Skąd pochodzi spin protonu?

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Plan

- Wstęp
- Wyniki DIS i SIDIS; polaryzacja kwarków i gluonów
- “Tomografia” nucleonu
- Orbitalny moment pędu
- QCD na sieciach
- Podsumowanie

Spin

- Spin is a fundamental degree of freedom originated from space-time symmetry quantum and relativistic object - survives high-energy limit
- Spin plays a critical role in determining the basic structure of fundamental interactions
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The driving question for QCD spin physics is where the nucleon spin comes from?

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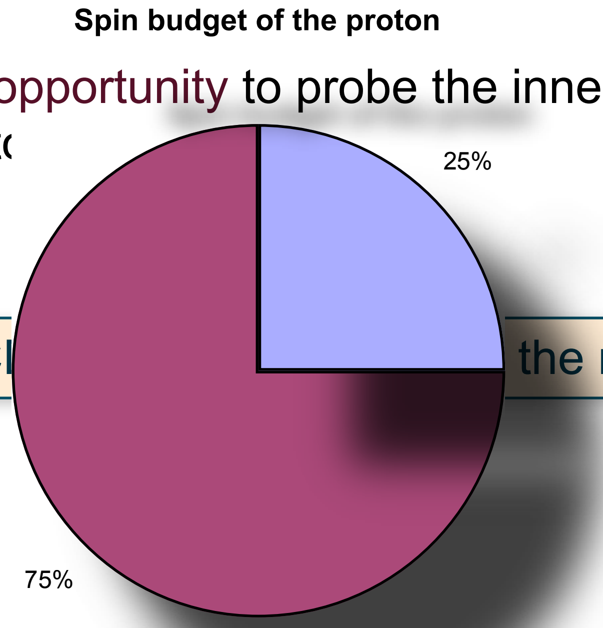
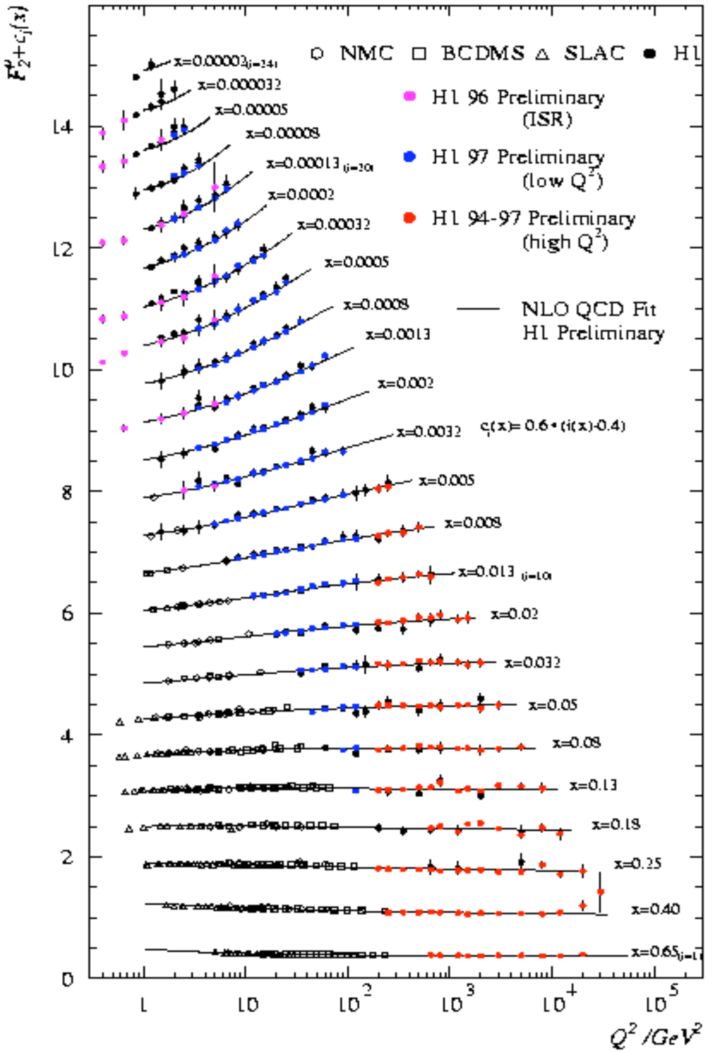
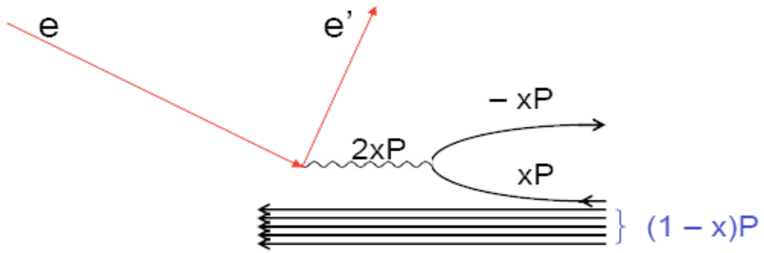
The driving question for QC the nucleon spin comes from?



“Dark” angular momentum

Model partonów





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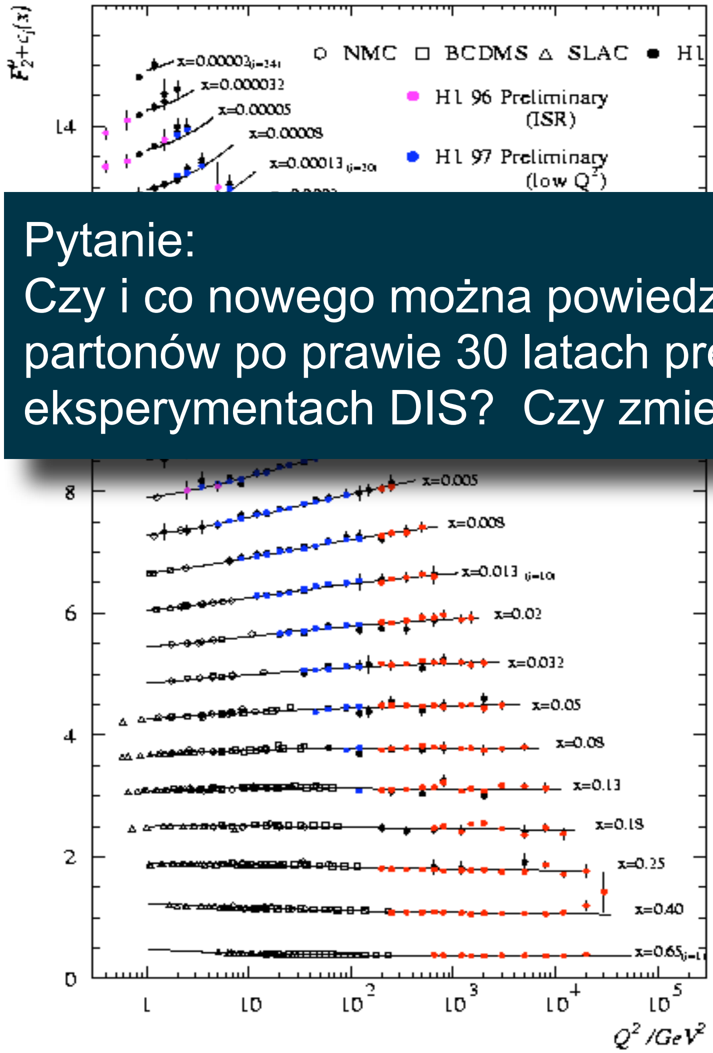

Pytanie:
Czy i co nowego można powiedzieć studentom o budowie protonu i o modelu partonów po prawie 30 latach precyzyjnych pomiarów w spolaryzowanych eksperymentach DIS? Czy zmieniło to jakościowo “obraz” budowy nukleonu

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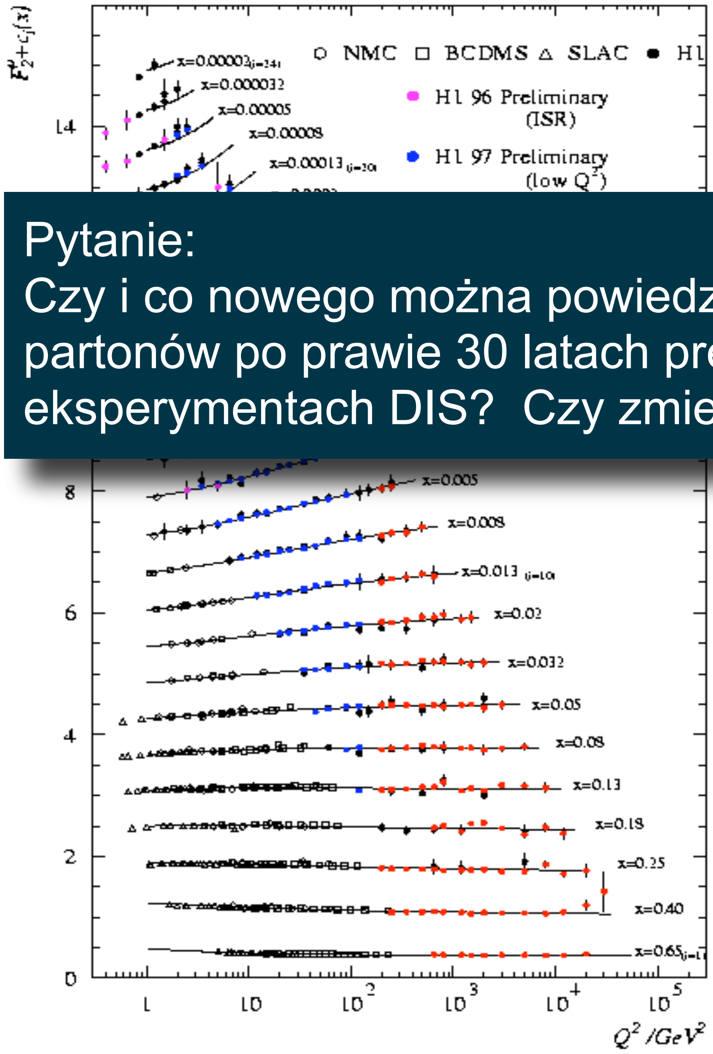

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Odpowiedź: “odkrycie”, że szybko poruszający się nukleon jest nadal 3-wymiarowym obiektem

QPM - jednowymiarowy opis struktury; Funkcje struktury $F_{1,2}(x)$ opisują tylko rozkład pędów partonów wzdłuż kierunku ruchu nukleonu

Eksperymenty

Experiments:

- Inclusive spin-dependent DIS
  EMC, SMC, COMPASS (CERN),E142,E143,E154,E156,
  HERMES (DESY) ,JLAB-Hall,A,B(CLAS)
- Semi-inclusive DIS: SMC, COMPASS,HERMES
- Polarized pp collision RHIC-PHENIX & STAR, BRAHMS (Brookhaven)
- ee: BELLE (KEK) (Fragmentation functions)

COMPASS II, JLAB-12, EIC, eLHC

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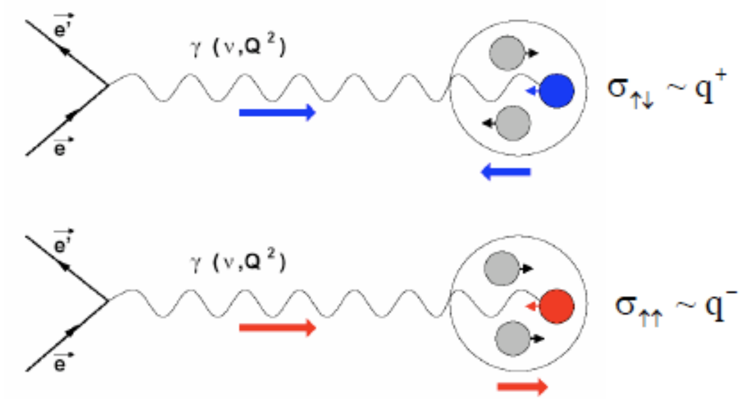
inclusive DIS and structure functions

inclusive DIS:

$$\sigma \sim F_1(x) = \tfrac{1}{2}\sum_i e_q^2 q_i(x) \qquad F_2(x) \approx 2xF_1$$

$$\Delta\sigma = \overset{\leftrightarrows}{\sigma} - \overset{\rightrightarrows}{\sigma} \sim g_1(x) = \tfrac{1}{2}\sum_i e_q^2 \Delta q_i(x) \qquad g_2$$



$\Delta q(x) = q(x)^{+} - q(x)^{-}$

$q(x) = q(x)^{+} + q(x)^{-}$

+ quark ↑↑ nucleon

− quark ↑↓ nucleon

Asymetrie

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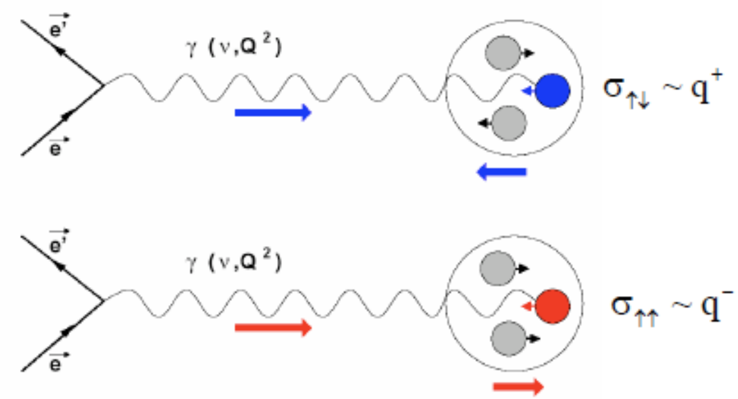

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$$A_1(x,Q^2) = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\uparrow\uparrow}} \approx \frac{\sum_q e_q^2\, \Delta q(x,Q^2)}{\sum_q e_q^2\, q(x,Q^2)} = \frac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

First moment of g₁ structure functions

Sum rules

first moment $$\Gamma_1 = \int g_1(x)dx$$

Bjorken s.r. $$\Gamma_1^p - \Gamma_1^n = \frac{g_A}{6g_V} C_1^{NS}$$

Ellis-Jaffe s.r. $$\Gamma_1^{p,n} = \left(\pm a_3 + \frac{a_8}{3}\right)\frac{C_1^{NS}}{12} + a_0 \frac{C_1^S}{9}$$

$a_3, a_8, g_{A,V}$ measuered in weak β decays (+Su(3)$_f$)

$C_1^{S,NS}$ calculable in pQCD

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$$a_0 = \Delta\Sigma = \Delta u + \Delta d + \Delta s$$

Quark contribution to nucleon helicity

First moment of g_1 structure functions

Compass only Phys. Lett. B 647 (2007) 8

$$\Gamma_1^N\left(Q^2\right) = \frac{1}{9}\left(1 - \frac{\alpha_s(Q^2)}{\pi} + O(\alpha_s^2)\right)\left(a_0(Q^2) + \frac{1}{4}a_8\right)$$

from Y. Goto *et al.*, PRD62 (2000) 034017:
(SU(3)$_f$ assumed for weak decays)
a_8 = 0.585 ± 0.025

$$a_{0|Q_0^2=3(GeV/c)^2} = 0.35 \pm 0.03(stat) \pm 0.05(syst)$$

QCD NLO

$$\Gamma_1^N\left(Q^2\right) = \frac{1}{9}C_1^S(Q^2)\hat{a}_0 + \frac{1}{36}C_1^{NS}(Q^2)a_8$$

beyond NLO

$$\hat{a}_{0|Q^2\to\infty} = 0.33 \pm 0.03(stat) \pm 0.05(syst)$$

C_1 calculated behind 3 loops app.
S.A.Larin *et al.*,Phys.Lett.B404(1997)153

$$(\Delta s + \Delta\bar{s}) = \frac{1}{3}(\hat{a}_0 - a_8) = -0.08 \pm 0.01(stat) \pm 0.02(syst)$$

Anomaly and the role of gluons

“Switching on” spin leads us to two complications:

$$q^{-} \sim \psi \tfrac{1}{2}(1-\gamma_5)\gamma_\mu \psi \qquad q^{+} \sim \psi \tfrac{1}{2}(1+\gamma_5)\gamma_\mu \psi$$

$$\Delta q = u^{+} - u^{-} \sim \psi \tfrac{1}{2}\gamma_\mu \gamma_5 \psi$$

- axial current is not conserved due to Adler-Bell-Jackiw triangle anomaly
- there is no local, gauge invariant dimension-3 axial operator for gluons

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Question is if we need conserved current?

$\overline{\text{MS}}$ scheme - no but a_0 depends on the scale
AB scheme - yes - a_0 does not depend on the scale but now

$$a_0 = \Delta\Sigma - \tfrac{3}{2}\tfrac{\alpha_S}{\pi}\Delta G$$

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△ anomaly gives a possible interpretation of the measured value of a_0 - if gluon polarization is large enough then E-J sum rule can be restored and quark contribution to the nucleon spin is significant as expected in simple QPM

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Nucleon spin decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L$$

naive scenario - ΔΣ=1, the rest is 0 (e.g. SU(6) static model)
relativistic corrections change 1 to ~0.65

Bag model

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example: MIT bag model: relativistic quarks confined in sphere with radius R

$$\Psi = \begin{pmatrix} f \\ i\hat{\sigma}\hat{r}g \end{pmatrix} \rightarrow \begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix} \Rightarrow (f^2 - \tfrac{1}{3}g^2) = \sim 0.65$$

lower spinor (“p” state) effectively transfer spin (helicity) to orbital angular momentum and the contribution to the nucleon spin is smaller

Some remarks

1. Large gluon polarization allows to restore E-J sum rule but automatically enforces presence of the large OAM to balance nucleon spin to 1/2
   The simplest solution would be if gluon polarization is large enough to satisfy 1/2 without presence of angular momentum. In such a case E-J sum rule is violated.

2. Problem with ‘locality” of the gluon helicity operator in QCD can be solved in a specific gauge: e.g. in the axial gauge or light-cone gauge such a local operator can be well-defined.
   Then the problem of gauge invariance appears but one should remember that what is measured is not pure QCD effects but the convolution with QPM. QPM “is defined” in the light-cone gauge

$$\partial^\mu A^0_{5\mu} = 2i \sum_{i=1}^{f} m_i \bar{q}_i \gamma_5 q_i + 2n_f \partial^\mu K_\mu,$$

where

$$K_\mu = \frac{\alpha_s}{4\pi} \epsilon^{\mu\nu\rho\sigma} \left[A^\nu_a \left(\partial^\rho A^\sigma_a - \frac{1}{3} g f_{abc} A^\rho_b A^\sigma_c \right) \right],$$

Longitudinal spin asymmetry

Semi-inclusive asymmetry

Inclusive asymmetry:

$$A_1(x,Q^2)=\frac{\sigma_{\uparrow\downarrow}-\sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\downarrow}+\sigma_{\uparrow\uparrow}}\approx\frac{\sum_q e_q^2\,\Delta q(x,Q^2)}{\sum_q e_q^2\,q(x,Q^2)}=\frac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

Semi-inclusive asymmetry:

$$A_1^h(x,z,Q^2)=\frac{\sigma^h_{\uparrow\downarrow}-\sigma^h_{\uparrow\uparrow}}{\sigma^h_{\uparrow\downarrow}+\sigma^h_{\uparrow\uparrow}}\approx\frac{\sum_q e_q^2\,\Delta q(x,Q^2)D_q^h(z,Q^2)}{\sum_q e_q^2\,q(x,Q^2)D_q^h(z,Q^2)}$$

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inclusive and semi-inclusive asymmetry measured on proton and neutron or deuteron allows to full flavour separation

2002-2006 deuteron data taking in COMPASS:
2007: proton data; DIS events - $Q^2 > 1\ (\mathrm{GeV}/c)^2$

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Difference asymmetry

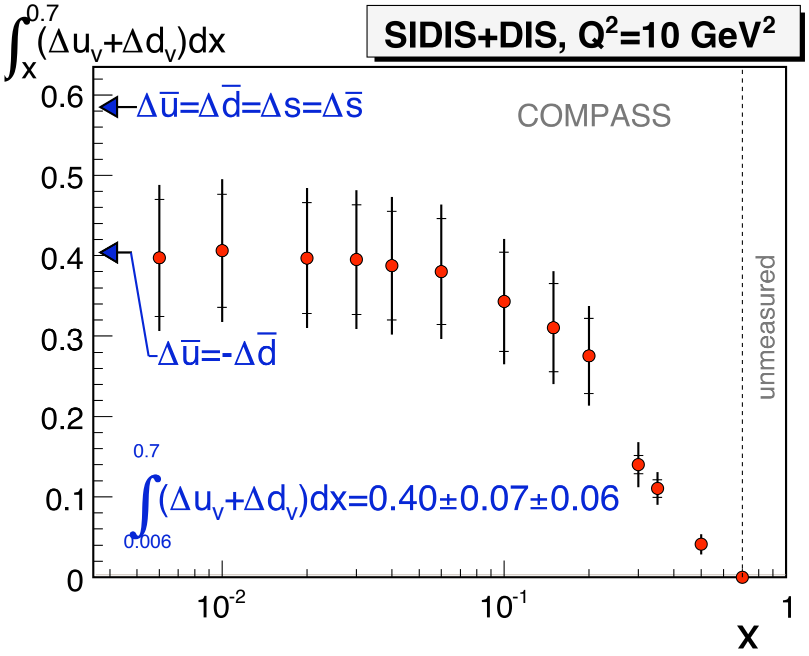
Idea: Phys.Lett.B230(1989)141,
SMC:Phys.Lett.B369(1996)93,
COMPASS: Phys.Lett.B660(2008)458

$$A^{+-} = \frac{(\sigma^{h^+}_{\uparrow\downarrow} - \sigma^{h^-}_{\uparrow\downarrow}) - (\sigma^{h^+}_{\uparrow\uparrow} - \sigma^{h^-}_{\uparrow\uparrow})}{(\sigma^{h^+}_{\uparrow\downarrow} - \sigma^{h^-}_{\uparrow\downarrow}) + (\sigma^{h^+}_{\uparrow\uparrow} - \sigma^{h^-}_{\uparrow\uparrow})}$$

$$A_d^{\pi^+-\pi^-}(x) = A_d^{K^+-K^-}(x) = \frac{\Delta u_v(x) + \Delta d_v(x)}{u_v(x) + d_v(x)} \longrightarrow$$

Only valence quarks!

Fragmentation functions cancel out in LO and under the assuption of independent fragmentation.



$$\Delta\bar{u} = \Delta\bar{d} = \Delta s = \Delta\bar{s}$$

$$\Delta\bar{u} = -\Delta\bar{d}$$

symmetric → scenario ← asymmetric

$$\Gamma_v = \int_0^1 (\Delta u_v(x) + \Delta d_v(x))dx$$

$$\Delta\bar{u} + \Delta\bar{d} = 3\Gamma_1^N - \tfrac{1}{2}\Gamma_v + \tfrac{1}{12}a_8 = (\Delta s + \Delta\bar{s}) + \tfrac{1}{2}(a_8 - \Gamma_v)$$

Transversely polarized target

$$\frac{1}{2}=\frac{1}{2}\sum_{q,\bar{q}}\int dx\cdot\Delta_T q(x)+\sum_{q,\bar{q},g}\langle L_z\rangle$$

1. No gluons!! (angular momentum conservation - different QCD evolution)
2. Transversity structure function - C-odd and chiral-odd. No accessible in DIS but measured in DY or SIDIS processes

Orbital Angular Momentum (OAM) becomes more important in this case as transversity gives smaller contribution

Nucleon spin decomposition again

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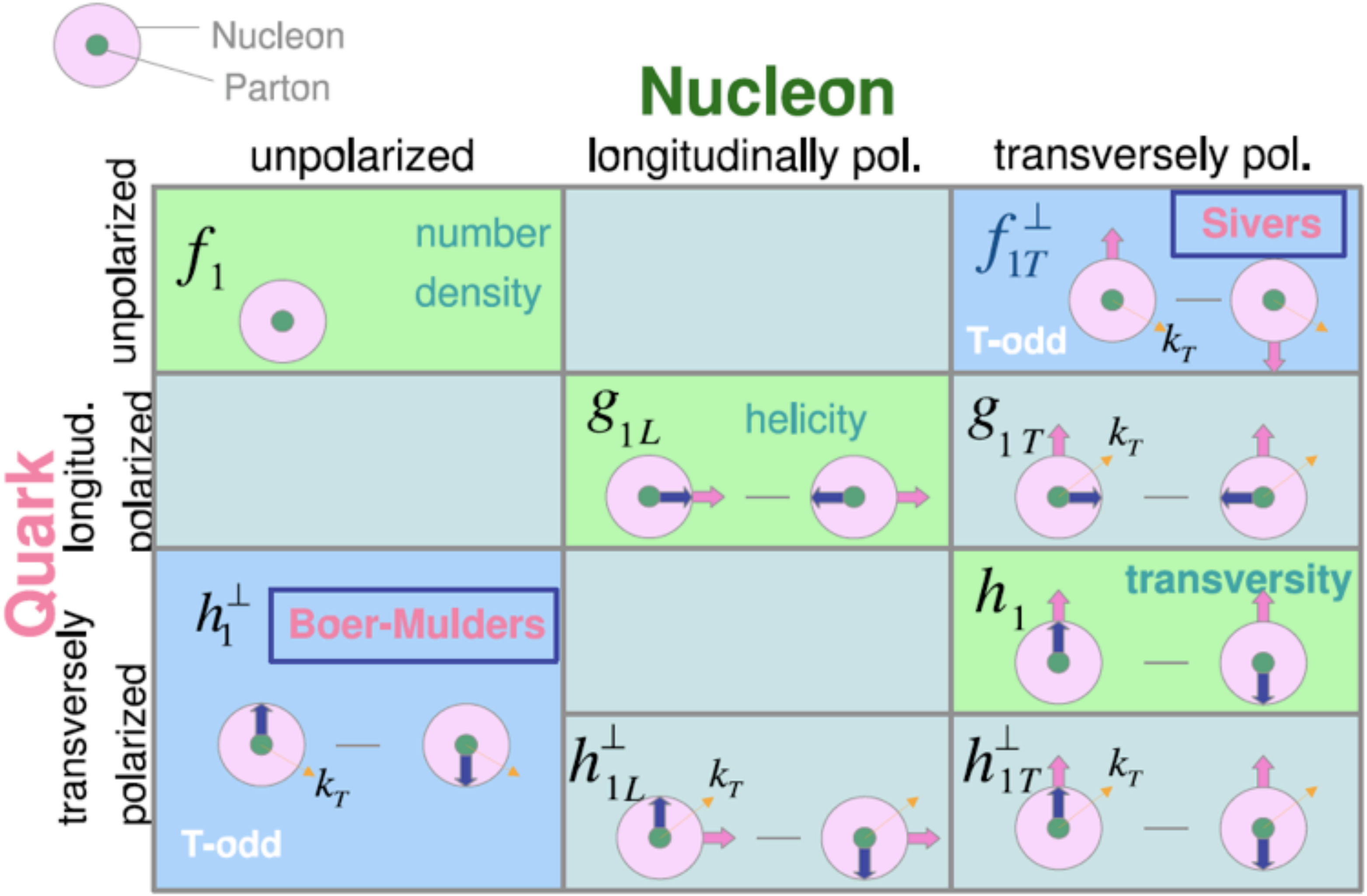
1. Nonrelativistic quarks - no differences between transversity and helicity - boosts and rotations commute
2. Relativistic quarks gives the difference - example: again MIT Bag model and “renormalization factor”

$$\Psi=\begin{pmatrix} f \\ i\hat{\sigma}\hat{r}g \end{pmatrix}\rightarrow\begin{pmatrix} \uparrow \\ \downarrow \end{pmatrix}\Rightarrow(f^2-\tfrac{1}{3}g^2)\ \sim 0.65$$

$$(f^2+\tfrac{1}{3}g^2)\sim 0.83$$

Beyond collinear approximation - k_T dependence



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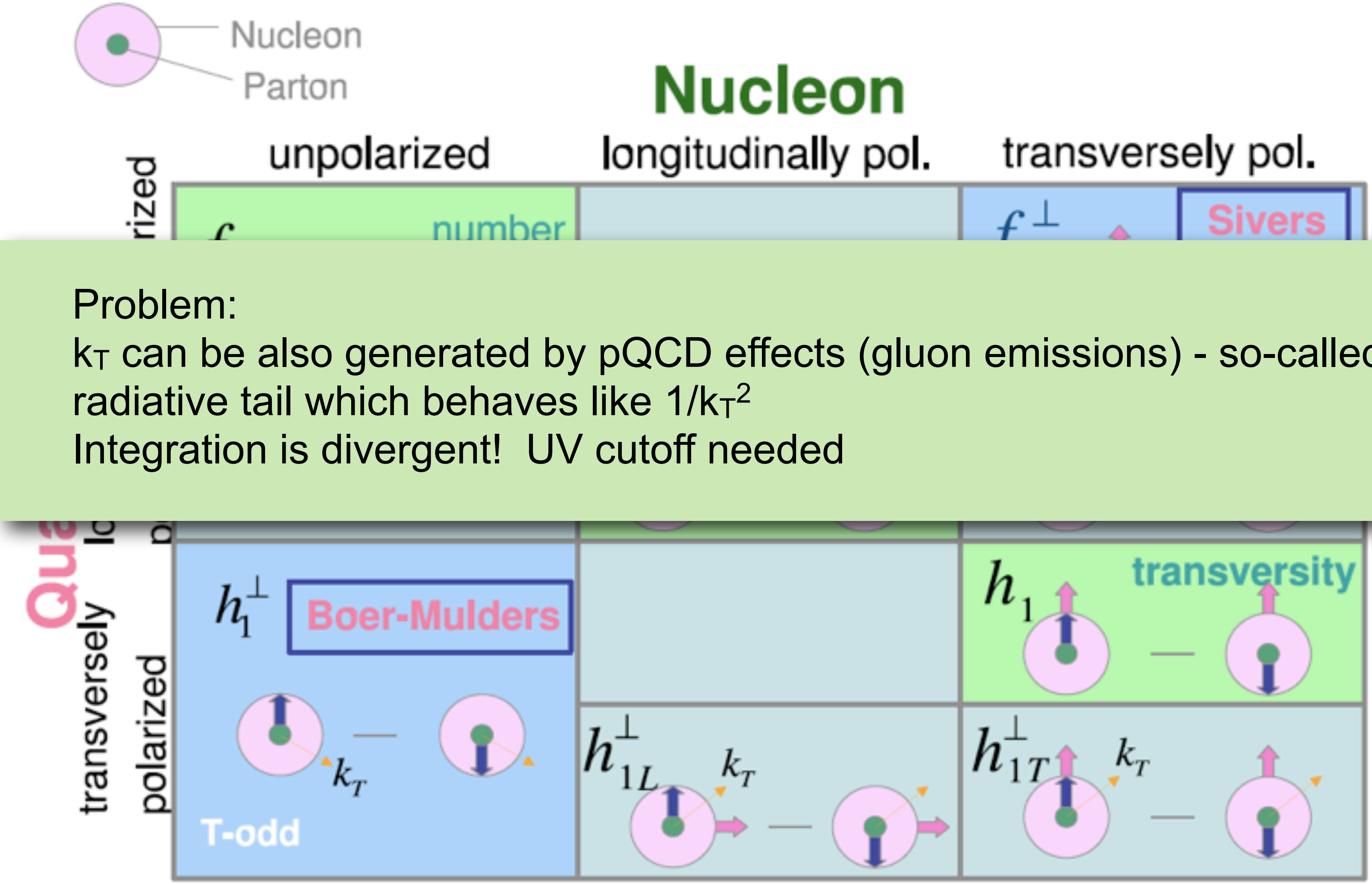

