Anomalous Soft Photons associated with Hadron Production in String Fragmentation

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1. Introduction
   -- What are anomalous soft photons?
   -- What are their properties?

2. Explanation of the anomalous soft photon phenomenon
   -- Bound QCD & QED states in the flux tube (string) environment
   -- Production of these bound states in the flux tube environment

3. Conclusions

- Anomalous soft photons are low-$p_T$ photons ($p_T<60$ MeV).
- They are in excess of what is expected from EM bremsstrahlung.
- They occur only when hadrons are produced.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Energy</th>
<th>Photon $p_T$</th>
<th>Photon/Brems ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ p$, SLAC, BC (1979)</td>
<td>10.5 GeV/c</td>
<td>$p_T &lt; 20$ MeV/c</td>
<td>1.25 $\pm$ 0.25</td>
</tr>
<tr>
<td>$K^+ p$, CERN WA27,BEBC (1984)</td>
<td>70 GeV/c</td>
<td>$p_T &lt; 60$ MeV/c</td>
<td>4.0 $\pm$ 0.8</td>
</tr>
<tr>
<td>$K^+ p$, CERN NA22, EHS (1993)</td>
<td>250 GeV/c</td>
<td>$p_T &lt; 40$ MeV/c</td>
<td>6.4 $\pm$ 1.6</td>
</tr>
<tr>
<td>$\pi^+ p$, CERN NA22, EHS (1997)</td>
<td>250 GeV/c</td>
<td>$p_T &lt; 40$ MeV/c</td>
<td>6.9 $\pm$ 1.3</td>
</tr>
<tr>
<td>$\pi^- p$, CERN WA83, OMEGA (1997)</td>
<td>280 GeV/c</td>
<td>$p_T &lt; 10$ MeV/c</td>
<td>7.9 $\pm$ 1.4</td>
</tr>
<tr>
<td>$\pi^- p$, CERN WA91, OMEGA (2002)</td>
<td>280 GeV/c</td>
<td>$p_T &lt; 20$ MeV/c</td>
<td>5.3 $\pm$ 0.9</td>
</tr>
<tr>
<td>$p p$, CERN WA102, OMEGA (2002)</td>
<td>450 GeV/c</td>
<td>$p_T &lt; 20$ MeV/c</td>
<td>4.1 $\pm$ 0.8</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow$ hadrons CERN DELPHI (2010) with hadron production</td>
<td>~91 GeV (CM)</td>
<td>$p_T &lt; 60$ MeV/c</td>
<td>~4.0</td>
</tr>
<tr>
<td>$e^+e^- \rightarrow \mu^+ \mu^-$ CERN DELPHI (2008) with no hadron production</td>
<td>~91 GeV (CM)</td>
<td>$p_T &lt; 60$ MeV/c</td>
<td>~1.0</td>
</tr>
</tbody>
</table>

(Table compiled by V. Perepelitsa)
Bremsstrahlung calculations

\[
\frac{dN_\gamma}{d^3k'} = \frac{\alpha}{(2\pi)^2} \frac{1}{E_\gamma} \int d^3p_1 \ldots d^3p_N \sum_{i,j} \eta_i \eta_j \frac{-(P_i P_j)}{(P_i K)(P_j K)} \frac{dN_{\text{hadrons}}}{d^3p_1 \ldots d^3p_N},
\]

where $K$ and $k$ denote photon four- and three-momenta, $P$ are the four-momenta of all the charged particles participating in the reaction. $\eta = +1$ for negative incoming and for positive outgoing particles, $\eta = -1$ for positive incoming and negative outgoing particles, and the sum is extended over all the $N + 2$ charged particles involved. The last factor in the integrand is a differential hadron production ratio.
Anomalous soft photons, in excess of what is expected from EM bremsstrahlung, have been observed in $K^+p$, $\pi^+p$, $\pi^-p$, $pp$, and $e^+e^-$ collisions at high energies.

