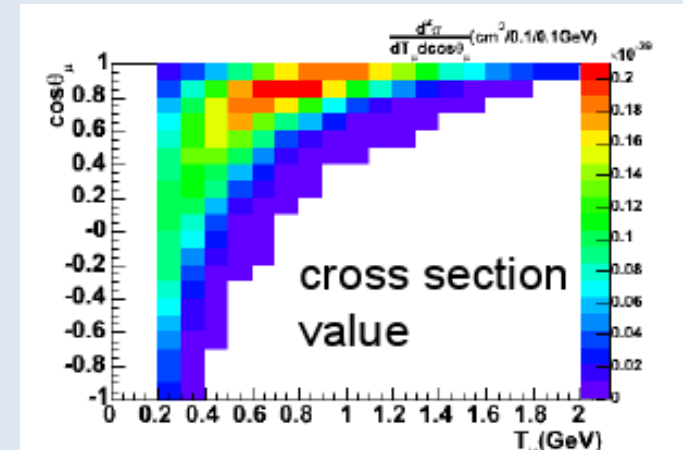


FIG. 13: (Color online). Flux-integrated double differential cross section per target neutron for the  $\nu_\mu$  CCQE process. The dark bars indicate the measured values and the surrounding lighter bands show the shape error. The overall normalization (scale) error is 10.7%. Numerical values are provided in Table VI in the Appendix.

arXiv: 1002.2680 [hep-ex],  
to be published in Phys. Rev.



It is important to confront with Martini's double differential cross section !

# Quasi-elastic low Q<sup>2</sup> problem

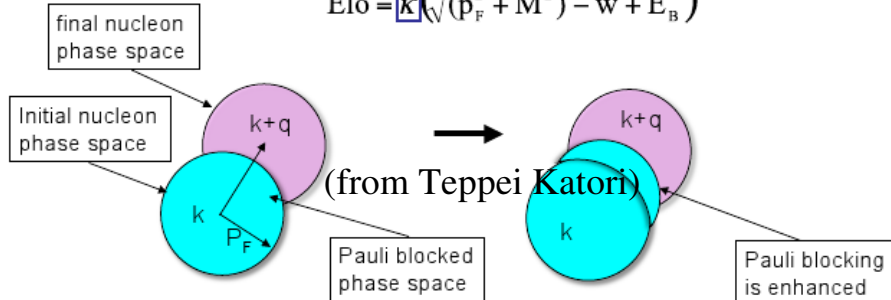


MiniBooNE and Minos introduced *ad hoc* parameters to correct for low Q<sup>2</sup> behavior.

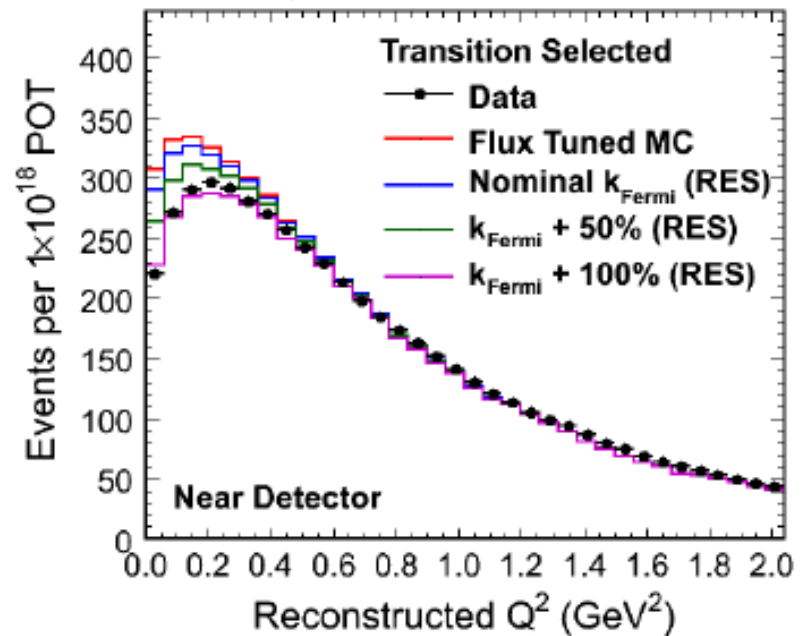
## Pauli blocking parameter "kappa", $\kappa$

To enhance the Pauli blocking at low Q<sup>2</sup>, we introduced a new parameter  $\kappa$ , which is the energy scale factor of lower bound of nucleon sea in RFG model in Smith-Moniz formalism, and controls the size of nucleon phase space

$$E_{lo} = \kappa \left( \sqrt{p_F^2 + M^2} - w + E_b \right)$$



## MINOS Preliminary



# How well do we understand the flux?

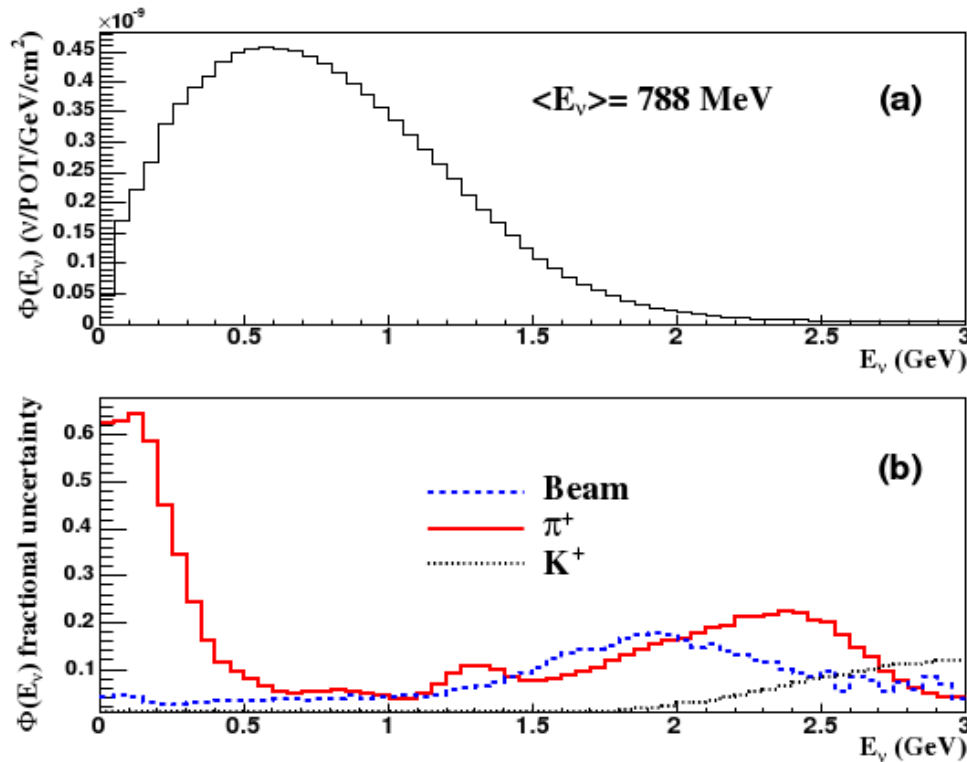
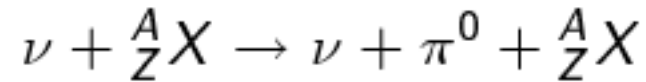
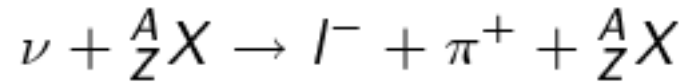


FIG. 2: (color online) Predicted  $\nu_\mu$  flux at the MiniBooNE detector (a) along with the fractional uncertainties grouped into various contributions (b). The integrated flux is  $5.16 \times 10^{-10} \nu_\mu/\text{POT}/\text{cm}^2$  ( $0 < E_\nu < 3$  GeV) with a mean energy of 788 MeV. Numerical values corresponding to the top plot are provided in Table [V](#) in the Appendix.

# Coherent pion production



Reaction is (nucleus  $X$   
remains in the ground state):



This is a small fraction of the overall single pion production cross section, but there has been recently a lot of experimental and theoretical activity.

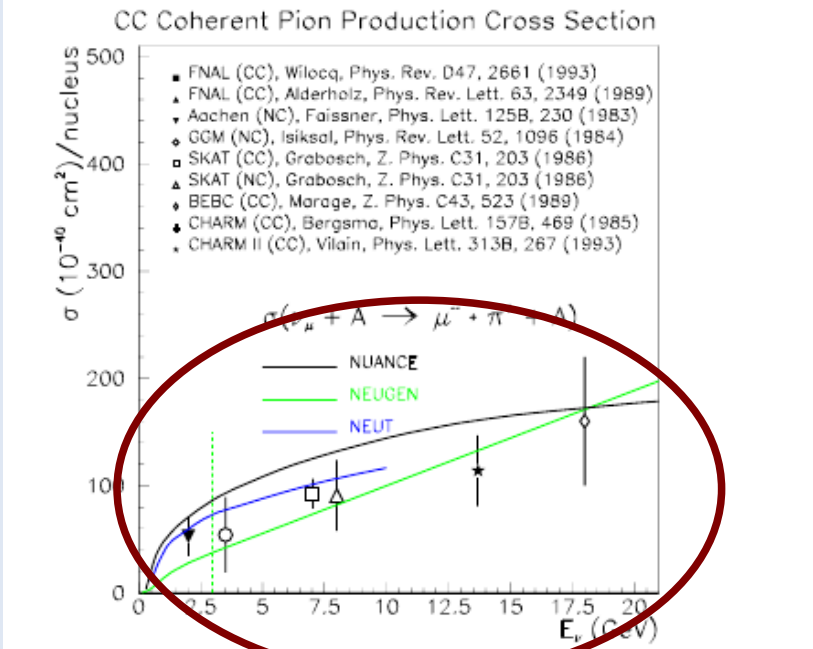
(dominant mechanism for pion production is via resonance excitation)

# Coherent pion production



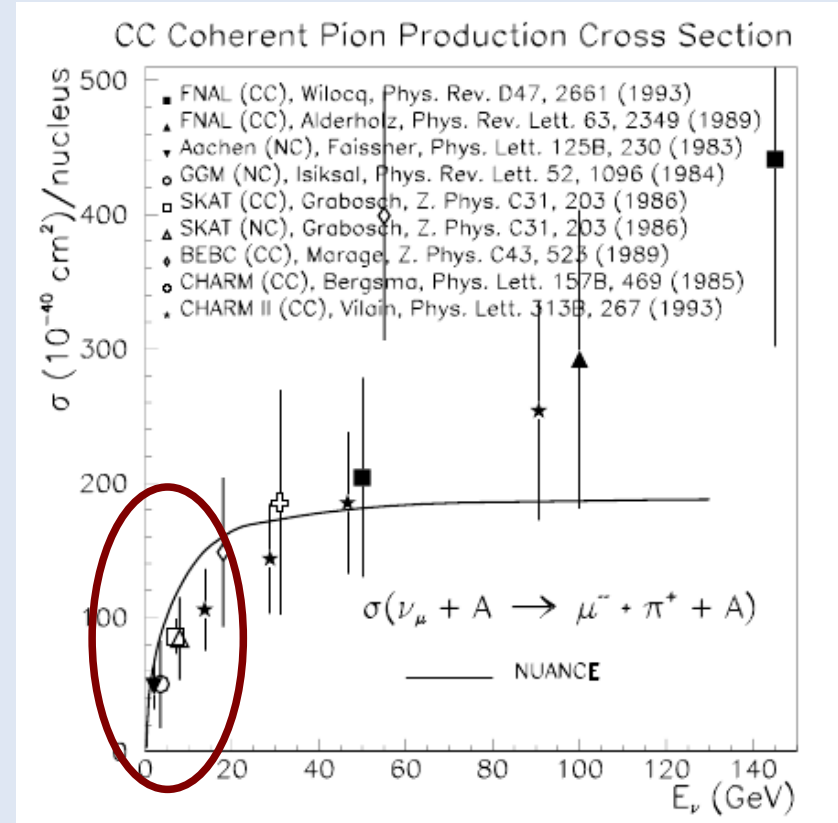
Well established at higher neutrino energies:

To allow comparison between experiments on different nuclear targets, assume  $A^{1/3}$  scaling (corrected to  $A = 16$ )



(from Sam Zeller, NuInt02)

All the data are put together !



$$\sigma_{NC}(\text{coh}) = 1/2 \sigma_{CC}(\text{coh})$$

$$\sigma^\nu(\text{coh}) = \sigma^{\bar{\nu}}(\text{coh})$$

# Coherent pion production

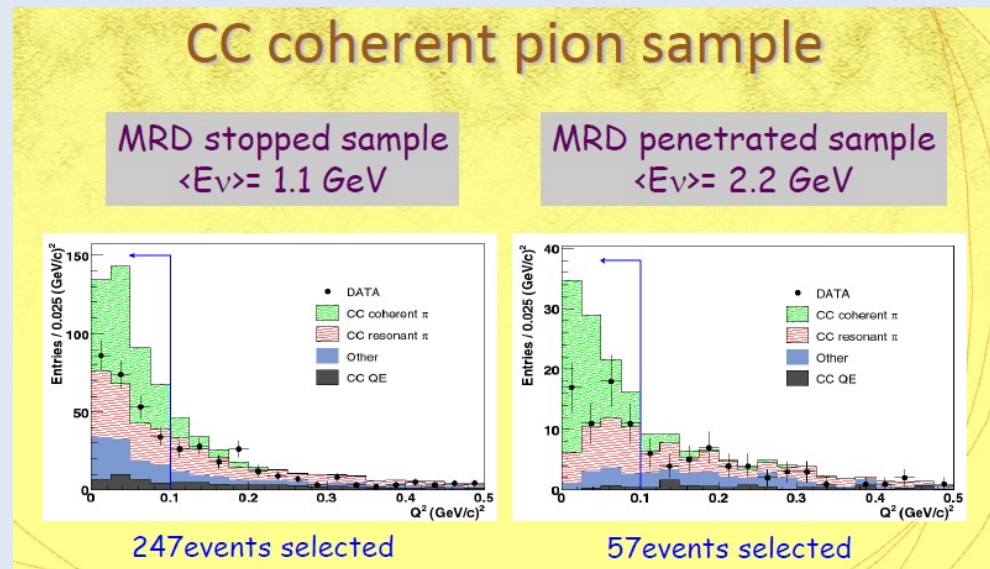


Experimentally the situation with low( $\sim 1$  GeV) energy coherent pion production is little puzzling:

- for NC reaction K2K and MiniBooNE reported a nonzero coherent contribution to the cross section
- for CC reaction K2K and SciBooNE reported no coherent signal

After imposing suitable cuts:

- QE rejection
- RES rejection (forward going pions are kept)



(from K. Hiraide)

# Coherent pion production



SciBooNE's conclusions:

MRD stopped sample

$\langle E_\nu \rangle = 1.1 \text{ GeV}$

$$\sigma(\text{CC coherent } \pi) / \sigma(\text{CC})$$

$$= (0.16 \pm 0.17(\text{stat})_{-0.27}^{+0.30}(\text{sys})) \times 10^{-2}$$

MRD penetrated sample

$\langle E_\nu \rangle = 2.2 \text{ GeV}$

$$\sigma(\text{CC coherent } \pi) / \sigma(\text{CC})$$

$$= (0.68 \pm 0.32(\text{stat})_{-0.25}^{+0.39}(\text{sys})) \times 10^{-2}$$

No evidence of CC coherent pion production is found



90% CL upper limit (Bayesian)

$$\begin{aligned} \sigma(\text{CC coherent } \pi) / \sigma(\text{CC}) &< 0.67 \times 10^{-2} && \text{for } \langle E_\nu \rangle = 1.1 \text{ GeV} \\ &< 1.36 \times 10^{-2} && \langle E_\nu \rangle = 2.2 \text{ GeV} \end{aligned}$$

K. Hiraide et al, PRD78,112004 (2008)

(from K. Hiraide)

The COH signal refers to what is expected from MC...



# Coherent pion production

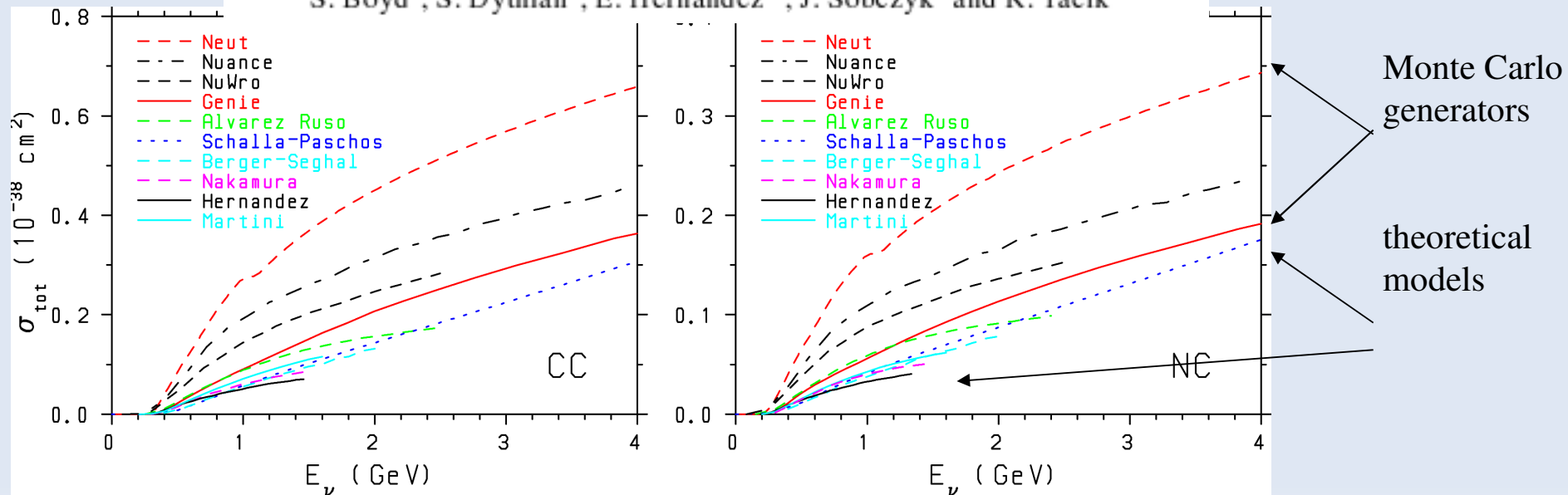


Current MC describe coherent pion production using the Rein-Sehgal model. They (we!) all claim to use the same model...

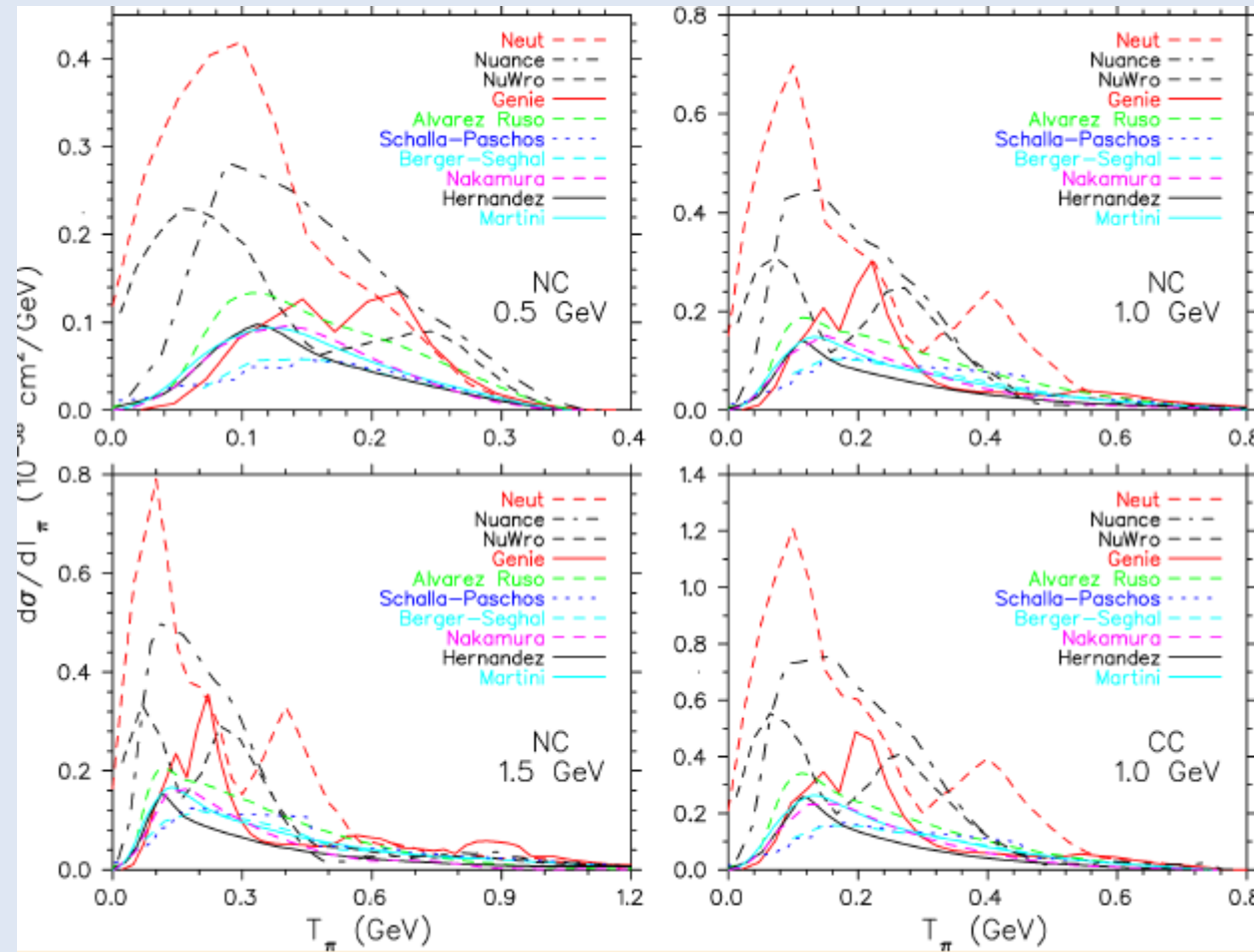
The few plots below come from the comparison project done for the last year NuInt09.

