Tachyons

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9.03.2012

What is a tachyon?

A tachyon is a particle moving with velocity

always v>c as seen in any reference frame

➔emission and detection of a tachyon are

separated by a space-like interval

In Special Relativity one would write:

$$E^2 - p^2 = -\kappa^2$$

Why about tachyons?

Regards fundamental aspects of Nature

- why only $v<c$, $v=c$? why not $v>c$? curiosity
- (OPERA – only opportunity to talk about) luck
- ν_e mass measurement (^3T decay) $m^2 < 0$
- ν_μ mass measurement (π^+ decay) affects imagination

$$m^2 = -\,0.016 \pm 0.023\ \mathrm{MeV}^2$$

- surprisingly **rich consequences** of theoretical investigations

What tachyons are NOT

(1):

TACHYONS ARE NOT PARTICLES THAT MAY MOVE, DEPENDING ON CIRCUMSTANCES, WITH VELOCITIES SMALLER OR GREATER THAN c !!!

FALSE COMMON IDEAS

c remains the limiting velocity

(2):

TACHYONS ARE NOT PARTICLES THAT:

- **-- have negative or imaginary mass**
- **-- have negative kinetic energy**
- **-- allow to reach into the past**

FALSE COMMON IDEAS

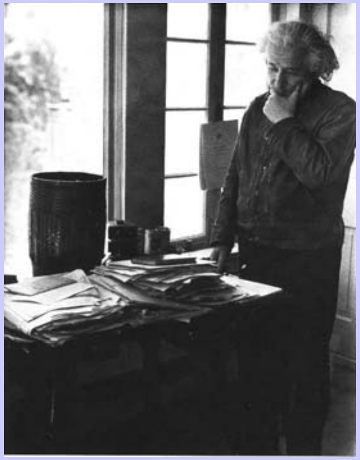
(3):

If TACHYONS existed, that would NOT imply that Einstein's Special Relativity be invalidated

History of the concept

Sudarshan, Bilanyuk, Deshpande 1962: serious difficulties to describe tachyons within Einstein's SR:

- **Causality violation**
- **Negative energies, backward time arrow**
- **Kinematical singularities**
- **Vacuum instability**
- **Infinite dimensional spinors for half-integer spin tachyons**
- **Problems with quantisation**



Focus on causality violation in SR

Time transformation in SR reads:

$$\Delta t' = \gamma \Delta t - V \gamma \Delta x$$

- **simultaneity ($\Delta t = 0$) is not absolute**
- **$\Delta t'$ may change sign for space-like intervals**

(interval = emission and detection of a tachyon)

SR does not describe tachyons

Hint:

LT: $\Delta t' = \gamma \Delta t - V \gamma \Delta x$

If one could only achieve:

$\Delta t' = (\ldots) \, \Delta t$

causality would be preserved

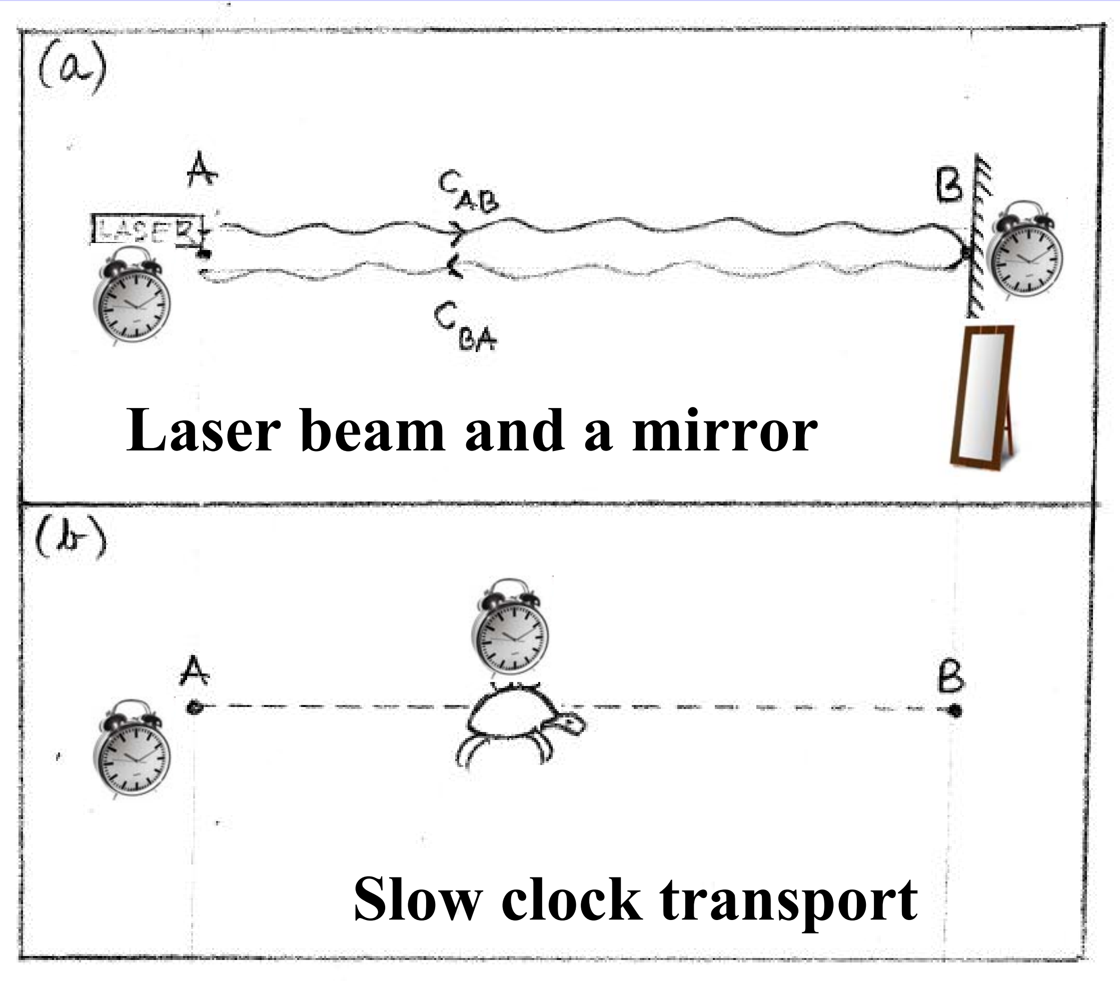
if $(\ldots) > 0$

Possible -- owing to a different clock synchronisation procedure

Clock synchronisation and the velocity of light

- **Reichenbach, Grunbaum, Winnie**
- **SR test theories: Robertson, Mansouri & Sexl, Will, Lammerzahl, Zhang**
- **Chang, Tangherlini**

Examples of clock synchronisation in Einstein's SR



Easy to show:

$$c_{AB} = c_{BA} = c$$

$$\frac{1}{2}\left(\frac{1}{c_{AB}} + \frac{1}{c_{BA}}\right) = \frac{1}{c}$$

Notation:

c_{AB} – one way velocity of light

from A to B

Velocity of light in SR

Over a closed path

- **one clock (no conventions for synchronisation)**
- **result: c**

Over an open path (from A to B): c_{AB}

- **two clocks**
- **clock synchronisation procedure needed**
- **result c**
- **one-way velocity c_{AB} is not measurable using light**

In Nature, one-way velocity of light
A→ B may be different from B→A
we are not able to measure neither
-- but c over a closed path
(using light or v<c particles)

Either we obtain directly c or

we must admit additional assumptions !

