Implications of CMS searches for the Constrained MSSM – A Bayesian approach

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Outline

1. Constrained MSSM (CMSSM)

2. Bayesian statistics in a nutshell

3. Impact of CMS alphaT 1.1/fb and XENON100 limits

4. Summary

Fowlie, Kalinowski, Kazana, Roszkowski, Tsai (arXiv:1111.6098)

Roszkowski, Sessolo, Tsai (arXiv:1202.1503)

Constrained Minimal Supersymmetric Standard Model

G. L. Kane, C. F. Kolda, L. Roszkowski and J. D. Wells, Phys. Rev. D 49 (1994) 6173

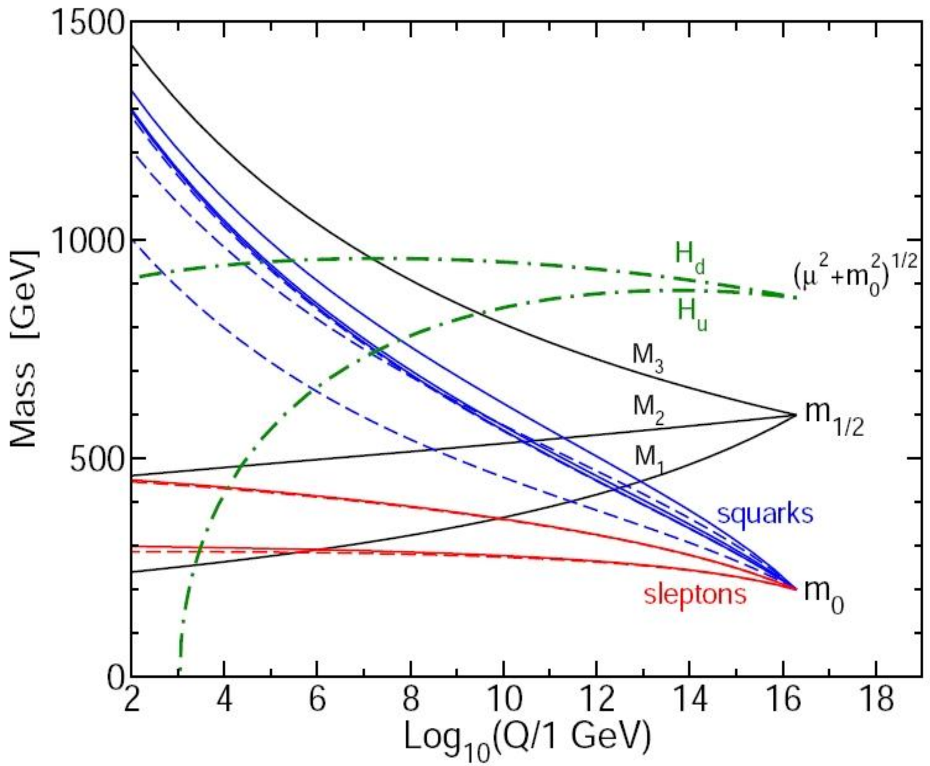


figure from hep-ph/9709356

At $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV:

- gauginos $M_1 = M_2 = m_{\tilde{g}} = m_{1/2}$
- scalars
  $m^2_{\tilde{q}_i} = m^2_{\tilde{l}_i} = m^2_{H_b} = m^2_{H_t} = m^2_0$
- 3–linear soft terms $A_b = A_t = A_0$
- radiative EWSB
  $\mu^2 = \frac{m^2_{H_b} - m^2_{H_t} \tan^2\beta}{\tan^2\beta - 1} - \frac{m^2_Z}{2}$
- five independent parameters:
  $m_{1/2},\ m_0,\ A_0,\ \tan\beta,\ \mathrm{sgn}(\mu)$
- well developed machinery to compute masses and couplings
- neutralino χ mostly bino

Comparison of experimental data and model prediction

Traditional chi-square method:

$$\chi^2 = \sum \frac{(\text{prediction} - \text{measurement})^2}{(\text{error})^2}.$$

As long as experimental central value and error are given, one can use chi-square method to compare the experimental data and model predictions.

Alternatively...

Bayesian Statistics...

Bayes theorem:

$$\text{Posterior} = \frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}.$$

- **Likelihood**: the probability of obtaining data if hypothesis is true.
- **Prior**: what we know about hypothesis BEFORE seeing the data.
- **Evidence**: normalization constant, crucial for model comparison.
- **Posterior**: the probability about hypothesis AFTER seeing the data.

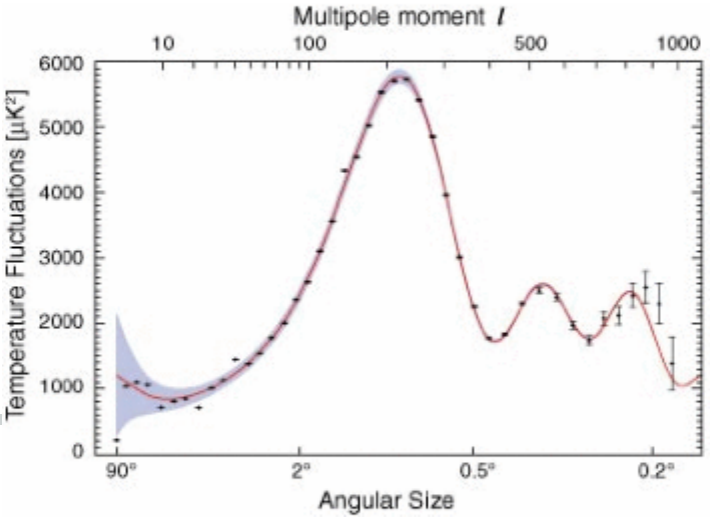
There is no single, ``right'' statistics...

Frequentist: "probability is the number of times the event occurs over the total number of trials, in the limit of an infinite series of equiprobable repetitions"

Bayesian: "probability is a measure of the degree of belief about a proposition"

Bayesian statistics is very popular in many branches of science (astronomy, cosmology, etc.).