## Comparison of Models of Neutrino-Nucleus Interactions

S. Boyd\*, S. Dytman†, E. Hernández\*\*, J. Sobczyk‡ and R. Tacik§



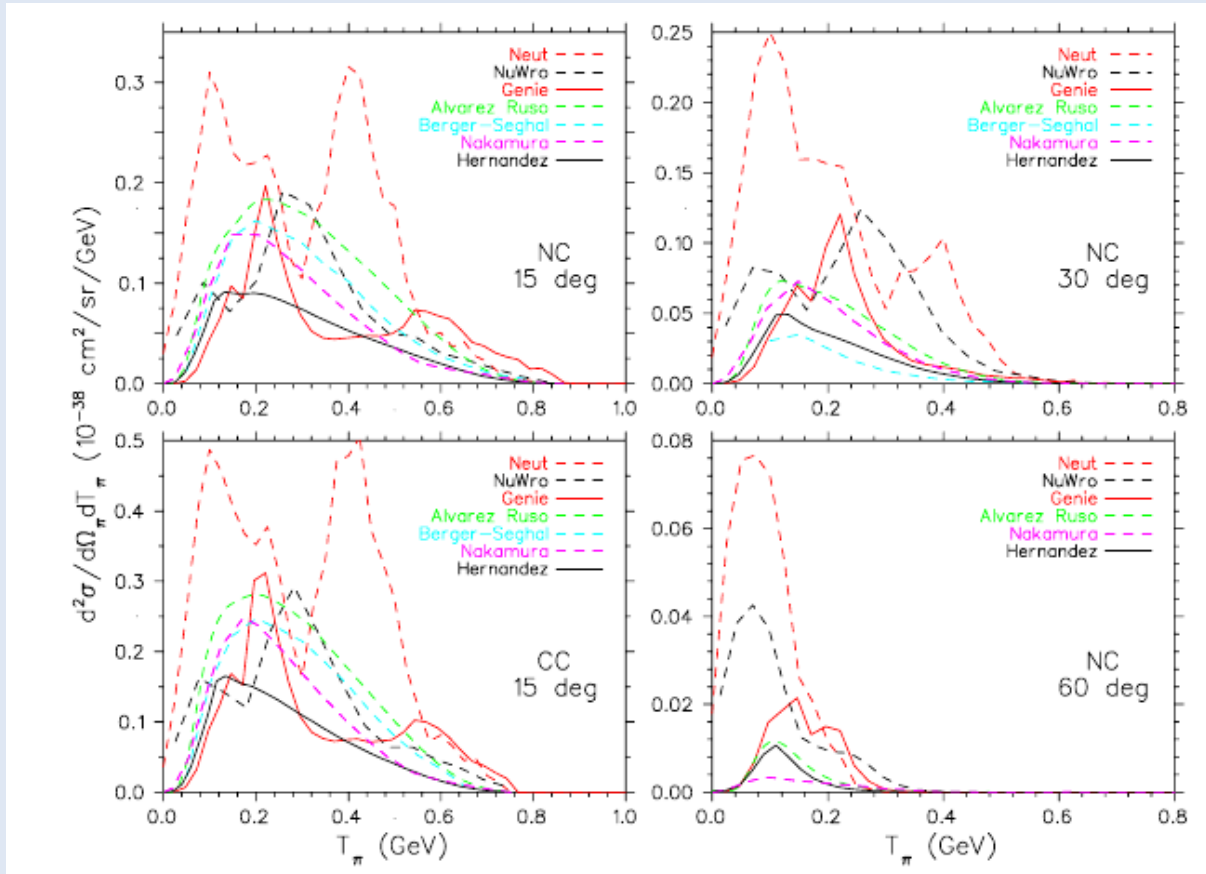
# Coherent pion production



Predictions for distributions of pions kinetic energy.

Monte Carlo's produce a lot of structure not seen in modern theoretical computations!

# Coherent pion production



Double differential cross sections at fixed pion production angle.

Neutrino energy is always 1 GeV.

# Coherent pion production



There are three main theoretical approaches:

- PCAC relates neutrino coherent process to elastic pion-nucleus scattering
- microscopic computations with  $\Delta$  resonance
- Martini-Marteau model, RPA many body computations to cover both quasi-elastic and  $\Delta$  excitation.

# Coherent pion production



Rein&Sehgal founded their model on the Adler's PCAC based theorem relating

$$\nu + |\alpha\rangle \rightarrow l^- + |\beta\rangle \quad \text{and} \quad \pi + |\alpha\rangle \rightarrow |\beta\rangle$$

$$q_\mu q^\mu \rightarrow 0 \implies |M(\nu + |\alpha\rangle \rightarrow l^- + |\beta\rangle)|^2 =$$

$$= 16 G^2 \cos^2 \theta_c f_\pi^2 \frac{E_\nu E_l}{(E_\nu - E_l)^2} |M(\pi + |\alpha\rangle \rightarrow |\beta\rangle)|^2$$

It is enough to choose:  $|\alpha\rangle = \frac{A}{Z} X \quad \wedge \quad |\beta\rangle = \pi + \frac{A}{Z} X$

and coherent pion production becomes related to elastic pion-nucleus scattering!

Further improvements and clarifications:

- a form-factor to extrapolate to nonzero  $Q^2$
- lepton mass corrections (Berger & Sehgal)
- kinematics
- precise pion-nucleus elastic scattering data

# NC1Pi0



**Motivation:** dangerous background in the electron neutrino appearance measurement in SK. Needs good theoretical control.

There are 4 **different** (but not *completely* independent) measurements:

**Beams:** K2K, MiniBooNE neutrinos, MiniBooNE antineutrinos

**Targets:**  $H_2O$ ,  $CH_2$ ,  $C_8H_8$  (*different ratios of carbon to hydrogen*)

**Events:** NC1Pi0 with FSI, NCPi0 with some (?) cuts (SciBooNE)

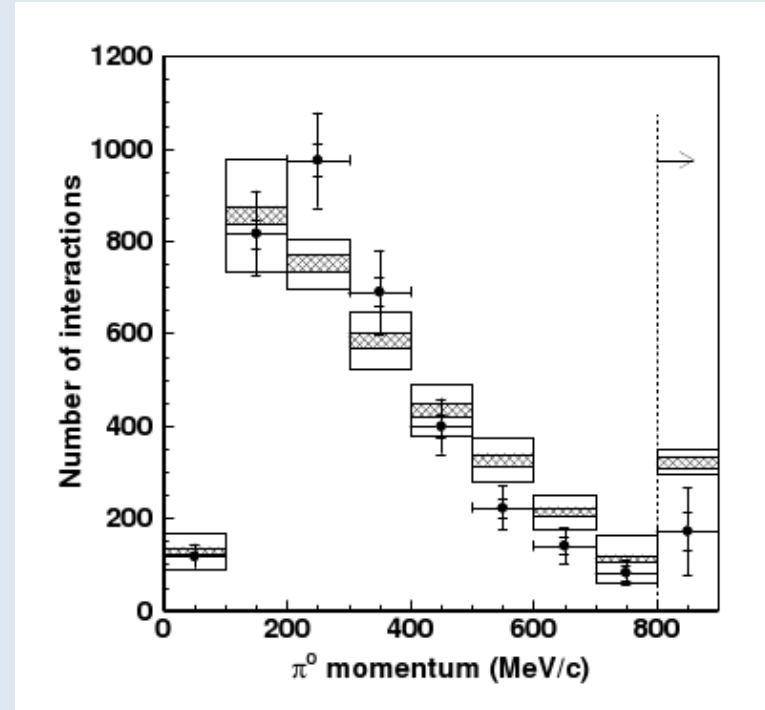
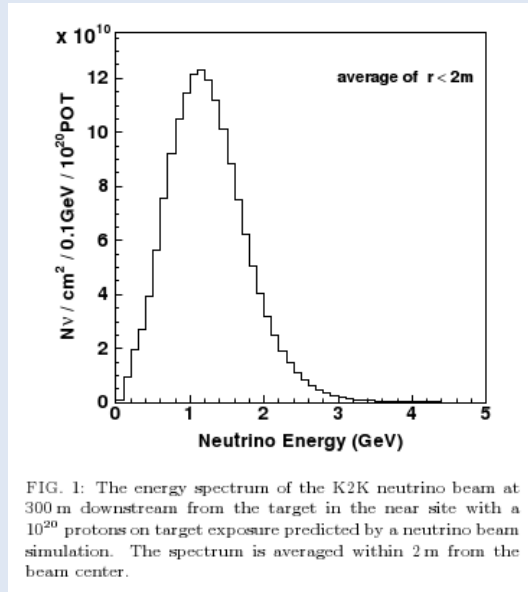
**Cross section:** normalized (MiniBooNE),  
ratio NC1Pi0/CC (K2K, SciBooNE).

# NC1Pi0



**K2K: Nakayama et al, PLB619 (2005) 255**

**Definition:  
1Pi0 &&  
no other  
pions**



By taking the ratio, the relative cross section for  $\text{NC}1\pi^0$  interactions to the total  $\nu_\mu\text{CC}$  cross section is measured to be  $0.064 \pm 0.001(\text{stat.}) \pm 0.007(\text{sys.})$ .

Target:  $\text{H}_2\text{O}$

# NC1Pi0

## MiniBooNE (and SciBooNE) beams:

Neutrino mode

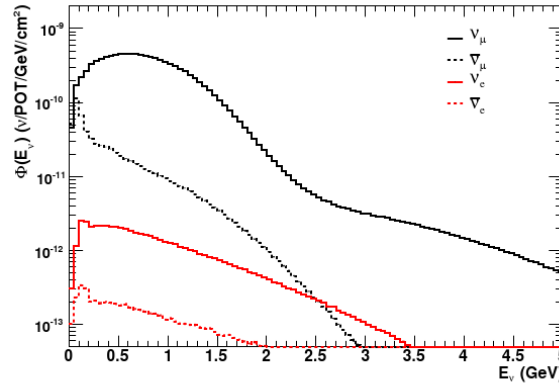


FIG. 27: Total predicted flux at the MiniBooNE detector by neutrino species with horn in neutrino mode.

Antineutrino mode

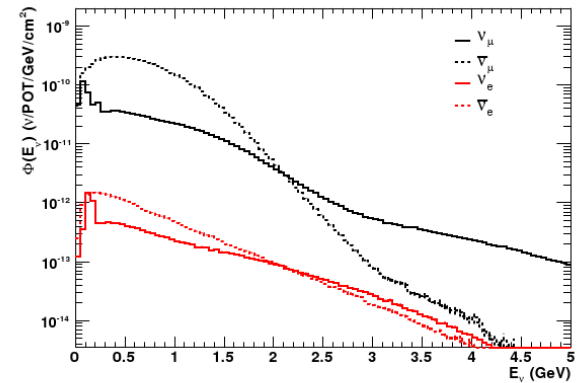


FIG. 28: Total predicted flux at the MiniBooNE detector by neutrino species with horn in anti-neutrino mode.



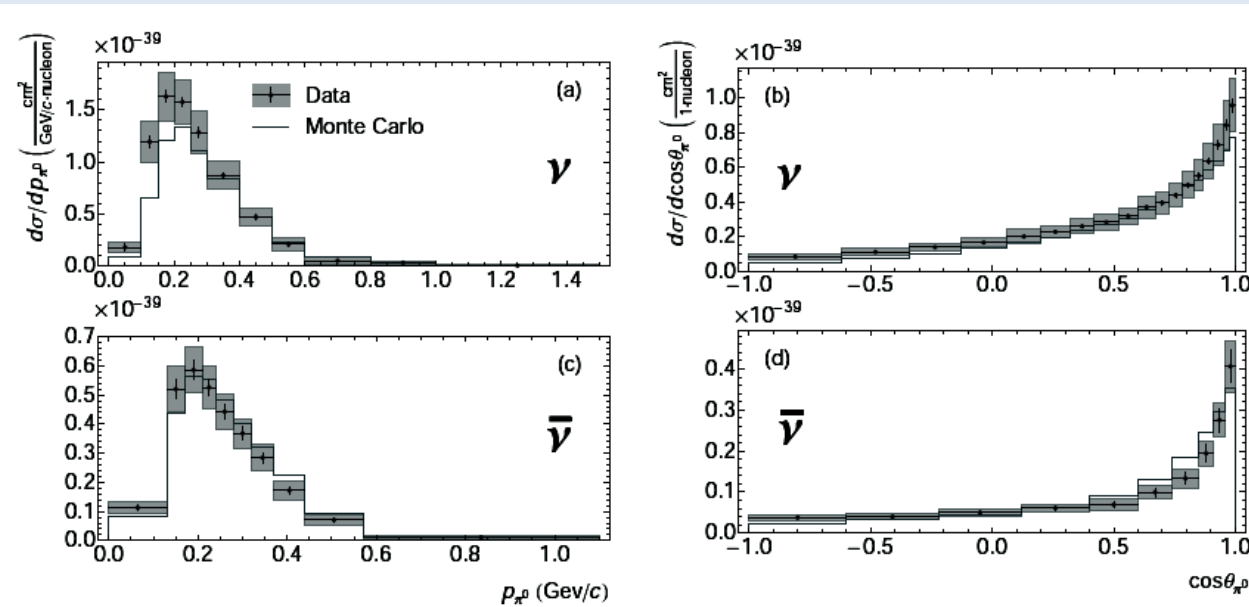
# NC1Pi0



MiniBooNE:

Phys. Rev. D81 (2010) 013005

scattering. We define signal NC  $1\pi^0$  events to be NC interactions wherein only one  $\pi^0$  and no additional meson exits the target nucleus (no requirement on the number or identity of outgoing nucleons is made). This definition is consistent with that used at K2K[22]. It is specifically



target:  $CH_2$

FIG. 7: Flux-averaged absolute differential cross sections for NC  $1\pi^0$  production on  $CH_2$  including the effects of FSI. Data are shown as black dots with statistical error bars and systematic error boxes. The dark-gray line is the Monte Carlo prediction[26] using R-S models of single pion production[2, 5] modified as described in the text. (a)  $\frac{d\sigma}{dp_{\pi^0}}$  for  $\nu_\mu$ -induced production. (b)  $\frac{d\sigma}{d\cos\theta_{\pi^0}}$  for  $\nu_\mu$ -induced production. (c)  $\frac{d\sigma}{dp_{\pi^0}}$  for  $\bar{\nu}_\mu$ -induced production. (d)  $\frac{d\sigma}{d\cos\theta_{\pi^0}}$  for  $\bar{\nu}_\mu$ -induced production. The numerical values for the cross sections appear in Appendix C and are also available at the MiniBooNE website[36].