Roemer …

Beyond Einstein's synchronisation

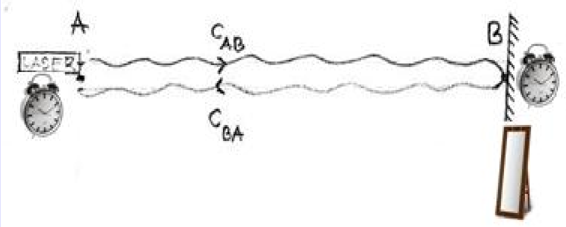
Derivation of transformation laws in 1+1 dimension

(simple demonstration of the idea)

Notation: subscript E

t_E – coord. time in Einstein synchronisation (also v_E, γ_E)

t -- coord. time in arbitrary synchronisation



$$t_E(B) = t_E(A) + \frac{1}{2}\Delta t_{ABA}$$

$$t_{\varepsilon_R}(B) = t_{\varepsilon_R}(A) + \varepsilon_R \Delta t_{ABA}$$

$$\Delta t_{ABA} = \frac{2\Delta x}{c}$$

$$0 < \varepsilon_R < 1 - \text{Reichenbach coefficient}$$

its value tells of synchronisation procedure

$$\Delta t_E = \Delta t_{\varepsilon_R} + (1 - 2\varepsilon_R)\frac{\Delta x}{c}$$

redefine coefficient

$$\Delta t_E = \Delta t + \varepsilon \frac{\Delta x}{c} \qquad -1 < \varepsilon < 1$$

Relations between velocities:

$$v_E = \frac{\Delta x}{\Delta t_E} \qquad v = \frac{\Delta x}{\Delta t}$$

$$v_E = \frac{v}{1 + \varepsilon v/c} \qquad v = \frac{v_E}{1 - \varepsilon v_E/c}$$

Derive transformation laws for Δt and Δx
(in arbitrary synchronisation)

Start from the Lorentz transformation ($\Delta x_E = \Delta x$):

$$\Delta t'_E = \gamma_E(\Delta t_E - V_E \Delta x/c^2) \qquad \Delta x'_E = \gamma_E(\Delta x - V_E \Delta t_E)$$

Note: $\Delta t'_E = \Delta t' + \varepsilon' \frac{\Delta x'}{c}$ (ε **transforms too**)

Substitute above relations to obtain:

$$\Delta t' = \gamma \Delta t \left[1 + \frac{\varepsilon' V/c}{1 + \varepsilon V/c} \right] + \gamma \frac{\Delta x}{c} \left[\varepsilon - \varepsilon' - (1 - \varepsilon\varepsilon') \frac{V/c}{1 + \varepsilon V/c} \right]$$

$$\Delta x' = \gamma \frac{\Delta x - V \Delta t}{1 + \varepsilon V/c} \qquad \gamma = \frac{1 + \varepsilon V/c}{\sqrt{(1 + \varepsilon V/c)^2 - (V/c)^2}}$$

To satisfy ABSOLUTE SIMULTANEITY, $\Delta t' \propto \Delta t$, request:

$$\left[\varepsilon - \varepsilon' - (1 - \varepsilon\varepsilon') \frac{V/c}{1 + \varepsilon V/c} \right] = 0$$

In consequence we have the following transformation laws:

$$\Delta t' = \Delta t \sqrt{(1 + \varepsilon V/c)^2 - (V/c)^2}$$

$$\Delta x' = \frac{\Delta x - V \Delta t}{\sqrt{(1 + \varepsilon V/c)^2 - (V/c)^2}}$$

$$\varepsilon' = \varepsilon - (1 - \varepsilon^2) V/c$$

$$v' = \frac{v - V}{(1 + \varepsilon V/c)^2 - (V/c)^2}$$

Velocity of light

In a moving frame:

- $\varepsilon' = -V/c$ defines the procedure of clock synchronisation
- velocity of light is direction dependent:
  $c_+ = c/(1+V/c)$ and $c_- = c/(1-V/c)$ satisfying:
  $1/c = \frac{1}{2}(1/c_+ + 1/c_-)$
  i.e. average velocity over a closed path equals c