For example, The Wilkinson Microwave Anisotropy Probe (WMAP) analysis of cosmic microwave background (CMB) spectrum:

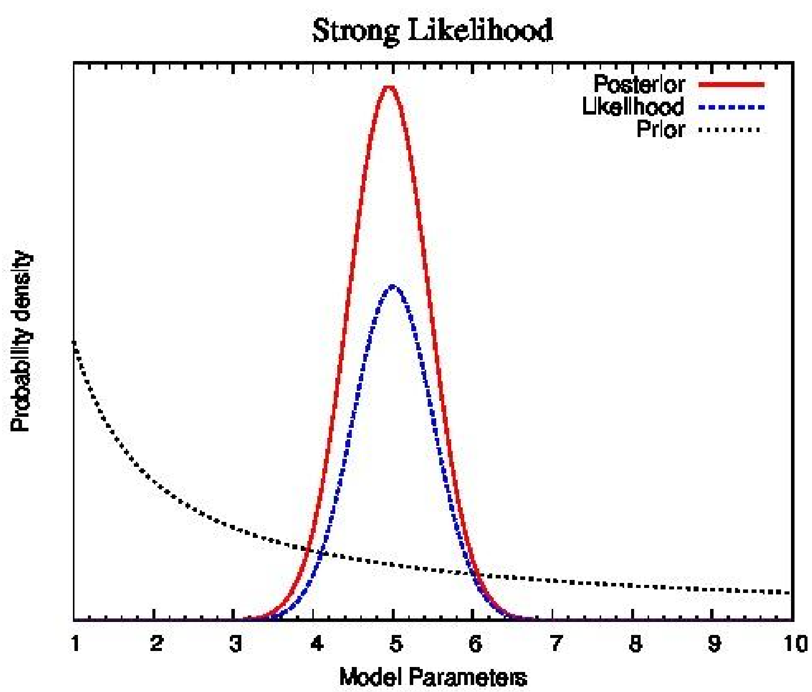


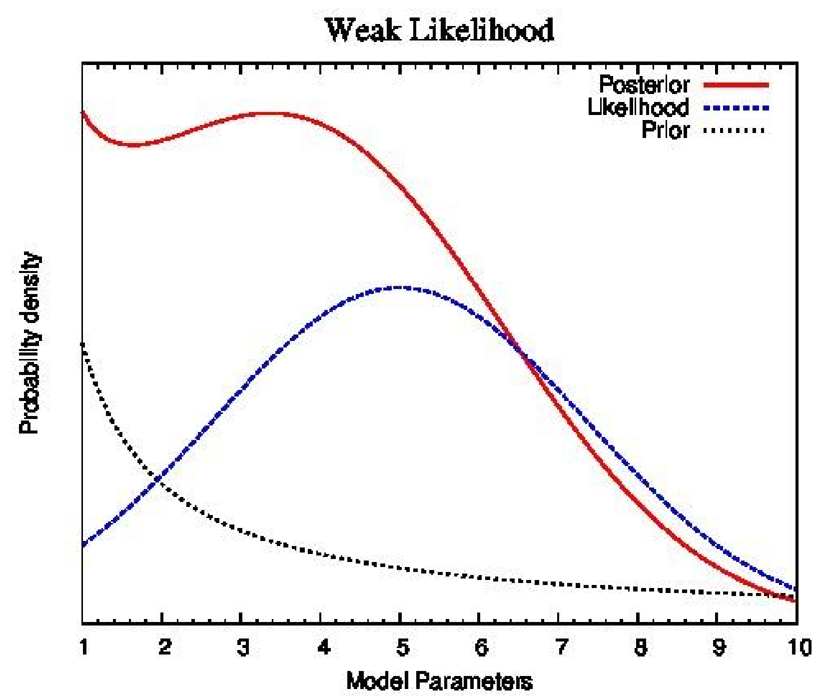
Prior dependence

$$\text{Posterior} = \frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}.$$

Prior dependence is two-fold:

- ○ **Prior range.**
- ○ **Prior distribution.**





- **If the Likelihood is well-peaked, the posterior follows the Likelihood**
- **Otherwise, it follows the prior.**

$$\text{Posterior} = \frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}.$$

Gaussian Likelihood

Take a single observable $\xi(m)$ that has been measured

(e.g., M_W)

- c – central value, σ – standard exptal error
- define

$$\chi^2 = \frac{[\xi(m)-c]^2}{\sigma^2}$$

- assuming Gaussian distribution ($d \to (c, \sigma)$):

$$\mathcal{L} = p(\sigma, c|\xi(m)) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{\chi^2}{2}\right]$$

- when include theoretical error estimate τ (assumed Gaussian):

$$\sigma \to s = \sqrt{\sigma^2 + \tau^2}$$

TH error "smears out" the EXPTAL range

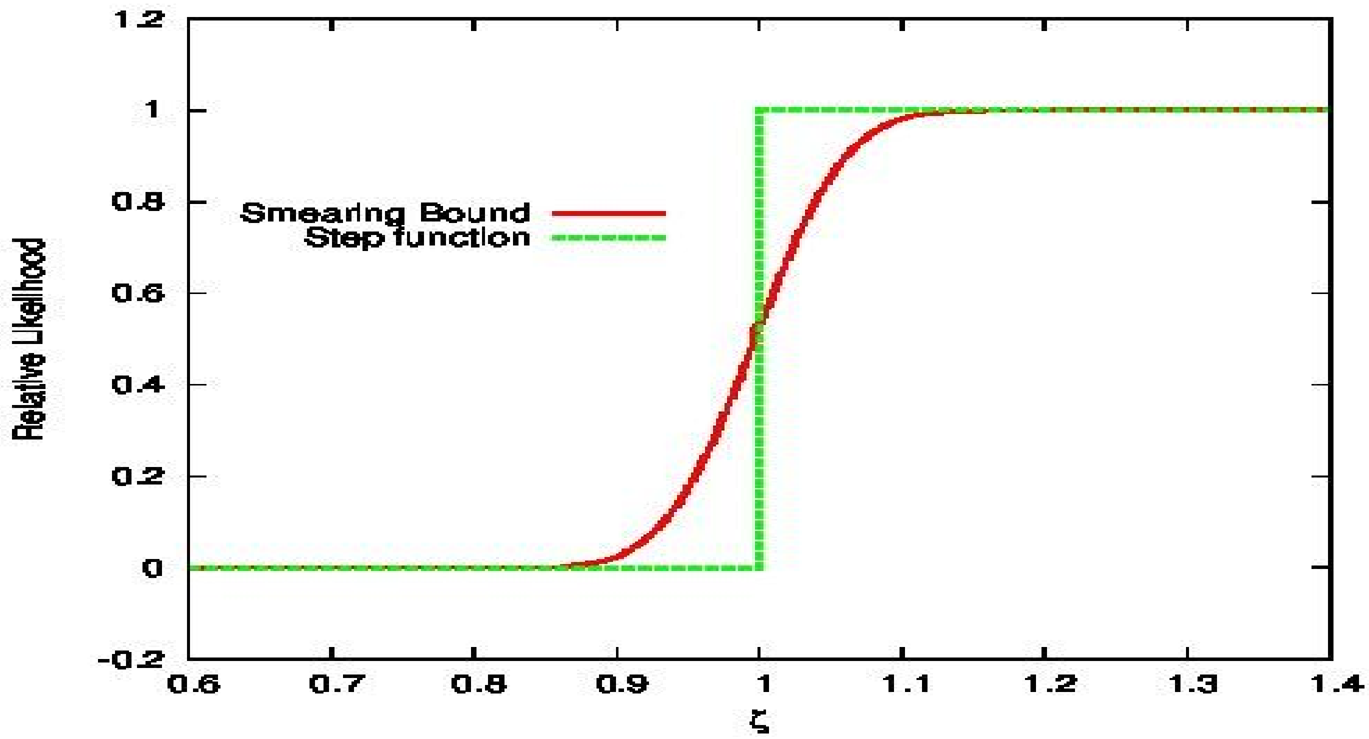
- for several uncorrelated observables (assumed Gaussian):