# NC1Pi0



SciBooNE:

Phys. Rev. D81 (2010) 033004

neutrino beam on a polystyrene target ( $C_8H_8$ ). We obtain  $(7.7 \pm 0.5(\text{stat.}) \pm 0.5(\text{sys.})) \times 10^{-2}$  as the ratio of the neutral current neutral pion production to total charged current cross section; the

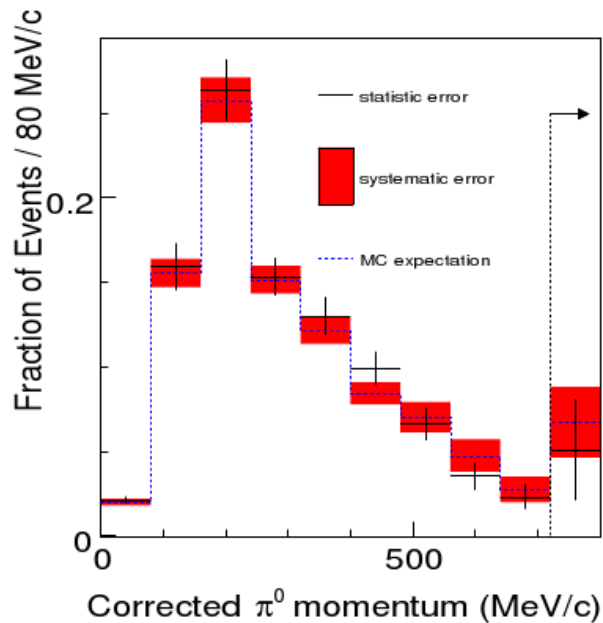


FIG. 21: The  $\pi^0$  momentum distribution after all corrections described in the text, with statistical (error bars) and systematic (red boxes) uncertainties. The dashed line shows the Monte Carlo expectation based on the Rein and Sehgal model.

We define an NC $\pi^0$  interaction as an NC neutrino interaction in which at least one  $\pi^0$  is emitted in the final state from the target nucleus,  $\nu_\mu C \rightarrow \nu_\mu \pi^0 X$  where  $X$  represents the nuclear remnant and any combination of nucleons and mesons. According to our MC simulation,

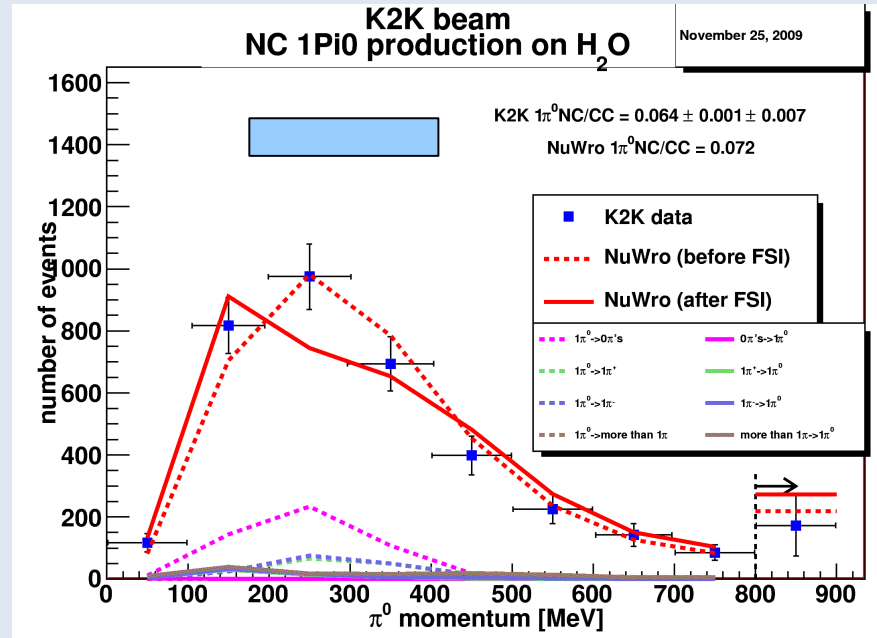
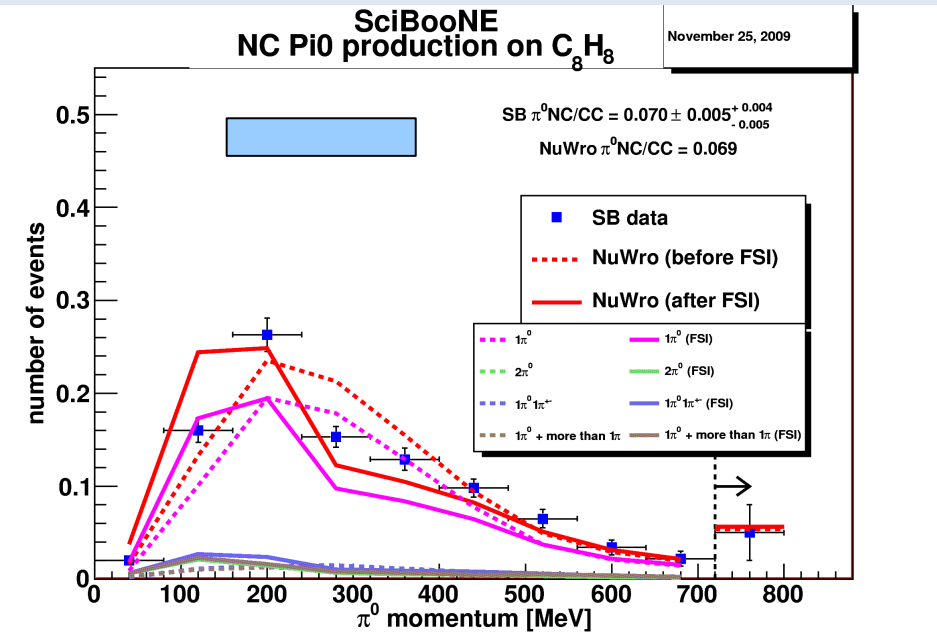
target:  $C_8H_8$

# NC 1 Pi0 production

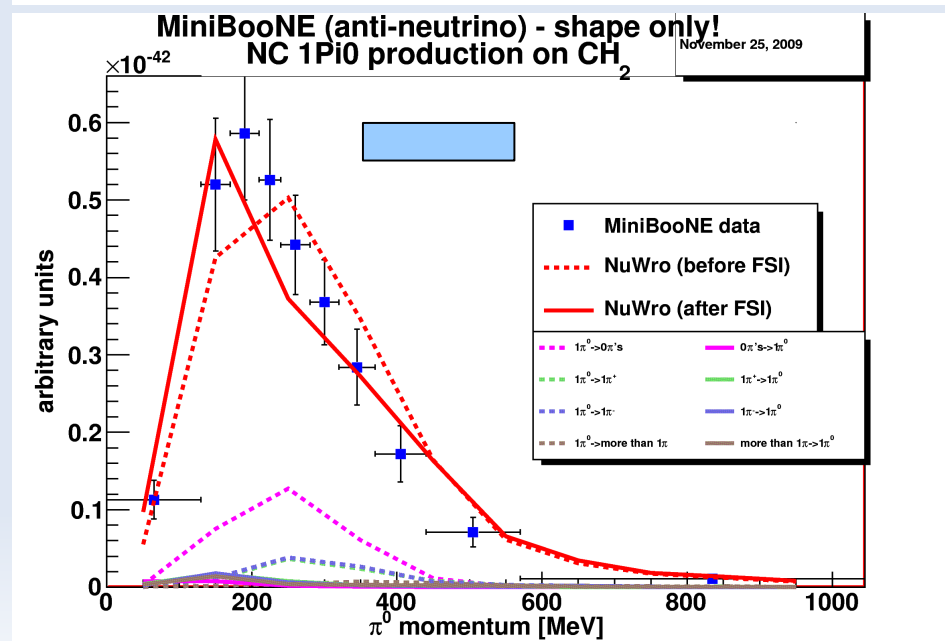
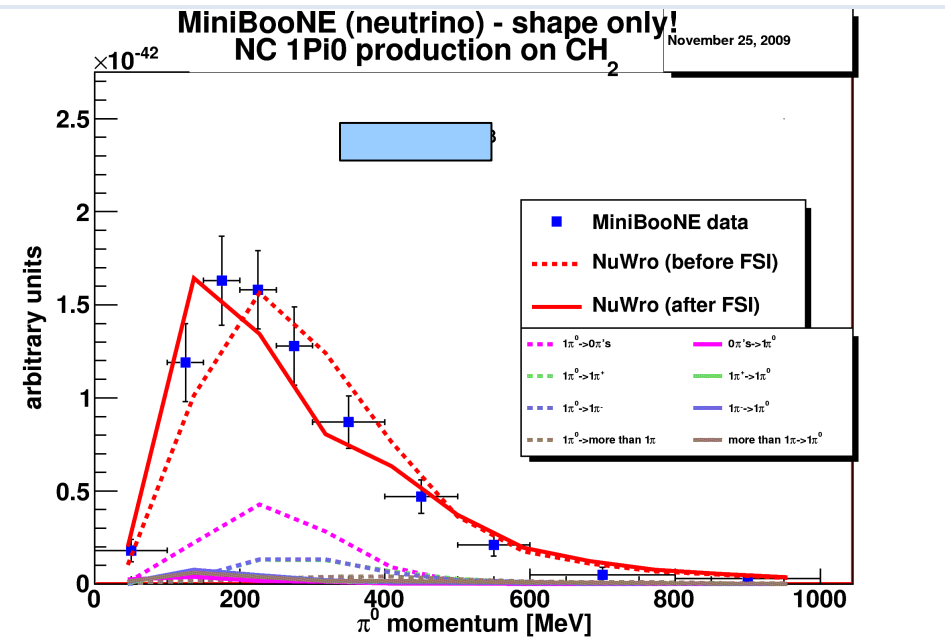
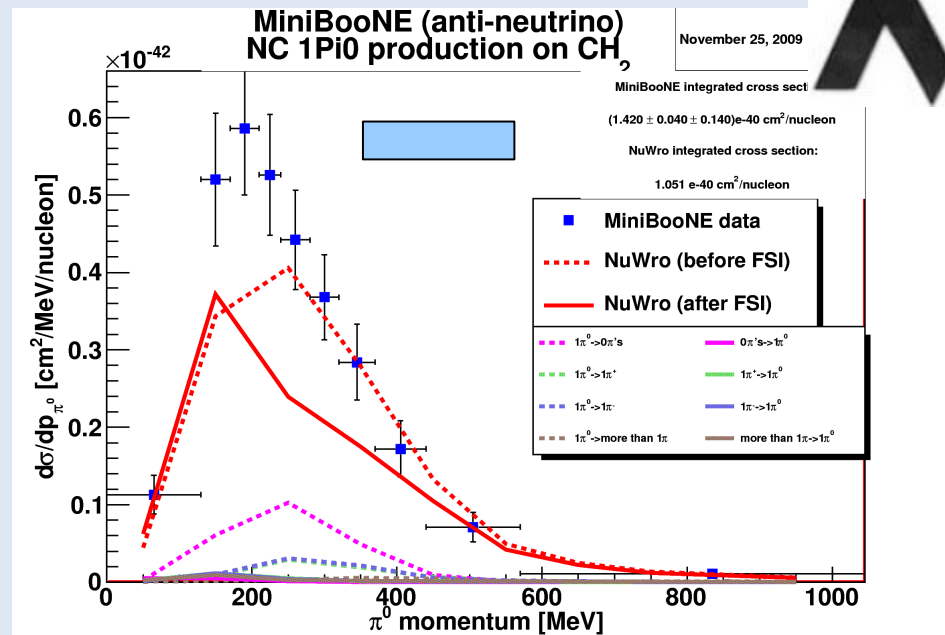
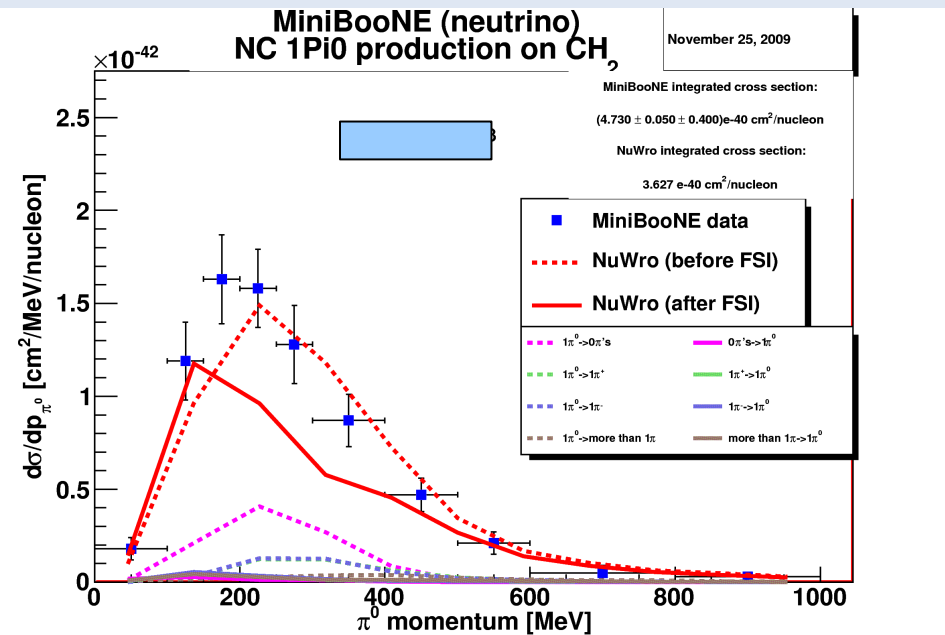