Problem:
k_T can be also generated by pQCD effects (gluon emissions) - so-called radiative tail which behaves like $1/k_T^2$
Integration is divergent! UV cutoff needed

Beyond collinear approximation - k_T dependence

$$W(x,p) = \int \Psi^*(x - \eta/2)\Psi(x + \eta/2)e^{ip\eta}d\eta,$$

$$\langle O(x,p)\rangle = \int O(x,p)W(x,p)dxdp.$$

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A similar concept in QCD

$$\hat{W}(x,\vec{k}_\perp,\vec{S})_\eta = \int e^{ikz} < \vec{P},\vec{S}|\bar{\Psi}(0)W_\eta(0,z)\Psi(z)[\vec{P},\vec{S}| > |_{z^+=0}\frac{dz^-d^2z_\perp}{(2\pi)^3}.$$

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a gauge-link operator - key point here

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$$\begin{aligned}
\frac{1}{2}Tr(\gamma^+\hat{W}(x,\vec{k}_\perp,\vec{S}) &= f_1(x,\vec{k}_\perp) - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M}f_{1T}^\perp(x,\vec{k}_\perp),\\
\frac{1}{2}Tr(\gamma^+\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S}) &= S_L g_{1L}(x,\vec{k}_\perp) + \frac{\vec{k}_\perp\vec{S}_T}{M}g_{1T}(x,\vec{k}_\perp),\\
\frac{1}{2}Tr(i\sigma^{j+}\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S}) &= S_T^j h_1(x,\vec{k}_\perp) + S_L\frac{k_\perp^j}{M}h_{1L}^\perp(x,\vec{k}_\perp)\\
&+ \frac{(k_\perp^j k_\perp^k - \frac{1}{2}\vec{k}_\perp^2)S_T^k}{M^2}h_{1T}^\perp(x,\vec{k}_\perp) + \frac{\varepsilon^{jk}k_\perp^k}{M}h_1^\perp(x,\vec{k}_\perp).
\end{aligned}$$

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A similar concept in QCD

Niektóre z TMD są związane z orbitalnym momentem pędu partonów
Opisują trójwymiarową strukturę nukleonu w przestrzeni pędów

$$\frac{1}{2}Tr(\gamma^+\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S})) = S_L g_{1L}(x,\vec{k}_\perp) + \frac{\vec{k}_\perp\vec{S}_T}{M}g_{1T}(x,\vec{k}_\perp),$$

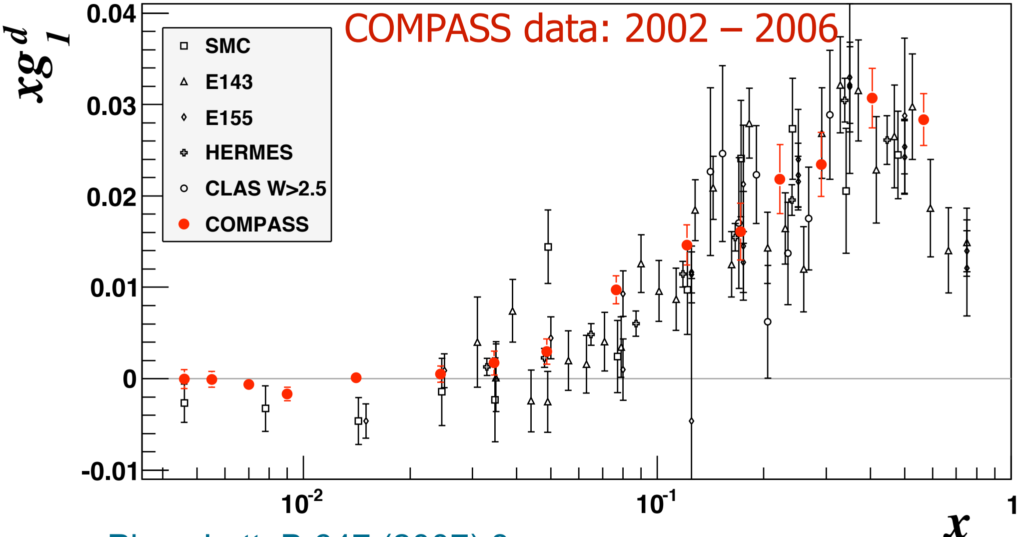
$$\frac{1}{2}Tr(i\sigma^{j+}\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S})) = S_T^j h_1(x,\vec{k}_\perp) + S_L\frac{k_\perp^j}{M}h_{1L}^\perp(x,\vec{k}_\perp) + \frac{(k_\perp^j k_\perp^k - \frac{1}{2}\vec{k}_\perp^2)S_T^k}{M^2}h_{1T}^\perp(x,\vec{k}_\perp) + \frac{\varepsilon^{jk}k_\perp^k}{M}h_1^\perp(x,\vec{k}_\perp).$$

DIS & SIDIS data
g_1 & g_2, quark & gluon polarisation

mainly COMPASS data

Deuteron and proton g_1 structure function

COMPASS data



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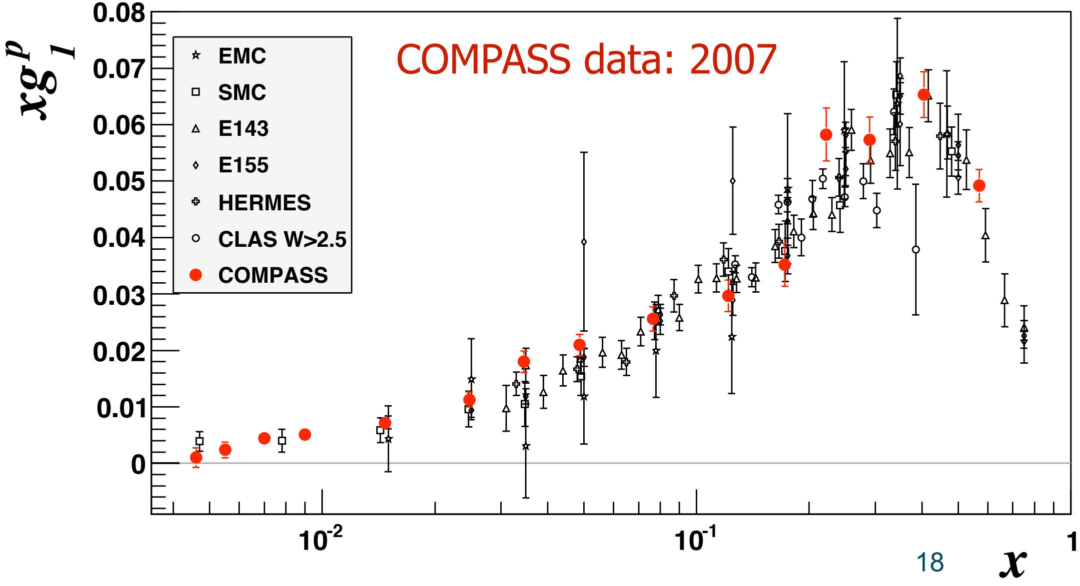
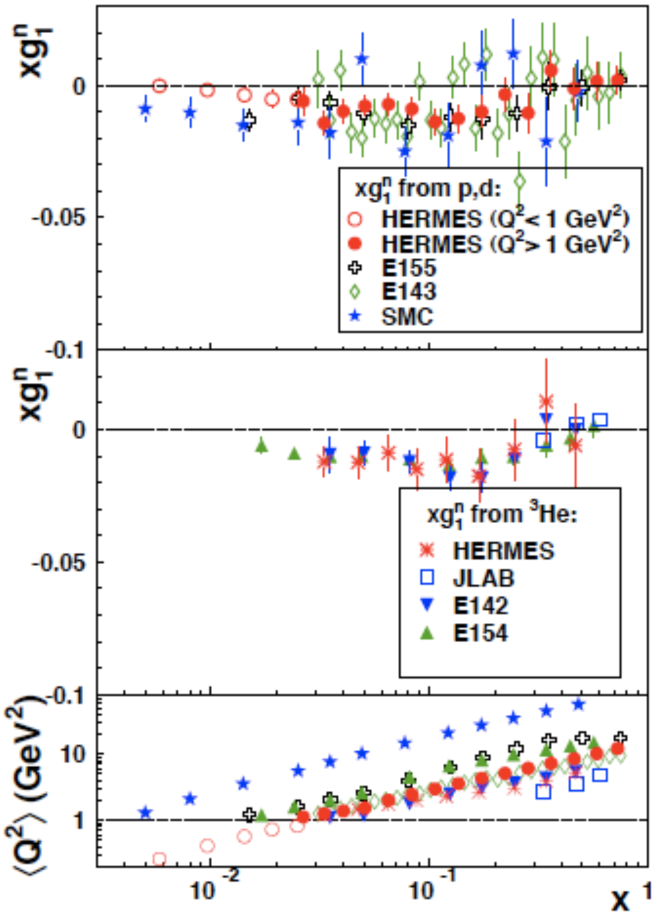


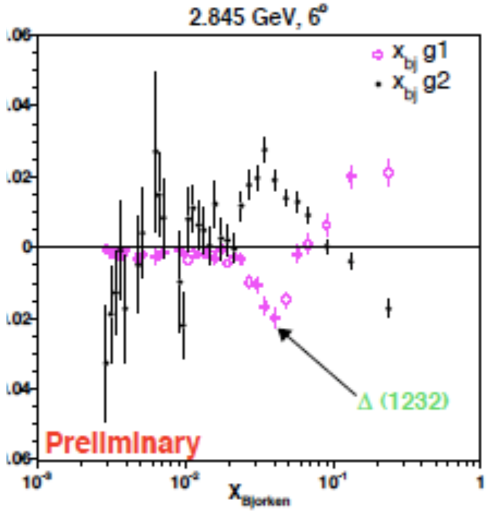
Very good agreement between experiments. Points evolved to $Q^2 = 3\ (\mathrm{GeV}/c)^2$

Neutron g_1 & g_2 structure function

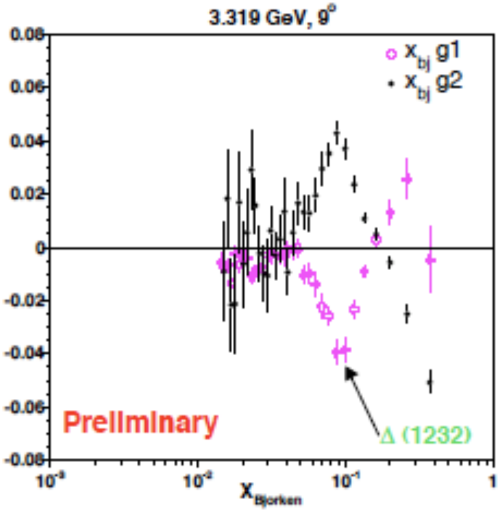
HERMES, SMC, E155, E143 data

JLAB E97-100 data





$Q^2(\Delta) = 0.077$ GeV^2



$Q^2(\Delta) = 0.23$ GeV^2

Hall A.

COMPASS data

- **SIDIS**

$$A_1^h = \frac{\sum e_q^2[\Delta q(x)\int D_q^h(z)dz + \Delta\bar{q}(x)\int D_{\bar{q}}^h(z)dz]}{\sum e_q^2[q(x)\int D_q^h(z)dz + \bar{q}(x)\int D_{\bar{q}}^h(z)dz]}$$

- $D_q^h \neq D_{\bar{q}}^h$
  yields quark and antiquark separation

- **measured:**
  $A_1^d, A_{1d}^{K\pm}, A_{1d}^{\pi\pm}, A_1^p, A_{1p}^{K\pm}, A_{1p}^{\pi\pm}$

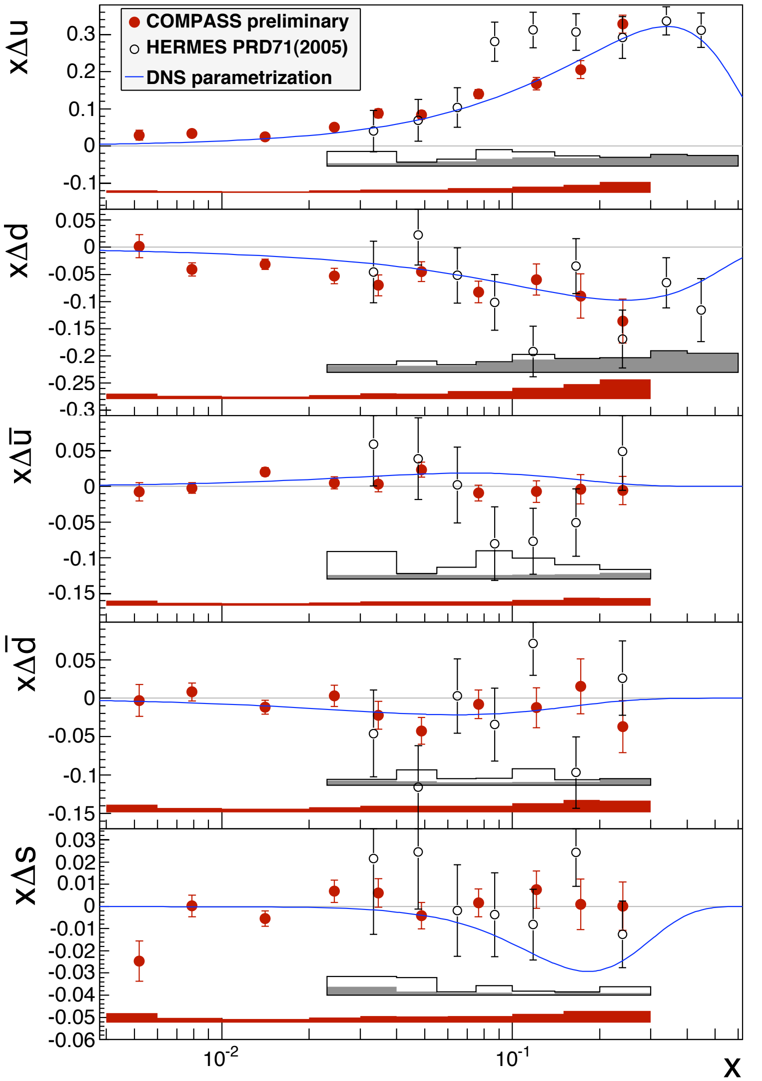
- **determined:** $\Delta u, \Delta\bar{u}, \Delta d, \Delta\bar{d}, \Delta s \equiv \Delta\bar{s}$

- system of linear equations in LO

- **input:** MRST04 unpolarised PDFs, DSS parametrisation of FFs (e^+e^-, DIS, hadron-hadron)

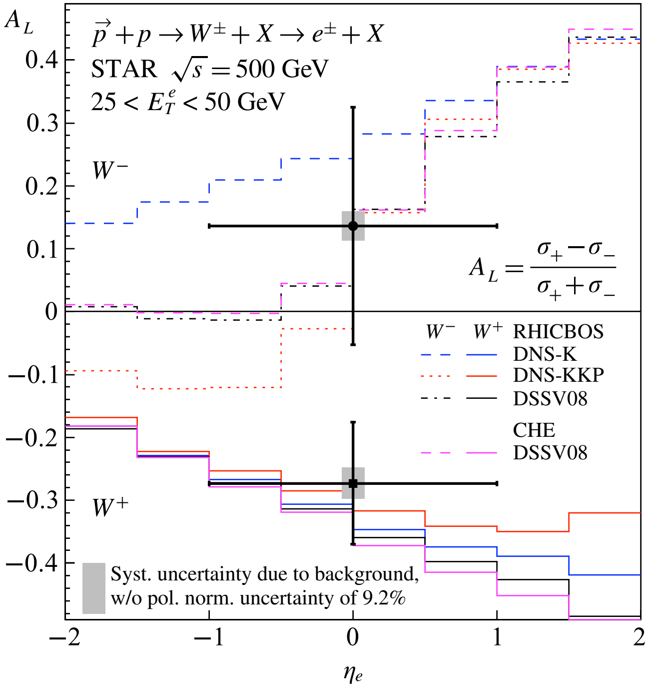
$$\int_{0.004}^{0.3} \Delta s(x)dx = -0.01 \pm 0.01 \pm 0.01$$

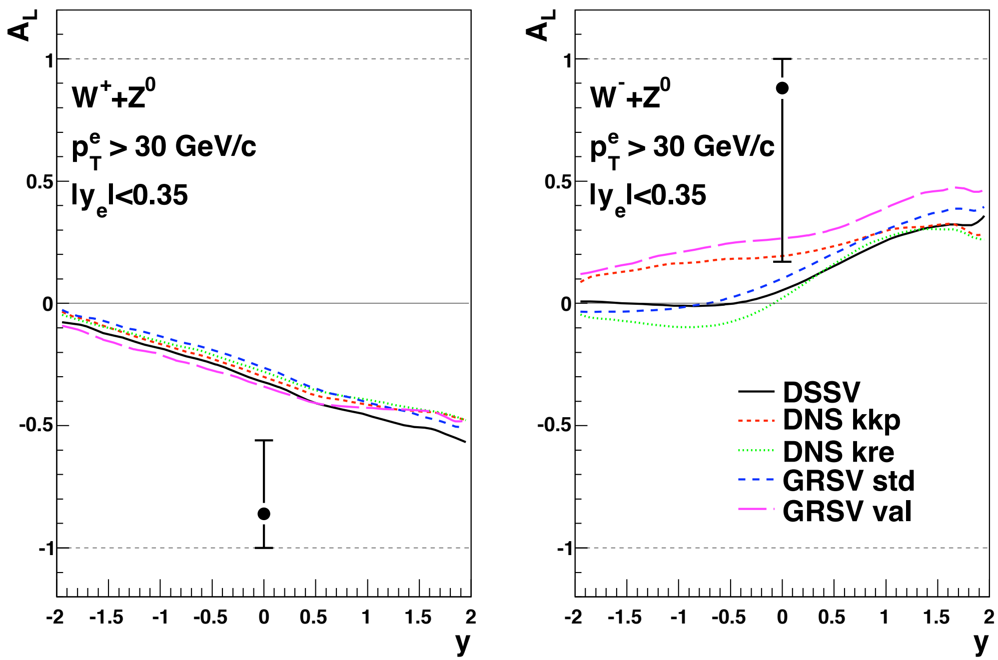
DNS: De Florian, Navarro, Sassot, Phys. Rev. D71, 2005



Flavour separation: single W asymmetry

RHIC data Left: STAR righ: PHENIX





$$A_L^{W^+} = \frac{\Delta u(x_1)\overline{d}(x_2) - \Delta\overline{d}(x_1)u(x_2)}{u(x_1)\overline{d}(x_2) + \overline{d}(x_1)u(x_2)},$$

$$A_L^{W^+} \to \Delta u/u$$

STAR, M. M. Aggarwal *et al.*, Phys. Rev. Lett. **106**, 062002 (2011).

$$A_L^{W^-} \to \Delta\overline{d}/\overline{d}$$

PHENIX, A. Adare *et al.*, Phys. Rev. Lett. **106**, 062001 (2011).

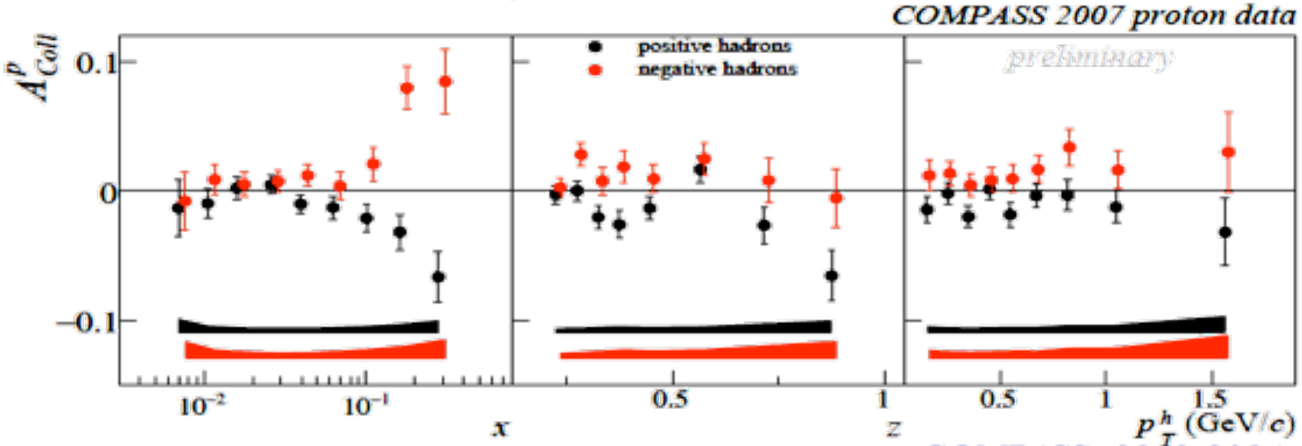
COMPASS data: Collins asymmetry

Couple $\Delta_T q$ to chiral odd Collins FF

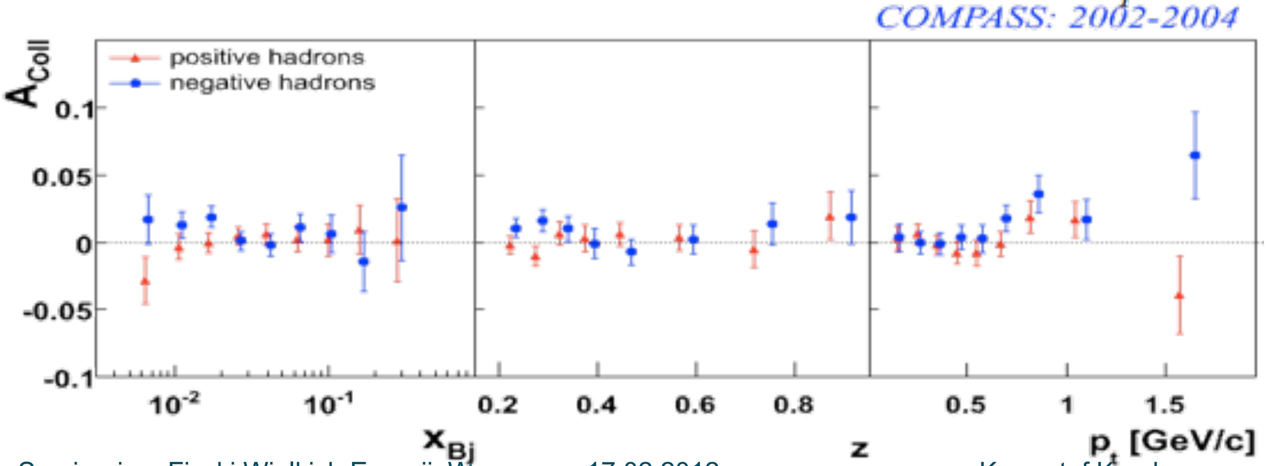
$$A_{Coll} = \frac{\sum_q e_q^2 \Delta_T q(x) \Delta_T^0 D_q^h(z, p_T^h)}{\sum_q e_q^2 q(x) D_q^h(z, p_T^h)}$$

$$\frac{\Delta\sigma}{\sigma} \propto A_{Coll} \sin \Phi_C$$

$$\Phi_C = \phi_h - \phi_s - \pi$$



Proton



Deuteron

COMPASS data: Collins asymmetry

Couple $\Delta_T q$ to chiral odd Collins FF

$$A_{Coll} = \frac{\sum_q e_q^2 \Delta_T q(x) \Delta_T^0 D_q^h(z, p_T^h)}{\sum_q e_q^2 q(x) D_q^h(z, p_T^h)}$$

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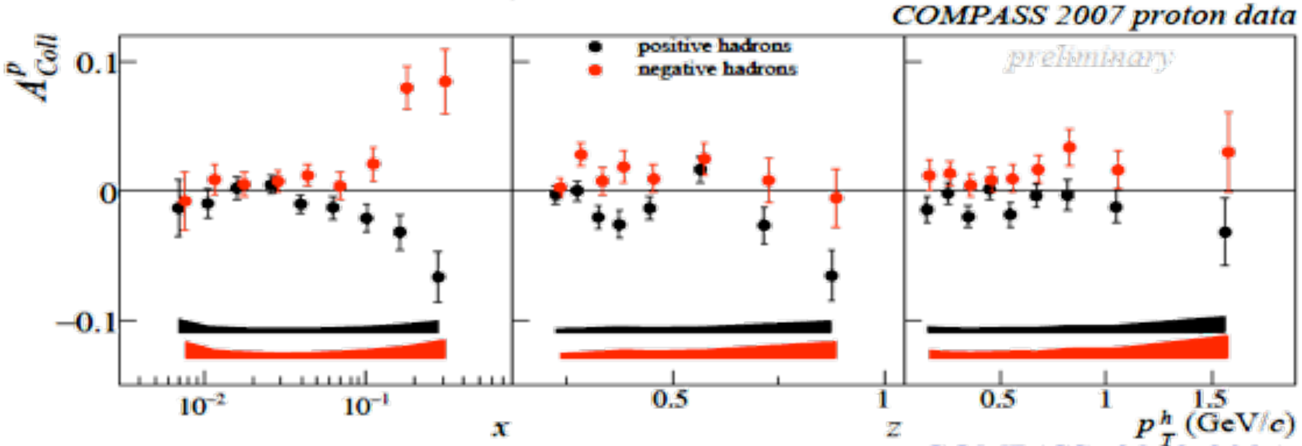
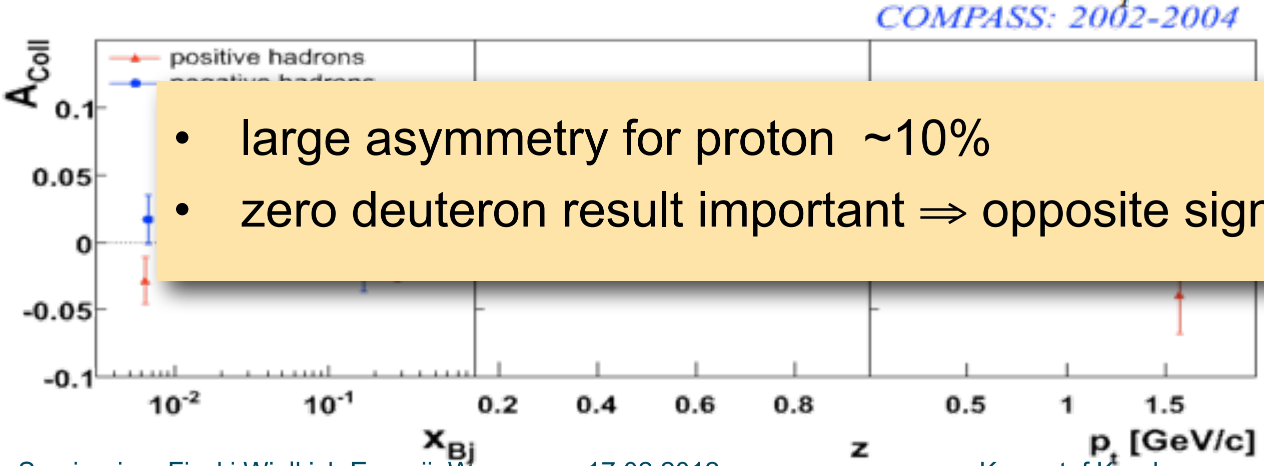