$$\mathcal{L} = \exp\left[-\sum_i \frac{\chi_i^2}{2}\right]$$

$$\text{Posterior} = \frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}.$$

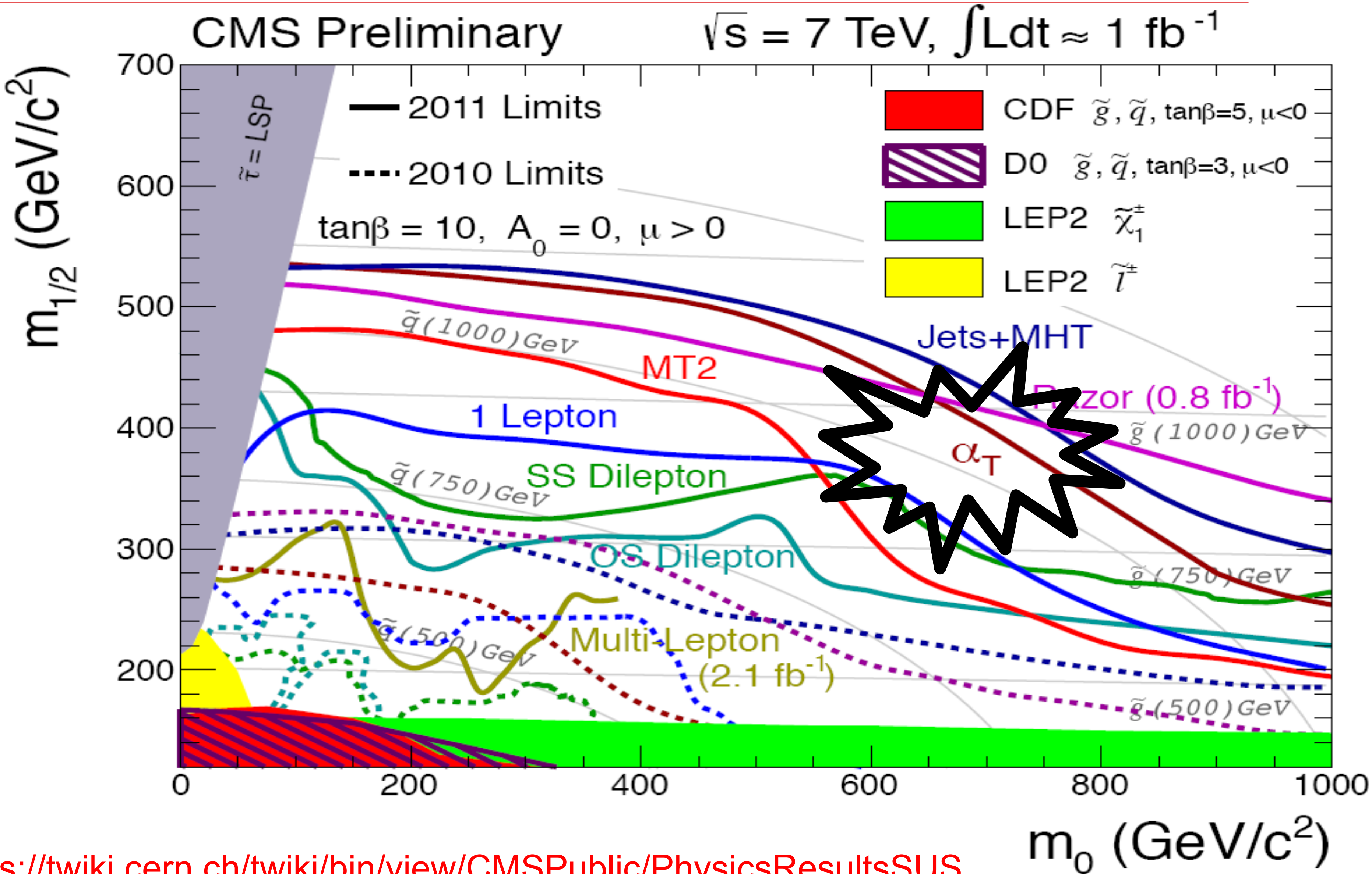
Likelihood from limits

Upper/lower limit for null result: exclusion.



- Use error function to smear the bound!
- Can add theory error as well.

SUSY search at CMS



https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUS

How could we go beyond the CMS published alphaT m_0 and m_1/2 range?