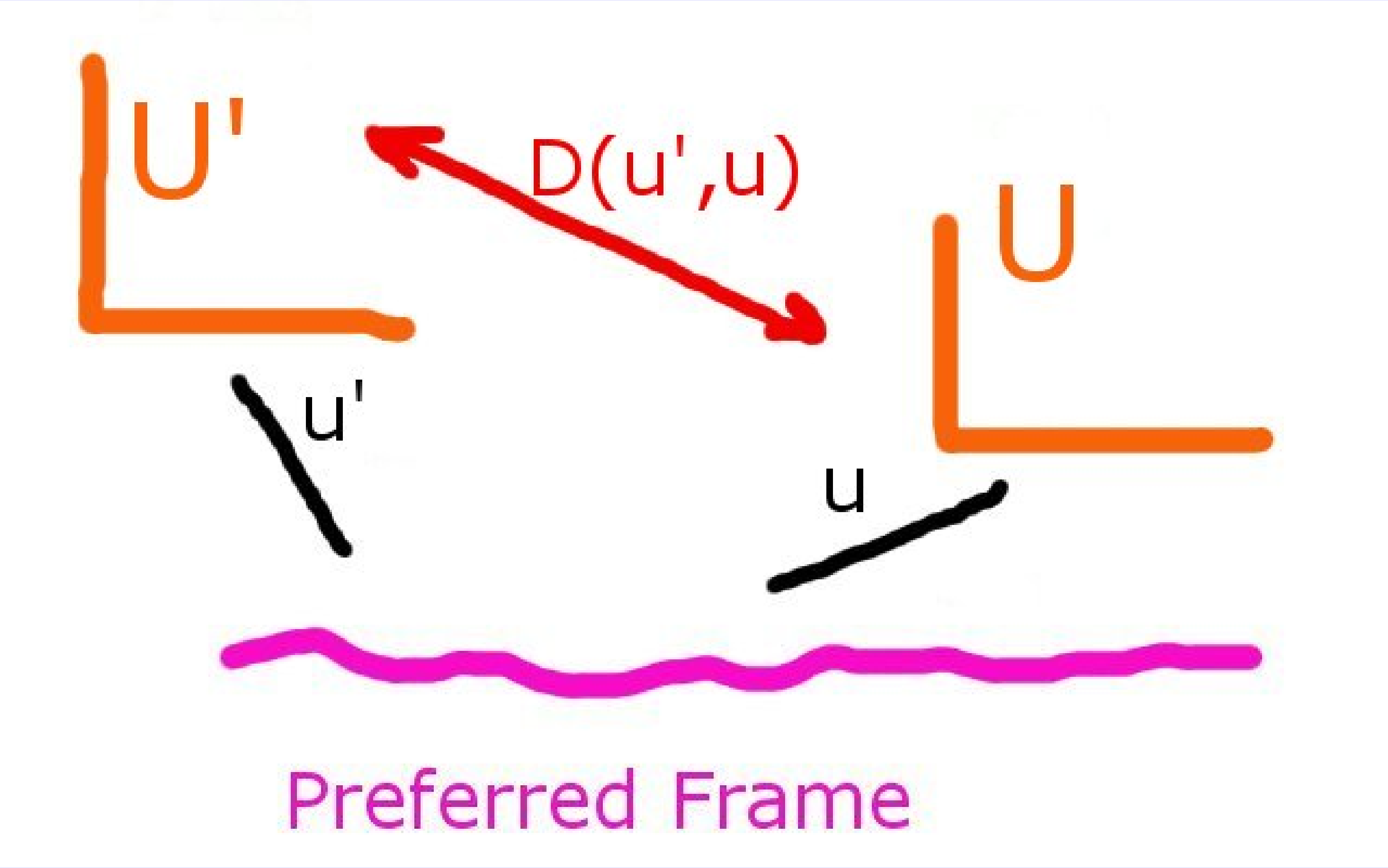
Definition of the preferred frame

Notice that in a reference frame in which $\varepsilon = 0$, one has:

- Einstein clock synchronisation applies $t_E(B) = t_E(A) + \frac{1}{2}\Delta t_{ABA}$
- Velocity of light is constant and isotropic

Call this reference frame: **Preferred Frame**

Preferred Frame Mechanics



U'
D(u',u)
U
u'
u
Preferred Frame

Fully covariant description – basic formulae

J. Rembieliński, Int. J. Mod. Phys. A12 (1997) 1677

$u = (u^0, \vec{u})$ – four-velocity of PF w.r.t. $\mathcal{O}$

$$\varepsilon(\vec{n}, \vec{u}) = \frac{1}{2}\left[1 - \vec{n}\vec{u}u^0\right]$$

Preferred synchronisation

$$\vec{c}(u) = \frac{c\vec{n}}{1 - \vec{n}\vec{u}u^0} \qquad \frac{1}{u^{0\,2}} - \vec{u}^2 = 1.$$

$\vec{n}$ – direction of light propagation

Transformation of coordinates between $\mathcal{O}$ and $\mathcal{O}'$:

$$\mathrm{d}x'^{\mu}(u') = \mathbf{D}^{\mu}{}_{\nu}(W,u)\,\mathrm{d}x^{\nu}(u),$$

$$[\mathbf{D}^{\mu}{}_{\nu}]\,(W,u) = \left(\begin{array}{c|c} 1/W^0 & 0 \\ \hline -\overrightarrow{W} & I + \frac{\overrightarrow{W}\otimes\overrightarrow{W}^T}{1+\sqrt{1+\overrightarrow{W}^2}} - \overrightarrow{W}\otimes\overrightarrow{u}^T\,u^0 \end{array}\right),$$

W **– four-velocity of** $\mathcal{O}'$ **w.r.t.** $\mathcal{O}$ **and** $(\overrightarrow{a}\otimes\overrightarrow{b}^T)_{ij} = a_i b_j$**.**

$$W^0(u,u') = \frac{u^0}{u'^0} \qquad \overrightarrow{W}(u,u') = \frac{(u^0+u'^0)(\vec{u}-\vec{u}')}{1+u^0\,u'^0(1+\vec{u}\,\vec{u}')}$$

Relativity Principle broken but Lorentz covariance valid

Velocity of $\mathcal{O}'$ as seen from $\mathcal{O}$:

$$\overrightarrow{V} = \frac{\overrightarrow{W}}{W^0}$$

Velocity of $\mathcal{O}$ as seen from $\mathcal{O}'$:

$$\overrightarrow{V'} = -W^0\overrightarrow{W}$$

$V' \neq V$ – **breaking of the Reciprocity Principle** $(V' = -V)$

Inverse transformation:

$$\mathbf{D}_\mu{}^\nu(W,u) = \left(\begin{array}{c|c} W^0 & 0 \\ \hline \overrightarrow{W} & I - \frac{\overrightarrow{W}\otimes\overrightarrow{W}^T}{W^0\left(1+\sqrt{1+\overrightarrow{W}^2}\right)} + \frac{u^0}{W^0}\left(\overrightarrow{W}\otimes\overrightarrow{u}^T\right) \end{array}\right)$$

$$\mathbf{D}(W',u') = \mathbf{D}^{-1}(W,u)$$

Metric tensor depends on u

$$g(u) = \left(\begin{array}{c|c} 1 & u^0 \vec{u}^T \\ \hline u^0 \vec{u} & -I + \vec{u} \otimes \vec{u}^T {u^0}^2 \end{array} \right)$$

Diagonal in PF

u=(1,0,0,0)

Invariants for covariant and contravariant four-vectors:

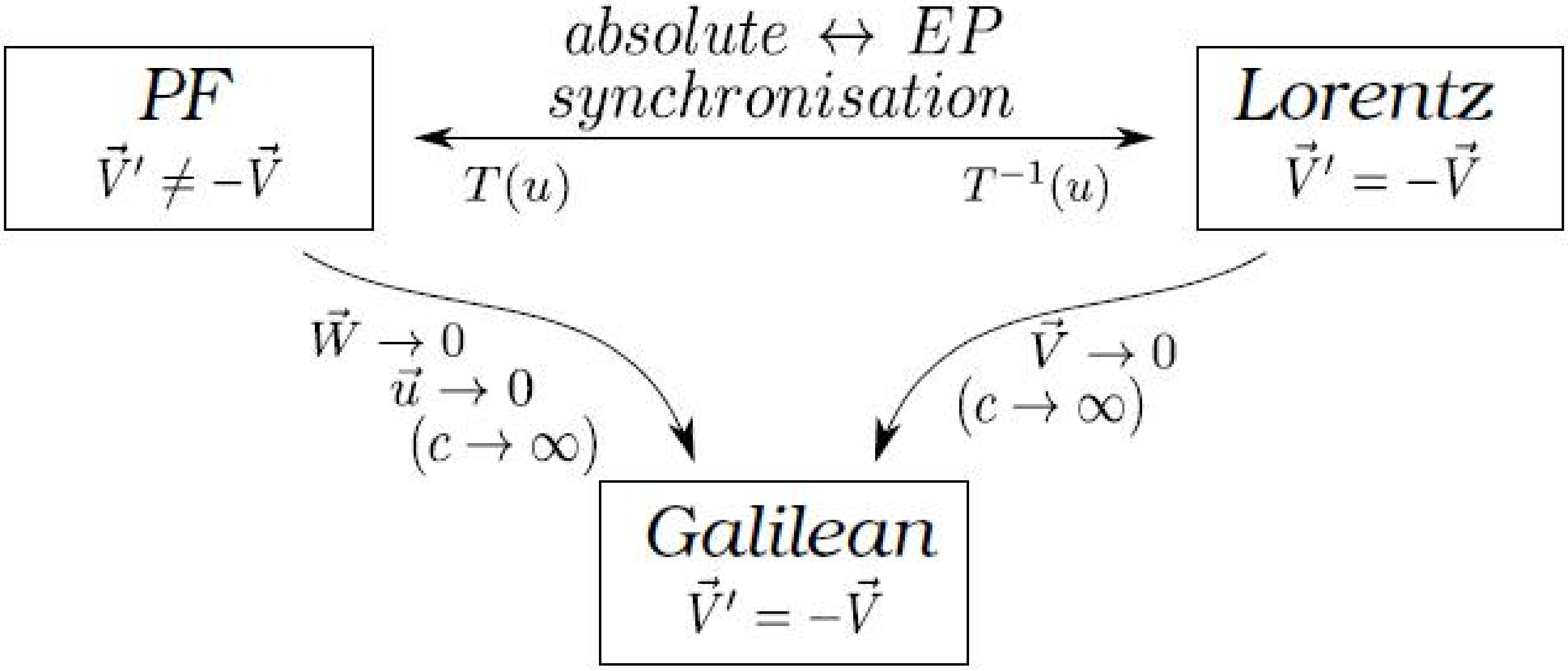
$$a_0^2 - \left(\underset{\rightarrow}{a} - u^0 \vec{a} a_0 \right)^2 = a^2 \qquad \left[b^0 + u^0 \left(\vec{u} \vec{b} \right) \right]^2 - \vec{b}^2 = b^2$$

Resolving SR problems

In kinematics:

- **NO causality violation** $D^0_0 > 0 \quad D^0_k = 0$
- **NO negative energies, NO backward time arrow**
- **NO kinematical singularities**
- **NO vacuum instability**

Correspondence



A new principle of relativity

For light and v<c particles:

Any inertial reference frame

may be assumed the preferred frame

If tachyons*** existed

There would exist a preferred frame of reference in Nature as well as the preferred clock synchronisation procedure

***** or other superluminal phenomena**

PFM offers a description mathematically equivalent to SR for light and v<c particles

DESCRIPTION SUMMARY

Tachyons -- PFM only

Bradyons, light -- PFM or SR equivalently

(but for practical reasons choose SR)

Einstein's SR remains valid

Tachyon in PF

Dispersion relation:

A simple example

$$E^2 - \vec{p}^2 = -\kappa^2$$

κ – tachyonic mass

E – tachyon energy $\equiv$ kinetic energy (no rest energy)

$\vec{p}$ – tachyon momentum, rest-momentum $|\vec{p}| = \kappa$ when $v \to \infty$

$$E(v) = \frac{\kappa}{\sqrt{v^2 - 1}} \to 0 \quad \text{as} \quad v \to \infty$$

$$E(v) \to \infty \quad \text{as} \quad v \to c \quad (v > c)$$

Velocity of light if tachyons*** existed

Over a closed path - as in SR: c

Over an open path (from A to B): c_{AB}

- two clocks
- synchronised using tachyons with v → infinity

one-way velocity of light would be measurable

using tachyons

No obstacles for superluminal transmission of information (if technically feasible)

How to discover the PF?

- **measure one way velocity of light using absolute clock synchronisation (tachyons)**
- **measure processes sensitive to the four-vector u**

Natural candidate:

Cosmic Background Radiation ?

Velocity of Earth w.r.t. CMBR -- order of 10^{-3}

Advantages of the PFM

- Covariant position operator, spin operator, probability current -- can be defined
- Non-local QM phenomena can be covariantly described

(e.g. quantum spin correlations) **IMPORTANT**

- Covariant statistical classical and quantum thermodynamics can be formulated

Other superluminal phenomena? YES

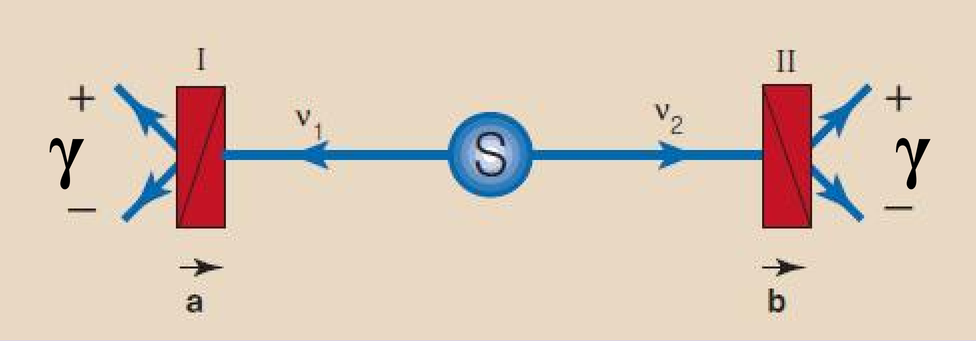
EPR quantum spin correlations

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)



Correlated although separated by a space-like interval

Fermionic tachyons of helicity 1/2

Fermionic tachyons ($\lambda = \frac{1}{2}$)

'Dirac'- like equation as in relativistic QM

Similar method of derivation

Important differences in results!!!

γ matrices in absolute synchronisation:

$$\gamma^{\mu} = T(u)^{\mu}_{\nu}\gamma^{\nu}_{E}$$

γ^{ν}_{E} – **standard Dirac matrices** ($\gamma^5 = \gamma^5_E$).

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g_{\mu\nu}$$

The Klein-Gordon equation to be fulfilled:

$$\left[g_{\mu\nu}(u)\partial_{\mu}\partial_{\nu} - \kappa^2\right]\psi = 0.$$

Helicity condition to be fulfilled:

$$\hat{\lambda}(u)\psi(u,k) = \lambda\psi(u,k)$$

$\hat{\lambda}(u)$ – **helicity operator**

Analogue of the Dirac equation:

$$\left[\gamma^5 \left(i\gamma\partial\right) - \kappa\right] \psi = 0.$$

Here: γ-matrices expressed by Dirac matrices

The bispinor field ψ is eigenvector of the helicity operator with eigenvalue 1/2:

tachyons $\lambda = -1/2$, antitachyons $\lambda = 1/2$

A candidate: neutrino ???