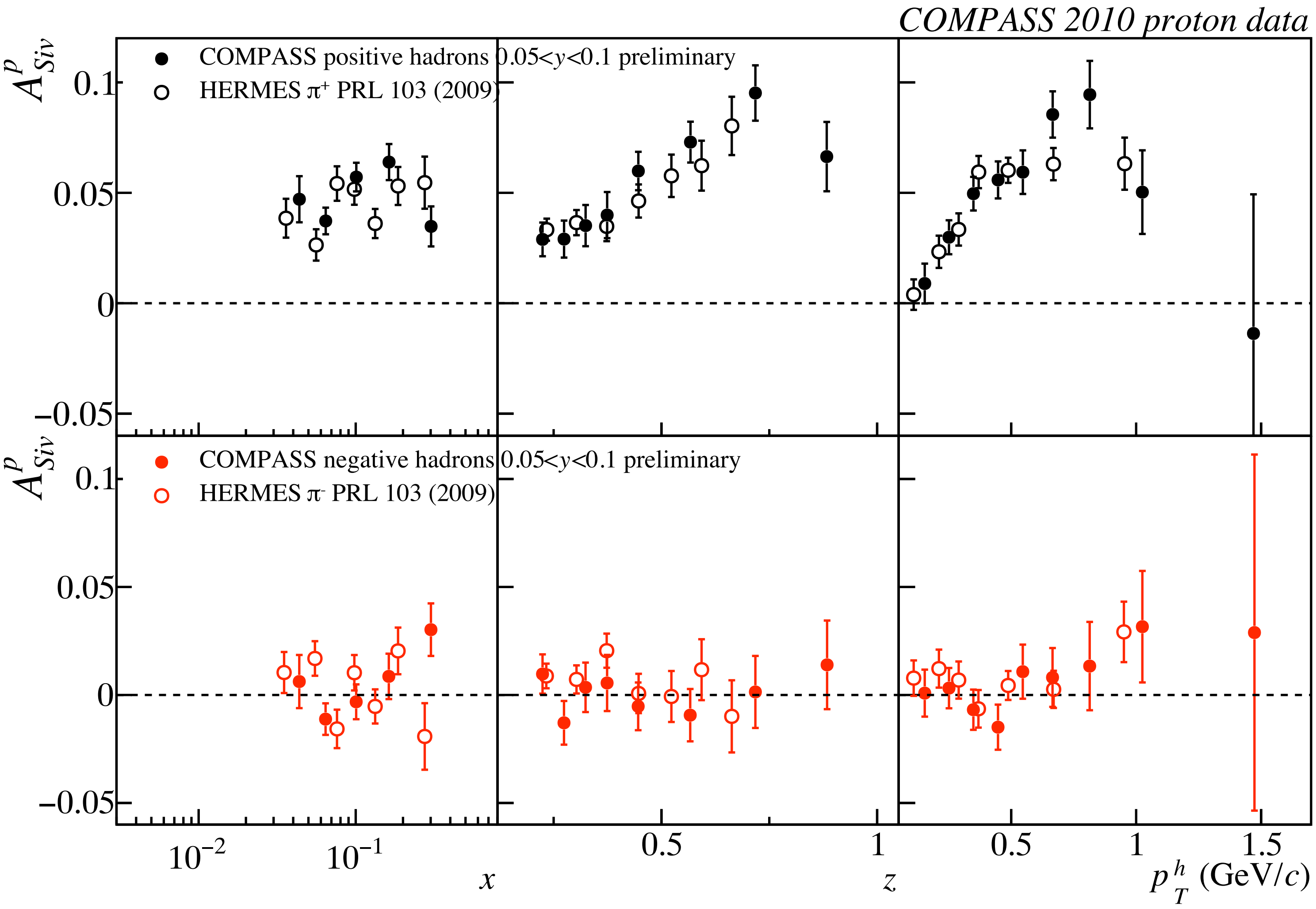
Proton



Deuteron

- large asymmetry for proton ~10%
- zero deuteron result important ⇒ opposite sign of u and d

COMPASS & HERMES data: Sivers asymmetry



Proton

h^+

h^-

deuteron: ~zero (again cancellation between u and d)

What is the lesson so far?

Helicty & transverse structure

1. Helicity:

$\Delta\Sigma \approx 1/3, \quad \Delta s \approx \Delta\bar{s} \approx 0$ shape disagrees with present QCD fits

$\Delta\bar{u} \approx -\Delta\bar{d}$ - asymmetric scenario with some indication that Δū ≥ Δđ ?

gluon polarization and OAM are important

2. Transversity and TMDs,

trasversity distribution small but non-zero as well as Collins FF function
Sivers non-zero on proton - non-trivial interpretation in the frame of pQCD
Rest of TMDs - very small

Some of TMDs are non-zero and they are related to orbital motion of partons.
TMDs plays an important role in the understanding of non-trivial pQCD foundations like factorization, twist-expansion etc. Also the lack of leading effect suggests the non-trivial higher-twist structure to be important as well as re-summations of soft gluon emissions

Gluon polarisation

▸QCD evolution

▸Semi-inclusive asymmetries compared to theory predictions based on gluon polarization “model” (method used in RHIC)

▸“Direct” measurement: idea - tag photon-gluon fusion (PGF) process

- Observation of the two hadrons with high-p_T in the final state
- Observation of the charmed mesons produced via open-charm mechanism

QCD evolution

- QCD fits - Idea:

$$g_1(x,Q^2) = \tfrac{1}{2}\langle e^2\rangle\left[C_q^s \otimes \Delta\Sigma + C_q^{NS} \otimes \Delta q^{NS} + 2n_f C_G \otimes \Delta G\right]$$

DGLAP eqs

$$\frac{d}{dt}\Delta q^{NS} = \frac{\alpha_S(t)}{2\pi} P_{qq}^{NS} \otimes \Delta q^{NS} \qquad t = \log\left(\frac{Q^2}{\Lambda^2}\right)$$

$$\frac{d}{dt}\begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix} = \frac{\alpha_S(t)}{2\pi}\begin{pmatrix} P_{qq}^S & 2n_f P_{qG}^S \\ P_{Gq}^S & P_{GG}^S \end{pmatrix} \otimes \begin{pmatrix}\Delta\Sigma \\ \Delta G\end{pmatrix}$$

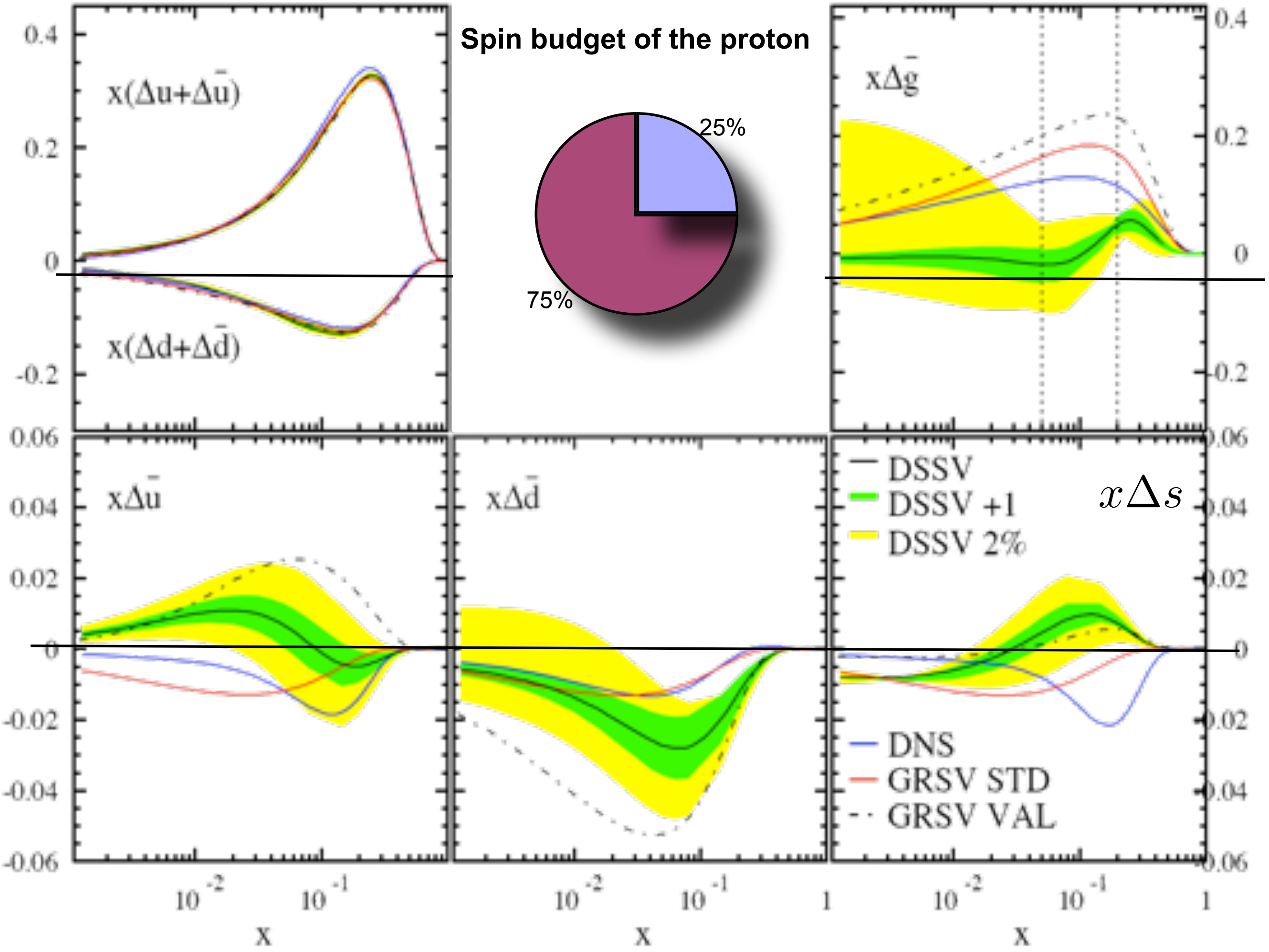
Initial parameterization in x at fixed Q²

$$(\Delta\Sigma,\Delta q_s,\Delta q_8,\Delta G) = \eta \frac{x^\alpha (1-x)^\beta (1+\gamma x)}{\int_0^1 x^\alpha (1-x)^\beta (1+\gamma x) dx}$$

Minimalization procedure

$$\chi^2 = \sum_{i=1}^{N} \frac{\left[g_1^{calc}(x,Q^2) - g_1^{\exp}(x,Q^2)\right]^2}{\left[\sigma_{stat}^{\exp}(x,Q^2)\right]^2}$$

QCD fits - DSSV

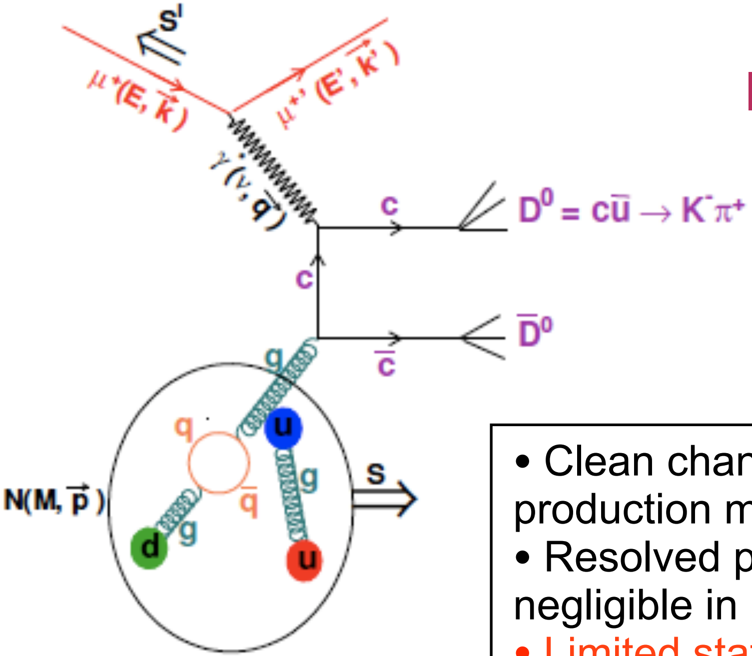


small gluon polarization - but large uncertainty

Observation of the charmed meson

COMPASS Phys. Lett. B 676 (2009) 31

Statistically weighting method based on NN approach used



Idea: tag $\gamma^* g \rightarrow\rightarrow c c_{bar}$ via open-charm production

- Clean channel (less MC dependent) under the assumption that production mechanism is PGF only (true in LO pQCD)
- Resolved photon and "intrinsic" charm production mechanism negligible in COMPASS kinematics
- Limited statistics (no vertex detector - long polarized target)
- Huge combinatorial background

$$\sigma^{PGF} = G \otimes \hat{\sigma}^{PGF} \otimes H$$

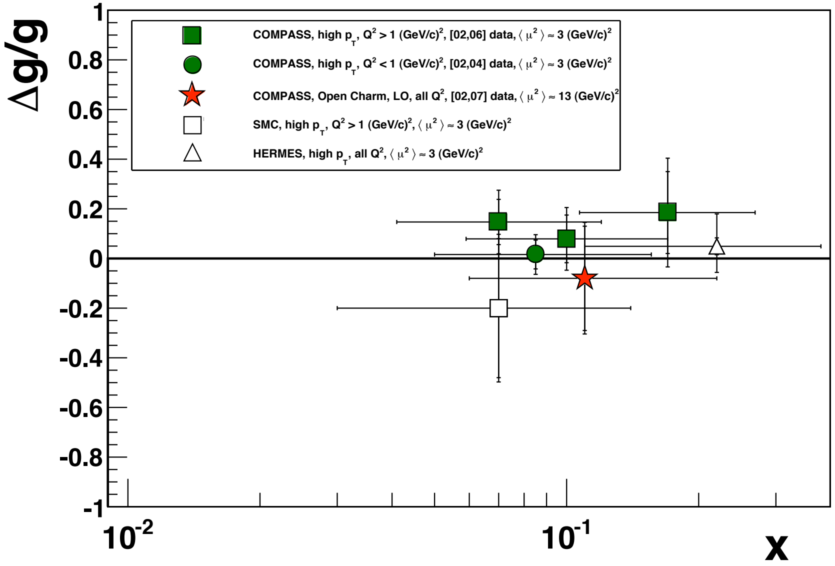
$$\Delta\sigma^{PGF} = \Delta G \otimes \Delta\hat{\sigma}^{PGF} \otimes H$$

from MC

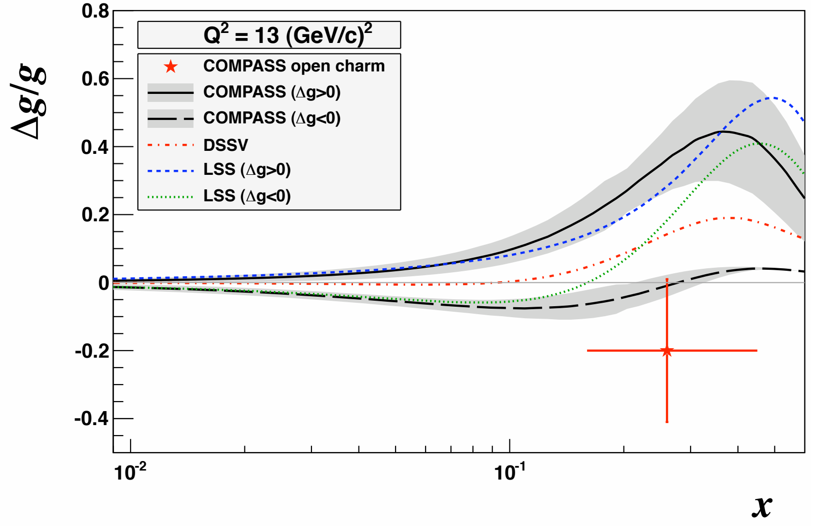
$$A \approx \boxed{\frac{\Delta G}{G}(\bar{x}_G)} < \hat{a}_{LL}^{PGF} >_G$$

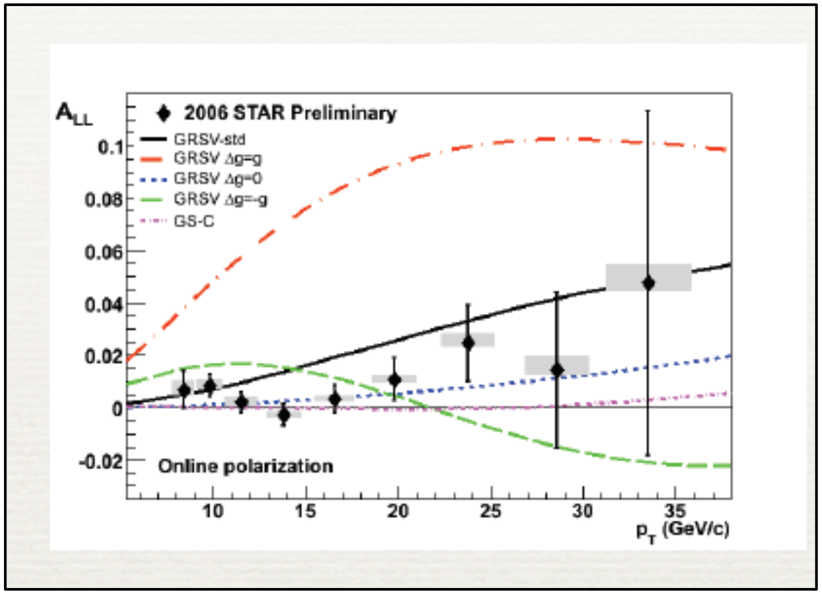
from data

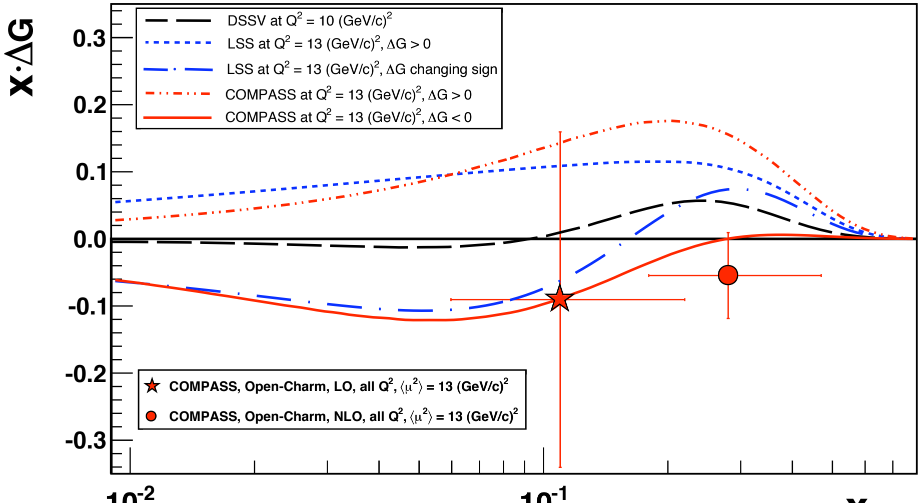
SMC,HERMES,COMPASS & RHIC data



NLO preliminary prediction

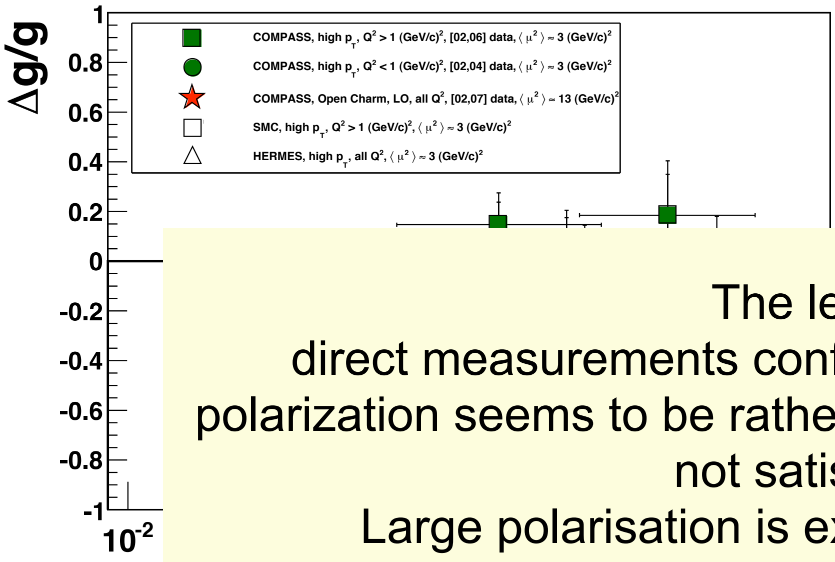




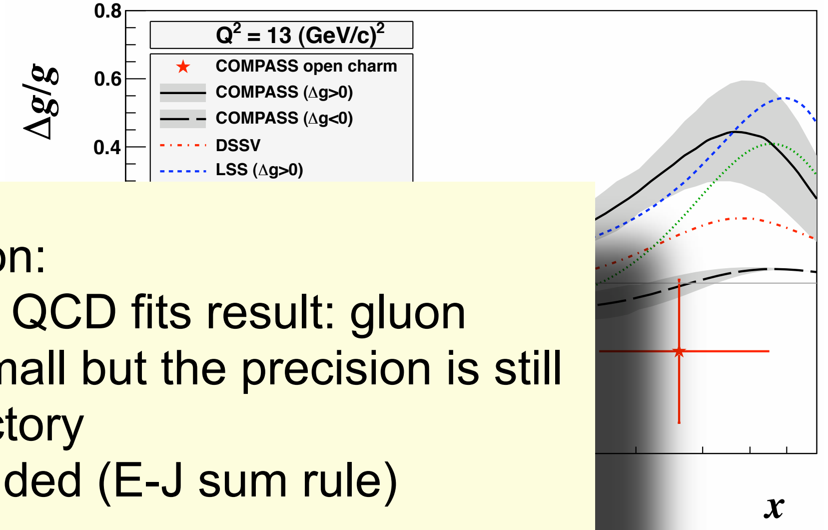


Gluon polarization@LO and NLO (charm)

SMC,HERMES,COMPASS & RHIC data

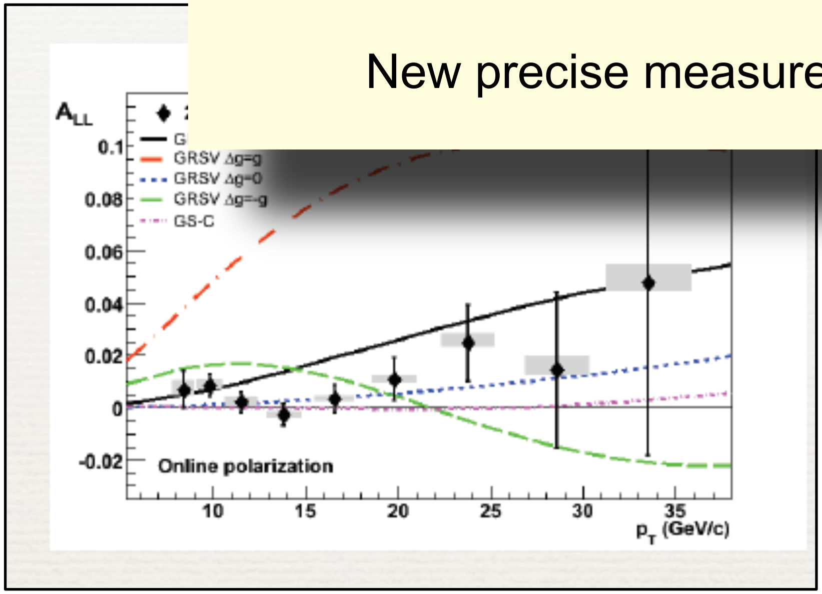


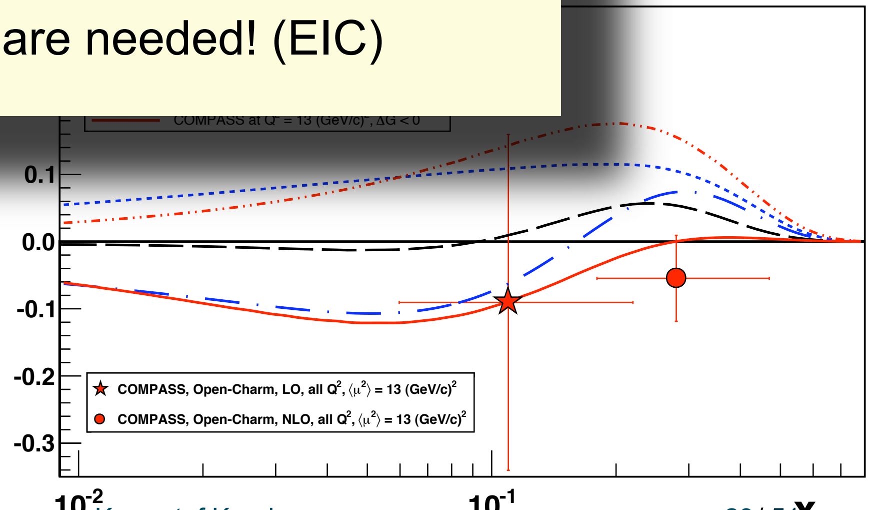
NLO preliminary prediction



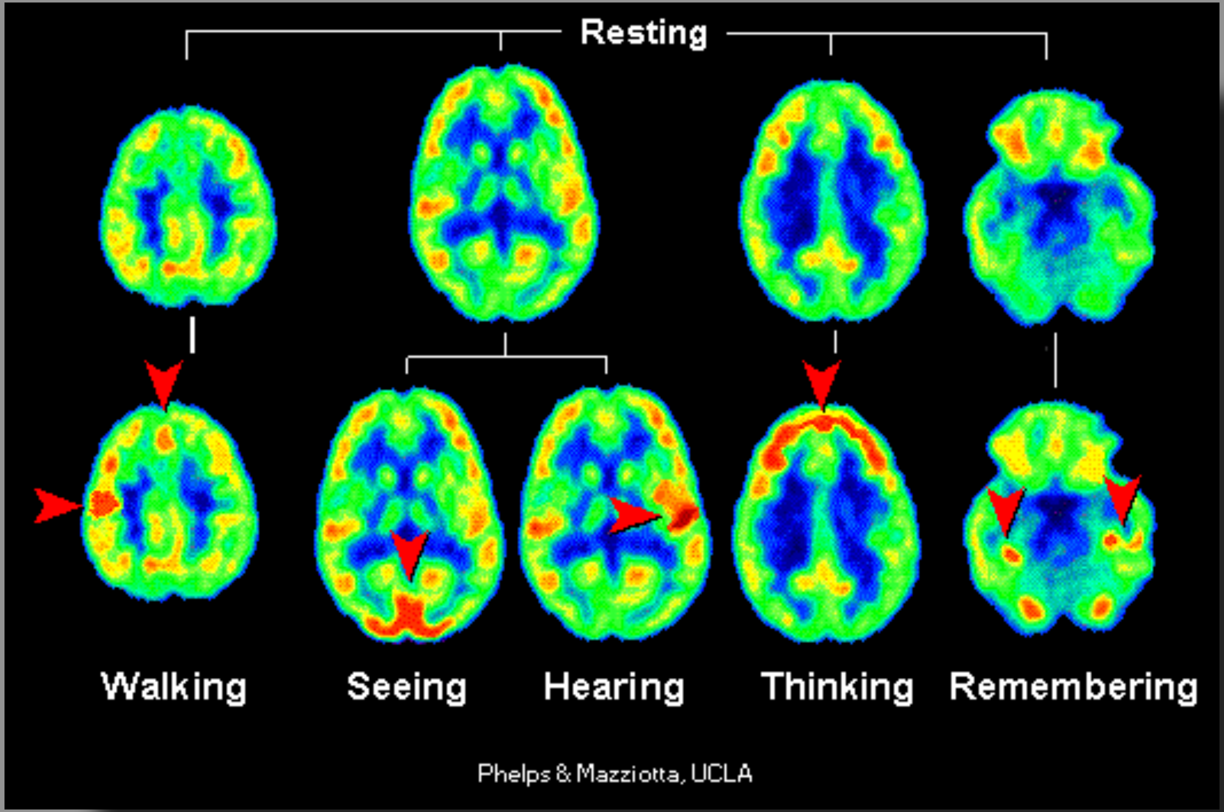
The lesson:
direct measurements confirm QCD fits result: gluon polarization seems to be rather small but the precision is still not satisfactory
Large polarisation is excluded (E-J sum rule)

New precise measurements are needed! (EIC)