How do we include information from CMS alphaT (1.1 /fb) analysis into our likelihood?

$$\text{Posterior} = \frac{\text{Prior} \times \text{Likelihood}}{\text{Evidence}}.$$

Poisson Likelihood

Poisson distribution to characterize counting experiments.

$$\mathcal{L} = \prod_i \frac{e^{-(s_i+b_i)} (s_i + b_i)^{o_i}}{o_i!}$$

o_i : observed events in LHC.

b_i : expected SM background events.

s_i : $s_i = \epsilon_i \times \sigma \times \int L.$

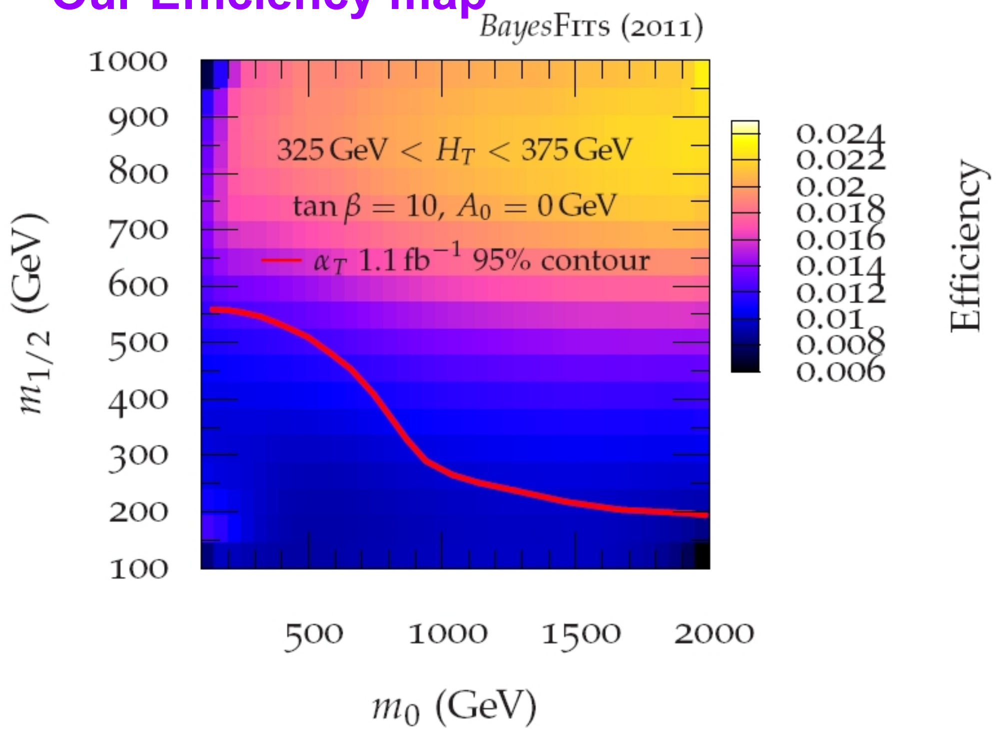
ϵ_i : $N_i(\alpha_T > 0.55)/N_{\text{total}}$

$i = 1, 2, 3..., 8.$

Our approximate Efficiency maps

$$\epsilon_i : N_i(\alpha_T > 0.55)/N_{\text{total}}$$

Our Efficiency map



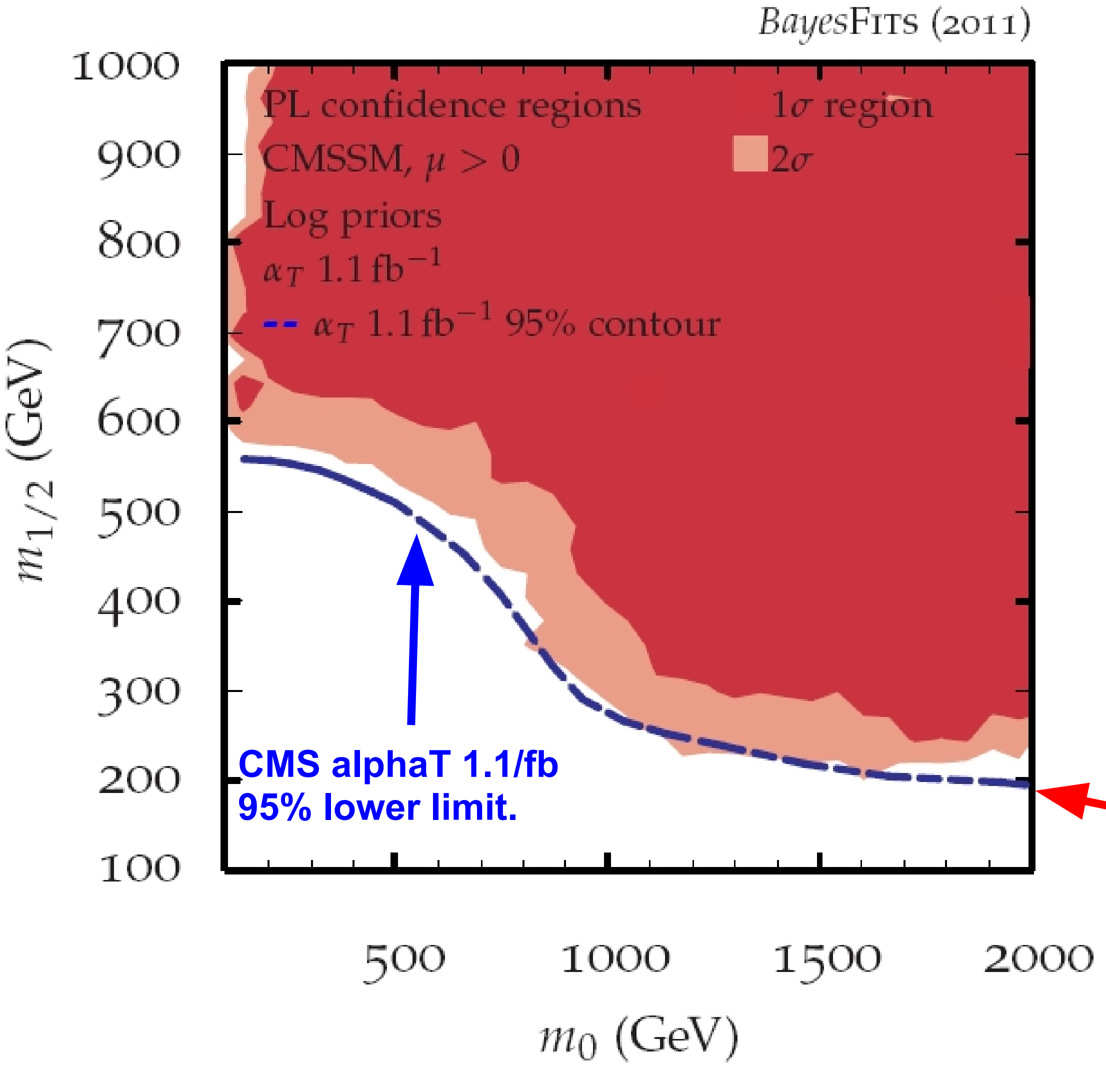
(b) The case of 325 GeV < H_T < 375 GeV.