Argument 1: excess of counts in Tritium decay

$$^3H \rightarrow ^3He + e^- + \bar{\nu}_e \quad (+18.6\ \mathrm{keV})$$

Electron differential energy spectrum near end-point:

$$\frac{\mathrm{d}N}{\mathrm{d}E} \propto (E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}$$

Linearised differential:

$$\propto \left[(E_0 - E)\sqrt{(E_0 - E)^2 - m_\nu^2}\right]^{1/2}$$

Electron integral energy spectrum near end-point:

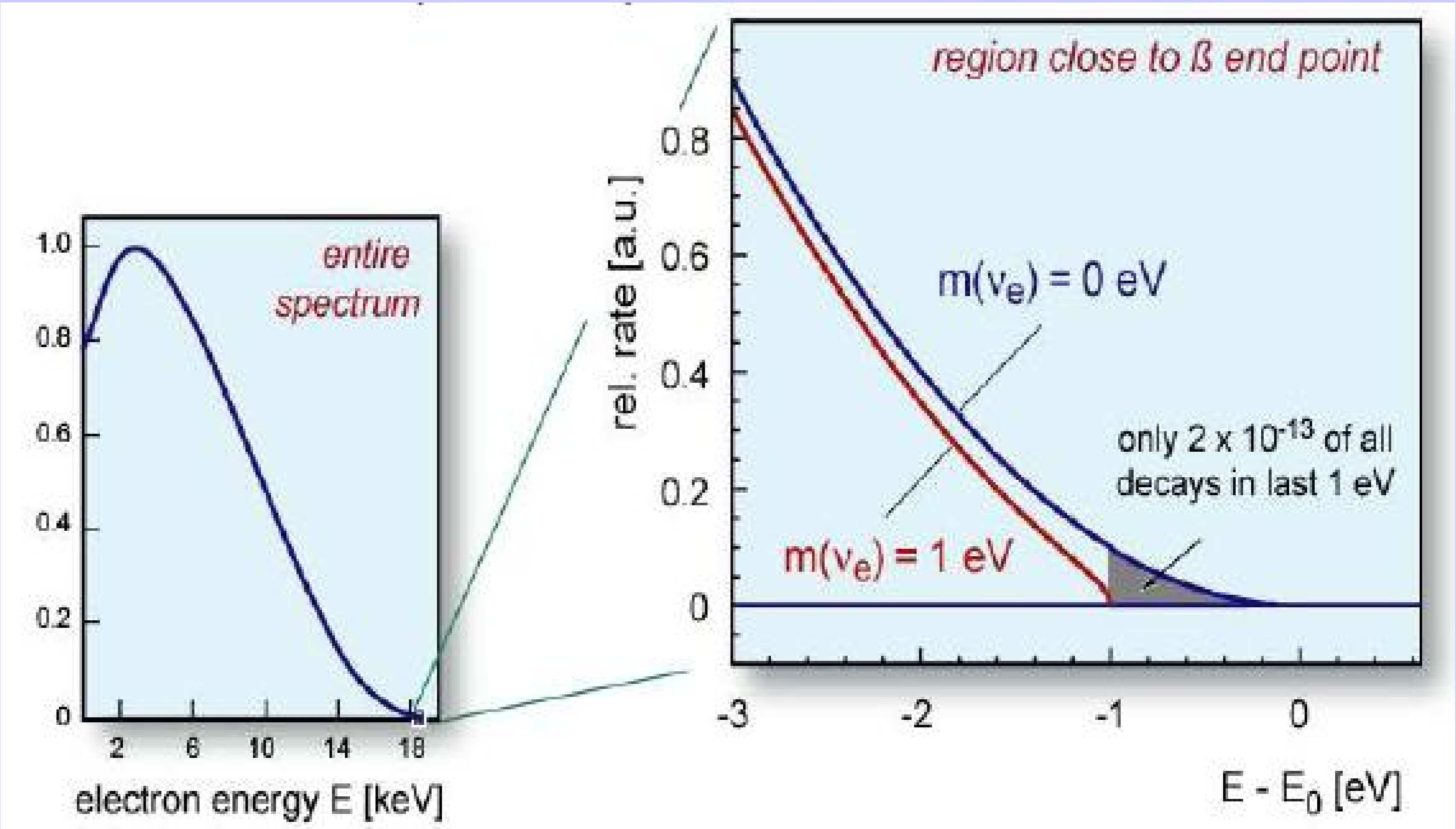
$$N \propto \int_E^{E_0} (E_0 - \xi)\sqrt{(E_0 - \xi)^2 - m_\nu^2}\,\mathrm{d}\xi$$

Linearised integral:

$$N^{1/3} \propto \left[\int_E^{E_0} (E_0 - \xi)\sqrt{(E_0 - \xi)^2 - m_\nu^2}\,\mathrm{d}\xi\right]^{1/3}$$