“Tomografia” nukleonu



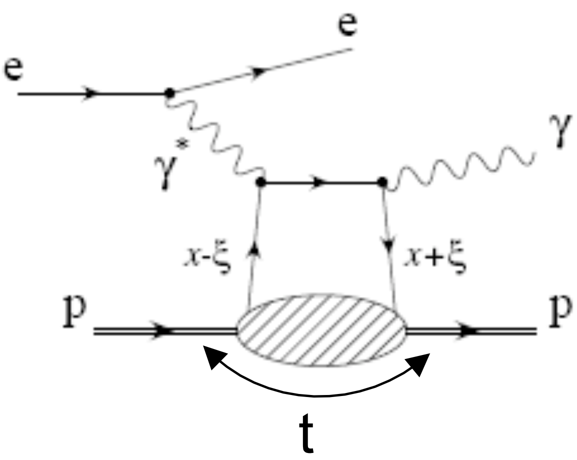
GPDs and DVCS

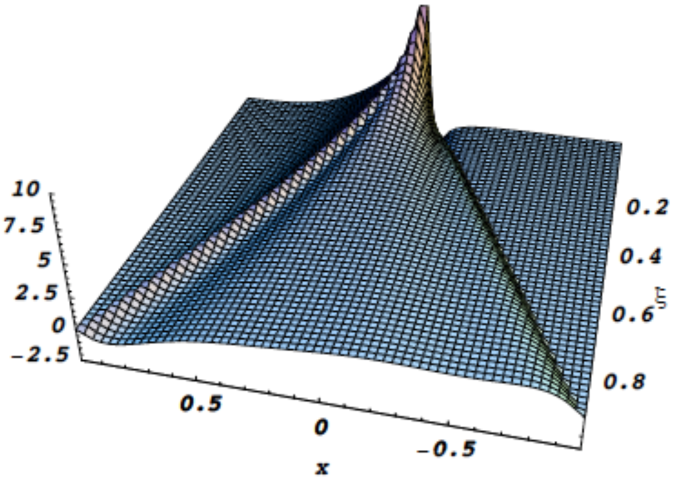
$H(x,\xi,t), \widetilde{H}, E, \widetilde{E}$

$H(x,0,0) = q(x)$

$\widetilde{H}(x,0,0) = \Delta q(x)$

$\int H(x,\xi,t)dx = F(t)$



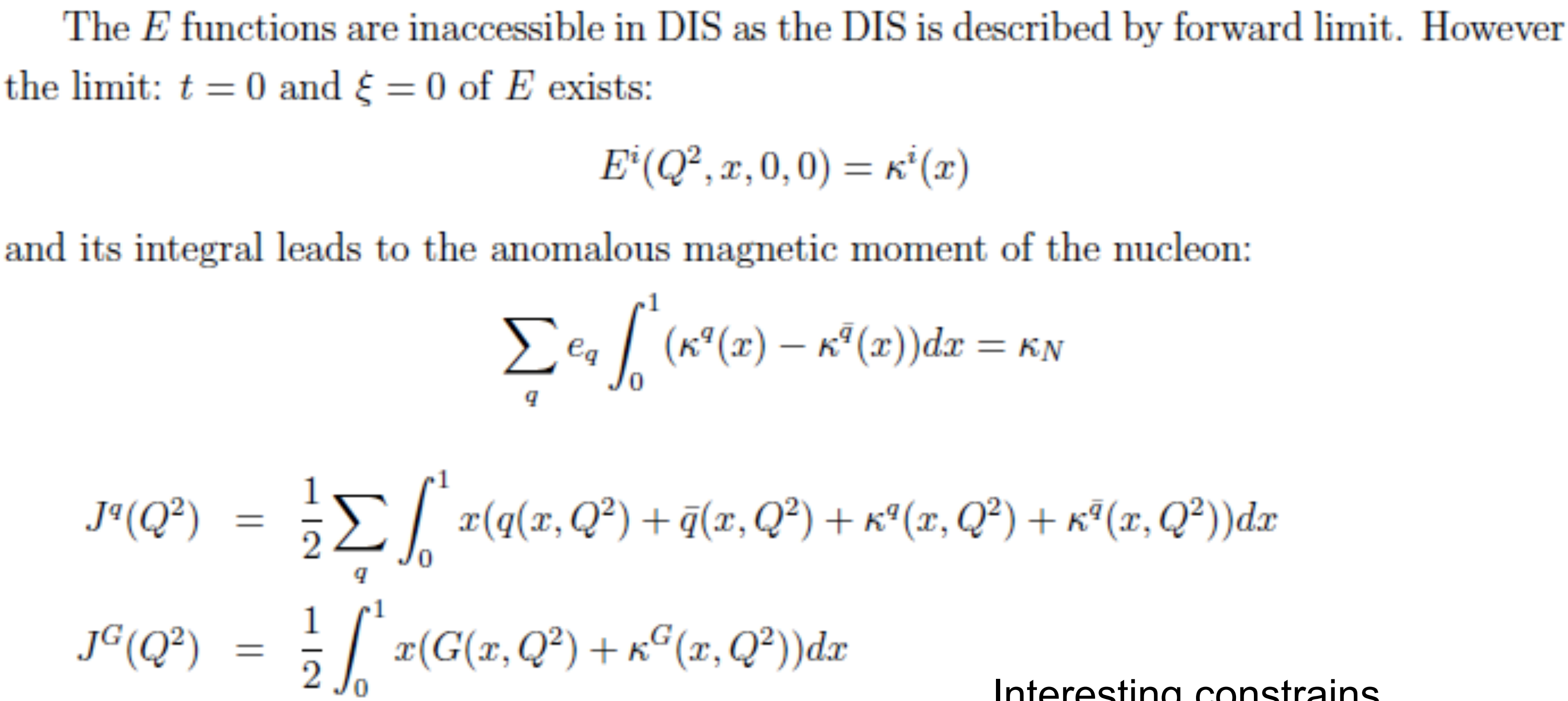


$$J^q(Q^2) = \frac{1}{2}\sum_{i=q,\bar{q}}\int_{-1}^{1} x(H^i(Q^2,x,\xi,0) + E^i(Q^2,x,\xi,0))dx$$

$$J^G(Q^2) = \frac{1}{2}\int_{-1}^{1} x(H^G(Q^2,x,\xi,0) + E^G(Q^2,x,\xi,0))dx$$

Ji’s sum rule

GPDs and anomalous magnetic moments



The E functions are inaccessible in DIS as the DIS is described by forward limit. However the limit: $t = 0$ and $\xi = 0$ of E exists:

$$E^i(Q^2, x, 0, 0) = \kappa^i(x)$$

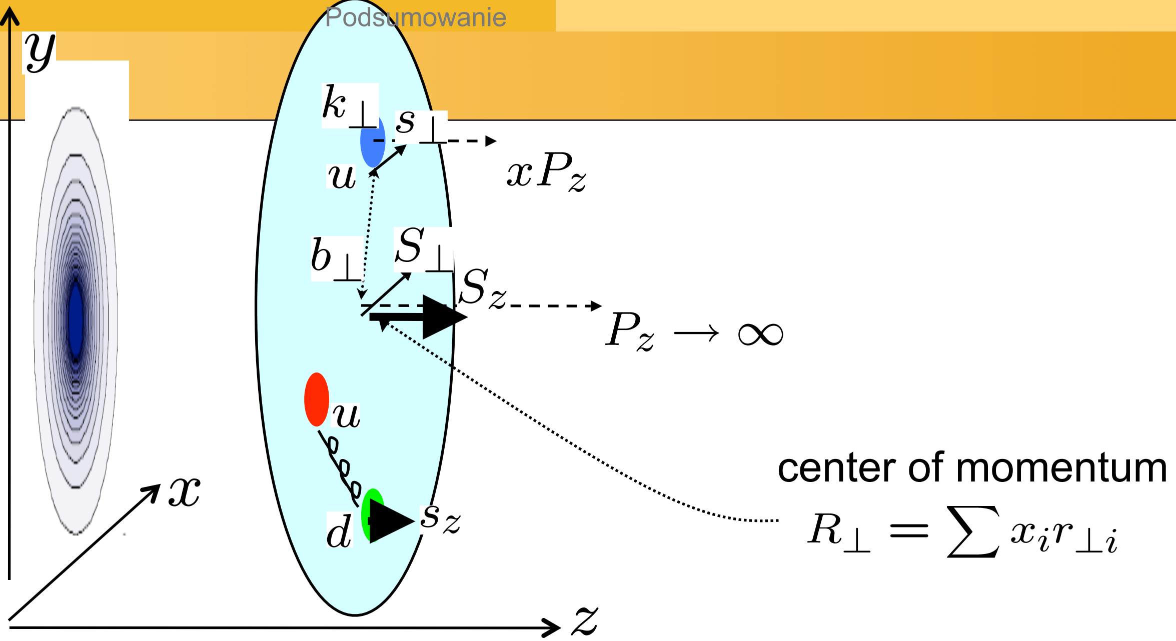
and its integral leads to the anomalous magnetic moment of the nucleon:

$$\sum_q e_q \int_0^1 (\kappa^q(x) - \kappa^{\bar{q}}(x))dx = \kappa_N$$

$$\begin{aligned} J^q(Q^2) &= \frac{1}{2}\sum_q \int_0^1 x(q(x,Q^2) + \bar{q}(x,Q^2) + \kappa^q(x,Q^2) + \kappa^{\bar{q}}(x,Q^2))dx \\ J^G(Q^2) &= \frac{1}{2}\int_0^1 x(G(x,Q^2) + \kappa^G(x,Q^2))dx \end{aligned}$$

Interesting constrains

GPDs and Impact Parameter PDFs (IPDs)



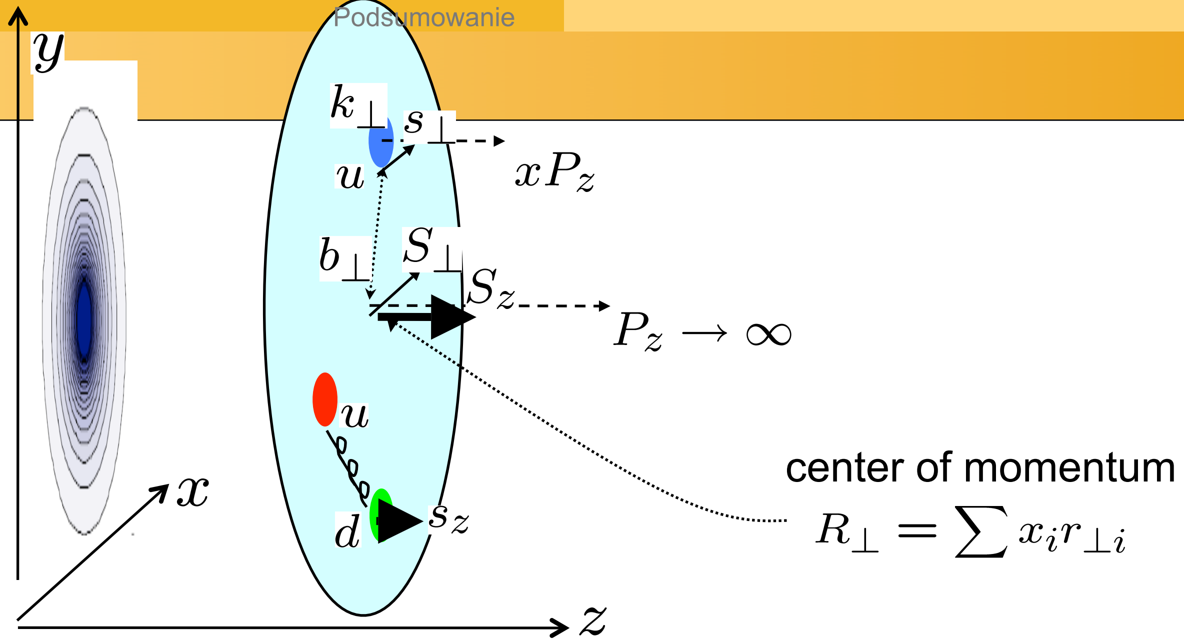
$$\mathcal{H}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} H(x,0,-\vec{\Delta}_\perp^2).$$

$$\tilde{\mathcal{H}}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} \tilde{H}(x,0,-\vec{\Delta}_\perp^2),$$

$$\mathcal{E}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} E(x,0,-\vec{\Delta}_\perp^2).$$

3-Dimensional image of nucleon in the mixed transverse plane-longitudinal momentum space

GPDs and Impact Parameter PDFs (IPDs)



center of momentum

$R_\perp = \sum x_i r_{\perp i}$

Komplementarnie do TMDs - inne “narzędzie” do trójwymiarowego obrazowania - tym razem 3D w przestrzeni pędowo (x)-położeniowej (b_T)

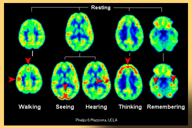
$\mathcal{H}$

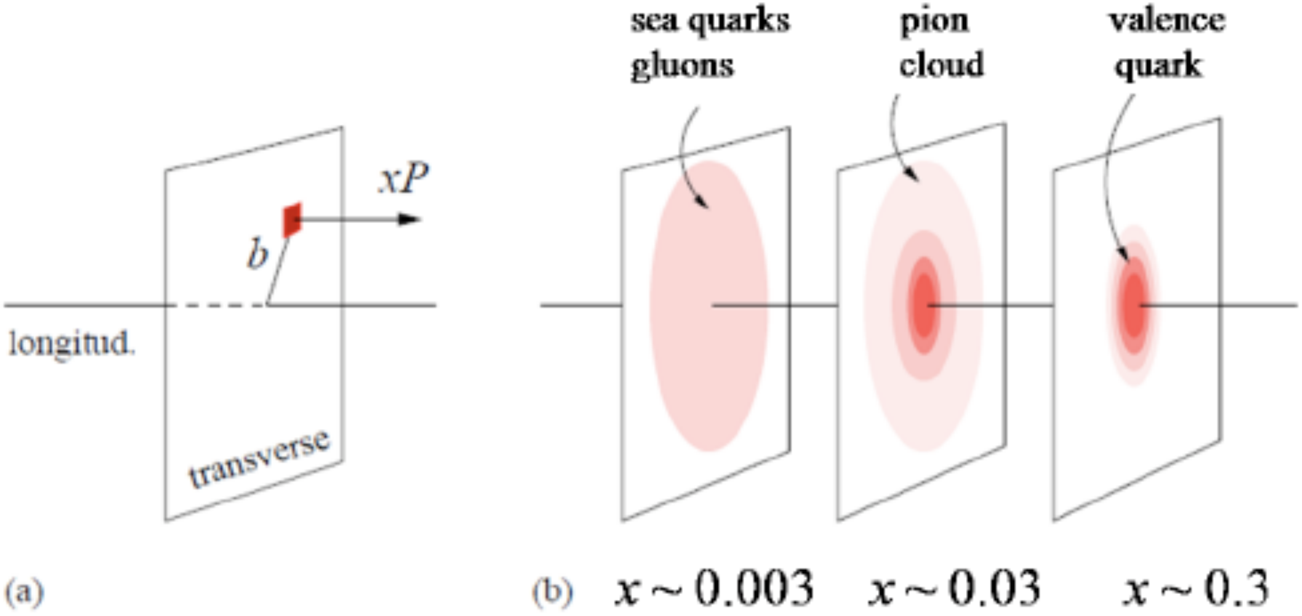
$\tilde{\mathcal{H}}$

$$\mathcal{E}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} E(x,0,-\vec{\Delta}_\perp^2).$$

3-Dimensional image of nucleon in the mixed transverse plane-longitudinal momentum space

Example of tomography

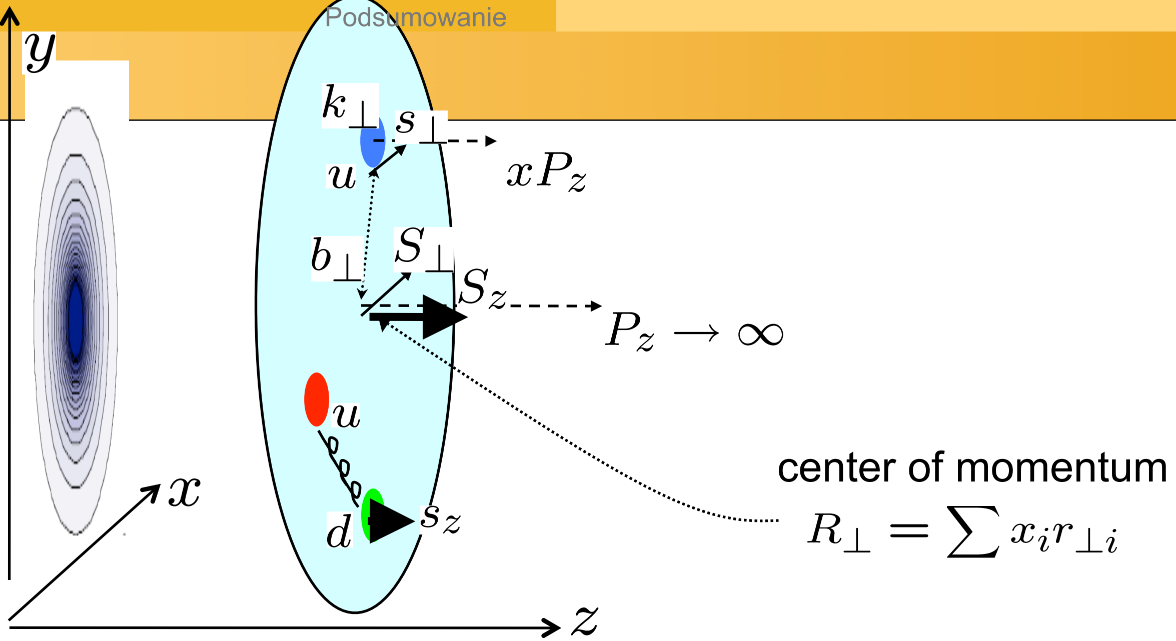




slices in longitudinal momentum x

TMDs and GPDs relation?

Burhard,2000



generalized parton distributions (GPDs)

$H(x, b_\perp) \overset{\text{FT}}{\leftrightarrow} H(x, \xi = 0, t)$

$b_\perp \leftrightarrow -\Delta_\perp \hat{=} |t|^{\frac{1}{2}}$

Burkardt PRD 2000

transverse momentum dependent PDFs

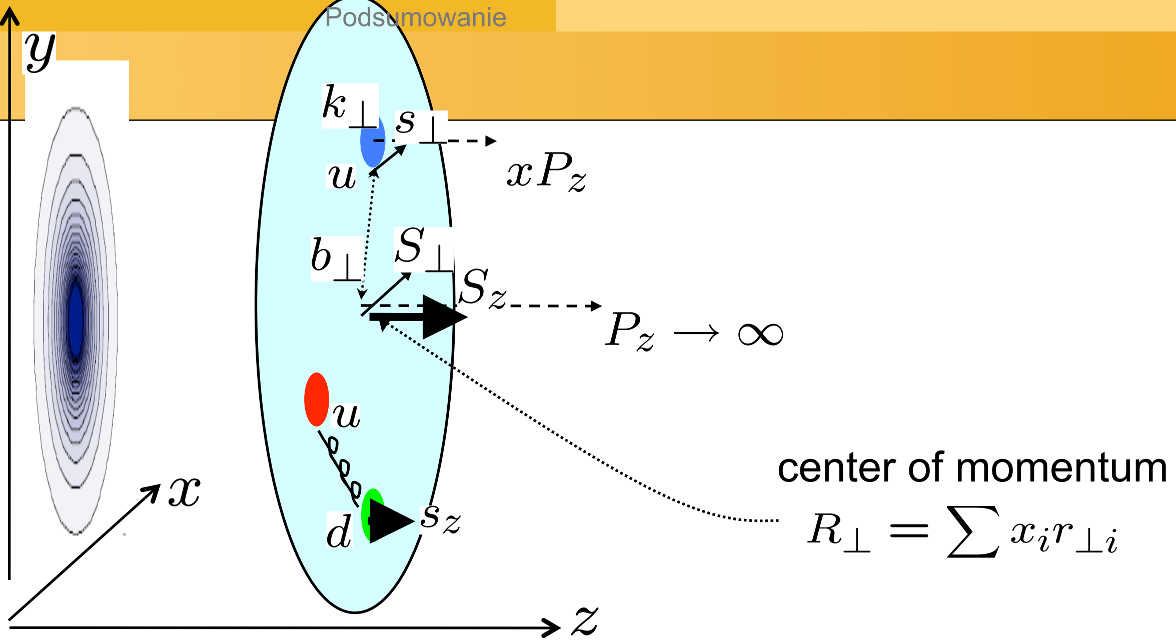
$f(x, k_\perp) \overset{\text{FT}}{\leftrightarrow} A(l \cdot P, l^2)$

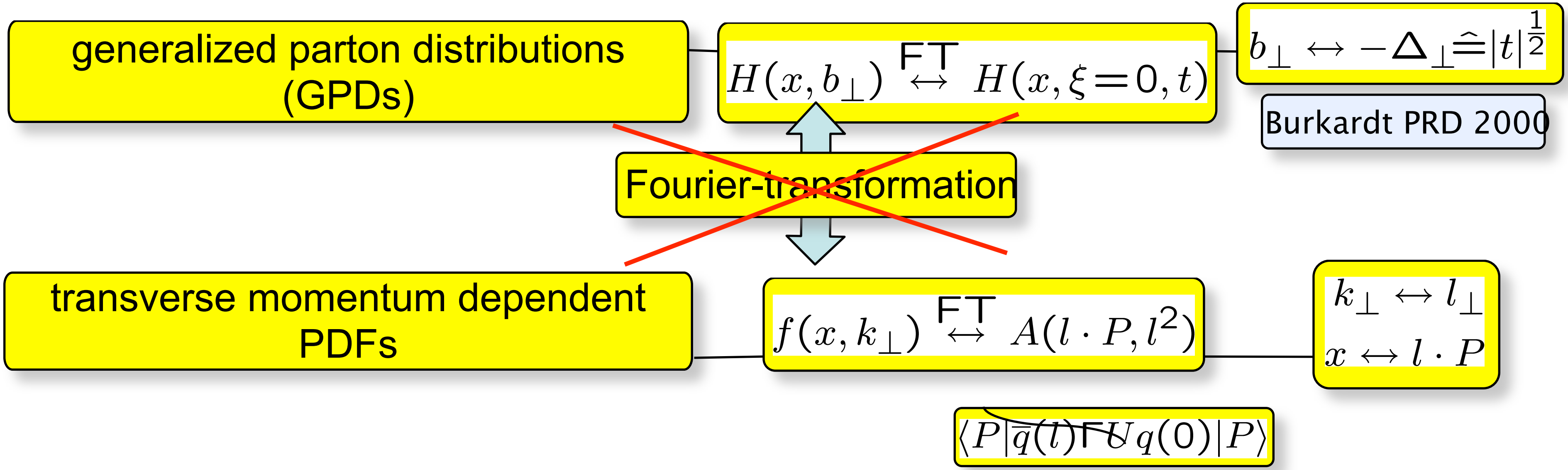
$k_\perp \leftrightarrow l_\perp$
$x \leftrightarrow l \cdot P$

$\langle P|\overline{q}(l)\Gamma U q(0)|P\rangle$

TMDs and GPDs relation?

Burhard,2000





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Burkardt PRD 2000

~~Fourier-transformation~~

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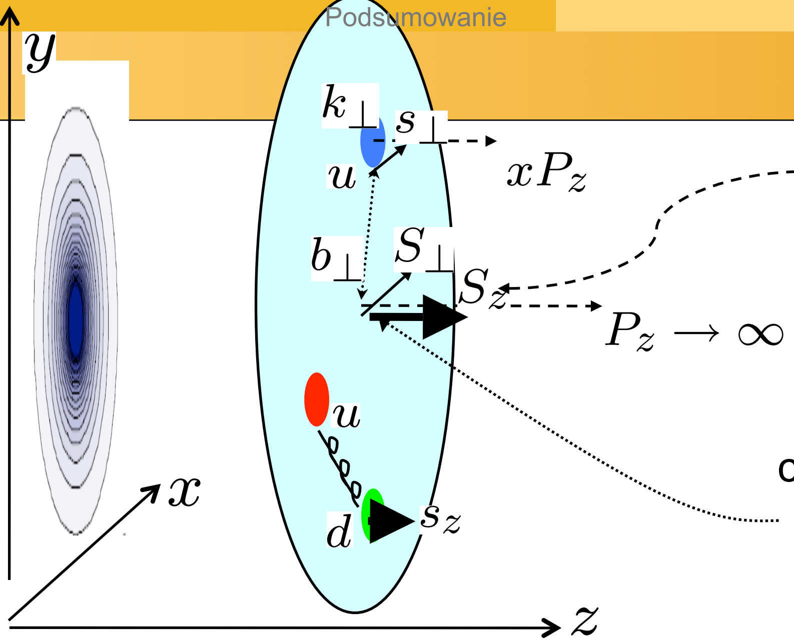
TMDs and GPDs relation?

Burhard,2000



$P_z \to \infty$

$$S_z = \frac{1}{2} = \sum_q J_q + J_g$$

center of momentum

$$R_\perp = \sum x_i r_{\perp i}$$

generalized parton distributions (GPDs)

$$H(x, b_\perp) \overset{\text{FT}}{\leftrightarrow} H(x, \xi = 0, t)$$

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Burkardt PRD 2000

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transverse momentum dependent PDFs

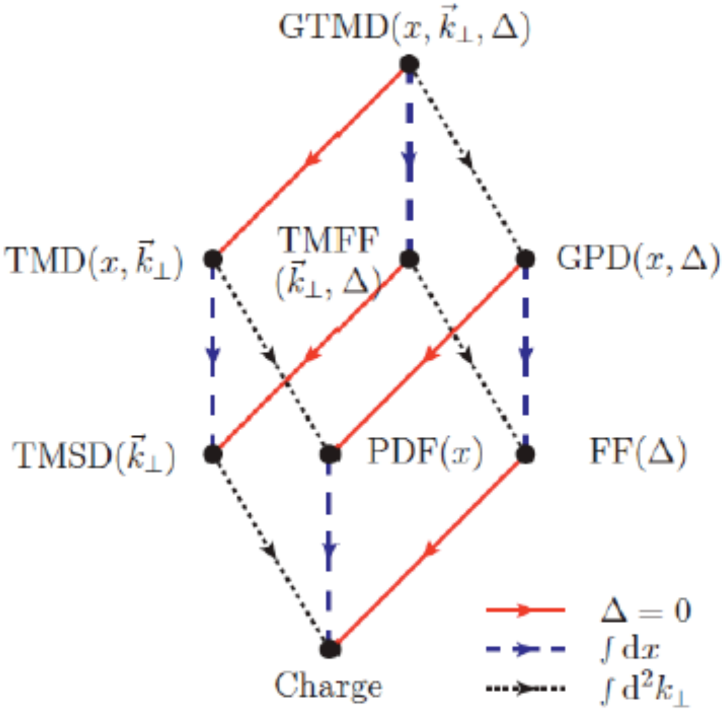
$$f(x, k_\perp) \overset{\text{FT}}{\leftrightarrow} A(l \cdot P, l^2)$$

$$k_\perp \leftrightarrow l_\perp$$
$$x \leftrightarrow l \cdot P$$

$$\langle P | \overline{q}(l) \Gamma U q(0) | P \rangle$$

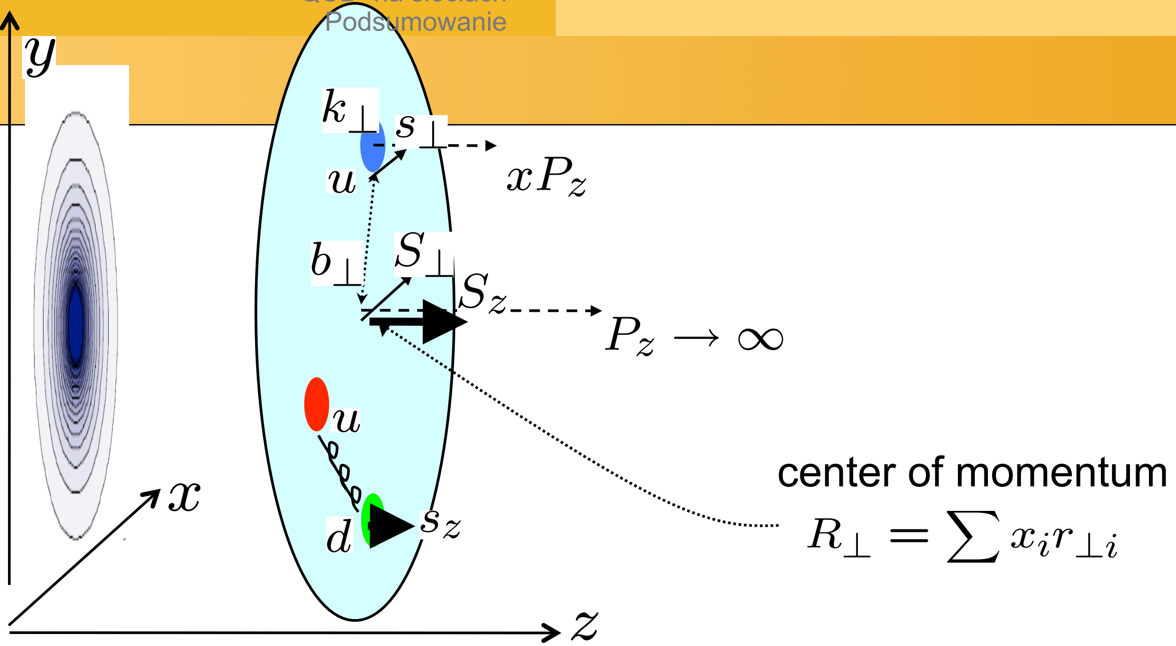
Generalized Wigner function

Generalized Parton Correlation Function GPCF - more general, gives GTMD



Transverse distortion

Burhard,2000

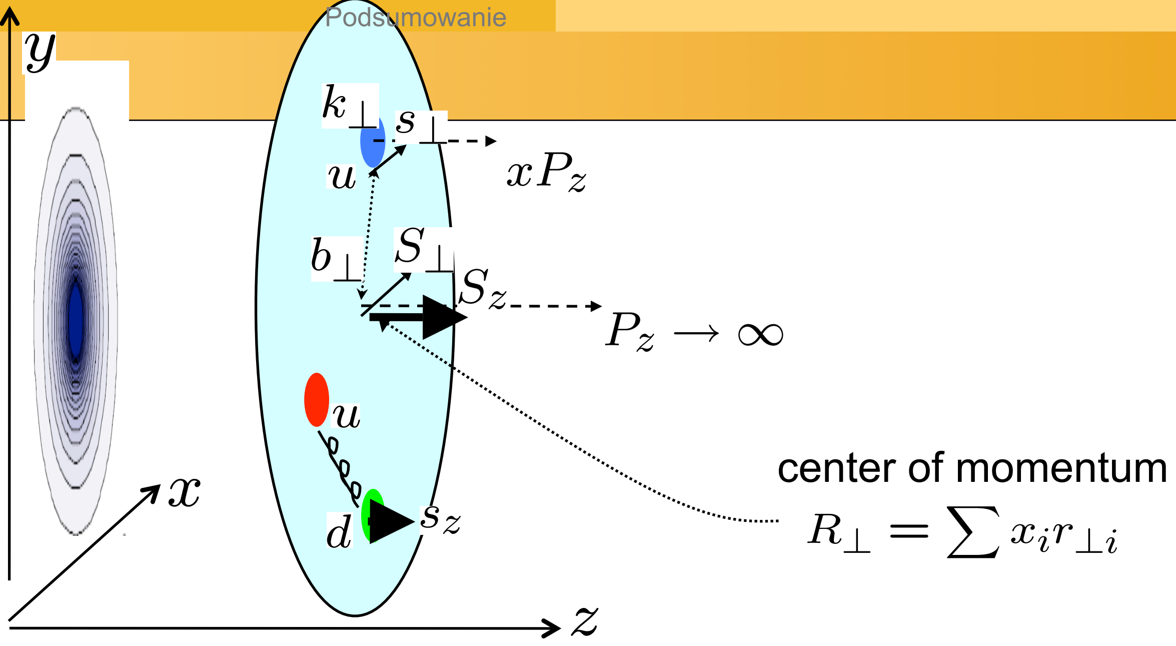


For a transversely polarized nucleon (e.g. polarized in the $+\hat{x}$-direction) the IPD $q_{\hat{x}}(x, \vec{b}_\perp)$ is no longer symmetric due to the non-zero value of the spin-flip GPD E. This deformation is described by the gradient of the Fourier transform of E:

$$q_{\hat{x}}(x, \vec{b}_\perp) = \mathcal{H}(x, \vec{b}_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \mathcal{E}(x, \vec{b}_\perp).$$