More selected events at large m_1/2 !

Fowlie, Kalinowski, Kazana, Roszkowski, Tsai (arXiv: 1111.6098)

Limit from CMS alphaT 1.1/fb

Fowlie, Kalinowski, Kazana, Roszkowski, Tsai (arXiv:1111.6098)



$$\mathcal{L} = \prod_i \frac{e^{-(s_i+b_i)}\,(s_i+b_i)^{o_i}}{o_i!}$$

$$s_i = \epsilon_i \times \sigma \times \int L.$$

VERY GOOD AGREEMENT WITH CMS 95% LIMIT!

We can extend the range of m_0 and m_1/2 to whatever we want.

SUSY: Summary of constraints

Dark Matter relic density

anomalous magnetic moment of the muon

Favour physics

We include all important constraints.

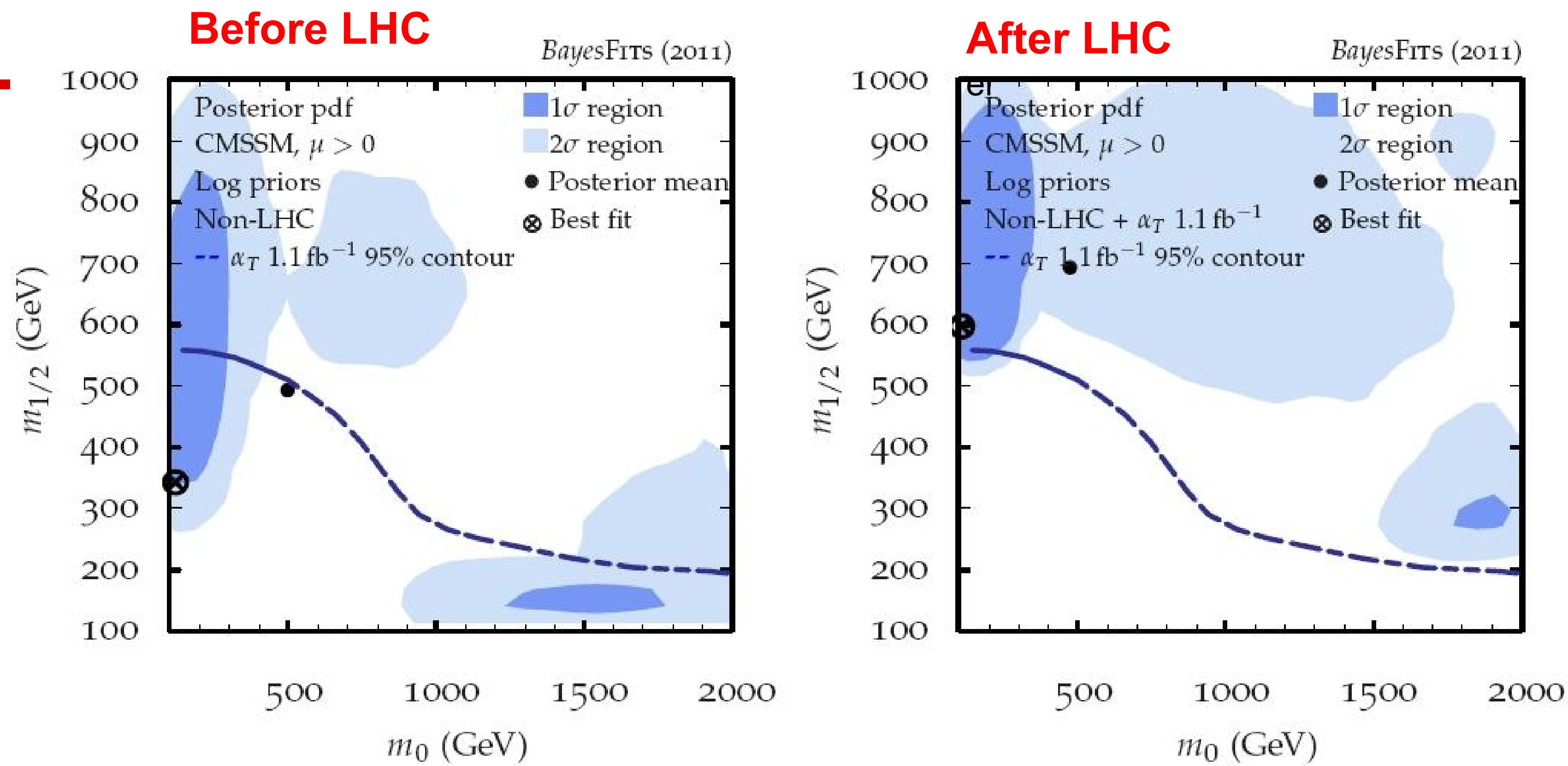
Dark matter search

Observable	Mean	Exp. Error	Theor. Error	Likelihood Distribution
Non-LHC:				
$\Omega_\chi h^2$	0.1120	0.0056	10%	Gaussian
$\sin^2\theta_{\rm eff}$	0.231160	0.00013	15.0×10^{-5}	Gaussian
M_W	80.399	0.023	0.015	Gaussian
$\delta(g-2)_\mu^{SUSY}\times10^{10}$	30.5	8.6	1	Gaussian
$\mathcal{BR}(\bar{B}\to X_s\gamma)\times10^4$	3.6	0.23	0.21	Gaussian
$\mathcal{BR}(B_u\to\tau\nu)\times10^4$	1.66	0.66	0.38	Gaussian
ΔM_{B_s}	17.77	0.12	2.4	Gaussian
$\mathcal{BR}(B_s\to\mu^+\mu^-)$	$<1.5\times10^{-8}$	0	14%	Upper limit — Error fn
LEP — 95% Limits				
m_h	>114.4	0	3	Lower limit — Error fn
ζ_h^2	$<f(m_h)$	0	0	Upper limit — Step fn
m_χ	>50	0	5%	Lower limit — Error fn
$m_{\chi_1^\pm}$	>103.5 (92.4)	0	5%	Lower limit — Error fn
$m_{\tilde{e}_R}$	>100 (73)	0	5%	Lower limit — Error fn
$m_{\tilde{\mu}_R}$	>95 (73)	0	5%	Lower limit — Error fn
$m_{\tilde{\tau}_1}$	>87 (73)	0	5%	Lower limit — Error fn
$m_{\tilde{\nu}}$	>94 (43)	0	5%	Lower limit — Error fn
LHC CMS α_T 1.1/fb analysis				
α_T	See text	See text	0	Poisson
XENON100				
$\sigma_p^{\rm SI}(m_\chi)$	$<f(m_\chi)$ — see text	0	1000%	Upper limit — Error fn
Nuisance				
$1/\alpha_{em}(M_Z)^{\overline{MS}}$	127.916	0.015	0	Gaussian
$m_t^{\rm pole}$	172.9	1.1	0	Gaussian
$m_b(m_b)^{\overline{MS}}$	4.19	0.12	0	Gaussian
$\alpha_s(M_Z)^{\overline{MS}}$	0.1184	0.0006	0	Gaussian