The data presents a challenge to Monte Carlo generators of events. FSI effects like pion absorption and „formation zone” are important.

I show predictions from NuWro MC generator of events.



# NC 1 Pi0 production

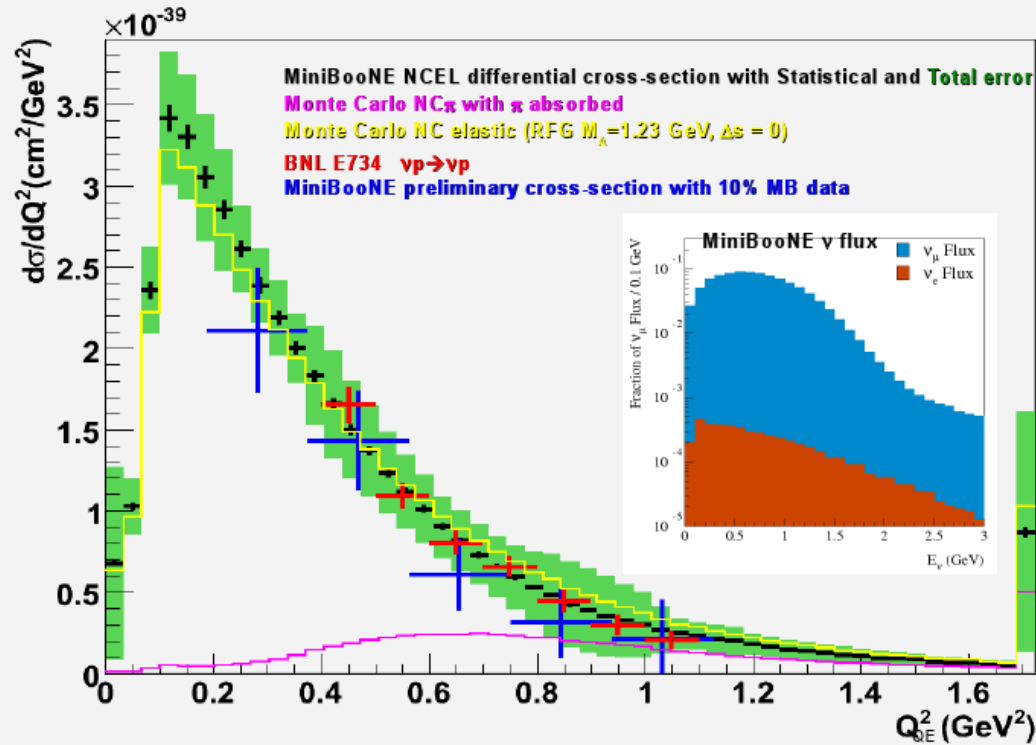


# Other measurements



## Neutral current elastic cross section

Results: Flux-averaged MiniBooNE NC elastic differential cross-section



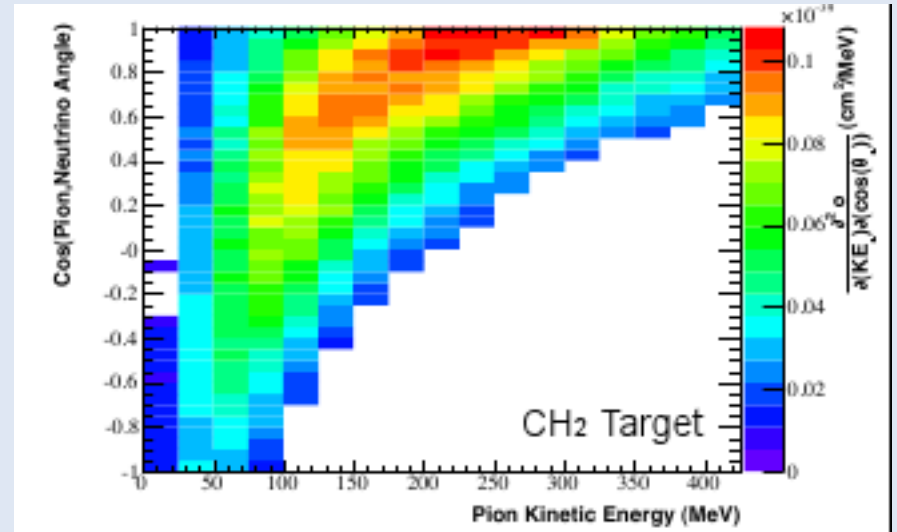
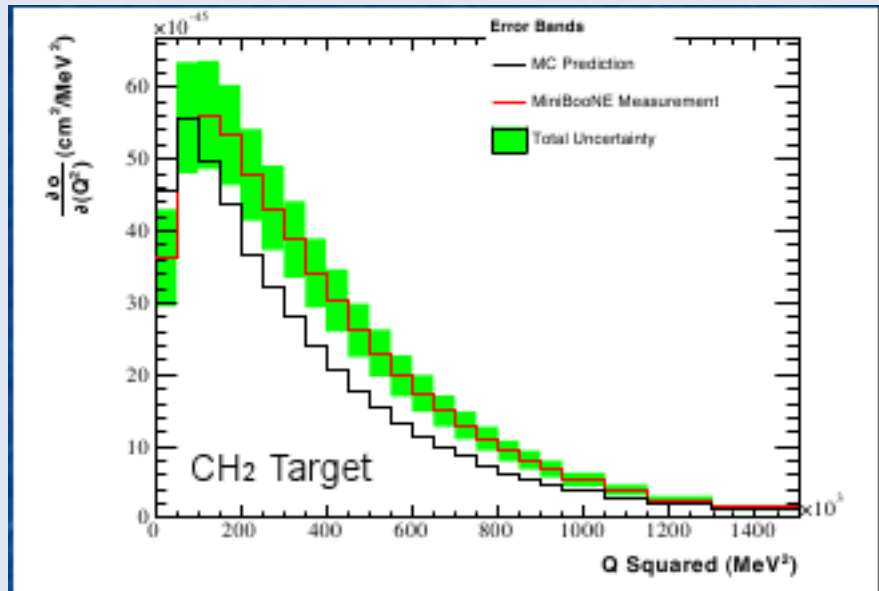
MiniBooNE measures both Cerenkov and scintillation light !

(from D. Perevalov)

# Other measurements



## Charge current $\text{Pi}^+$ production (MiniBooNE)



(from M. Wilking)

Note that the measured cross section is much larger than MC predictions !

# Other measurements



## Ratio CC1Pi+/CCQE (MiniBooNE)

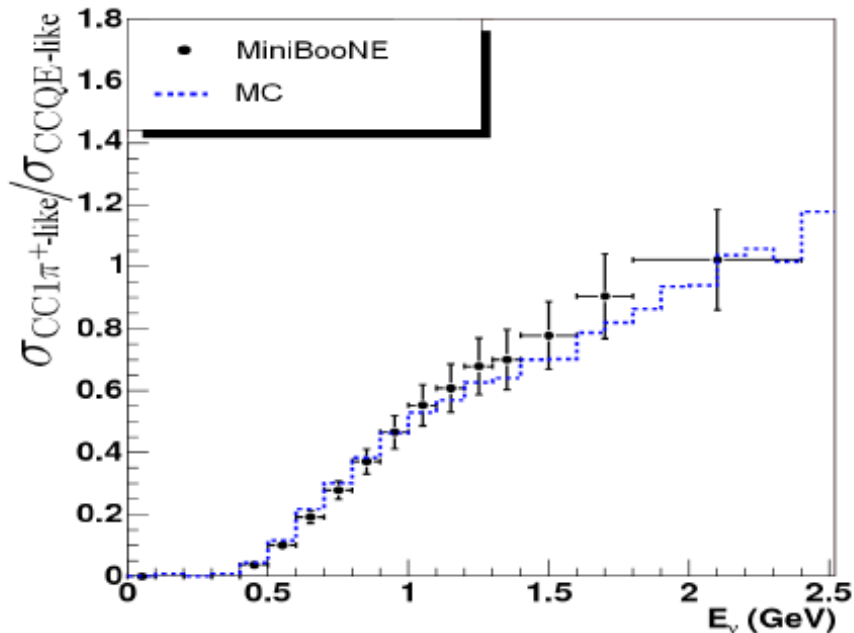


FIG. 1: Observed CC1 $\pi^+$ -like/CCQE-like cross section ratio on CH<sub>2</sub>, including both statistical and systematic uncertainties, compared with the MC prediction [6]. The data have not been corrected for hadronic re-interactions.

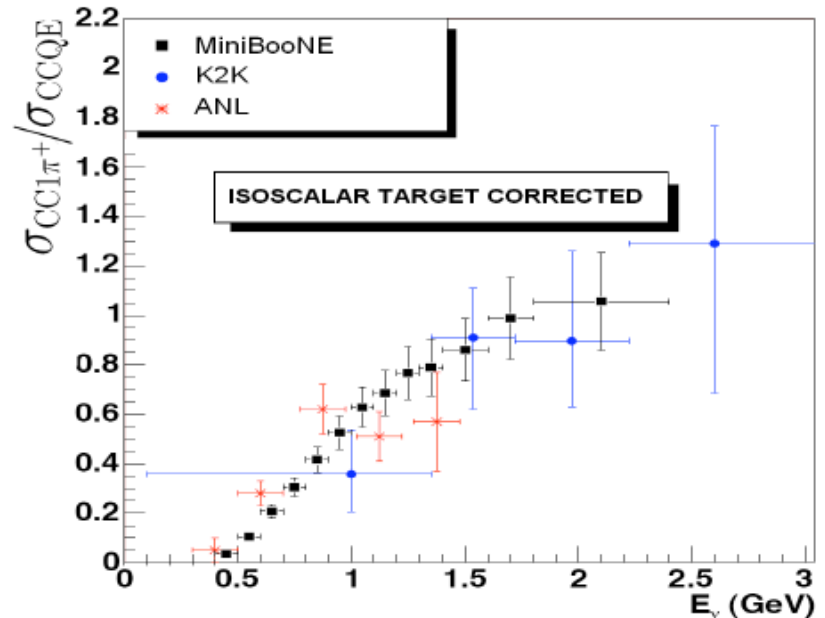


FIG. 2: FSI-corrected CC1 $\pi^+$  to CCQE cross section ratio on CH<sub>2</sub> compared with results from ANL ( $D_2$ ) [1] and K2K ( $C_8H_8$ ) [3]. The data have been corrected for final state interactions and re-scaled for an isoscalar target.