Transverse distortion

Burhard,2000



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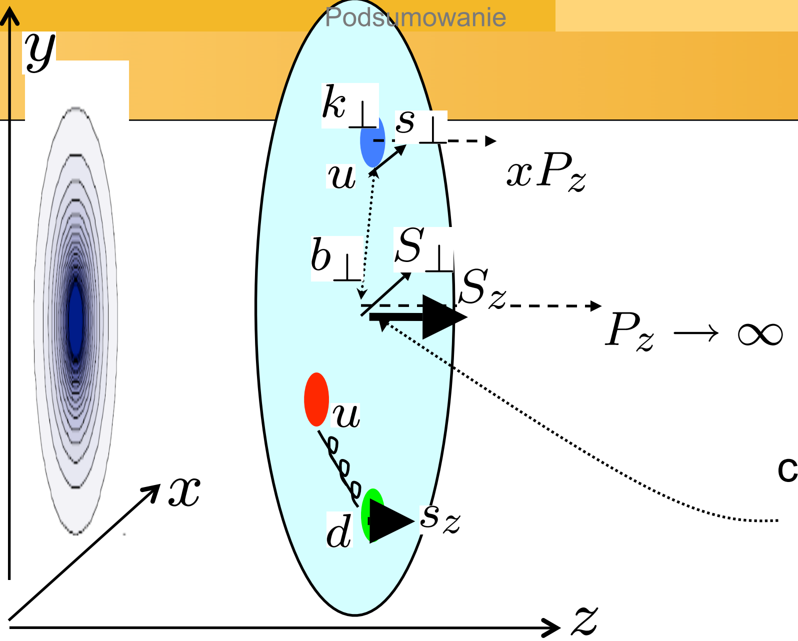
Transverse distortion

Burhard,2000



center of momentum

$R_\perp = \sum x_i r_{\perp i}$

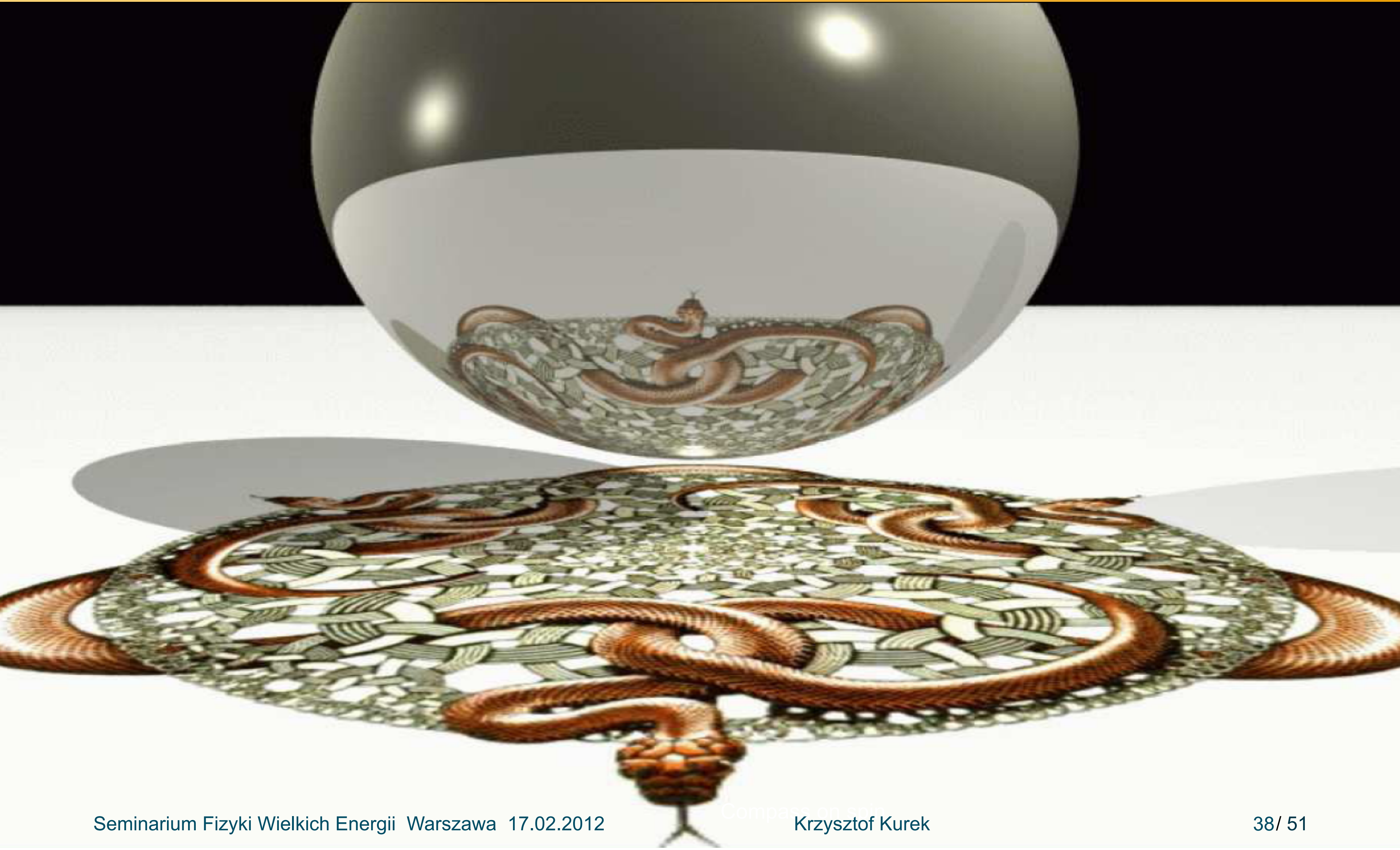
Zaburzenie rozkładu kwarków w płaszczyźnie poprzecznej spowodowane jest niezerową funkcją GPD E (spin-flip) a to oznacza niezerowy orbitalny moment pędu kwarków !

is

is

$$q_{\hat{x}}(x,\vec{b}_\perp) = \mathcal{H}(x,\vec{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\mathcal{E}(x,\vec{b}_\perp).$$

Orbitalny moment pędu



QCD evolution & Asymptotic solutions

Scale evolution equation:

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2) \end{pmatrix} = \frac{\alpha_s}{2\pi\cdot 9}\begin{pmatrix} -16, 3n_f \\ 16, -3n_f \end{pmatrix}\begin{pmatrix} J_q(\mu^2) \\ J_g(\mu^2) \end{pmatrix}$$

Asymptotic solution:

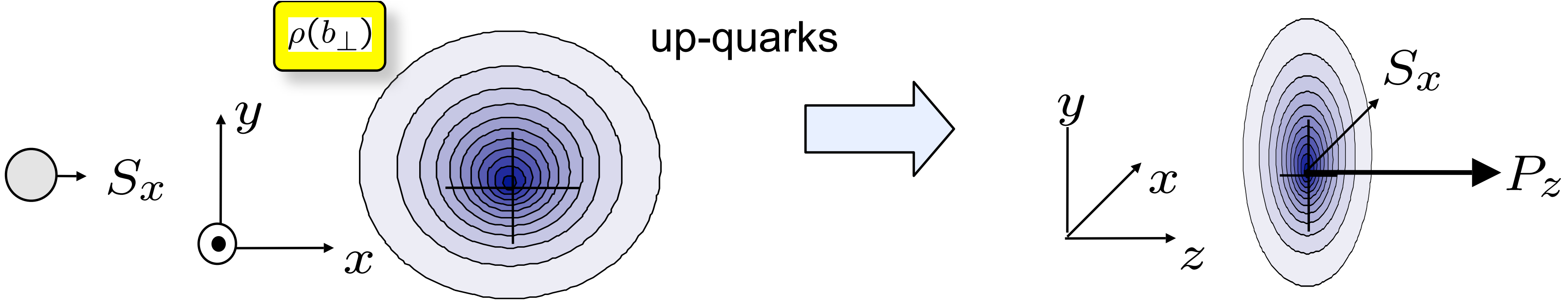
$$J_q(\infty) = \frac{1}{2}\frac{3n_f}{16+3n_f}, J_g(\infty) = \frac{1}{2}\frac{16}{16+3n_f}$$

Roughly 1/4

L_G must be important fpr gluons (ΔG small?)
L_q should be small?

Dynamical origin of quark transverse momentum

M.Burhardt 2002/2003 - Sivers function and GPD



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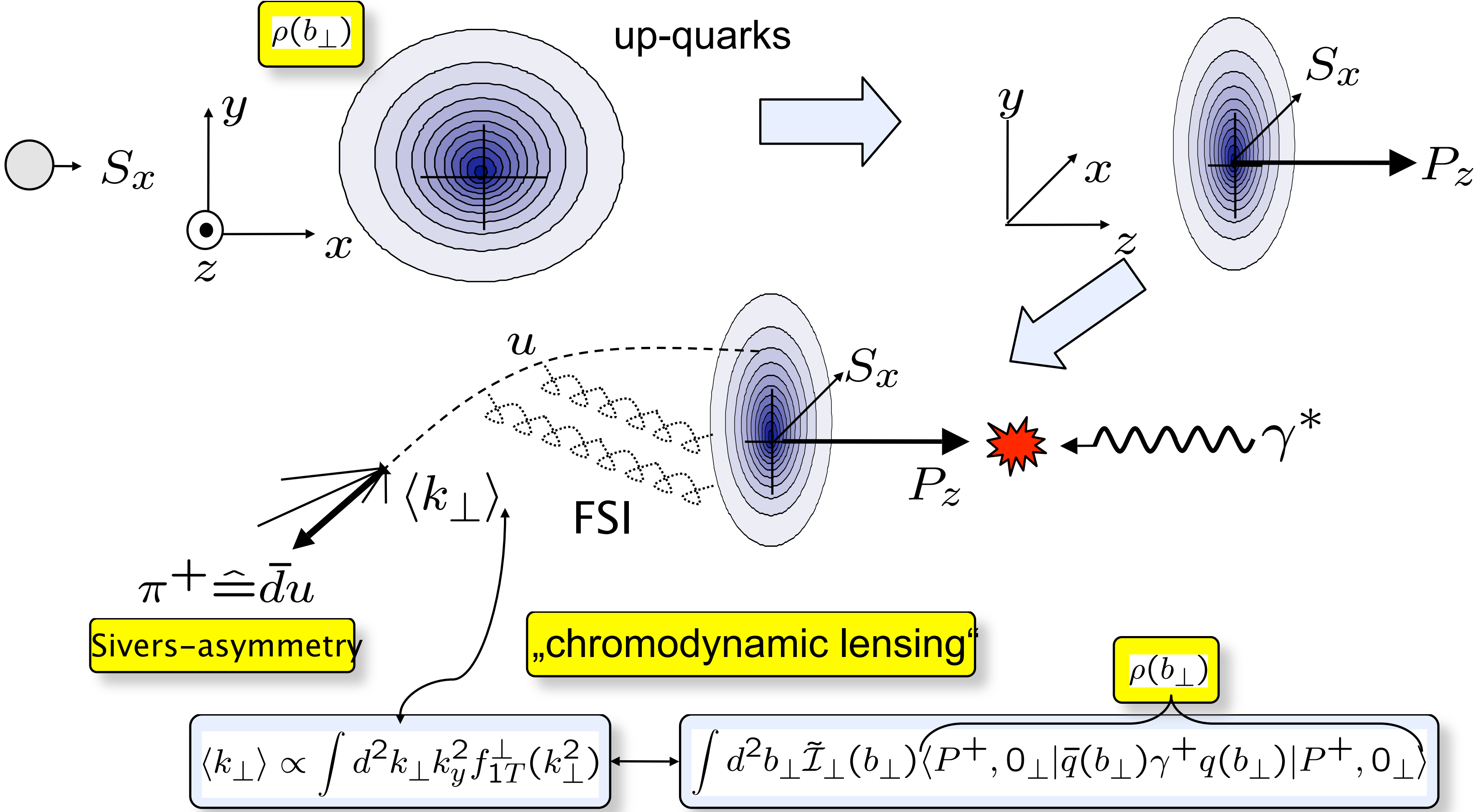

Sivers–asymmetry

„chromodynamic lensing“

$$\langle k_\perp\rangle \propto \int d^2k_\perp k_y^2 f_{1T}^\perp(k_\perp^2) \longleftrightarrow \int d^2b_\perp \tilde{\mathcal{I}}_\perp(b_\perp)\overbrace{\langle P^+,0_\perp|\bar{q}(b_\perp)\gamma^+ q(b_\perp)|P^+,0_\perp\rangle}^{\rho(b_\perp)}$$

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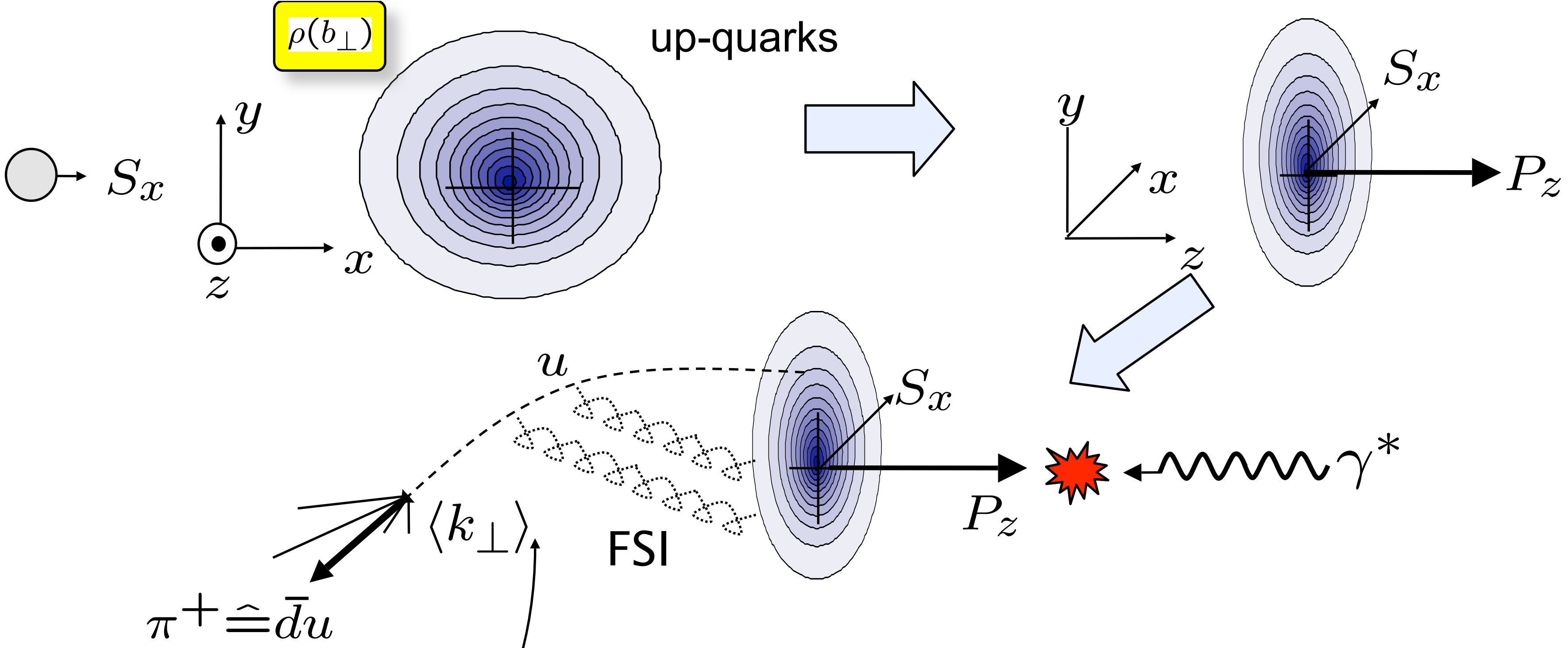

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alternative approach is based on „integrated“ twist-3 distributions
[Ji, Qui, Vogelsang, Yuan PRL, PRD, PLB 2006]

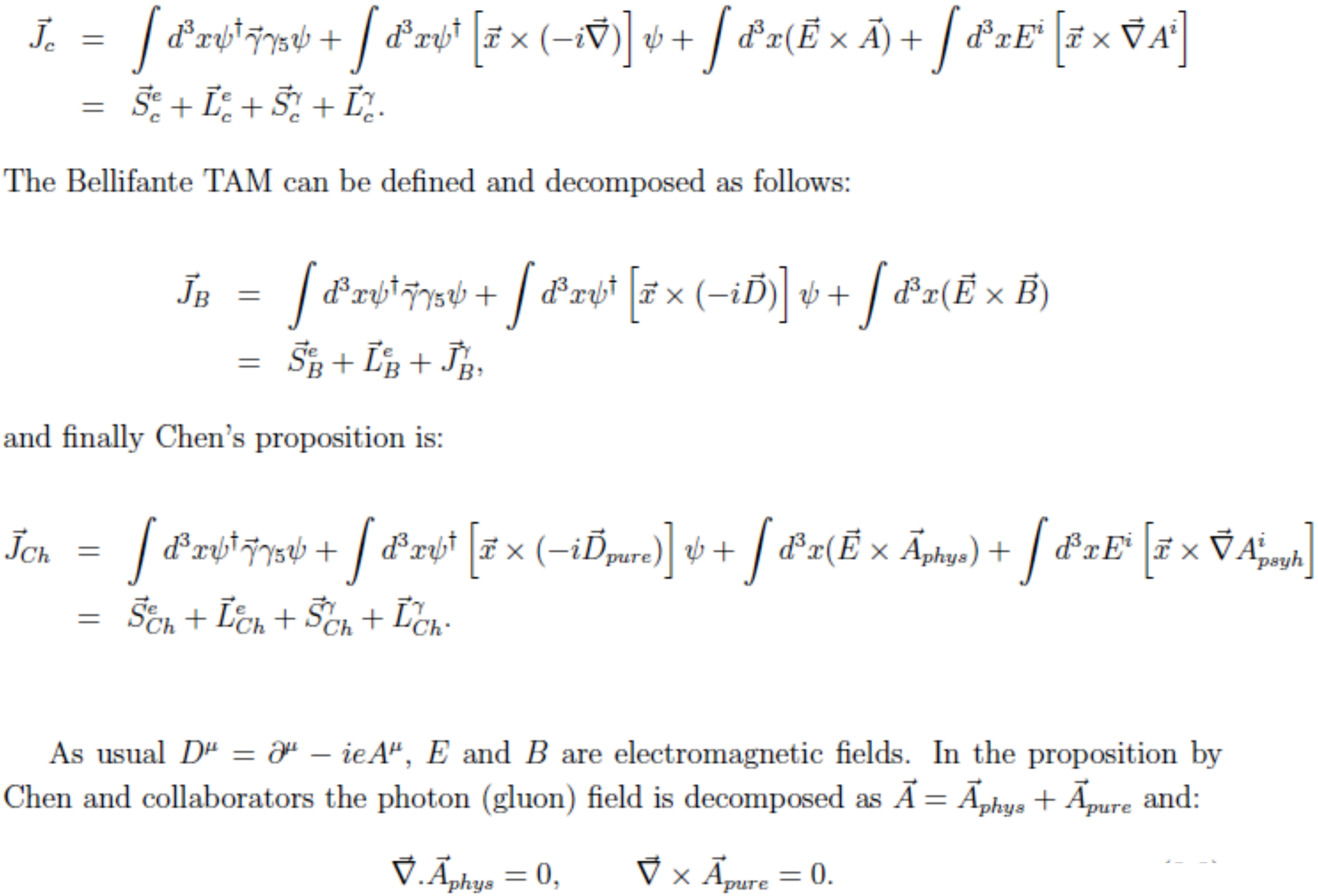
Controversy in QCD: how to split TOA into quark and gluon part?

X.Ji and Lattice: Gauge invariant, covariant and local operator.

There are 3 version so far:
1. Canonical one
2. Bellifante = Ji
3. recently proposed by Chen - also in QED

There are difference in asymptotic behavior:
Ji - 1/4
Chen -1/10 !

Controversy in QCD: how to split TOAM into quark and gluon part?



$$\begin{aligned}\vec{J}_c &= \int d^3x\psi^\dagger\vec{\gamma}\gamma_5\psi + \int d^3x\psi^\dagger\left[\vec{x}\times(-i\vec{\nabla})\right]\psi + \int d^3x(\vec{E}\times\vec{A}) + \int d^3xE^i\left[\vec{x}\times\vec{\nabla}A^i\right]\\ &= \vec{S}^e_c + \vec{L}^e_c + \vec{S}^\gamma_c + \vec{L}^\gamma_c.\end{aligned}$$

The Bellifante TAM can be defined and decomposed as follows:

$$\begin{aligned}\vec{J}_B &= \int d^3x\psi^\dagger\vec{\gamma}\gamma_5\psi + \int d^3x\psi^\dagger\left[\vec{x}\times(-i\vec{D})\right]\psi + \int d^3x(\vec{E}\times\vec{B})\\ &= \vec{S}^e_B + \vec{L}^e_B + \vec{J}^\gamma_B,\end{aligned}$$

and finally Chen's proposition is:

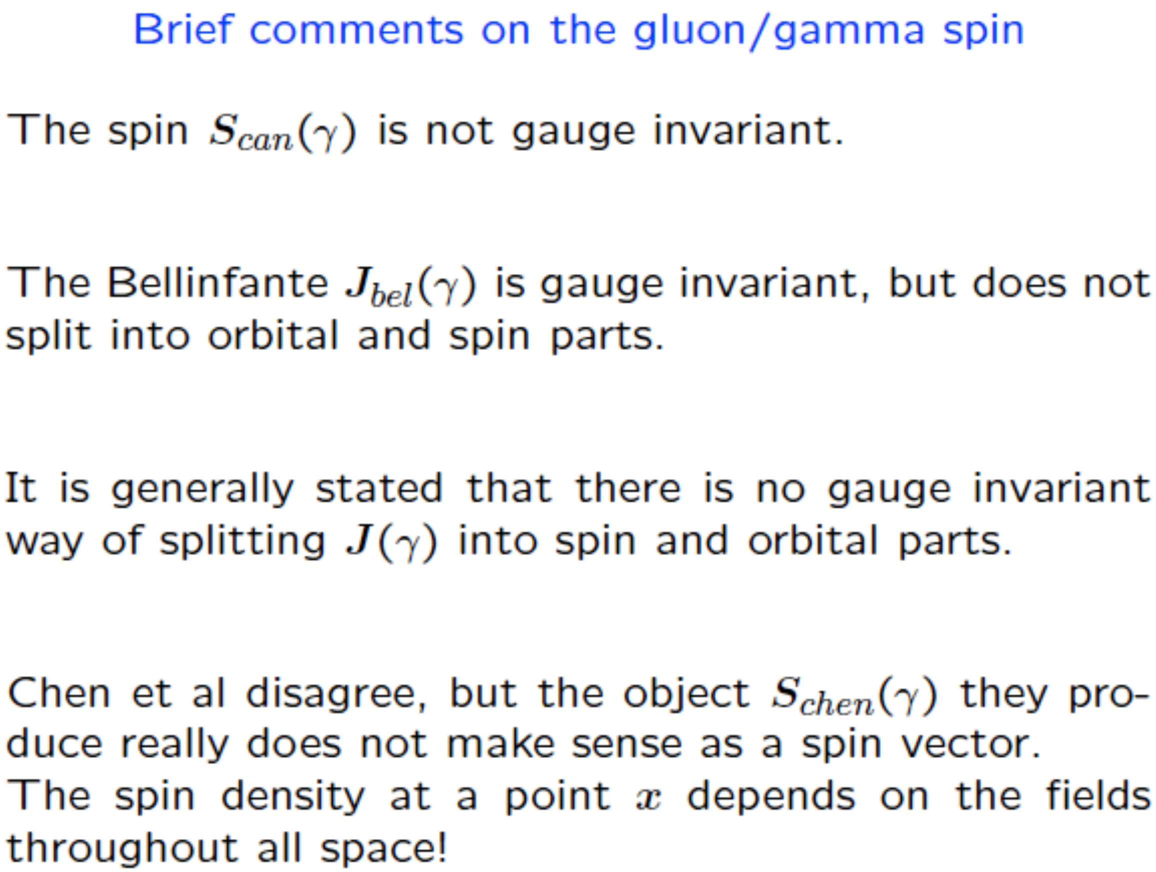
$$\begin{aligned}\vec{J}_{Ch} &= \int d^3x\psi^\dagger\vec{\gamma}\gamma_5\psi + \int d^3x\psi^\dagger\left[\vec{x}\times(-i\vec{D}_{pure})\right]\psi + \int d^3x(\vec{E}\times\vec{A}_{phys}) + \int d^3xE^i\left[\vec{x}\times\vec{\nabla}A^i_{psyh}\right]\\ &= \vec{S}^e_{Ch} + \vec{L}^e_{Ch} + \vec{S}^\gamma_{Ch} + \vec{L}^\gamma_{Ch}.\end{aligned}$$

As usual $D^\mu = \partial^\mu - ieA^\mu$, E and B are electromagnetic fields. In the proposition by Chen and collaborators the photon (gluon) field is decomposed as $\vec{A} = \vec{A}_{phys} + \vec{A}_{pure}$ and:

$$\vec{\nabla}.\vec{A}_{phys} = 0, \qquad \vec{\nabla}\times\vec{A}_{pure} = 0.$$

Controversy in QCD: how to split TOA into quark and gluon part?

E.Leader, Phys.Rev.D83, 096012 (2011)



Brief comments on the gluon/gamma spin

The spin $\boldsymbol{S}_{can}(\gamma)$ is not gauge invariant.