1. They are produced only in association with hadron production. They are not produced when there is no hadron production, in $e^+ + e^- \rightarrow \mu^+ + \mu^-$.  
2. Total anomalous soft photon yield is proportional to total hadron yield.  
3. Transverse momentum of anomalous soft photons $p_T \sim 2$ to $50$ MeV.  
4. Anomalous soft photon yield increase faster with increasing neutral hadron multiplicity $N_{\text{neu}}$ than with charged hadron multiplicity $N_{\text{ch}}$. 

Properties of anomalous soft photons:
WA91 pp at 450 GeV

\[ \frac{dN}{dp_T} \left( \frac{\text{GeV/c}}{\text{c}} \right) \]

- pp data, Belogianni et al.
- Phys. Lett. B546, 129 (02)
e⁺e⁻ annihilation at Z0 decay (~91 GeV)
Anomalous soft photons come in groups

\[ \frac{dN_\gamma}{dp_T} = \sum_\alpha N_\alpha \gamma \ 2\pi p_T \exp \left\{ -\frac{p_T^2}{(M_{\alpha T}^\gamma)^2} \right\}, \quad \frac{dN_\gamma}{dp_T^2} = \sum_\alpha N_\alpha \gamma \exp \left\{ -\frac{p_T^2}{(M_{\alpha T}^\gamma)^2} \right\} \]

**pp collisions at 450 GeV**

**e^+e^- annihilation at Z0 decay**

Experimental $\Delta p_T$ uncertainty $\sim 2$ MeV

Experimental $\Delta p_T$ uncertainty $\sim 10$ MeV
Anomalous soft photon yield is proportional to the particle (hadron) multiplicity

\[ \text{Rate, } 10^{-3} \text{ photons/jet} \]

\( e^+ e^- \) annihilation at Z0 decay (\( \sim 91 \text{ GeV} \))

DELPHI (EPJ 2010) arXiv:1004.1587
e+e- annihilation at Z0 decay (~91 GeV)

DELPHI (EPJ 2010) arXiv:1004.1587
Quantum field theory of meson and photon production

• When a quark pulls away from an antiquark at high energies, the vacuum is polarized
• Polarization causes the color charges of the quarks in the vacuum to oscillate
• Oscillations of the color charges of the quarks in the vacuum produces mesons
• Oscillations of the color charges of the quarks in the vacuum are accompanied by the oscillations of the electric charges of quarks in the vacuum
• Oscillations of the electric charges of the quarks in the vacuum produces photons
Such a model can explain:

1. Photon production accompanies by meson production
2. Photon yield is proportional to meson yield

We need to explain the other two features of the anomalous soft photon phenomenon:

3. Why $p_T \sim 10$-50 MeV
4. Why anomalous soft photon yield increase much faster with increasing neutral particle multiplicity than with charged multiplicity
The classical Lund model of particle production is only a part of a complete model. In the Lund model, the stability of the produced particles is determined by other theories.
Schwinger QED2 quantum field theory model is a complete model of particle production

• It shows how the produced particles with a mass $m = e / \sqrt{\pi}$ are stable quanta of the underlying QED2 quantum field

• It shows how particles are produced, when a quark pulls away from an antiquark at high energies
Schwinger QED2 quantum field theory

Quantum electrodynamics in 1+1 dimensions with massless fermions

\[ \gamma^\mu(p_\mu - eA_\mu)\psi = 0 \]

\[ \partial_\mu F^{\mu\nu} = \partial_\mu(\partial^\mu A^\nu - \partial^\nu A^\mu) = e j^\nu = e\overline{\psi}\gamma^\nu\psi \]

A small disturbance in \( A^\nu \) \( \Rightarrow \) A small disturbance in \( j^\nu \)

\[ \Rightarrow \text{A small disturbance in } A^\nu \]

Therefore, \( j^\nu \) is a self-consistent function of \( A^\nu \).