The results are very useful and widely used in comparisons because they are free from normalization controversy.

# Monte Carlo generators



The market of MCs:

**Neut** (K2K, SciBooNE, T2K)

**Nuance** (SK, Minos, MiniBooNE)

**GENIE/Neugen** (Minos, Minerva, T2K, Nova)

**FLUKA** (ICARUS)

Tools developed by theorists:

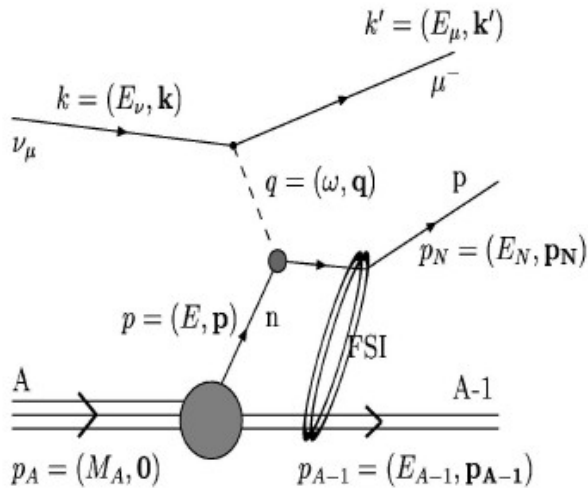
**GiBUU**

**NuWro**

It takes years to construct a MC and to test it.



# Impulse approximation



(from Ch. Maieron, XX Max Born Symposium)

- neutrino interacts with an individual (bound) nucleons
- followed by „final state interactions” (FSI);

**understood as:**

- a) only redistribution of the cross-section into exclusive final states (MCs)
- b) a modification of the inclusive cross section

The simplest realization:

**Fermi gas model** with 2 free parameters:  
Fermi momentum and binding energy.

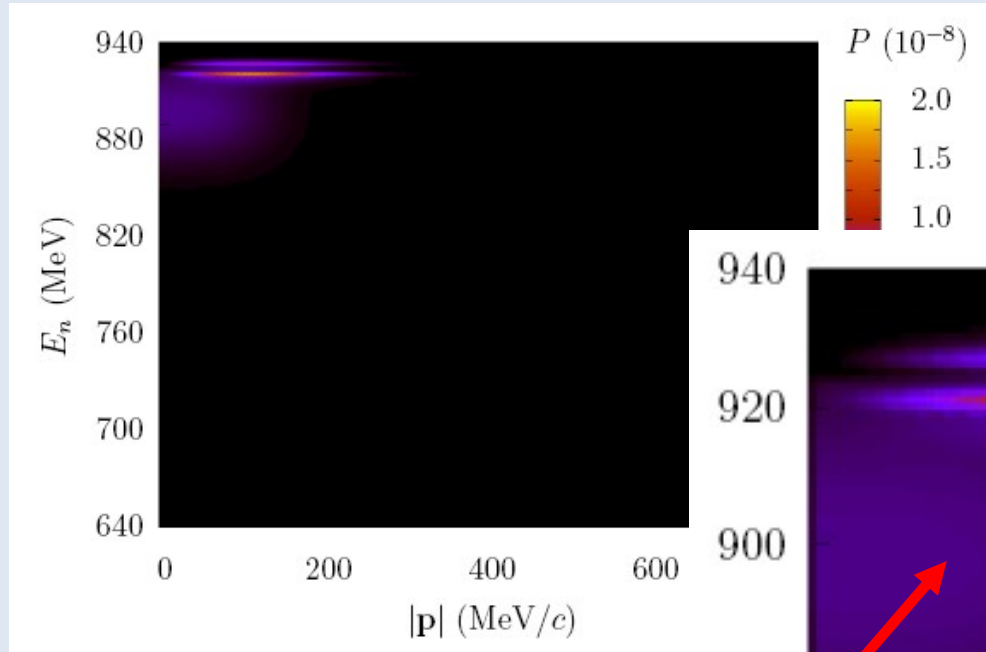
More realistic model: **spectral function**



# Spectral function

Spectral function for oxygen

	$1s_{1/2}$	$1p_{3/2}$	$1p_{1/2}$
$E$	45	18.44	12.11



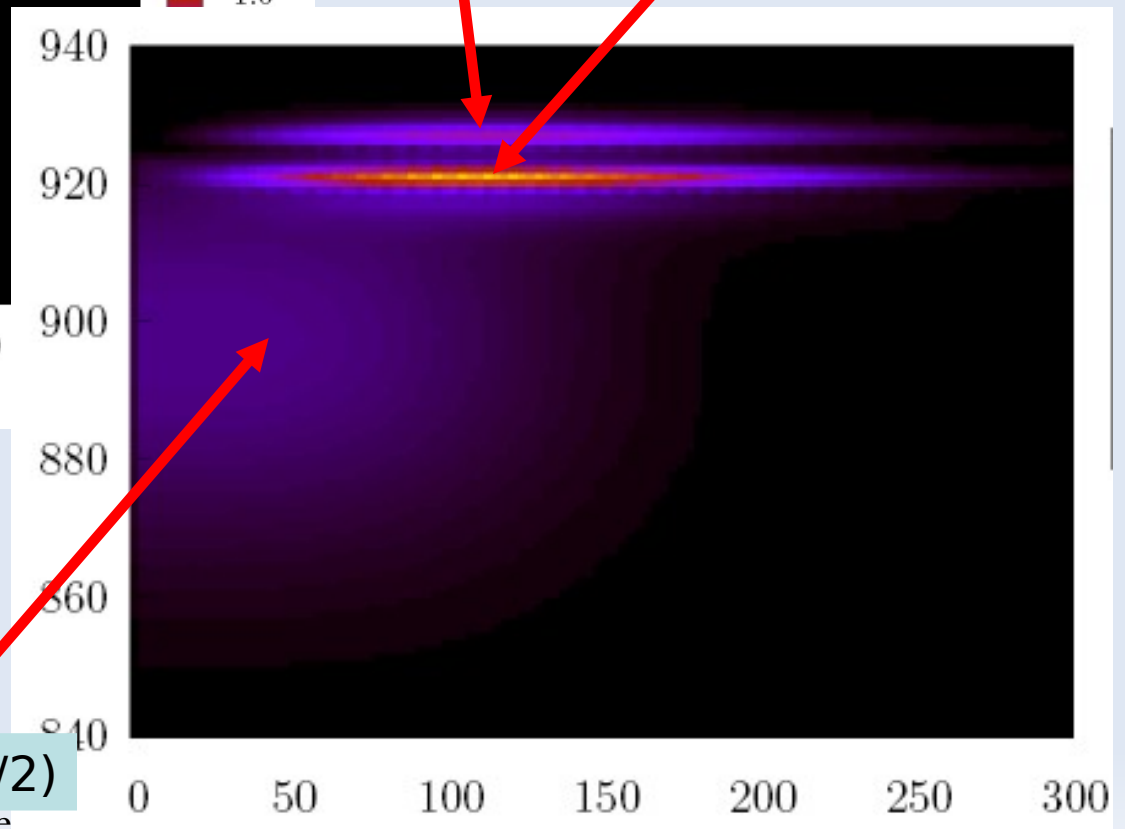
1p (3/2)

1p (1/2)

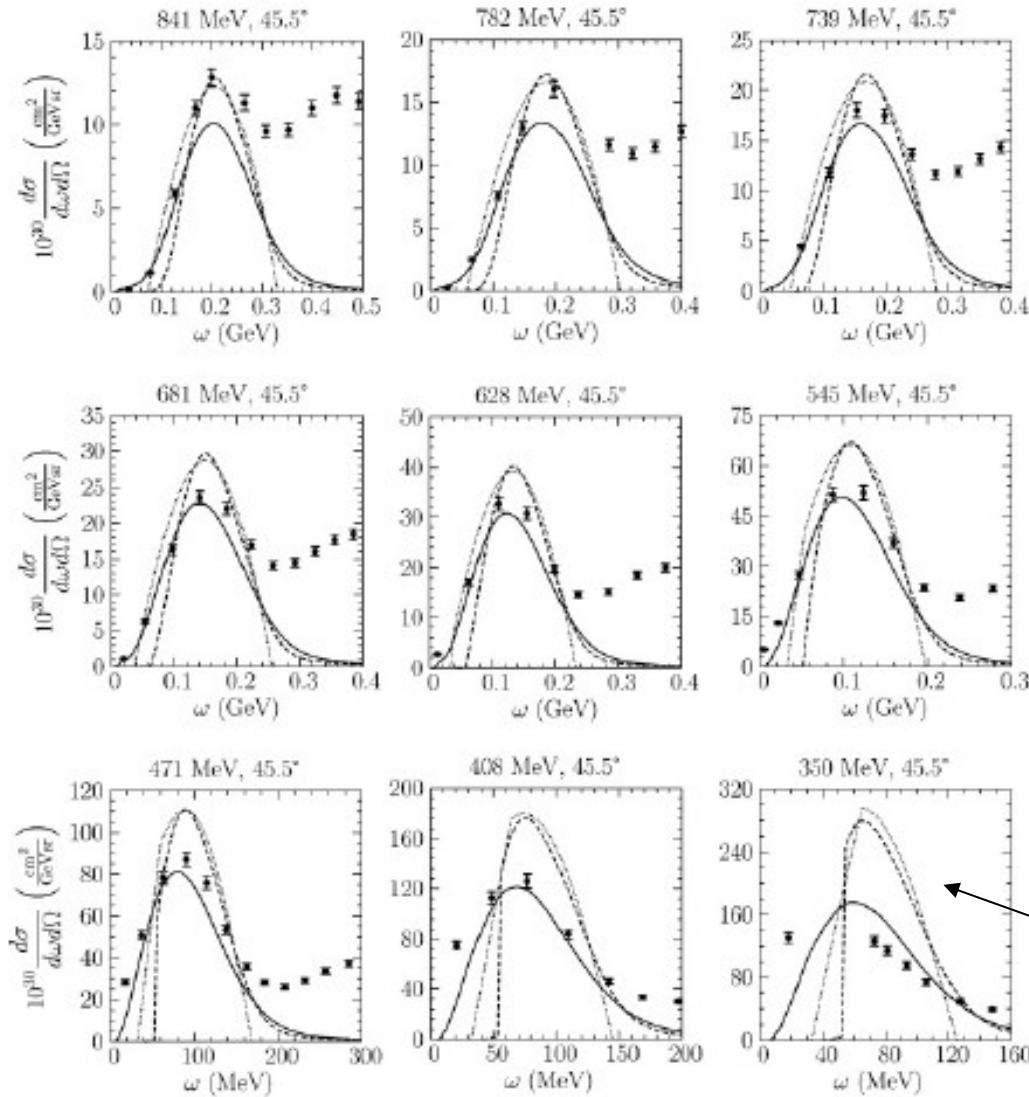
(O. Benhar)

Spectral function approach is parameter free. But FSI/Pauli blocking effects should be added.

1s(1/2)



# Spectral function



Calcium (Ca40) target:

- solid line** → the hole spectral function from the paper Ankowski, JTS, PRD77 (2008) 044311 (with FSI effects)
- dashed line** → Butkevich, Mikheyev model
- dotted line** → Fermi gas model

**Note that theoretical model do not include  $\Delta$  excitation dynamics.**

**Data is for the inclusive cross section!**

momentum transfer at the peak is 250 MeV !