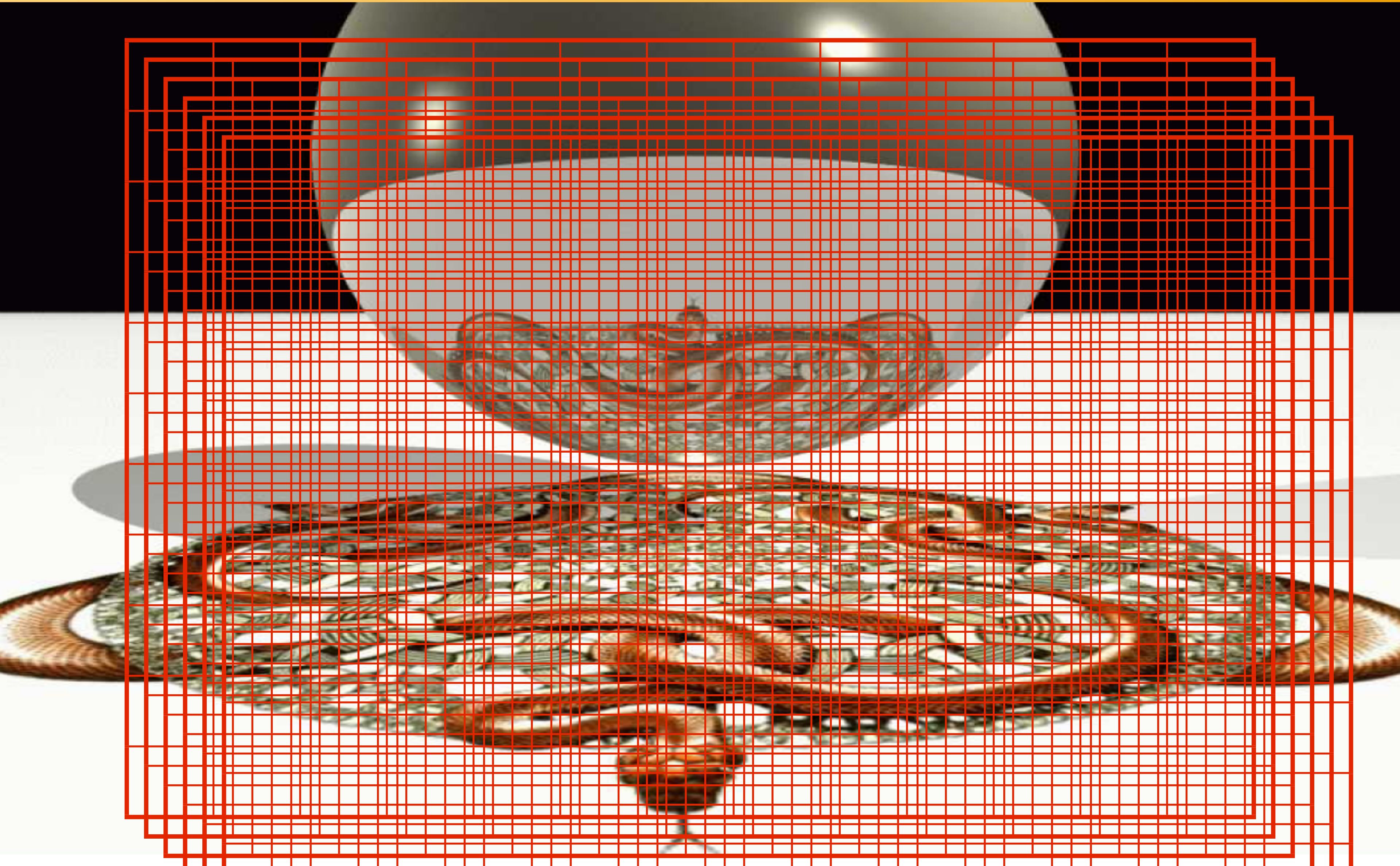
The Bellinfante $\boldsymbol{J}_{bel}(\gamma)$ is gauge invariant, but does not split into orbital and spin parts.

It is generally stated that there is no gauge invariant way of splitting $\boldsymbol{J}(\gamma)$ into spin and orbital parts.

Chen et al disagree, but the object $\boldsymbol{S}_{chen}(\gamma)$ they produce really does not make sense as a spin vector.
The spin density at a point x depends on the fields throughout all space!

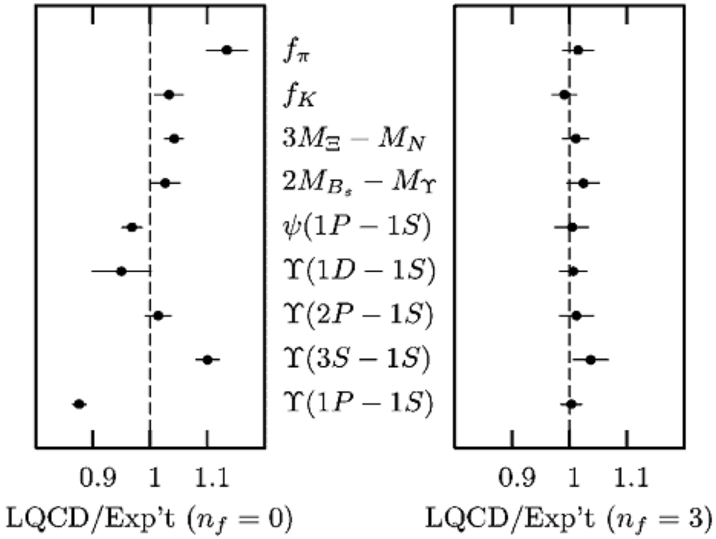
In addition: Chen's prediction for nucleon momentum carried by gluons is 1/5

Struktura spinowa na sieciach QCD



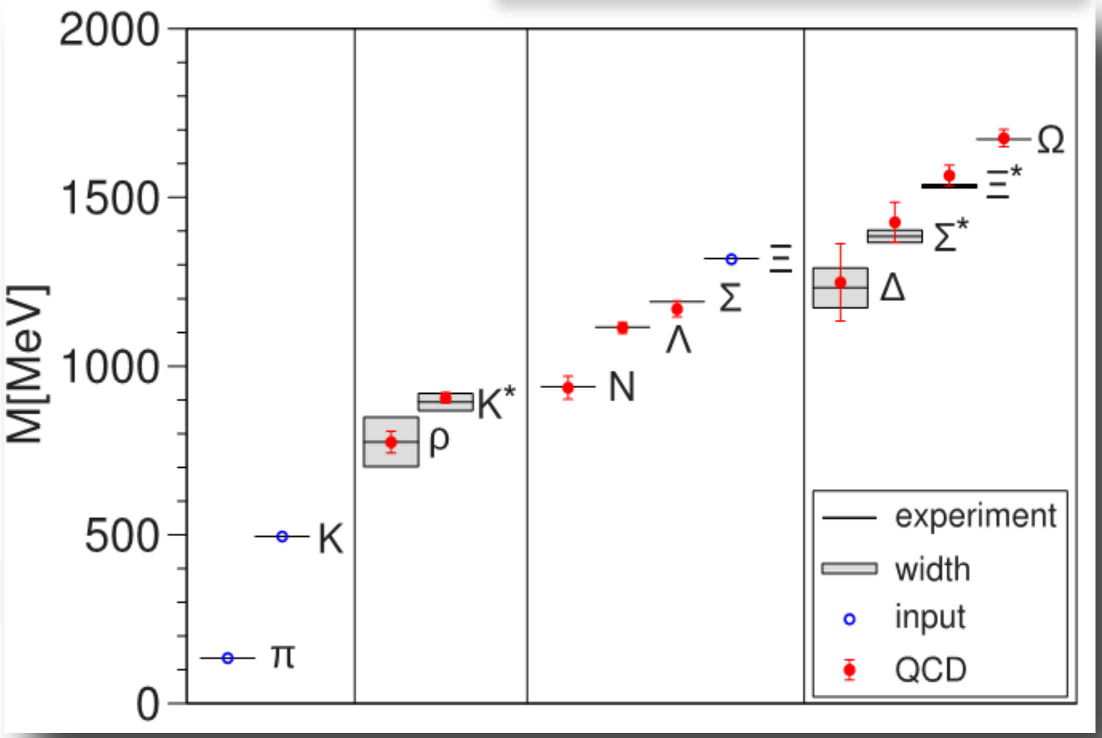
Lattice QCD “propaganda plots”

P. Haegler



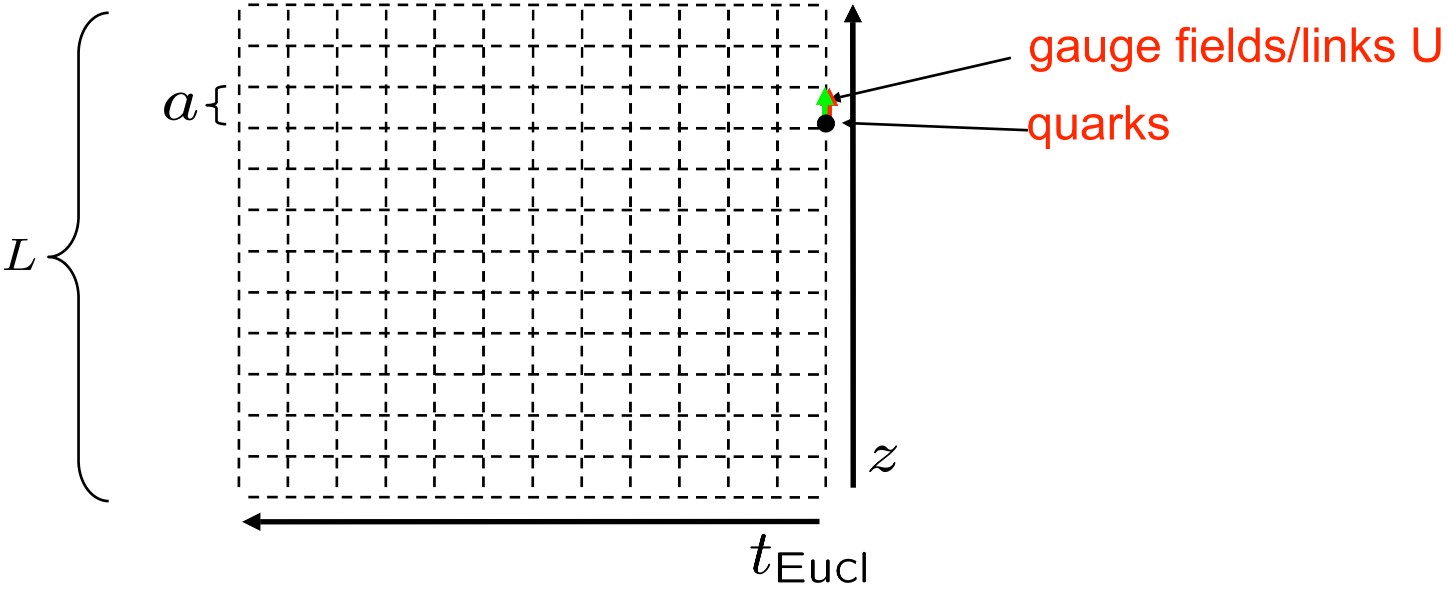
Davies, Lepage et al. [MILC] PRL 2004

BMW (Dürr et al.) Nature 2009



with lattice pion masses down to m_p~190 MeV

Lattice QCD calculations of hadron structure



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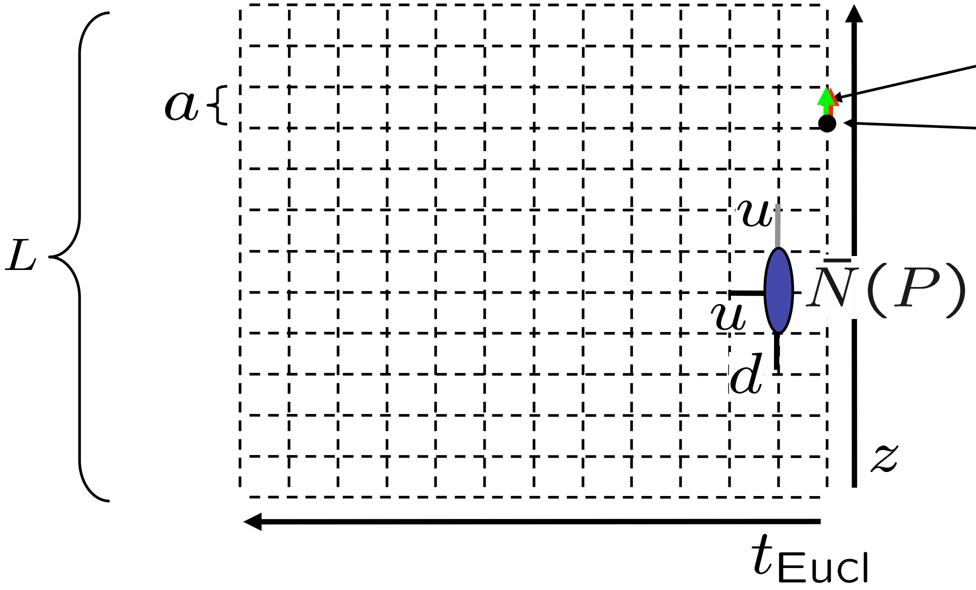

gauge fields/links U

quarks

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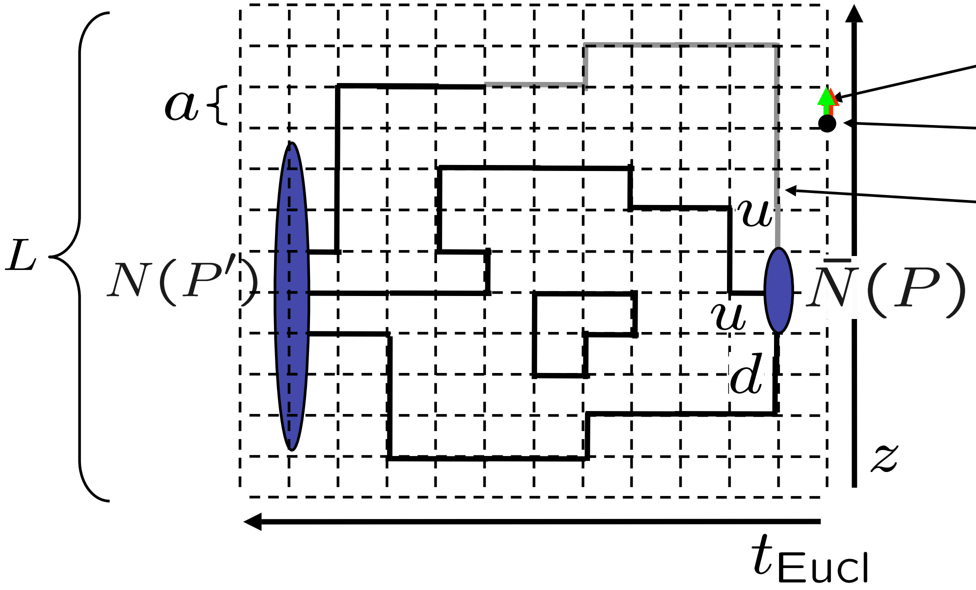

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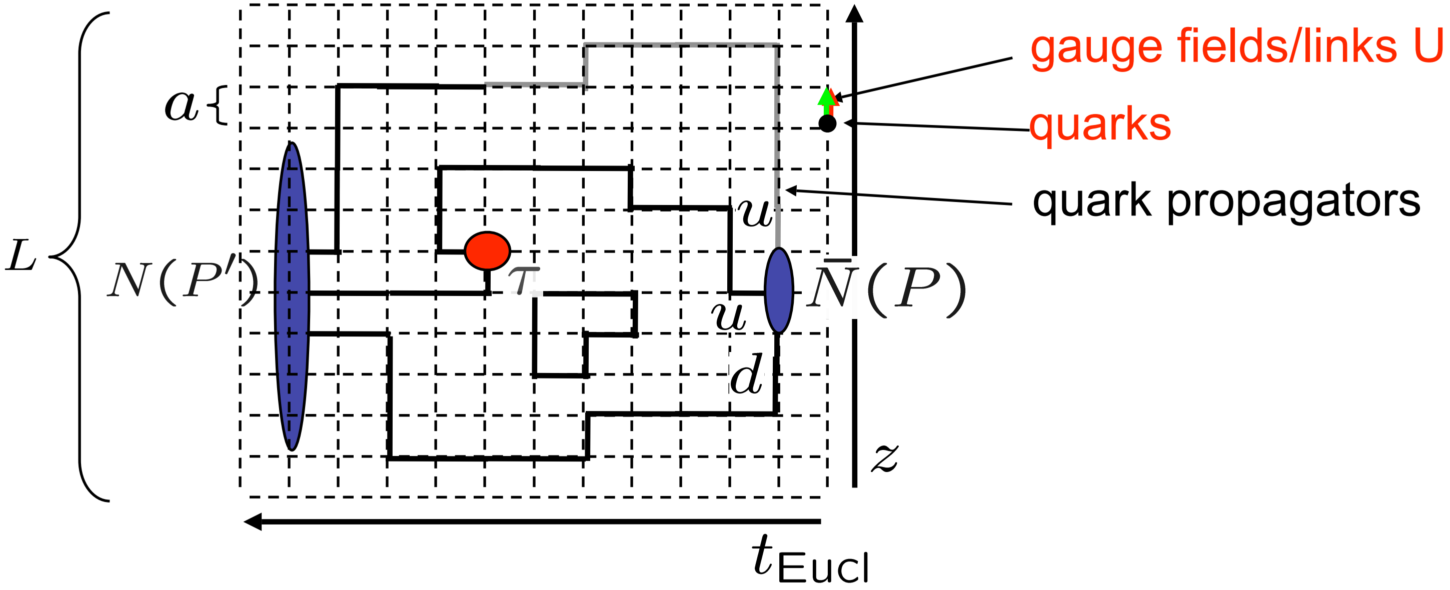

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quark propagators

● = vector-, axialvector-, *quark spin flip-,* (spin-2) *graviton-, „spin-n“* coupling

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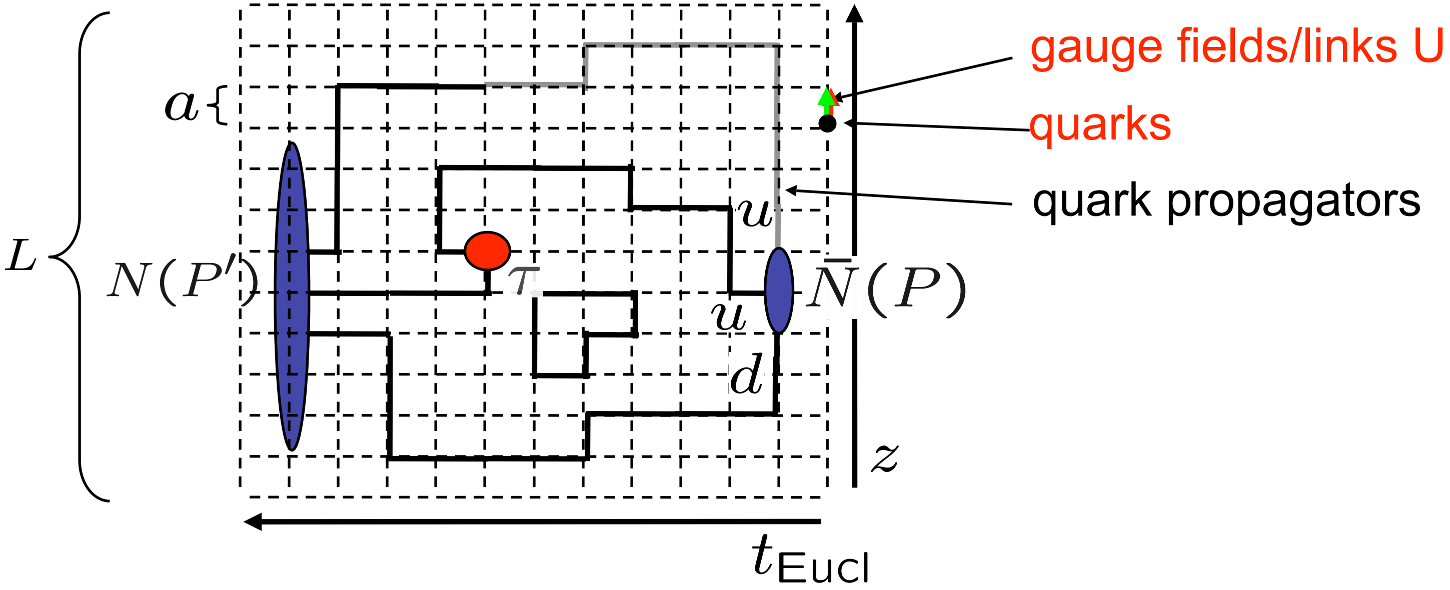

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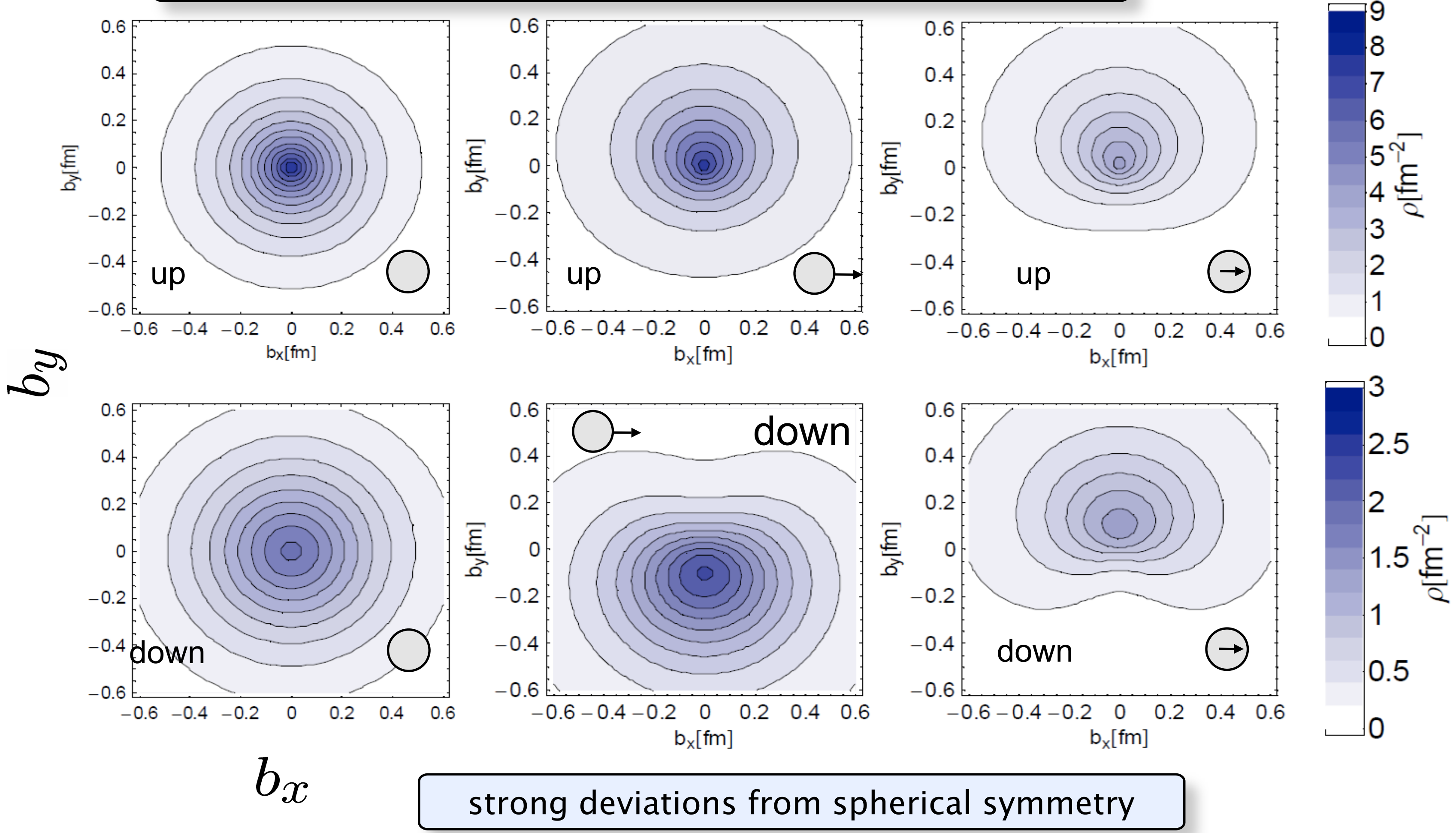
$$\langle q_2\bar{q}_1\rangle \propto \int DADqd\bar{q}e^{iS[q,\bar{q},A]} \rightarrow \left[\int DUe^{-S[U]}\,\mathrm{det}\,D[U]\right] D^{-1}_{1\rightarrow 2}[U] \approx \frac{1}{N}\sum_{i=1}^{N} D^{-1}_{1\rightarrow 2}[U_i]$$

compute the path-integral numerically

Lowest x-moments of quark densities in coordinate space

QCDSF n_f=2 Clover, Phys.Rev.Lett. 2007 [hep-lat/0612032]



strong deviations from spherical symmetry

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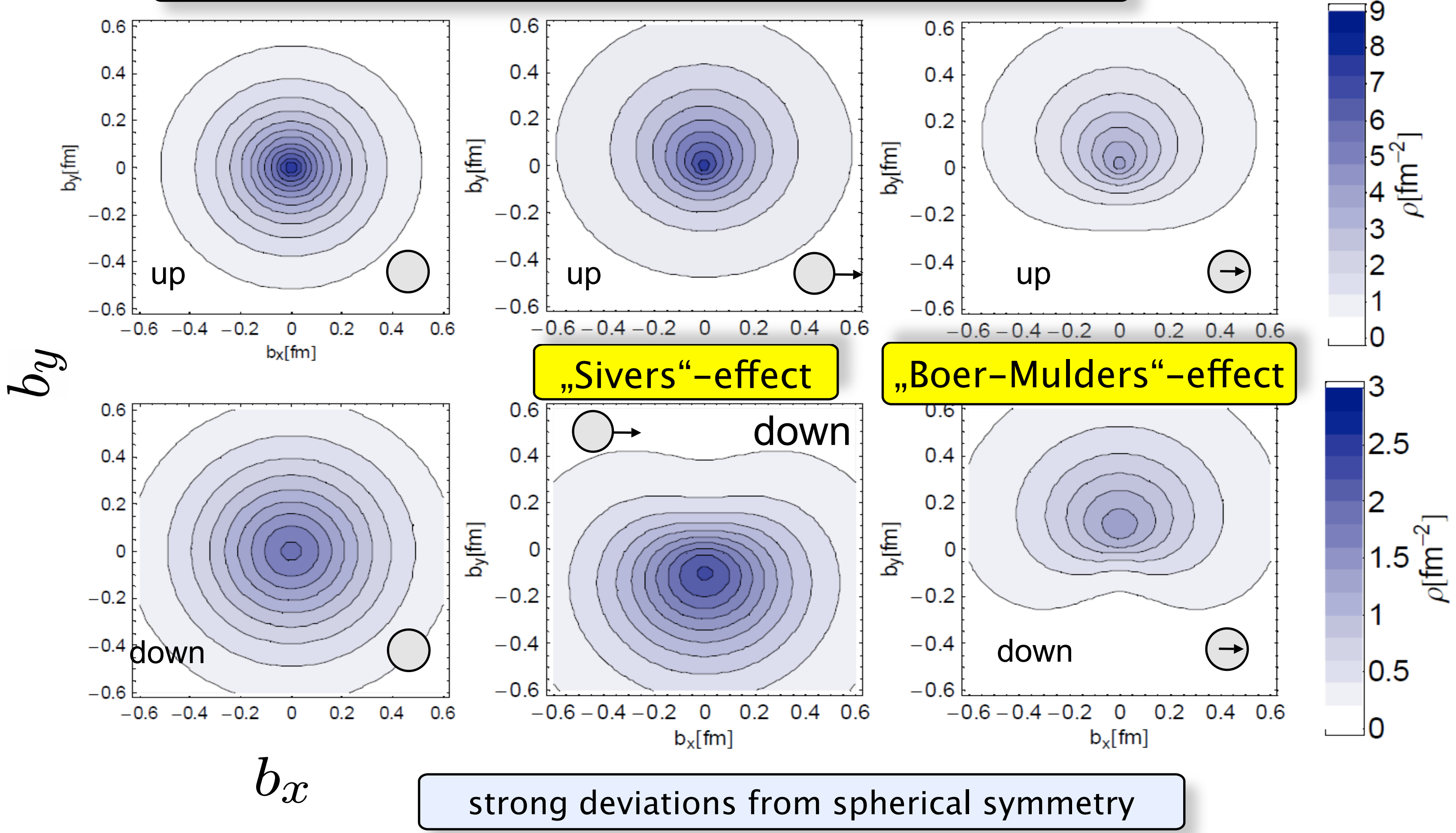