A gauge invariant relation between \( j^\nu \) and \( A^\nu \) is

\[ j^\nu = \frac{e}{\sqrt{\pi}} \left( A^\nu - \partial^\nu \frac{1}{\partial_\lambda \partial^\lambda} \partial_\mu \partial^\mu A^\nu \right) \]

When we substitute this into the Maxwell equation, we get

\[ \partial_\mu \partial^\mu A^\nu + \frac{e^2}{\pi} A^\nu = 0 \]

This is the Klein-Gordon equation for a boson with a mass

\[ m = \frac{e}{\sqrt{\pi}} \]
Schwinger particle production in QED2

Casher, Kogut, Susskind, Phy. Rev. D10, 732 (‘74)

Given the external source current

\[ j_{\text{ext}}^0(x,t) = -e\delta(x+t) + e\delta(x-t) \]
\[ j_{\text{ext}}^1(x,t) = +e\delta(x+t) + e\delta(x-t) \]

The produced fermion and boson densities are
Summary of QED2 particle production model

• Stable boson particles with mass $m = \frac{e}{\pi^{1/2}}$ are quanta of the system

• $dN/dy$ is boost invariant when a quark pulls from an antiquark at infinite energy

• For finite energy, $dN/dy$ becomes a rapidity plateau with the plateau distribution

$$\frac{dN}{dy} = \frac{\nu^2 \xi^2}{\sinh^2 \xi}$$

$$\xi = \frac{\nu^2 \pi^2 m \sinh \gamma}{3\sqrt{s}}$$
Comparison of QED2 model of particle production with data

QED2 model gives a good description of particle production

C.Y. Wong, Phys. Rev. C80, 054917 ('09)
Flux tube environment

$q \bar{q}$

When a $q$ pulls away from a $\bar{q}$ at high energies,
QCD$^4 \times$ QED$^4$ can be approximated by QCD$^2 \times$ QED$^2$,
with the formation of a flux tube between the $q$ and the $\bar{q}$.
The flux tube can be idealized as a string between the $q$ and the $\bar{q}$.
The coupling constants in the 4D and 2D theories are related by

$$g_{2D}^2 = \frac{g_{4D}^2}{\pi R_T^2}, \quad R_T = \text{flux tube radius}.$$  

We need to study the bound states and their production in QCD$^2 \times$ QED$^2$.

Bound states in QCD2×QED2 \(^{(1)}\)

1. \(\text{QCD2} \times \text{QED2}\) Lagrangian density is

\[
\mathcal{L} = \bar{\psi} \left[ \gamma^{\mu} \left( \not\! k_{\mu} + gA_{\mu} \right) - \not\! m_{\tau} \right] \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]

\(\text{QCD2} \times \text{QED2}\) can be studied with the \(\text{U}(3)\) group which is the product of \(\text{U}(1) \times \text{SU}(3)\).

2. The \(\text{U}(3)\) group has 9 generators:

\[
t^0 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad t^1, t^2, t^3, \ldots, t^8, \quad \text{tr} \left\{ t^\alpha t^\beta \right\} = \frac{\delta^{\alpha\beta}}{2}
\]

\(\text{QED}\) \(\text{U}(1)\) generator \(\text{SU}(3)\) generators

\[
(gA_{\mu} \psi)^b = \sum_{f=u,d} \sum_{c=1,2,3} \sum_{a=0}^{8} g_{f}^{a} A_{\mu}^{a} (t^{a})^{bc} \psi_{f}^{c}
\]

3. The different coupling constants \(g_{f}^{a}\) depend on the generator and on the flavor

\[
\begin{align*}
g_{u}^{0} &= -Q_{u} e_{\text{QED2}} \quad Q_{u} = \frac{2}{3} \quad \text{QED} \\
g_{d}^{0} &= -Q_{d} e_{\text{QED2}} \quad Q_{d} = -\frac{1}{3} \\
g_{u}^{(1,2,3,\ldots,8)} &= g_{d}^{(1,2,3,\ldots,8)} = g_{\text{QCD2}} \quad \text{QCD}
\end{align*}
\]
Bound states in QCD2XQED2 (2)