E_0 – end-point energy (unknown, fitted), E – electron energy

Electron energy spectrum

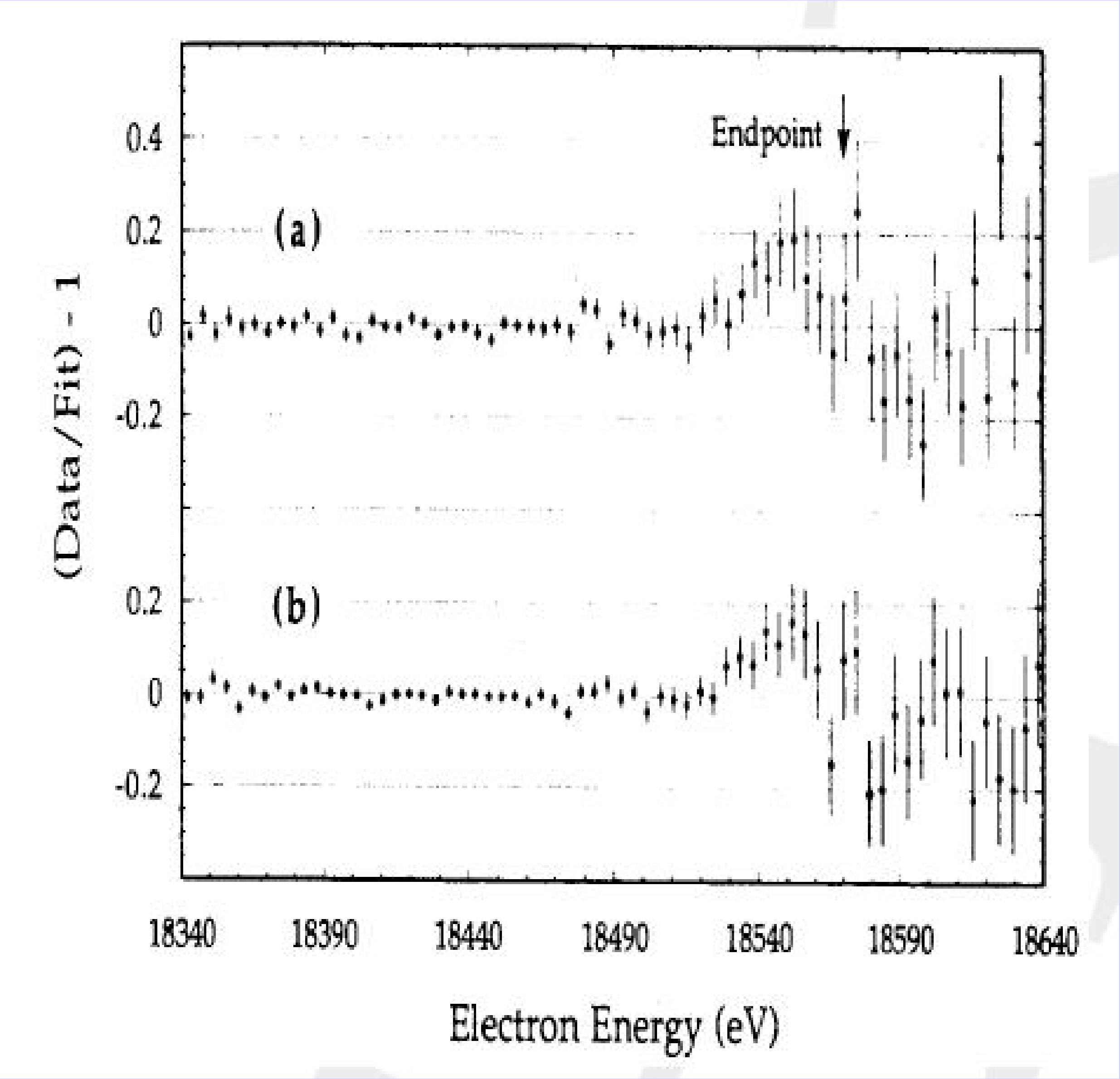


Credit: H. Drexlin

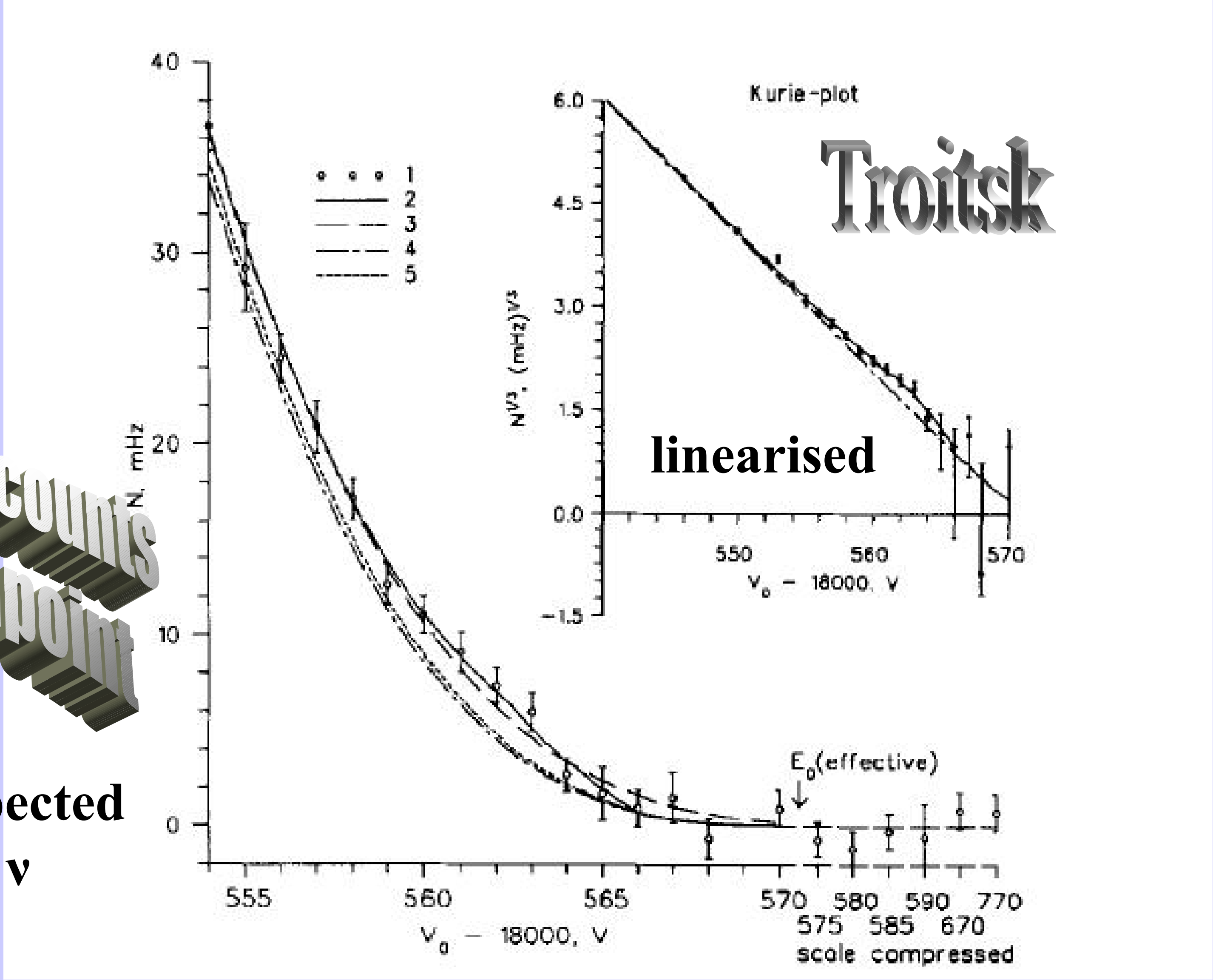
Differential spectrum

LLNL

1995



Integral spectrum

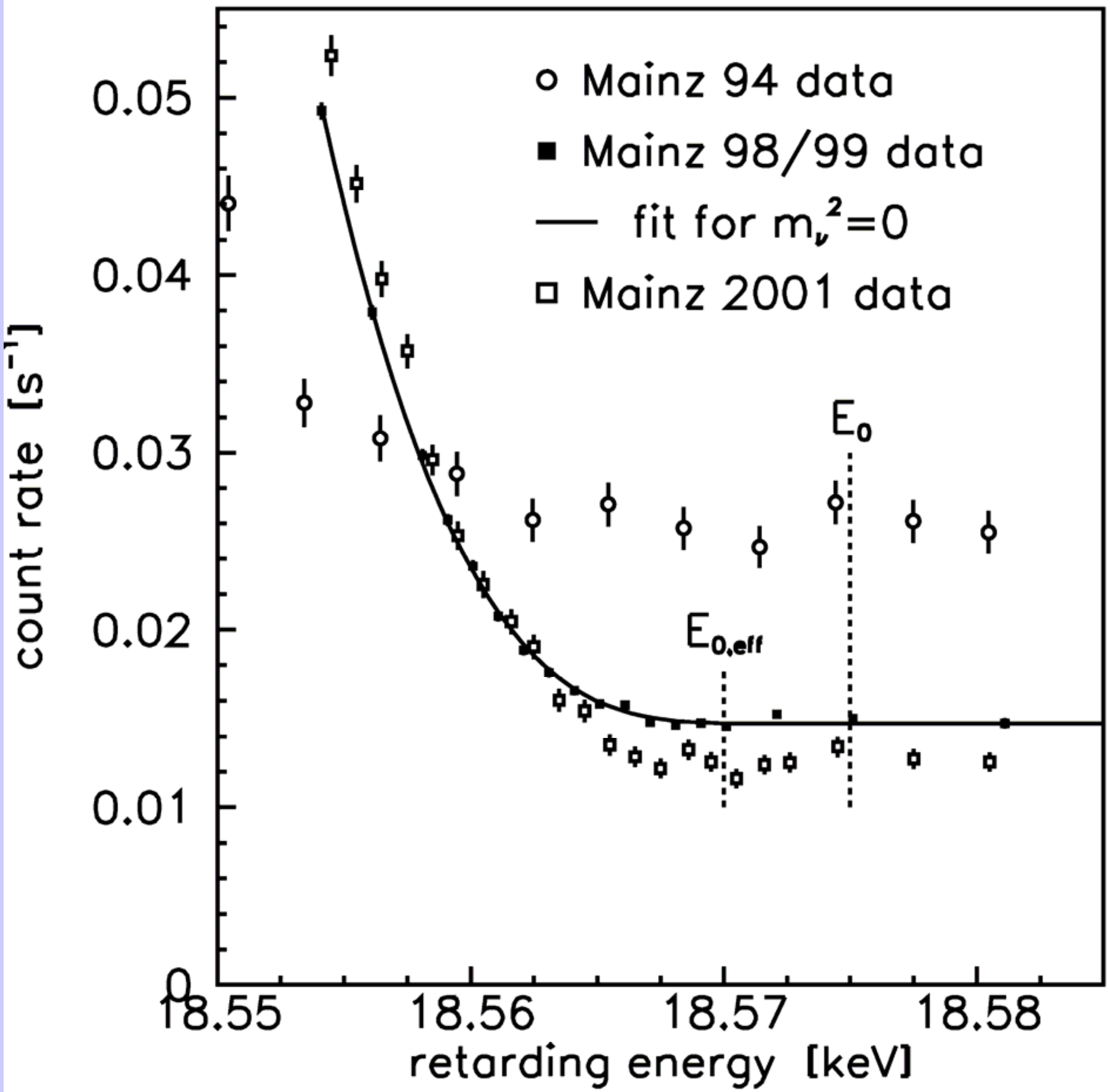


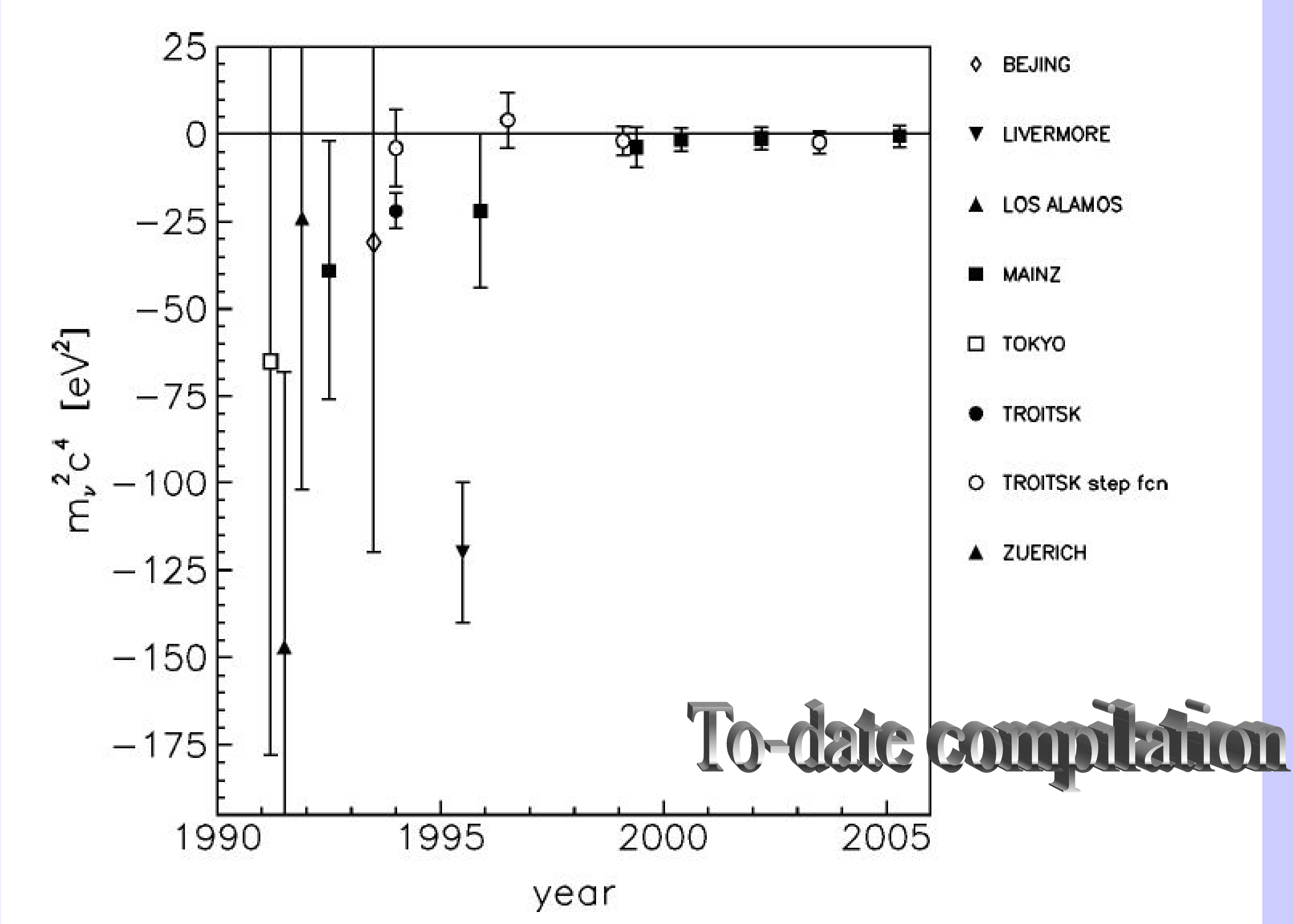
excess of counts near endpoint

Depletion expected
For massive ν

Integral spectrum

Mainz

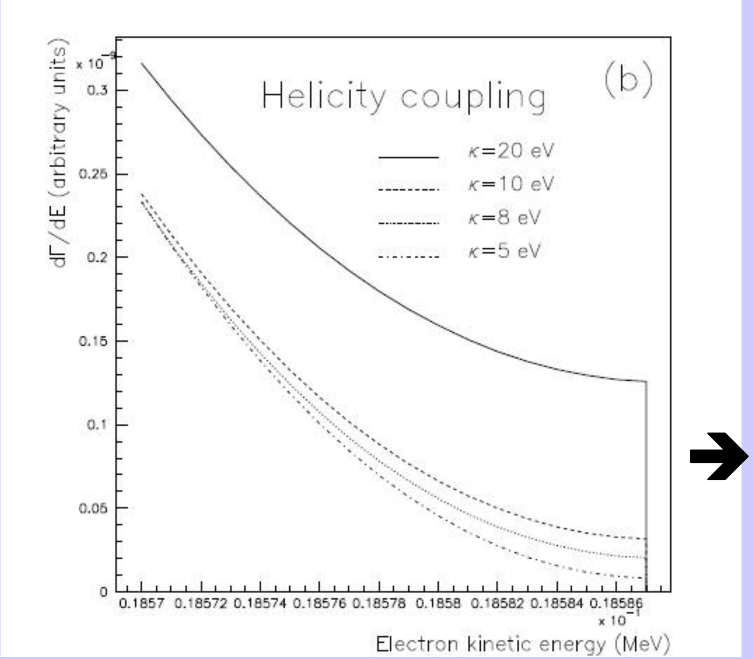




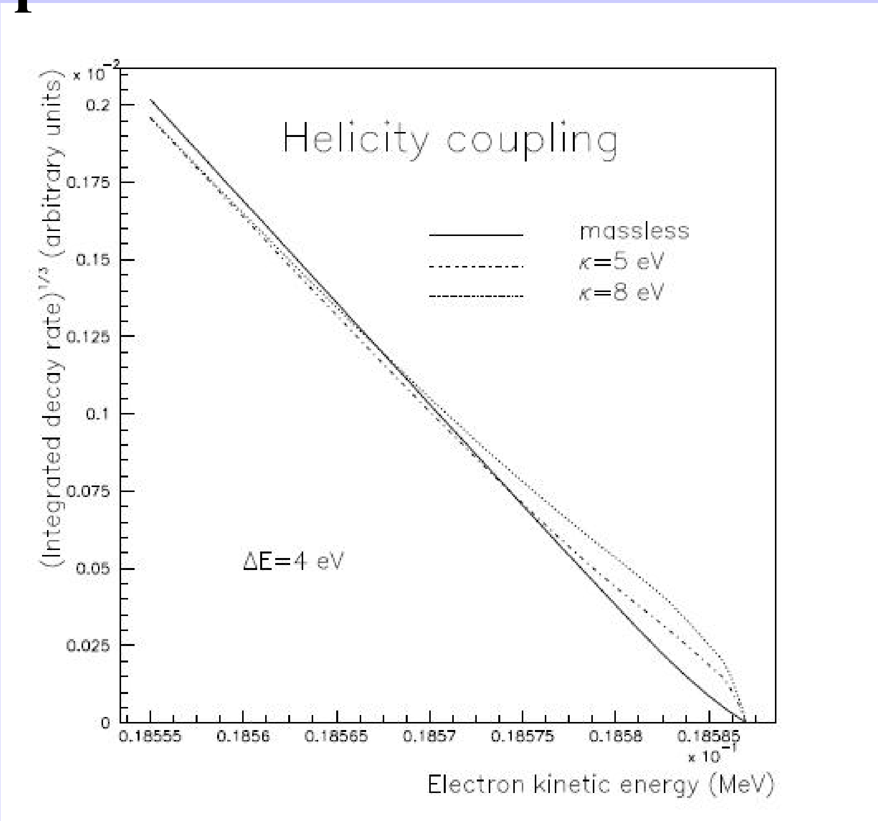
Final results: Mainz: -0.6 ±2.2 ±2.1 eV^2

Troitsk -2.3 ±2.5 ±2.0 eV^2

Explanation of excess if due to tachyonic neutrinos



➔ **exp. smeared**



Step due to rest-momentum of a tachyonic neutrino

PHASE SPACE factor

J. R, J. C.

Eur. Phys. J. C8 (1999) 157

Step at endpoint

- Tachyonic neutrino 'rest' momentum at endpoint = κ (not 0 like for massive neutrino)
- Very rapid – **step like --** rise of phase space with decreasing electron energy owing to non-vanishing tachyonic neutrino **momentum vector** (despite zero neutrino energy)