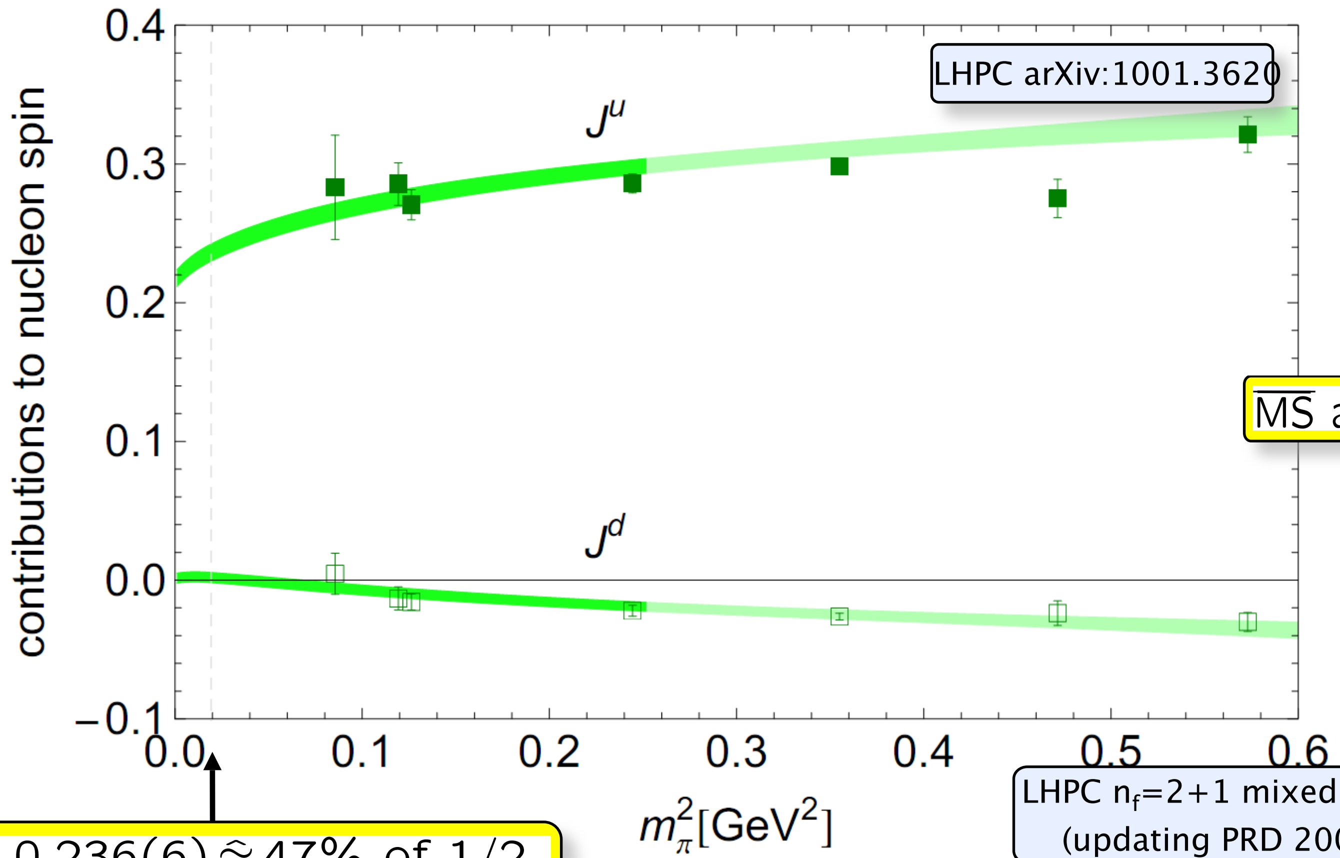
„Sivers“–effect

„Boer–Mulders“–effect

strong deviations from spherical symmetry

TAM in Lattice

P. Haegler



LHPC arXiv:1001.3620

$\overline{\text{MS}}$ at 4 GeV^2

LHPC n_f=2+1 mixed; arXiv:1001.3620
(updating PRD 2008, 0810.1933)

$$J^u = 0.236(6) \approx 47\% \text{ of } 1/2$$
$$J^d = 0.0018(37) \approx 1\% \text{ of } 1/2$$

$$J^{u+d} \approx 0.238 \pm 0.008 \approx 48\% \text{ of } 1/2$$

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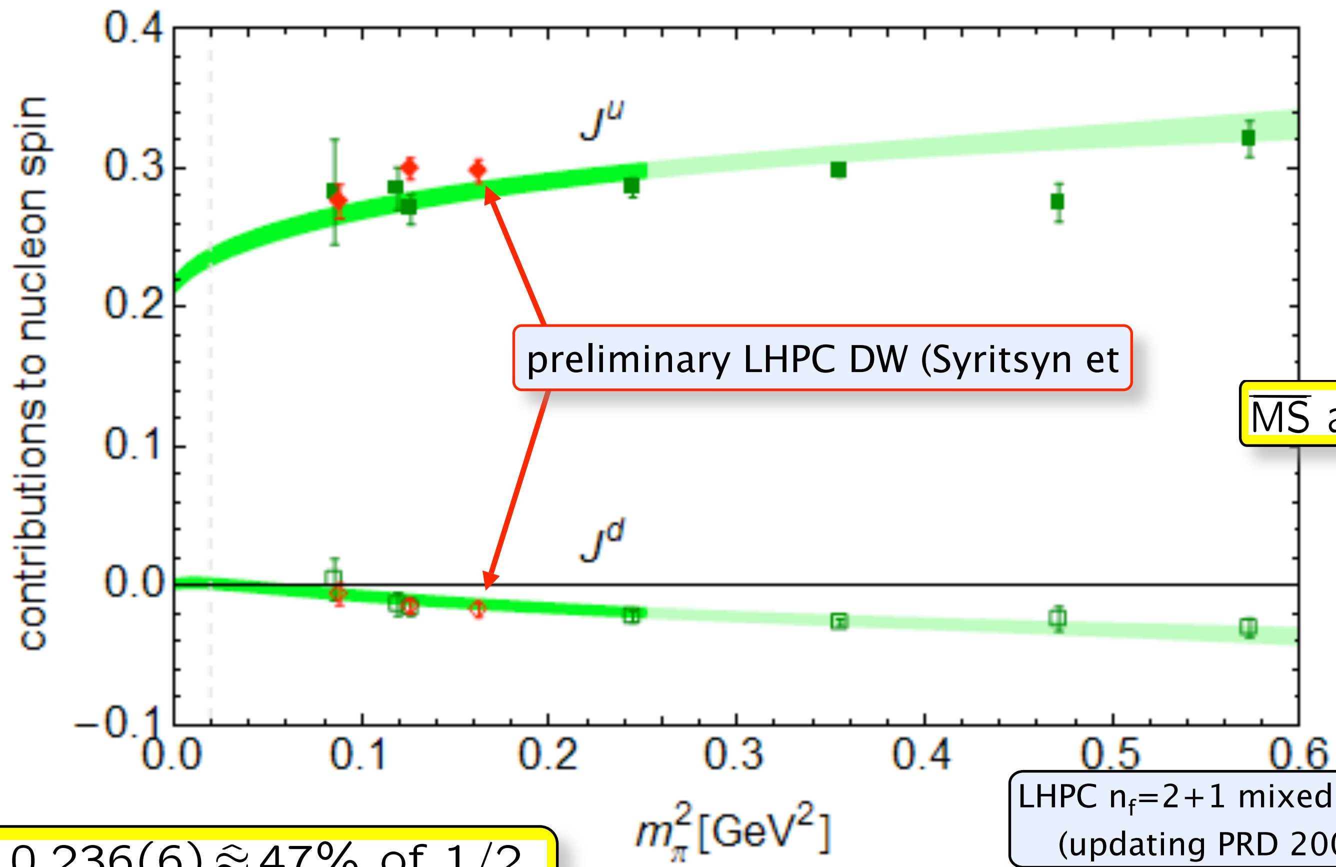

preliminary LHPC DW (Syritsyn et

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quark spin and OAM in Lattice

P. Haegler

$$L_q = J_q - \Delta\Sigma_q/2$$

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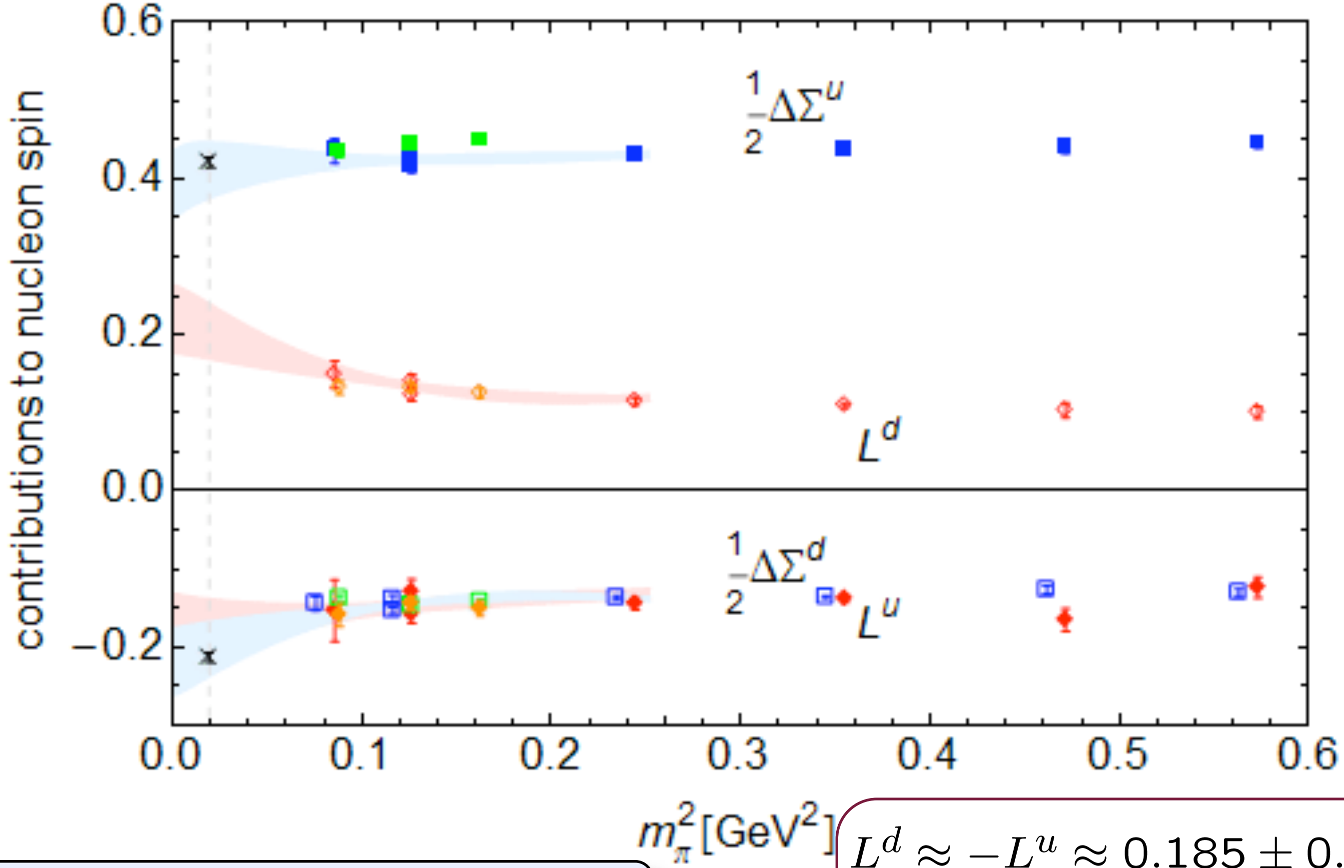
arXiv:1001.3620



$\overline{\text{MS}}$ at 4 GeV^2

$$L^d \approx -L^u \approx 0.185 \pm 0.06 \approx 36\% \text{ of } 1/2$$
$$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\% \text{ of } 1/2$$

$$J^u \approx 0.236 \pm 0.006 \mathrel{\widehat{=}} 48\% \text{ of } 1/2$$
$$J^d \approx 0.002 \pm 0.004$$

$$\kappa^{u+d} = 3\kappa^{p+n} = -0.36$$

quark spin and OAM in Lattice

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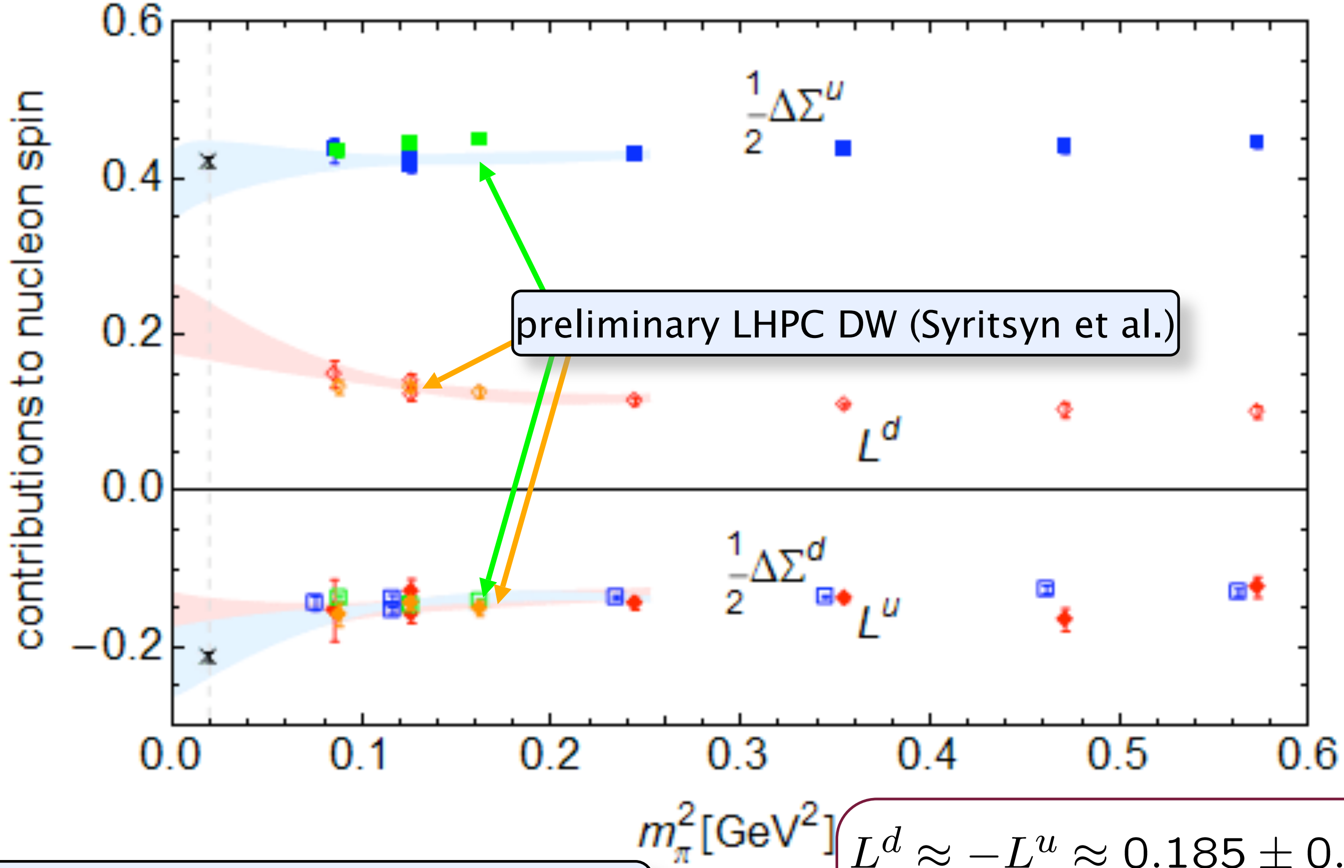

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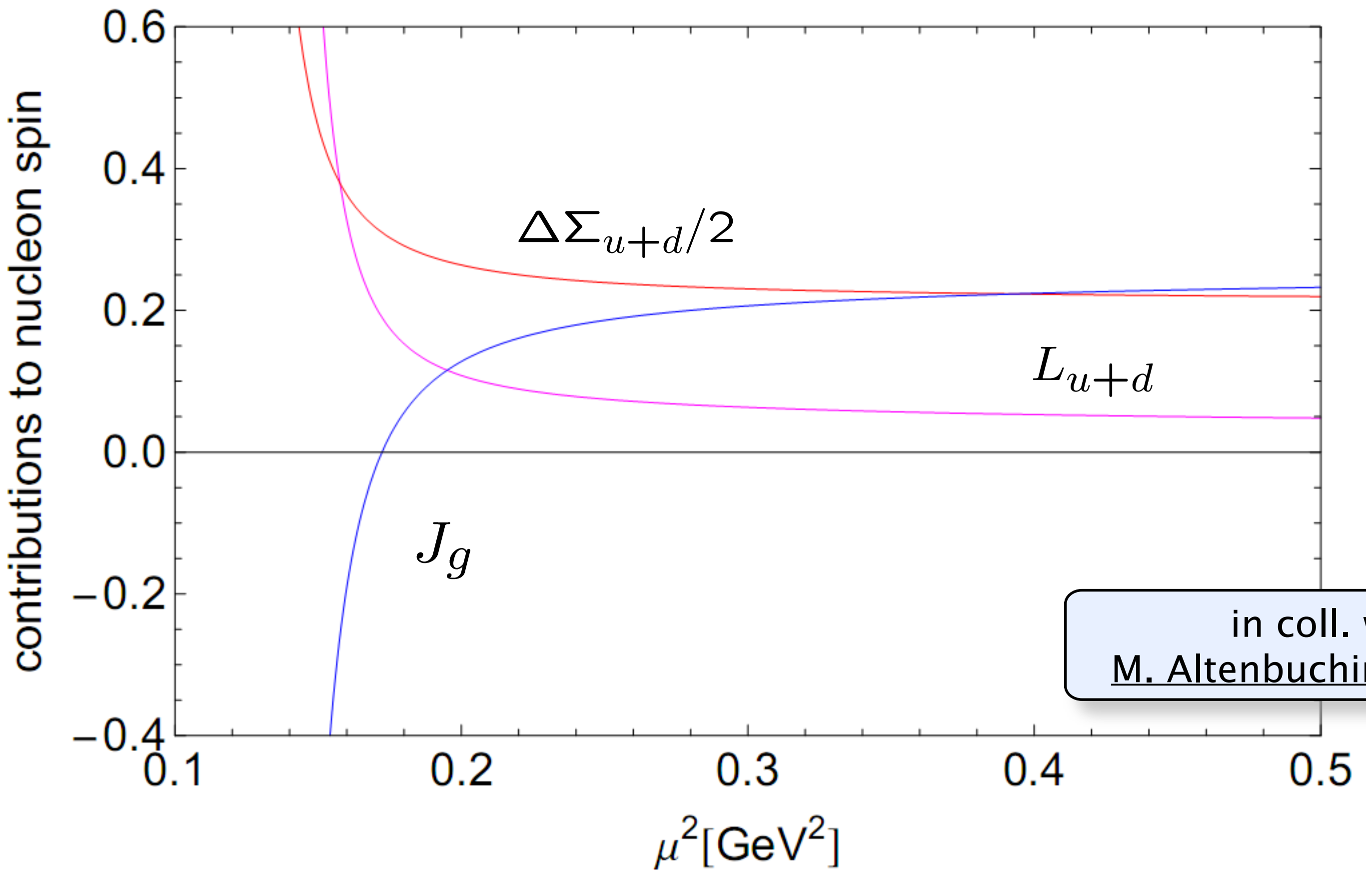
Lattice vs relativistic quark model

P.Haegler BNL summer talk - QCD evolution

(Wakamatsu 2005; Thomas, PRL 2008)



← 4 GeV^2

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in coll. with LHPC and
M. Altenbuchinger, W. Weise (TUM)

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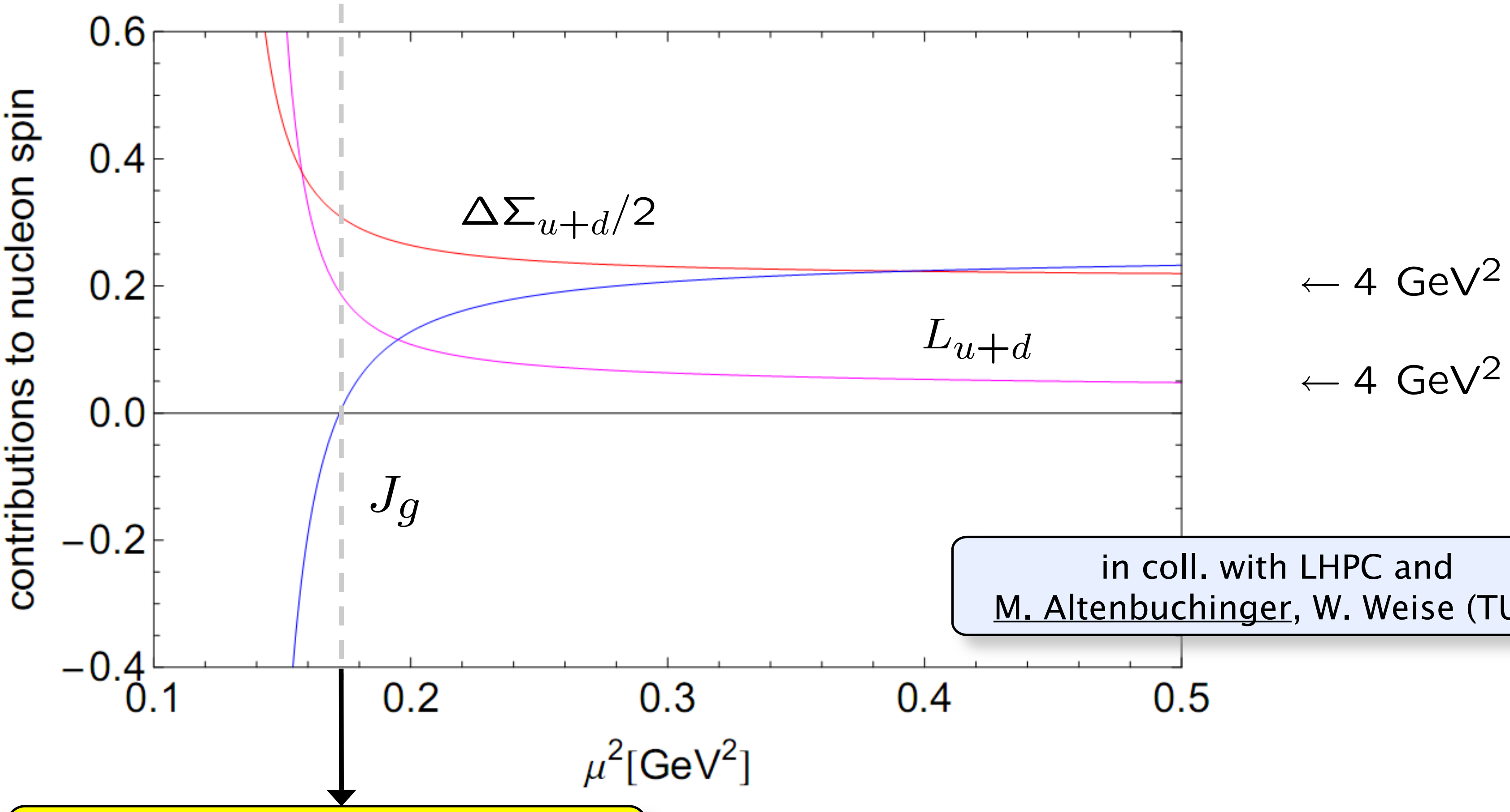

in coll. with LHPC and
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$L^{u+d} \approx 0.19 \approx 38\%$ of $1/2$
$\Delta\Sigma^{u+d}/2 \approx 0.31 \approx 62\%$ of $1/2$

lattice + evolution

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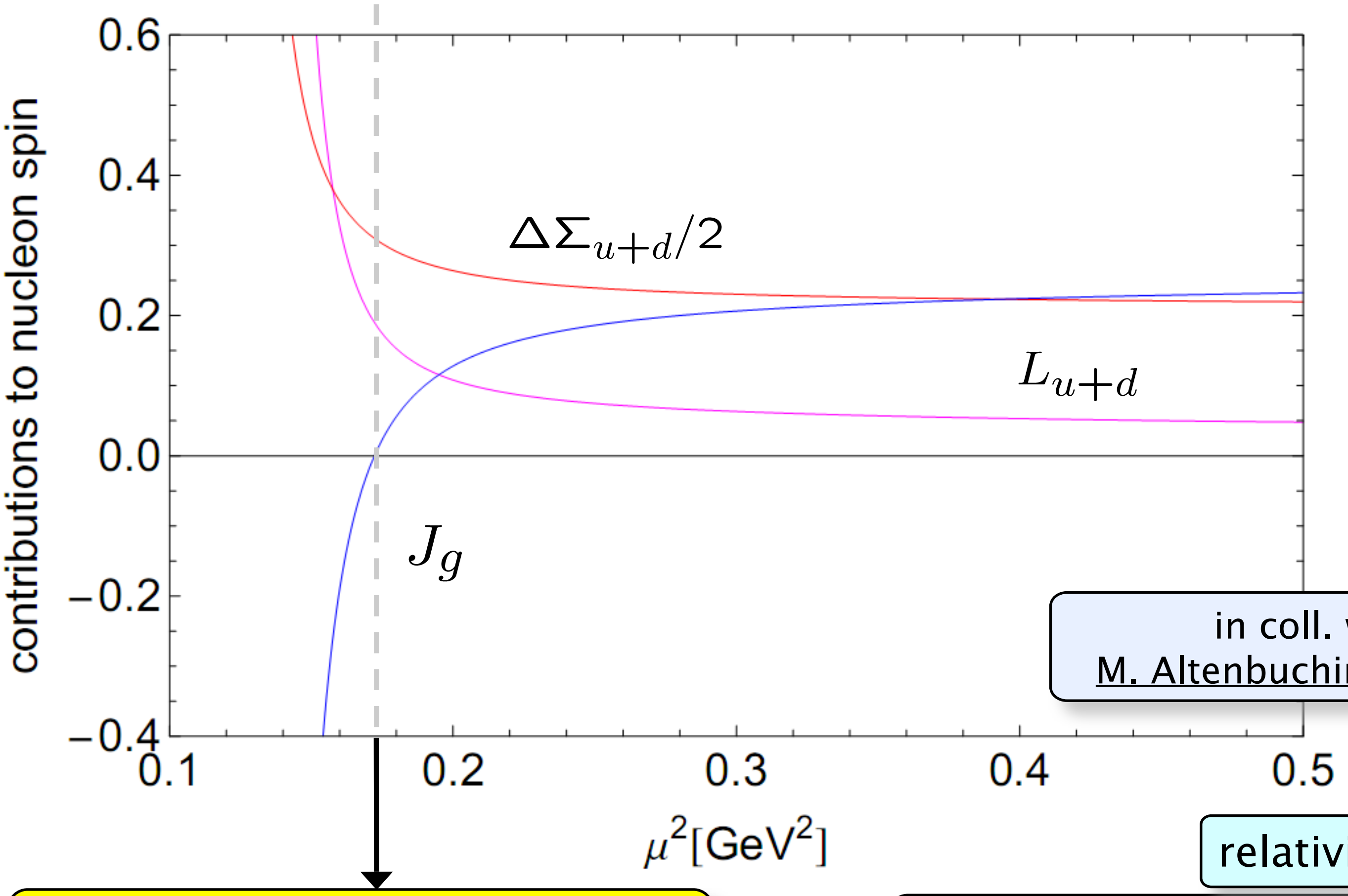

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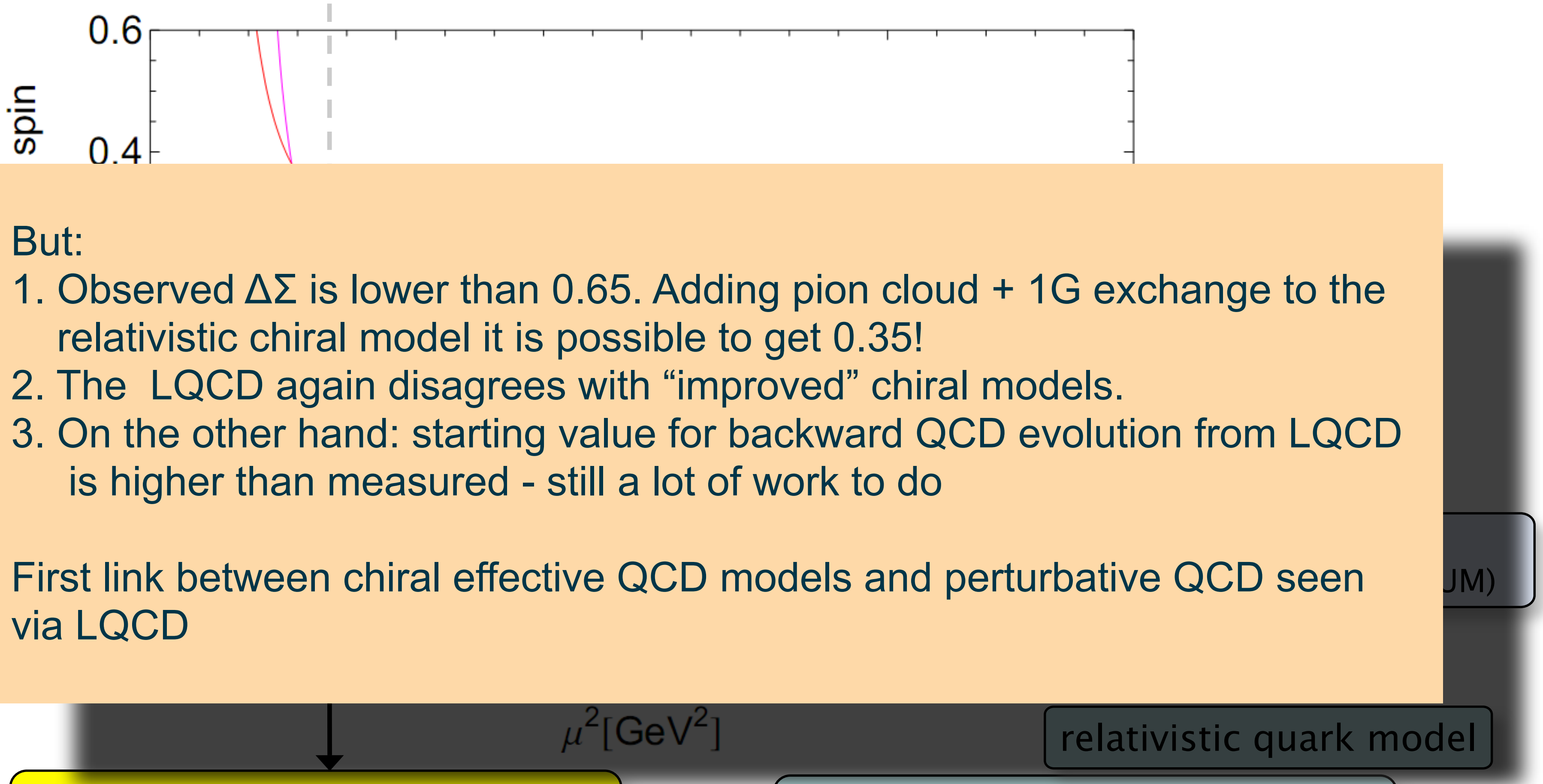

But:

1. Observed ΔΣ is lower than 0.65. Adding pion cloud + 1G exchange to the relativistic chiral model it is possible to get 0.35!
2. The LQCD again disagrees with “improved” chiral models.
3. On the other hand: starting value for backward QCD evolution from LQCD is higher than measured - still a lot of work to do

First link between chiral effective QCD models and perturbative QCD seen via LQCD

$L^{u+d} \approx 0.19 \approx 38\%$ of $1/2$

$\Delta\Sigma^{u+d}/2 \approx 0.31 \approx 62\%$ of $1/2$

lattice + evolution

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Nontrivial nucleon spin structure

1/1 - mass of the nucleon (in GeV)
1/2 - momentum carried by quarks/gluons
1/3 - spin carried by quarks
1/4 - orbital angular momentum of gluons
1/5 - ???