Bound state masses can be obtained by non-Abelian bosonization.
Bosonization should be carried out in such a way to give stable bosons.
We bosonize an element $u_f$ (flavor $f$) of the $U(3)$ group by $\phi_f^0$ and $\phi_f^1$

$$u_f = \exp \left\{ i \sqrt{2} \pi \sum_{a=0}^1 \phi_f^a t^a \right\}$$

We obtain the boson hamiltonian for $\phi_f^0$ and $\phi_f^1$,

$$2\mathcal{H} = N \sum_{a=0}^1 \left\{ \sum_{f=u,d} \left[ \frac{1}{2} \left( \Pi_f^a \right)^2 + \frac{1}{2} \left( \partial_t \phi_f^a \right)^2 \right] + \frac{1}{2} \left[ \sum_{f=u,d} g_f^a \phi_f^a \right]^2 \right\} + V_{m_r}$$

We construct isospin $I$ states with $(I_3 = 0)$,

$$\phi_I^a = \frac{1}{\sqrt{2}} \left[ \phi_u^a + (-1)^I \phi_d^a \right] \quad \text{and} \quad \Pi_I^a = \frac{1}{\sqrt{2}} \left[ \Pi_u^a + (-1)^I \Pi_d^a \right]$$

Then,

$$2\mathcal{H} = N \sum_{a=0}^1 \left\{ \sum_{I=0,1} \left[ \frac{1}{2} \left( \Pi_I^a \right)^2 + \frac{1}{2} \left( \partial_t \phi_I^a \right)^2 \right] + \frac{1}{2} \left[ \sum_{I=0,1} g_u^a + (-1)^I g_d^a \frac{\phi_I^a}{\sqrt{2} \pi} \right]^2 \right\} + V_{m_r}$$
**Meson and photon masses depend on isospin**

Isospin is a good quantum number in QCD2
- **isoscalar meson** -- $\eta^0$ \((I=0, I_3=0)\)
- **isovector mesons** -- $\pi^+, \pi^0, \pi^-$ \((I=1, I_3=1,0,-1)\)

Isospin is not a good quantum number in QED2
- **isoscalar photon** \((I=0, I_3=0)\)
- **isovector photon** \((I=1, I_3=0)\)
- **isovector QED** \((I=1, I_3=\pm 1)\) states unlikely to be stable
Meson and photon masses for $I_3=0$ states

$$(M_I^a)^2 = \left[ \frac{g_u^a + (-1)^I g_d^a}{\sqrt{2\pi}} \right]^2 + \begin{cases} 
1 & \text{for } a = 1 \text{(QCD2)} \\
\frac{2}{3} & \text{for } a = 0 \text{(QED2)} 
\end{cases} e^{\gamma} m_T \mu$$

$\gamma$ = the Euler constant $= 0.5772$

$m_T$ = quark transverse mass $\approx 1/R_T \approx 440$ MeV,

$\mu$ = normal-ordering mass scale (interaction-dependent)

$$\begin{cases} 
\mu(\text{QCD}) \approx \Lambda_{QCD} \approx m_T \approx 440 \text{ MeV} \\
\mu(\text{QED}) \approx \text{current quark rest mass} \approx O(1 \text{ MeV})
\end{cases}$$
Meson and photon masses for $I_3=0$ in QCD2\(\times\)QED2

\(g_{\text{QCD2}}^2=2\beta=0.4\,\text{GeV}^2,\) and \(R_T=0.35\,\text{fm}\)

<table>
<thead>
<tr>
<th>Coupling Constant</th>
<th>QCD2</th>
<th>QED2</th>
</tr>
</thead>
<tbody>
<tr>
<td>massless quarks</td>
<td>$g_{\text{QCD2}}=632.5,\text{MeV}$</td>
<td>$e_{\text{QED2}}=96,\text{MeV}$</td>
</tr>
<tr>
<td>isoscalar $I=0$</td>
<td>504.6 MeV</td>
<td>12.8 MeV</td>
</tr>
<tr>
<td>isovector $I=1$</td>
<td>0</td>
<td>38.4 MeV</td>
</tr>
<tr>
<td>$m_T=400,\text{MeV}$</td>
<td>$g_{\text{QCD2}}=632.5,\text{MeV}$</td>
<td>$e_{\text{QED2}}=96,\text{MeV}$</td>
</tr>
<tr>
<td>$\mu=m_T$</td>
<td>734.6 MeV</td>
<td>12.8 MeV</td>
</tr>
<tr>
<td>isoscalar $I=0$</td>
<td>533.8 MeV</td>
<td>38.4 MeV</td>
</tr>
<tr>
<td>isovector $I=1$</td>
<td></td>
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<td>$m_T=400,\text{MeV}$</td>
<td>$g_{\text{QCD2}}=632.5,\text{MeV}$</td>
<td>$e_{\text{QED2}}=96,\text{MeV}$</td>
</tr>
<tr>
<td>$\mu=m_q=O(1\text{ MeV})$</td>
<td>$O(25.3,\text{MeV})$</td>
<td>$O(44.1,\text{MeV})$</td>
</tr>
</tbody>
</table>
Meson and photon masses for $I_3=0$
Quantum field theory of particle production in QED