Including all the constraints into likelihood, we can conduct a random scan with prior range:

$$
\begin{aligned}
100 \text{ GeV} &\leq m_0 \leq 4000 \text{ GeV} \\
100 \text{ GeV} &\leq m_{1/2} \leq 2000 \text{ GeV} \\
-2000 \text{ GeV} &\leq A_0 \leq 2000 \text{ GeV} \\
3 &\leq \tan\beta \leq 62\,.
\end{aligned}
$$

Impact of the alphaT limit on CMSSM



(a) The CMSSM's parameters constrained by the non-LHC experiments only.

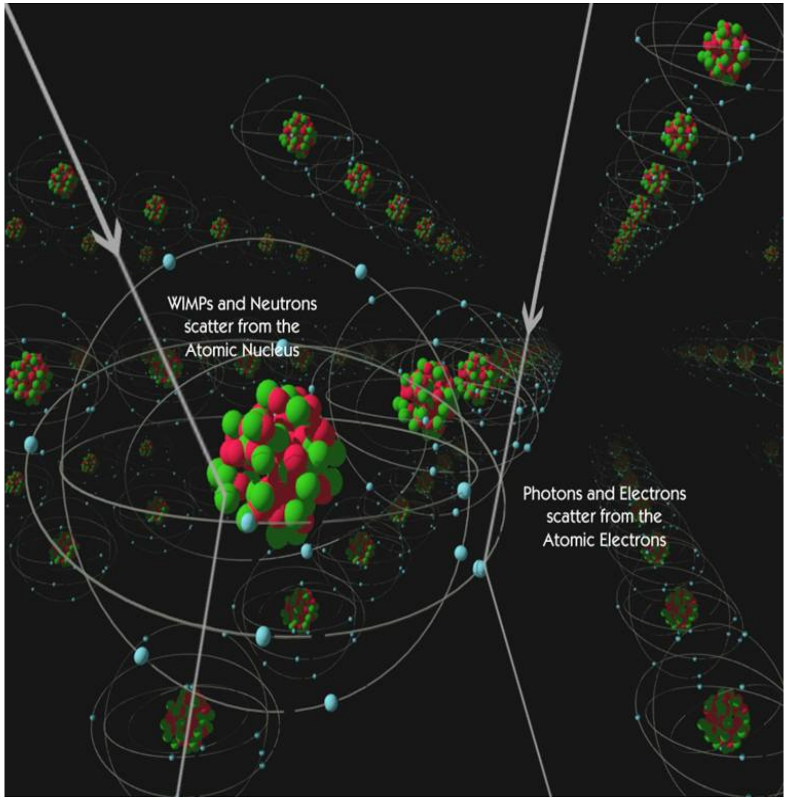
(b) The CMSSM's parameters constrained by the CMS α_T 1.1/fb analysis and by the non-LHC experiments.

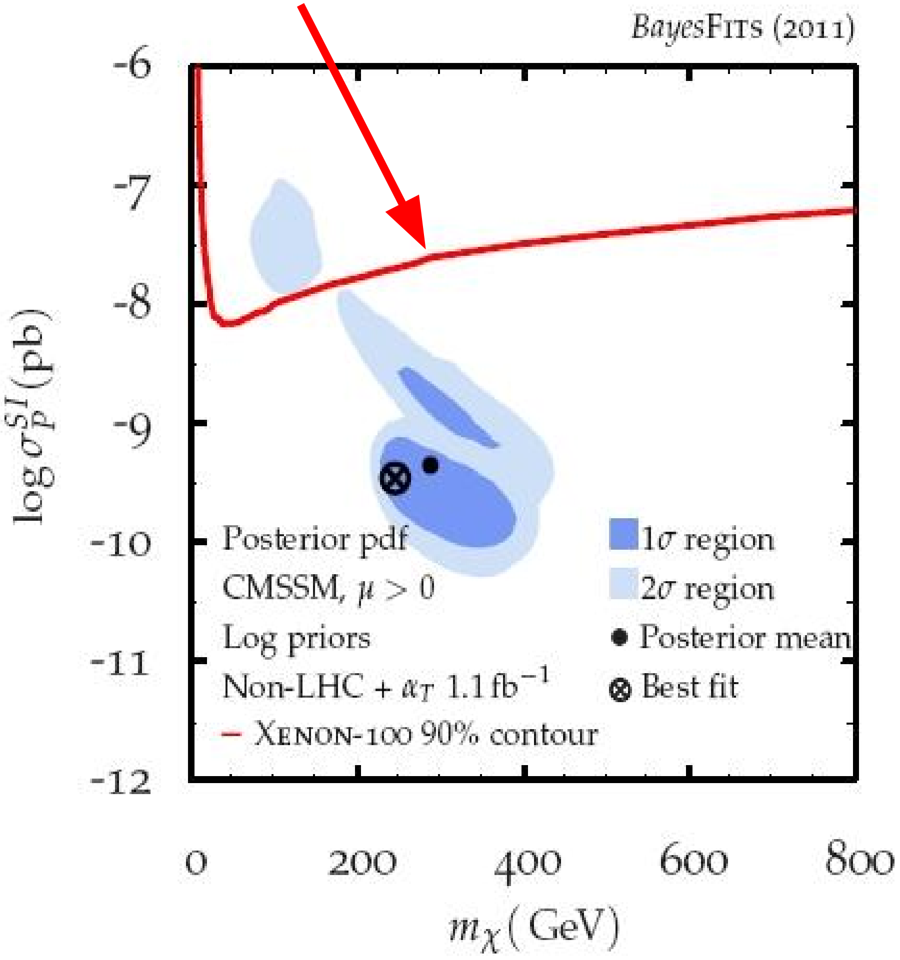
General trend:

- favoured ranges are now pushed up.
- poorer fit, but
- best fit point remains just above the CMS 95% CL limit

Direct searches of dark matter

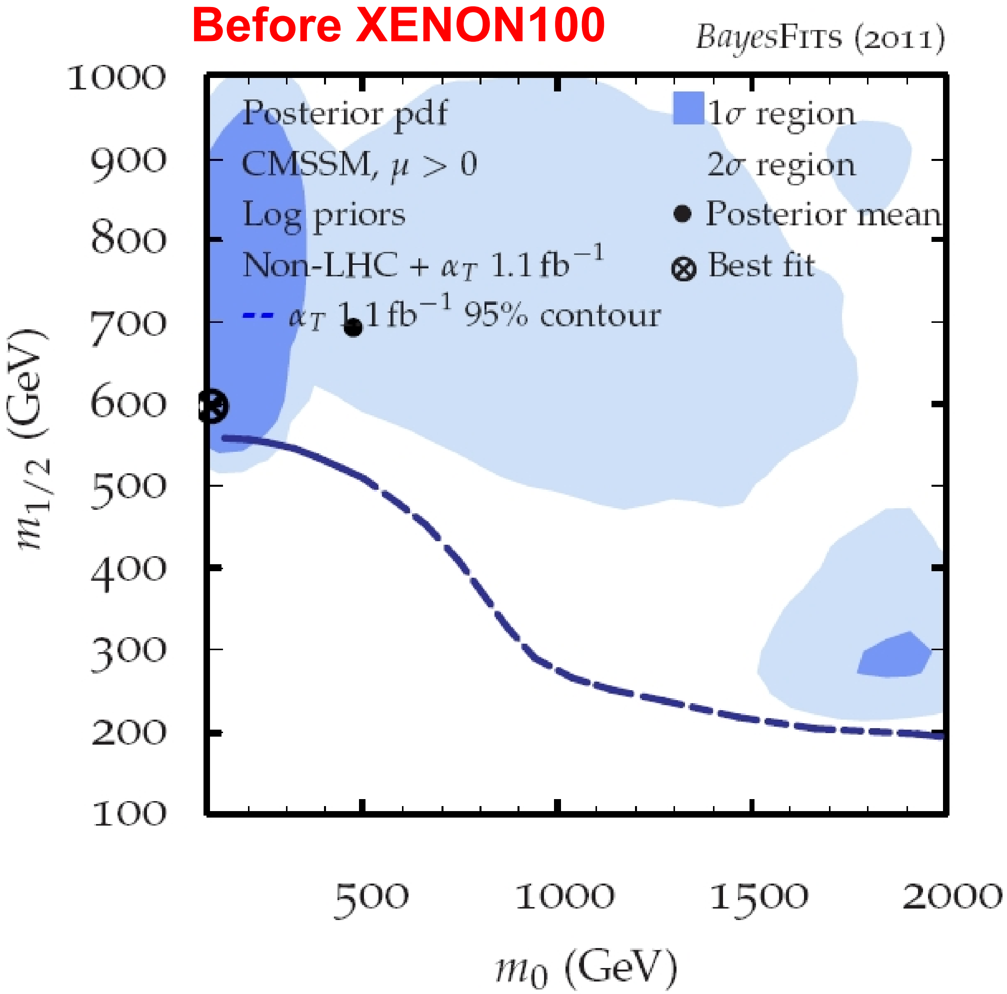
currently best limit from XENON100

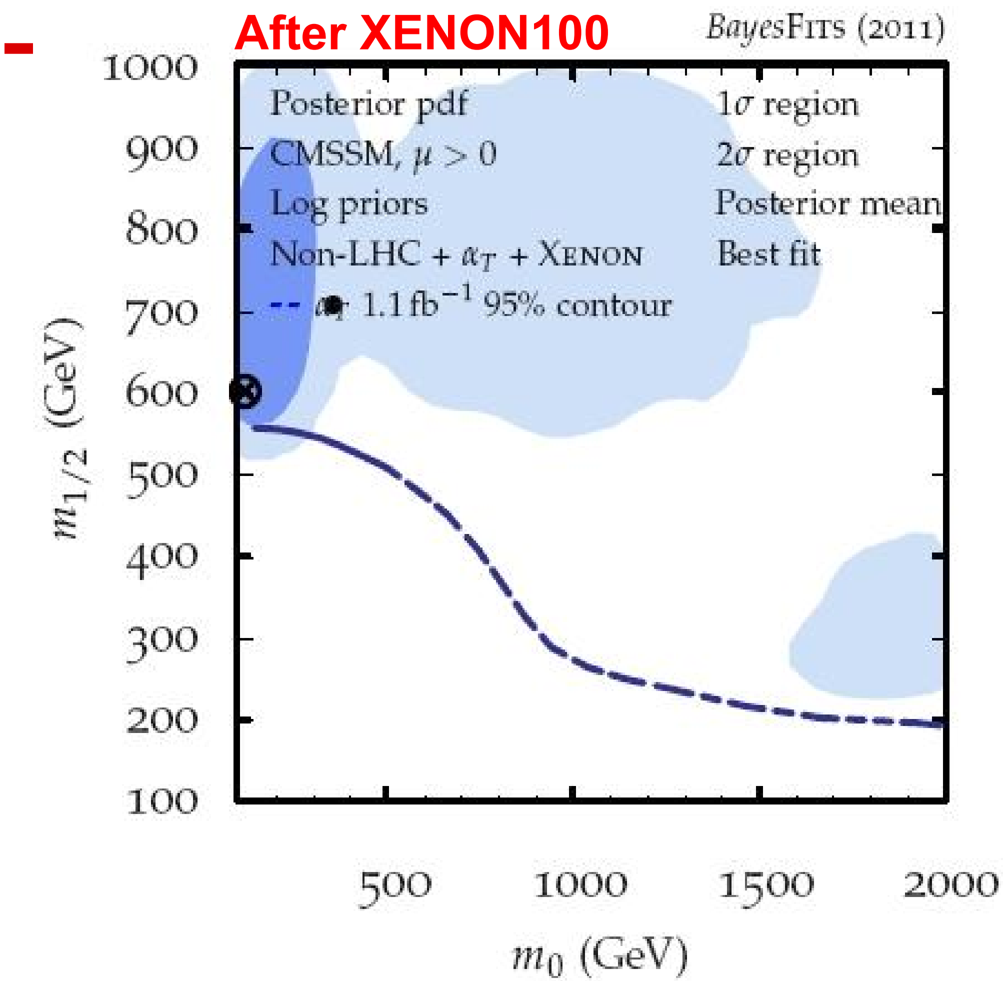




XENON100 limit not applied here

Impact on CMSSM



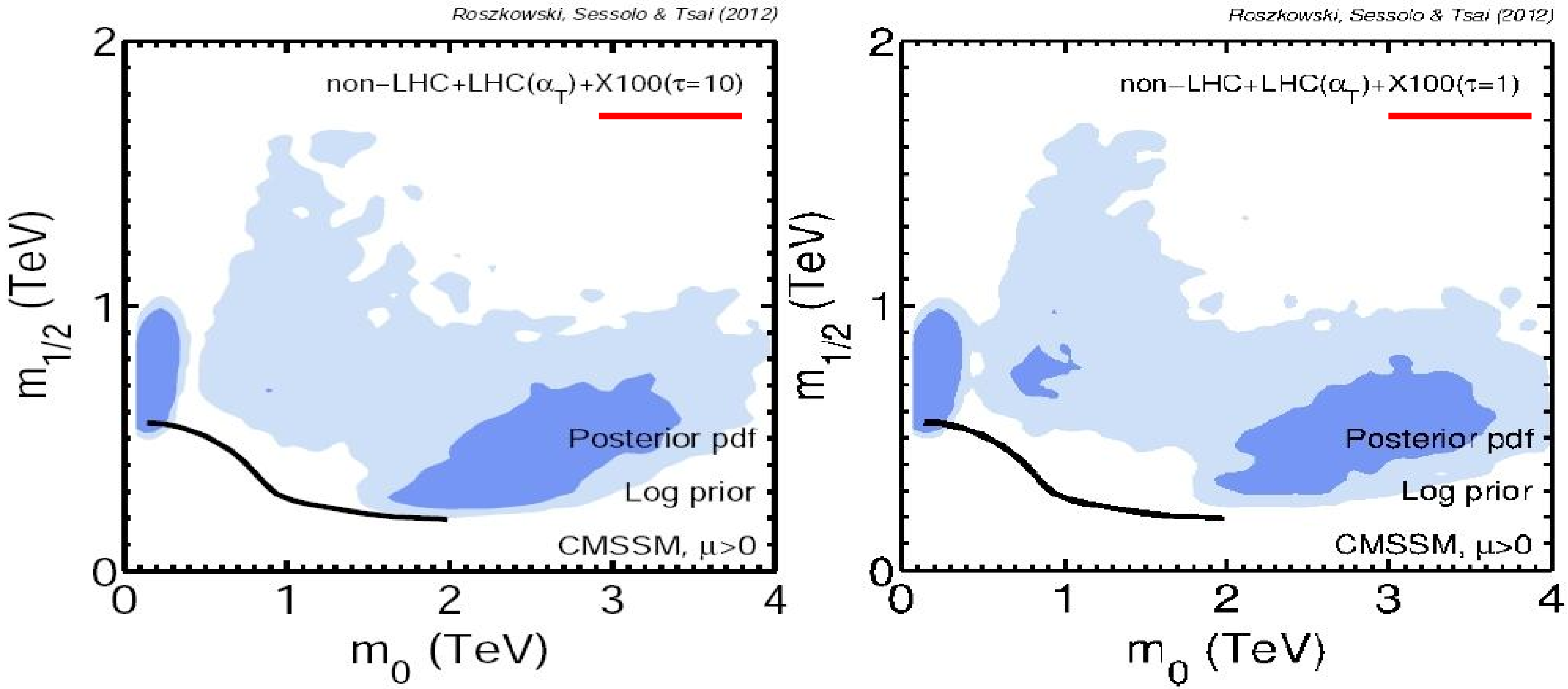


- DM XENON100 limit has small additional effect

→ LHC limits on CMSSM are stronger

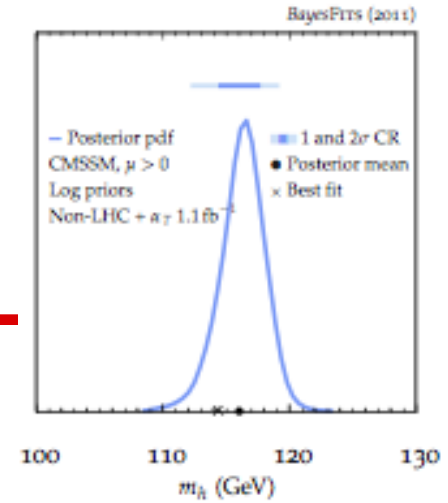
Also: large theoretical uncertainties (~ factor of 10) in evaluating sigma_p

Impact of Xenon100 detection



- The Xenon100 limit shows that DM searches are still marred by large (~ factor of 10) theoretical uncertainties.
- Even if reduce them to ~1 will make little difference.
- Current LHC limits are much stronger.

Roszkowski, Sessolo, Tsai (arXiv:1202.1503)



LHC: Posterior 68% and 95% C.L. ranges of sparticles and Higgs masses

Some examples:

Mass (GeV)	68%	95%	68%	95%
	Non-LHC		Non-LHC + CMS α_T 1.1/fb limit + XENON100	
m_h	(112.3, 116.5)	(110.1, 118.4)	(114.4, 117.8)	(112.2, 119.4)
m_χ	(56, 291)	(53, 356)	(250, 343)	(128, 390)
$m_{\chi_1^\pm}$	(110, 554)	(104, 676)	(475, 651)	(181, 738)
$m_{\tilde{q}}$	(326, 808)	(254, 1172)	(434, 761)	(398, 1302)
$m_{\tilde{g}}$	(403, 1576)	(384, 1885)	(1380, 1825)	(879, 2043)

Summary

- **With 1.1/fb at LHC: improved limits on SUSY particle masses.**
- **The CMSSM has become severely constrained but not excluded.**
- Constraints from direct detection of dark matter are currently weaker. One reason: still large theoretical uncertainties.
- **Our method is completely general. It can be applied to other models (SUSY or not).**
- **We have developed a framework based on Bayesian approach to include limits and future signals from the LHC.**
- **Work in progress: updated limits including Razor 5/fb and impact of possible Higgs signal.**

The End.
Thank you for your attention.