# Implementation



It was done by Cezary Juszczak based on Artur Ankowski code (verified to produce identical results).

## References

- A. Ankowski, J. Sobczyk, Argon Spectral Function and Neutrino Interactions, Phys. Rev. C74, 054316-1-10 (2006)
- A.M. Ankowski, J.T. Sobczyk Construction of spectral functions for medium-mass nuclei, Phys. Rev. C 77, 044311, 2008.

(we follow closely Omar Benhar approach)

# Energy reconstruction



The standard energy reconstruction formula:

$$E_{rec} = \frac{E'(M - B) + \frac{1}{2}(B^2 - 2MB + m^2)}{M - B - E' + k' \cos \theta}$$

$E'$  and  $k'$  are muon energy and momentum,  $M$  and  $m$  are nucleon and muon masses,  $B$  is the binding energy;  $\theta$  is the angle between incident neutrino's and muon's momenta.

uses only info about final muon and assumes the target nucleon to be at rest. Binding energy is used as in the Fermi gas model energy balance.

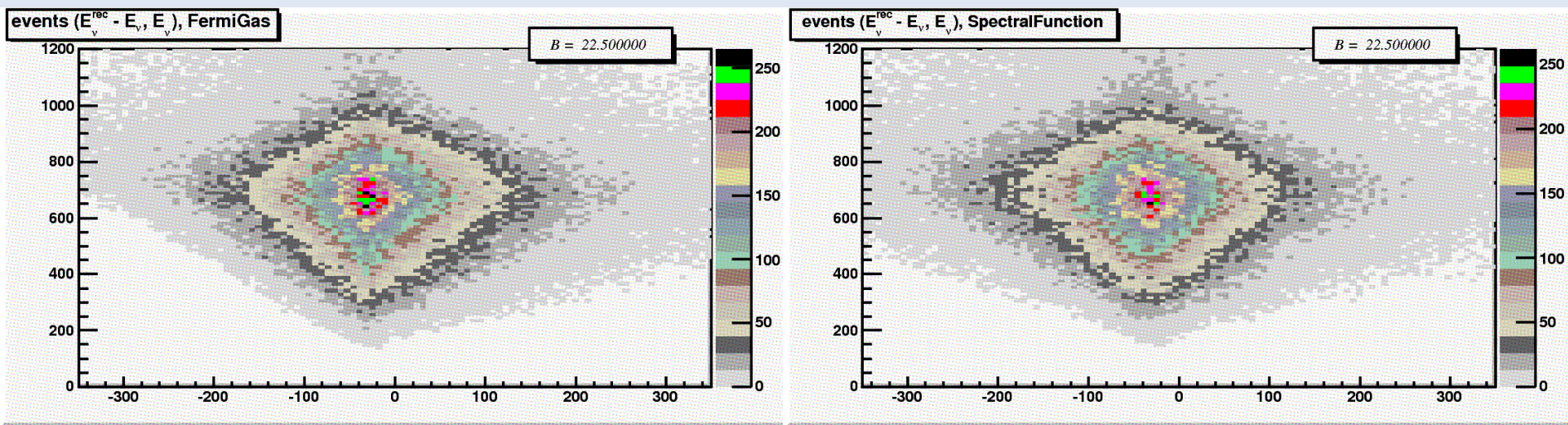
The estimation of the quality of the reconstruction formula should take into account proper description of the Fermi motion and this is what SF is designed to do correctly.

# Energy reconstruction



How much  $E_{rec}$  differ from  $E_{true}$ ? We can estimate it using MC based on the FG model. How much this evaluation differ from the results from MC based on the SF?

Two samples of CCQE events were produced by NuWro using the same T2K beam: one with FG model and the second one with the SF model.



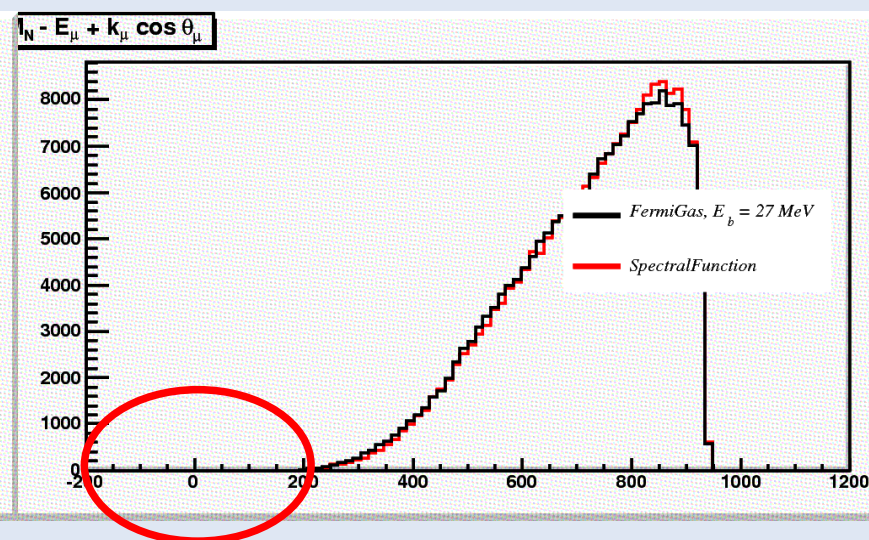
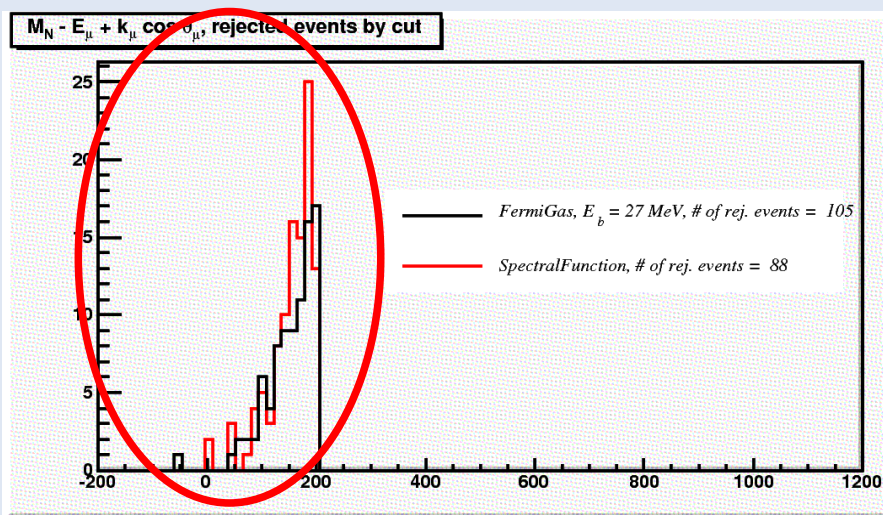
# Energy reconstruction



Warning: the denominator in the formula:

$$E_{rec} = \frac{E'(M - B) + \frac{1}{2} (B^2 - 2MB + m^2)}{M - B - E' + k' \cos \theta}$$

can be very small:



We introduce the cut:

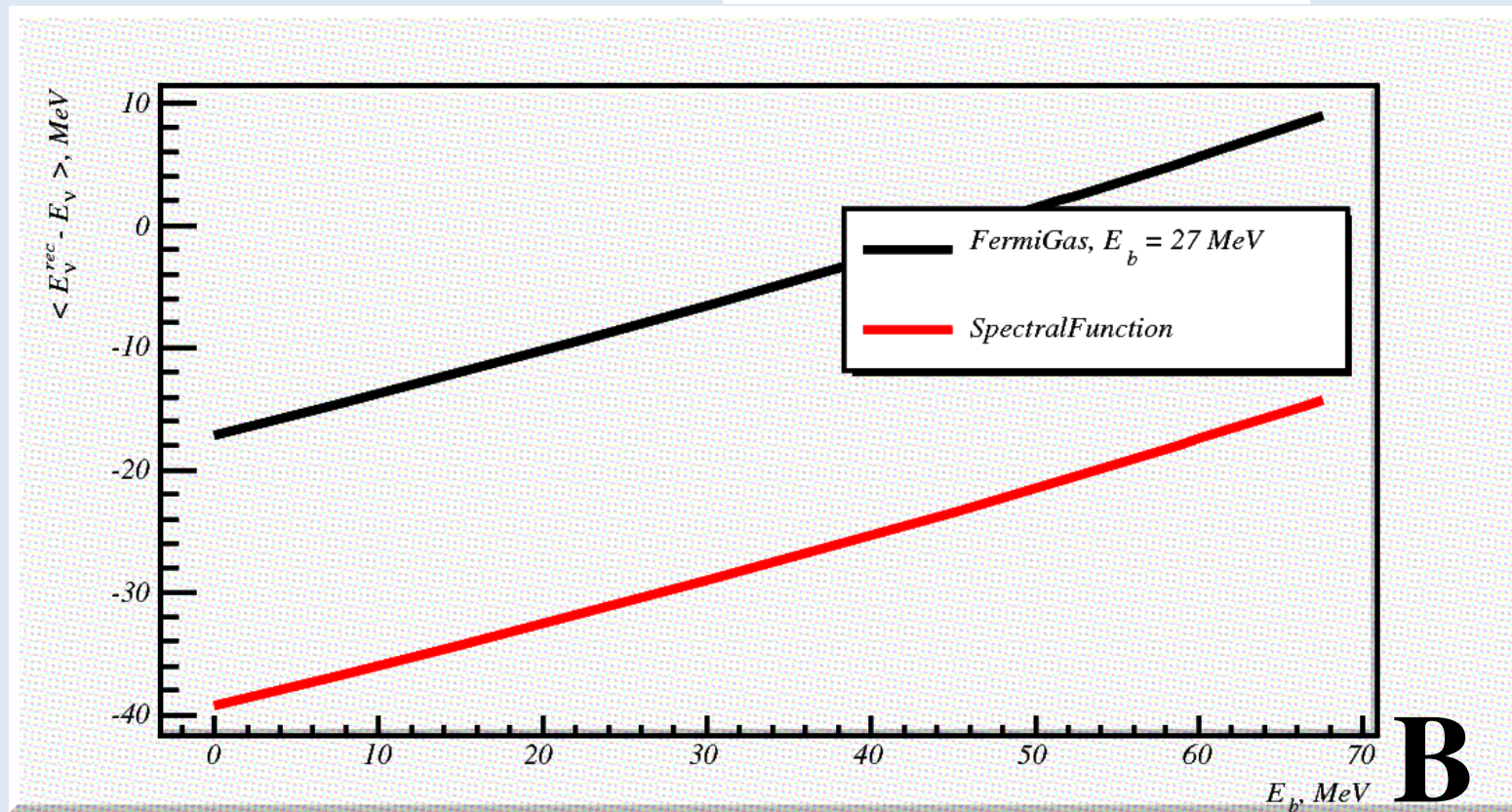
$$M - B - E' + k' \cos \theta > 200 \text{ MeV}$$

# Energy reconstruction



How does it work?