FUTURE: Experiment KATRIN (any time now)

Argument 2:
tachyon -- helicity 1/2
antitachyon -- helicity -1/2

- Coincides with experimental observations for neutrinos
- No need to introduce ad hoc the $(1-\gamma^5)$ term in the (weak) current -- the above property of spinors follows from the 'Dirac' equation for tachyons (i.e. from first principles)

Comment 3:

- **Tachyonic neutrino cannot be Majorana-type**
- **Observation of neutrinoless 2β decay would invalidate the tachyonic hypothesis**
- **Not discovered to date**

Summary

- **Tachyons cannot be described within Einstein's SR**
- **PFM given by J. Rembieliński is a unified description of v<c, c and v>c particles and phenomena**
- **PFM is equivalent to SR for light and v<c particles**
- **Tachyons can be described and quantised only within PFM**
- **If tachyons exist - the Preferred Frame and the preferred synchronisation procedure must have been chosen by Nature**
- **Signals (information) could be transmitted with superluminal velocity**

- **PFM allows to settle several unsolved problems in QM**
- **Tachyon has helicity - 1/2 , antitachyon + 1/2**
- **There are arguments favouring neutrinos as tachyonic candidates**
- **If tachyons do not exist, PF is still desirable:**

PFM remains a valid option in view of non-local quantum phenomena like EPR correlations

The end

Mainz runs 1997 - 2001

no.	Θ_{max}	t [d]	pt	ft [nm]	$\bar{b}$ [mHz]	$m^2(\nu_e)$ [eV^2]	U_0 [V]
Q1	45°	6		20.8		test measurement	
Q2	45°	26	50	96.7	16.7±0.3	−11.2±6.0	18573.5± 0.3
Q3	45°	24	64	49.3	12.7±0.2	−14.8±4.6	18574.0 ± 0.2
Q4	45°	38	64	49.5	11.7±0.2	−3.9±4.7	18574.5 ± 0.2
Q5	45°	46	64	47.5	21.6±0.2	−3.5±6.0	18574.4 ± 0.2
Q6	62°	38	33	43.0	12.5±0.2	+0.4±7.2	18575.7 ± 0.2
Q7	62°	29	33	43.2	14.3±0.2	−2.4±4.9	18575.4 ± 0.2
Q8	62°	54	39	45.5	16.5±0.2	−0.9±4.8	18576.2 ± 0.3
Q9	62°	56	39	44.4	18.6±0.3	−10.9±3.2	18575.1 ± 0.2
Q10	62°	35	45	45.5	16.6±0.3	−6.1±4.8	18574.6 ± 0.2
Q11	45°	31	45	48.2	12.6±0.2	+1.3±5.8	18576.7 ± 0.2
Q12	62°	19	45	48.5	12.6±0.2	−1.0±6.0	18576.6 ± 0.2

Selected for final results: Q5-Q8