Casher, Kogut, Susskind, Phy. Rev. D10, 732 (‘74)
Bjorken, Phy. Rev. D27, 140 (‘83)
Wong, Phys. Rev. C80, 054917 (‘09)

For a quark pulling away from an antiquark at infinite energies,

\[ j^0_{ext}(x,t) = +e_{\bar{q}} \delta(x+t) + e_q \delta(x-t) \]
\[ j^1_{ext}(x,t) = +e_{\bar{q}}v_{\bar{q}} \delta(x+t) + e_qv_q \delta(x-t), \]

stable bosons are produced.

dN/dy of the produced bosons is boost-invariant.

For a finite energy, dN/dy becomes a rapidity plateau.
Meson and photon production rates

Schwinger pair production mechanism:

\[
\frac{dN_I}{dz \, dt} = A \sum_{q\bar{q}} P_{q\bar{q}} \kappa_{q\bar{q}} \exp \left\{ - \frac{\pi (M_{IT} / 2)^2}{\kappa_{q\bar{q}}} \right\}
\]

where \( \kappa_{q\bar{q}} = g_{QCD}^2 / 2 \) for meson production
\( \kappa_{q\bar{q}} = g_{QED}^2 / 2 \) for photon production

<table>
<thead>
<tr>
<th>Quantities</th>
<th>QED2 Model</th>
<th>Experimental Anomalous Soft Photon Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_I^\gamma / N_I^\gamma ) ( _{I=0} / _{I=1} )</td>
<td>2.75</td>
<td>1.8 - 2.4</td>
</tr>
<tr>
<td>( N^\gamma / N_{par} )</td>
<td>( 7.91 \times 10^{-3} )</td>
<td>( 9.1 \times 10^{-3} )</td>
</tr>
<tr>
<td>( N^\gamma / N_{neu} )</td>
<td>( 49.1 \times 10^{-3} )</td>
<td>( 37.7 \times 10^{-3} )</td>
</tr>
<tr>
<td>( N^\gamma / N_{ch} )</td>
<td>( 3.71 \times 10^{-3} )</td>
<td>( 6.9 \times 10^{-3} )</td>
</tr>
</tbody>
</table>
Correlation of anomalous soft photon yield with $N_{\text{neu}}$ & $N_{\text{ch}}$

\[
\frac{(I = 0) \text{ photon yield}}{(I = 0) \text{ meson yield}} \gg \frac{(I = 1) \text{ photon yield}}{\pi^0 (I = 1) \text{ meson yield}}
\]

$\eta^0$ meson decay predominantly to neutral hadrons

$\pi^0$ meson is associated with the production of charged $\pi^+$ and $\pi^-$

Therefore, $(\text{photon yield}) / N_{\text{neu}} \gg (\text{photon yield}) / N_{\text{ch}}$. 
Predictions:

• Rapidity distribution of anomalous soft photons should have a plateau structure similar to hadron rapidity distribution

• The transverse momentum distribution of the isoscalar anomalous soft photons associated with a large $N_{\text{neu}}$ should be smaller (with $m_T \sim 15$ MeV) than those associated with large $N_{\text{ch}}$ (with $m_T \sim 50$ MeV)
Conclusion

Anomalous soft photons may arise from electric charge oscillations that accompany the color charge oscillations of the quarks in the vacuum, during the hadron production process.