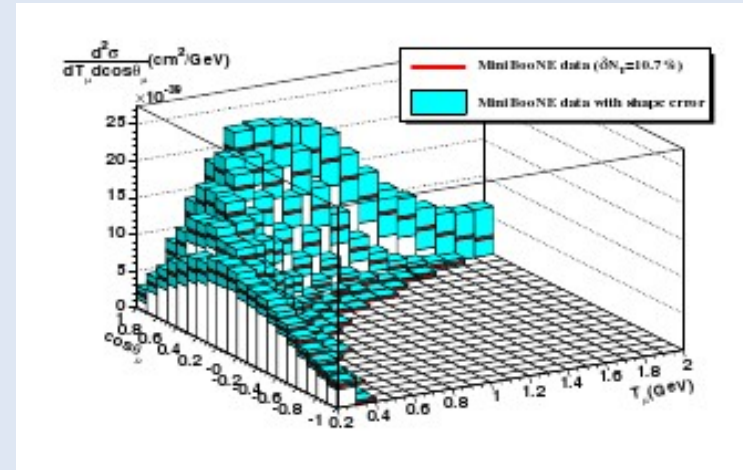
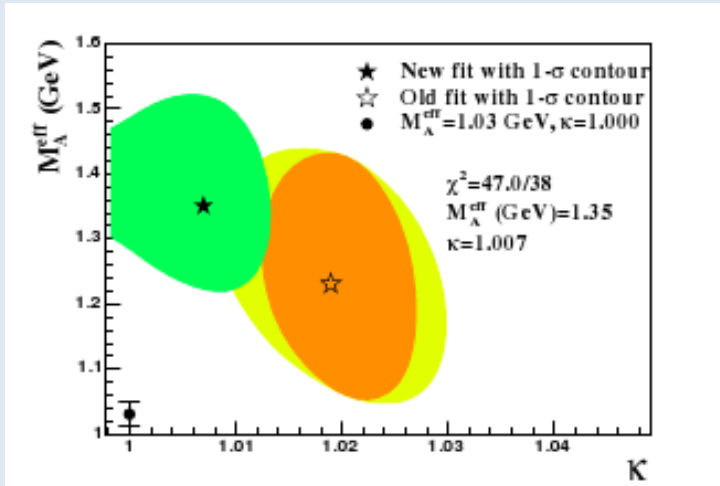
$$E_{rec} = \frac{E'(M - B) + \frac{1}{2}(B^2 - 2MB + m^2)}{M - B - E' + k' \cos \theta}$$



There is a B dependent bias. For SF it is larger by  $\sim 22$  MeV (the actual energy is on average **larger** then the reconstructed one).



# CCQE MiniBooNE data



$\cos\theta_\mu$ , $T_\mu$ (GeV)	0.2,0.3	0.3,0.4	0.4,0.5	0.5,0.6	0.6,0.7	0.7,0.8	0.8,0.9	0.9,1.0	1.0,1.1	1.1,1.2	1.2,1.3	1.3,1.4	1.4,1.5	1.5,1.6	1.6,1.7	1.7,1.8	1.8,1.9	1.9,2.0
+0.9,+1.0	190.0	326.5	539.2	901.8	1288	1633	1857	1874	1803	1636	1354	1047	794.0	687.9	494.3	372.5	278.3	227.4
+0.8,+0.9	401.9	780.6	1258	1714	2084	2100	2035	1620	1118	783.6	451.9	239.4	116.4	73.07	41.67	36.55	—	—
+0.7,+0.8	553.6	981.1	1501	1884	1847	1629	1203	723.8	359.8	156.2	66.90	26.87	1.527	19.50	—	—	—	—
+0.6,+0.7	681.9	1222	1546	1738	1365	909.6	526.7	222.8	81.65	35.61	11.36	0.131	—	—	—	—	—	—
+0.5,+0.6	765.6	1233	1495	1289	872.2	392.3	157.5	49.23	9.241	1.229	4.162	—	—	—	—	—	—	—
+0.4,+0.5	871.9	1279	1301	989.9	469.1	147.4	45.02	12.44	1.012	—	—	—	—	—	—	—	—	—
+0.3,+0.4	910.2	1157	1054	628.8	231.0	57.95	10.69	—	—	—	—	—	—	—	—	—	—	—
+0.2,+0.3	992.3	1148	850.0	394.4	105.0	16.96	10.93	—	—	—	—	—	—	—	—	—	—	—
+0.1,+0.2	1007	970.2	547.9	201.5	36.51	0.844	—	—	—	—	—	—	—	—	—	—	—	—
0.0,+0.1	1003	813.1	404.9	92.93	11.63	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.1, 0.0	919.3	686.6	272.3	40.63	2.176	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.2,-0.1	891.8	503.3	134.7	10.92	0.071	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.3,-0.2	857.5	401.6	79.10	1.947	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.4,-0.3	778.1	292.1	33.69	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.5,-0.4	692.3	202.2	17.42	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.6,-0.5	600.2	135.2	3.624	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.7,-0.6	497.6	85.80	0.164	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.8,-0.7	418.3	44.84	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-0.9,-0.8	348.7	25.82	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
-1.0,-0.9	289.2	15.18	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

TABLE VI: The MiniBooNE  $\nu_\mu$  CCQE flux-integrated double differential cross section in units of  $10^{-41}$   $\text{cm}^2/\text{GeV}$  in 0.1 GeV bins of  $T_\mu$  (columns) and 0.1 bins of  $\cos\theta_\mu$  (rows).

# CCQE MiniBooNE data



Using NuWro we make 2-dimensional fits:  
axial mass, overall normalization.

$$\chi^2(\lambda, M_A) = \sum_{j=1}^{bins} \frac{\left( \frac{d^2 \sigma_j^{MB}}{d \cos \theta d T_k} - \lambda \cdot \frac{d^2 \sigma_j^{NuWro}(M_A)}{d \cos \theta d T_k} \right)^2}{\left( \delta \frac{d^2 \sigma_j^{MB}}{d \cos \theta d T_k} \right)^2} + \frac{(1 - \lambda^{-1})^2}{(\delta \lambda^{MB})^2}$$

MiniBooNE data are given in the paper

$$\delta \lambda^{MB} = 10.7\%$$

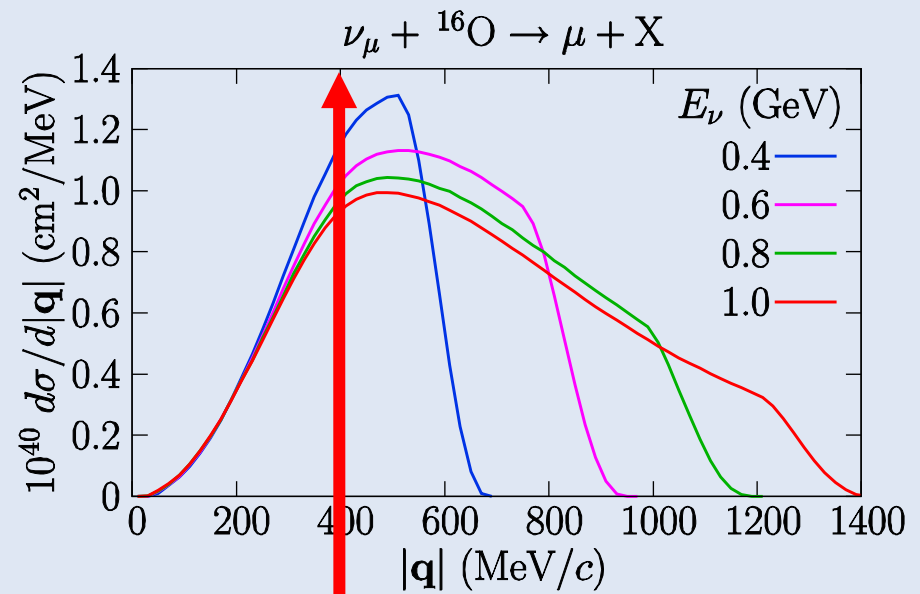
# Validity of impulse approximation



General conclusion from electron scattering data: the impulse approximation is reliable only for momentum transfers down to  $\sim 350$ - $400$  MeV.

Message for the neutrino physics:

The contribution from  $q < 400$  MeV region is always large, even for higher neutrino energies.



„safe” region

# CCQE MiniBooNE data



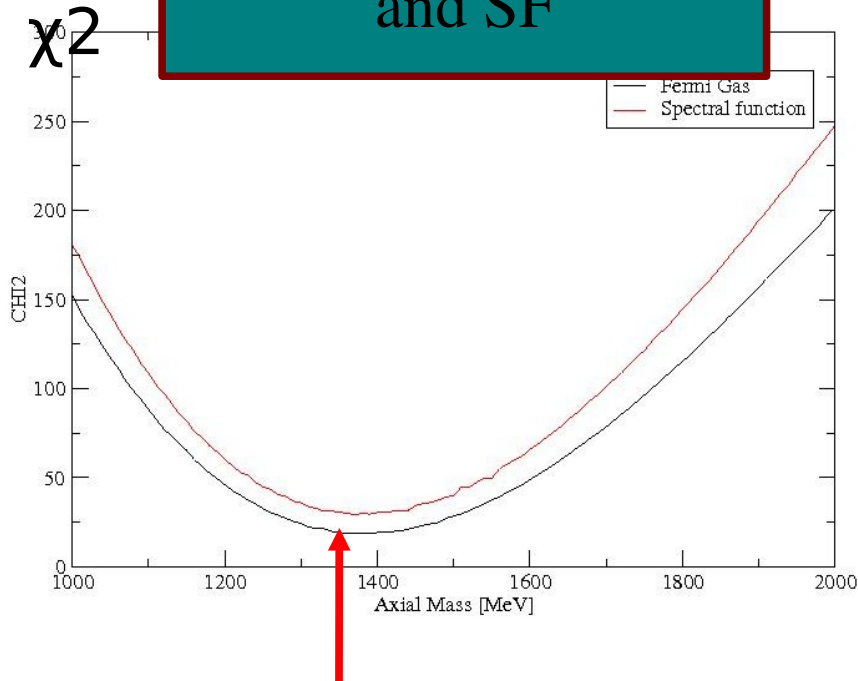
We make two fits:

- we use all the bins
- we eliminate bins with large ( $>50\%$ ) low momentum transfer contribution

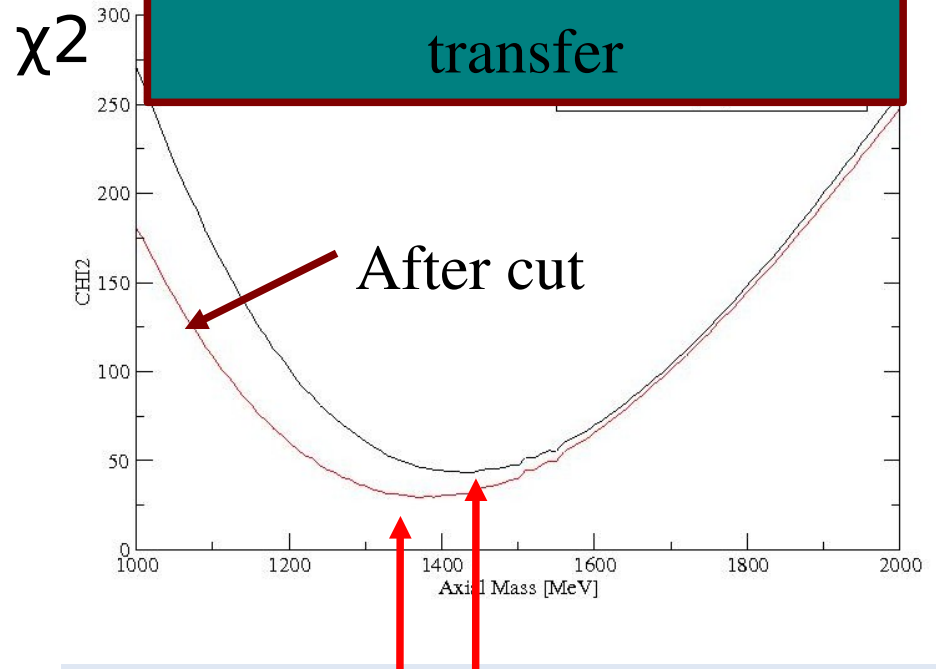
# CCQE MiniBooNE data



Comparison FG  
and SF



Impact of the cut on bins  
with small momentum  
transfer



**Very recent and preliminary results!**

# Wnioski



- oddziaływania neutrin o energiach rzędu 1 GeV to dziedzina dużej aktywności
- obszar małych wartości  $q$  (również  $Q^2$ ) wymaga bardziej wyrafinowanych modeli fizyki jądrowej
- ważne pytanie: jak istotny jest wkład 2p-2h? konieczne kolejne testy
- analiza koherentnej produkcji pionów wymaga ulepszeń w kodach Monte Carlo
- jest sporo nowych ciekawych wyników (np. NC 1Pi0) umożliwiających weryfikację opisu efektów jądrowych
- póki co funkcja spektralna nie wnosi wiele nowego, co nie oznacza, że nie powinno się używać jej zamiast gazu Fermiego
- **dalszy postęp wymaga nowych danych (Minerva!)**



Dziękuję  
za  
uwagę!