Nontrivial nucleon spin structure

1. Fast moving nucleon is a 3-D object
2. OAM of valence quarks have opposite sign and compensate ~0
3. If gluon polarisation is small then nucleon spin is composed of: spin of quarks and OAM of gluons

4. New measurements are needed:
   again gluon polarisation, GPDs & TMDs, DY
   COMPASS II, JLAB 12 and EIC

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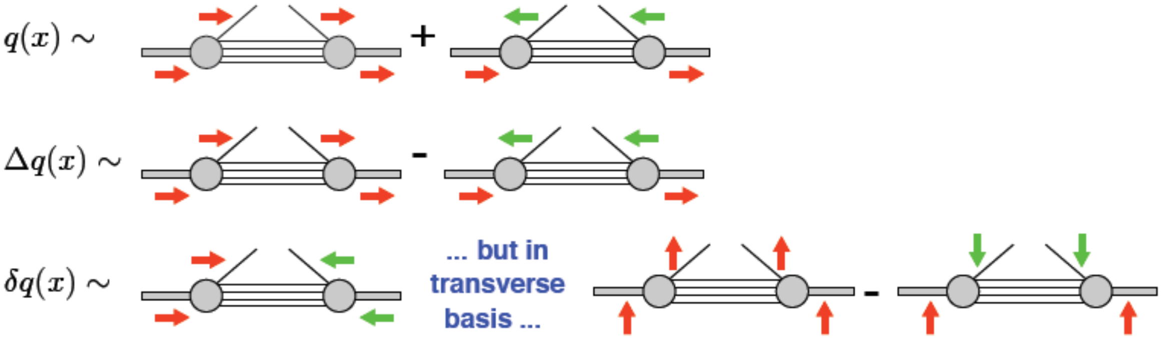
Spin provides a unique opportunity to probe the inner structure of a composite system such as the proton

Dziekuję za uwagę

Properties of transversity



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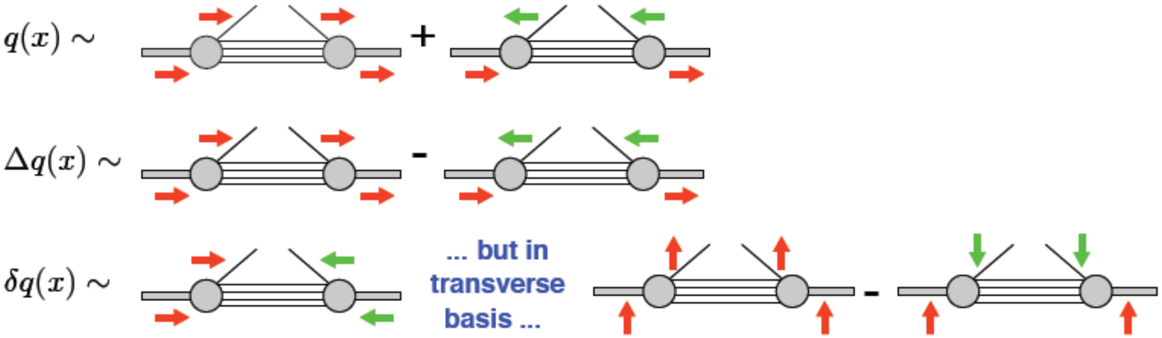
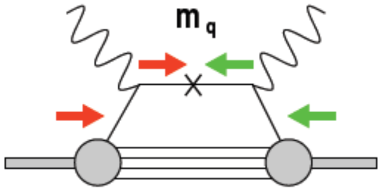
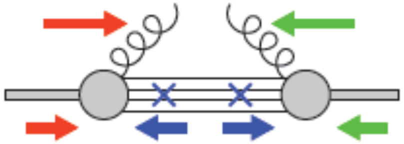


- **Chiral Odd**

Helicity flip amplitudes occur only at $\mathcal{O}(m_q/Q)$ in inclusive DIS ...



- **No Gluons**



Angular momentum conservation: $\Lambda - \lambda = \Lambda' - \lambda'$

$\Rightarrow$ transversity has *no gluon* component

$\Rightarrow$ different Q^2 *evolution* than $\Delta q(x)$

Test of Bjorken sum rule

COMPASS data

$$\Gamma_1^{NS}(Q^2) = \frac{1}{6}\frac{g_A}{g_V}C_1^{NS}(Q^2)$$

$C^{NS} = 0.89$ $Q^2 = 3\ (\mathrm{GeV/c})^2$

Non-singlet Q2 dependence is not dependent from gluon densities:

$$g_1^{NS}(x, Q^2) = \frac{1}{6}\int_x^1 \frac{dx'}{x'} C^{NS}\left(\frac{x}{x'}, \alpha_s(Q^2)\right) \Delta q_3(x', Q^2)$$

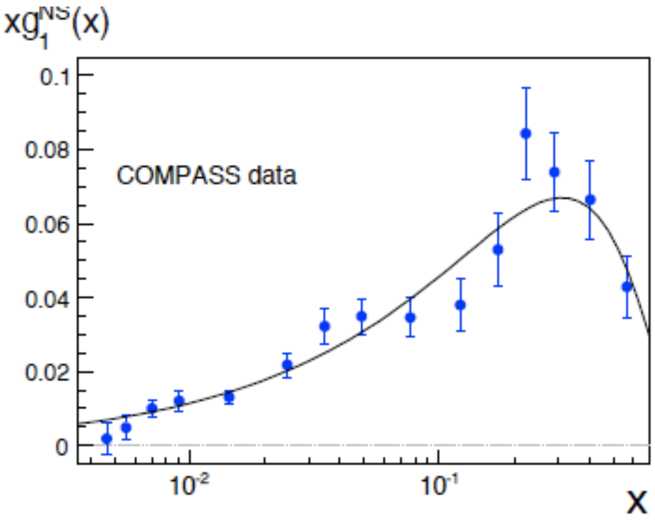
Parametrisation of the isovector spin density:

$$\Delta q_3(x) = \eta_3 \frac{x^{\alpha_3}(1-x)^{\beta_3}}{\int_0^1 x^{\alpha_3}(1-x)^{\beta_3}dx}$$

Fit with two different programs:

- $*$ $(x,\ Q^2)$ space
- $*$ space of moments

Param.	Value
η_3	1.28 ± 0.07
α_3	-0.22 ± 0.07
β_3	$2.2\pm^{0.5}_{0.4}$
χ^2/NDF	$14.4/12$
Prob.	0.27



$|g_A/g_V| = 1.28 \pm 0.07(\mathrm{stat.}) \pm 0.10(\mathrm{syst.})$

- dominant systematic error: beam and target polarisation
- PDG value: $g_A/g_V = 1.268 \pm 0.003$

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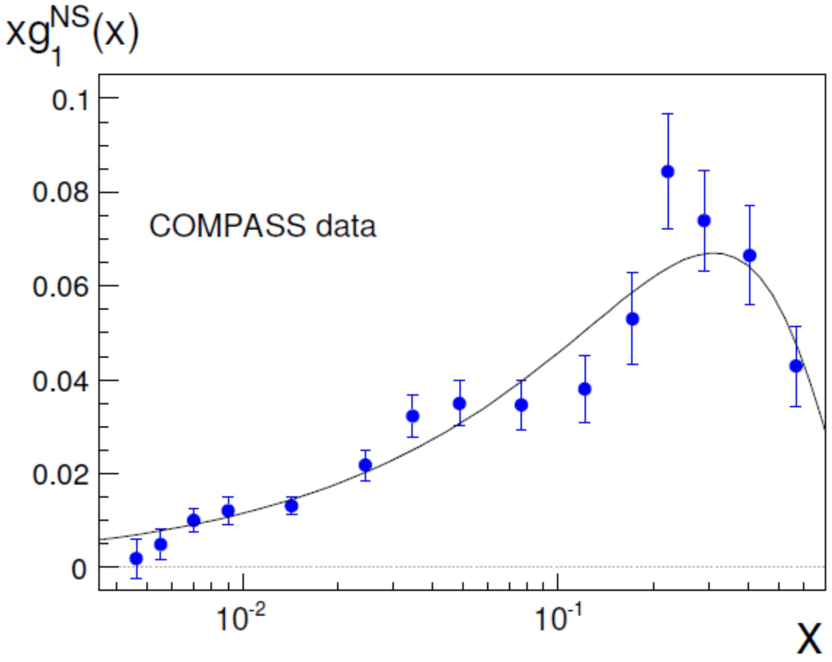
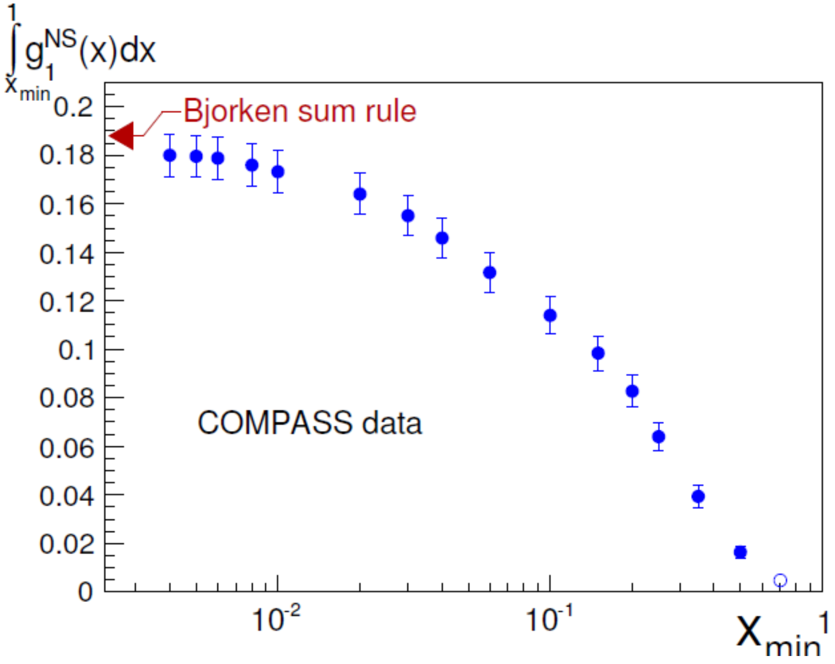
Phys. Lett. B 690 (2010) 466–472





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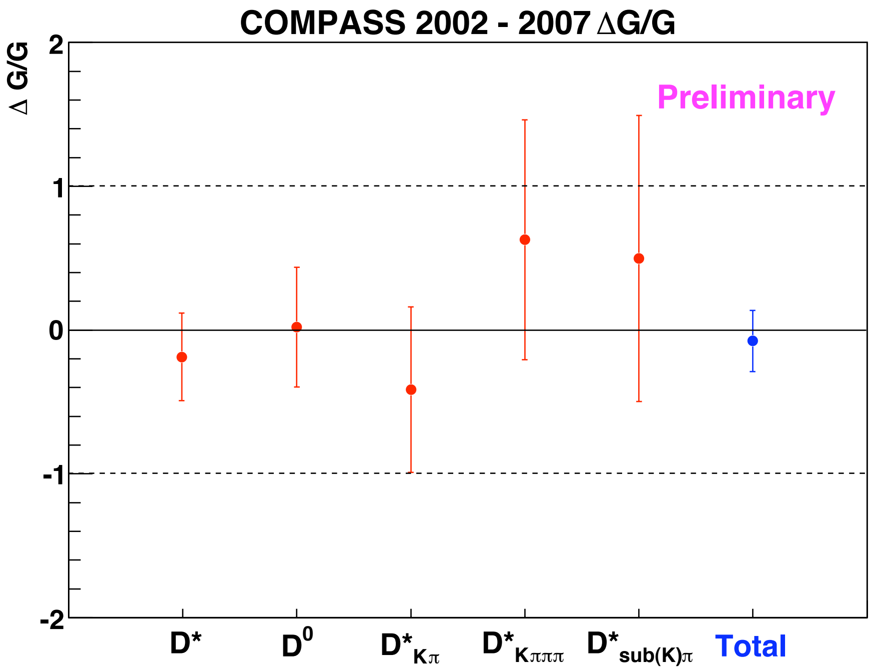
Final result in the LO QCD

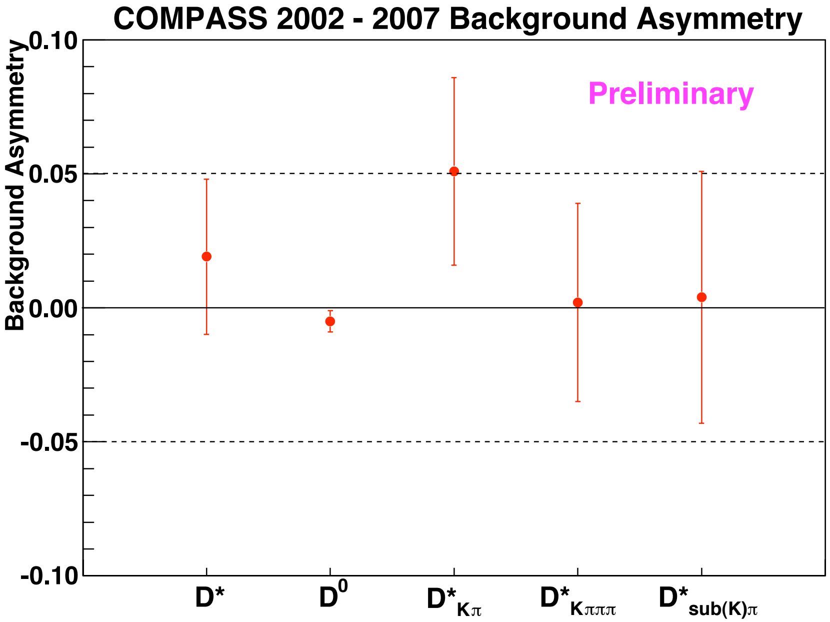


$$\frac{\Delta G}{G} = -0.08 \pm 0.21 \pm (0.11)$$

$$\langle x_G \rangle \approx 0.11 \qquad \mu^2 = 13 \frac{GeV^2}{c^2}$$

Notice: signal and background asymmetries are extracted in the same time

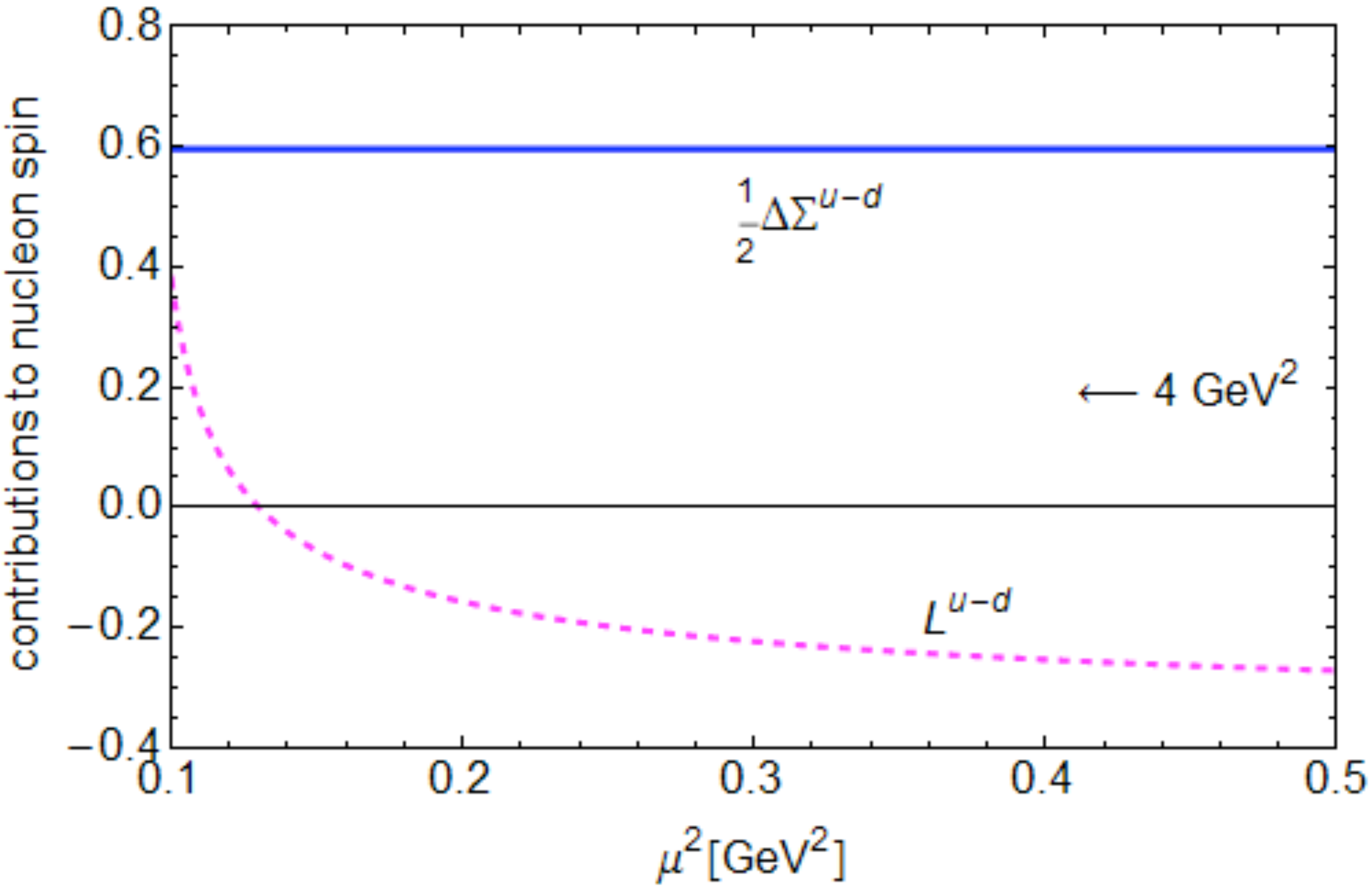
$$A^{measured} = fP_T P_b \left(\frac{S}{S+B} A^{signal} + \frac{B}{S+B} A^B\right)$$



Lattice QCD vs relativistic quark models – QCD evolution

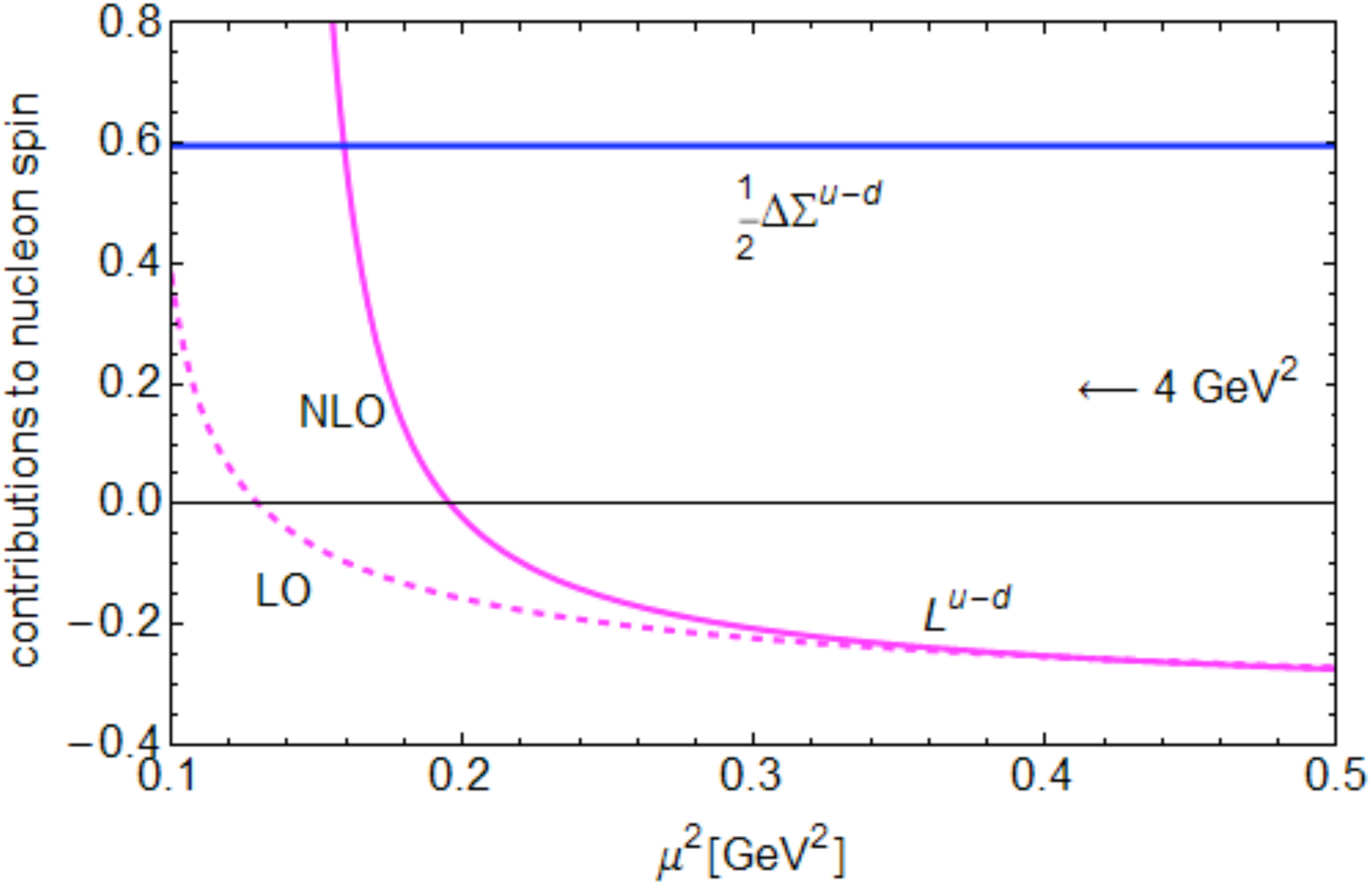
(Wakamatsu 2005; Thomas, PRL 2008)

$$L^{u-d}(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{\beta_0}\left(\frac{-16}{9}\right)} \left\{L^{u-d}(t_0) + \frac{1}{2}\Delta\Sigma^{u-d}\right\} - \frac{1}{2}\Delta\Sigma^{u-d}$$

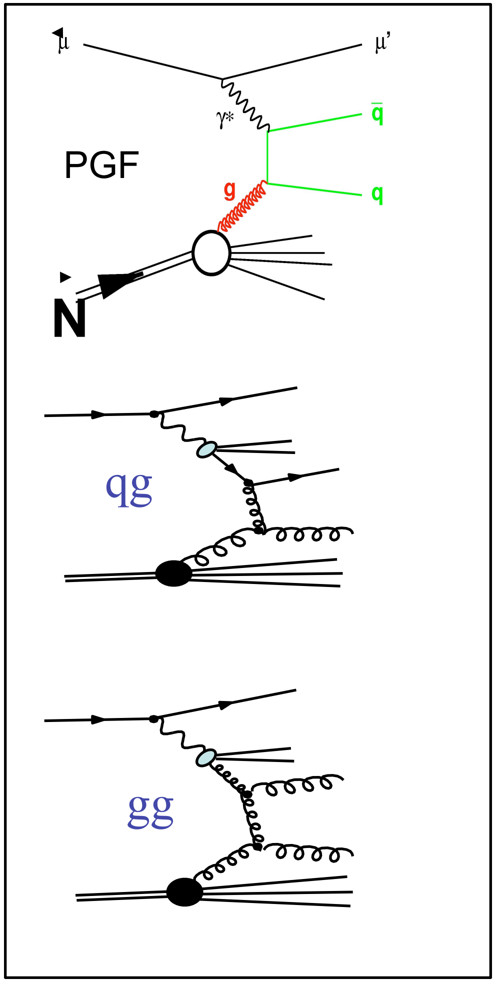


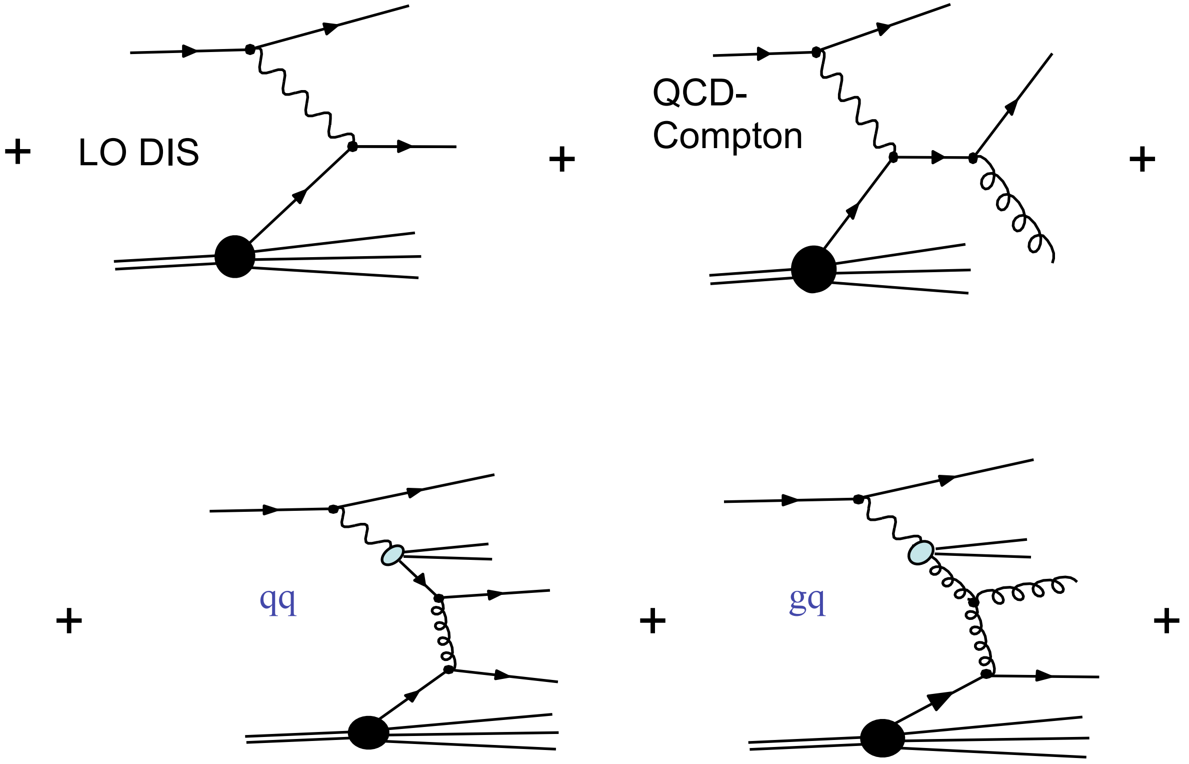
Lattice QCD vs relativistic quark models – QCD evolution

(Wakamatsu 2005; Thomas, PRL 2008)



Resolved photon contribution



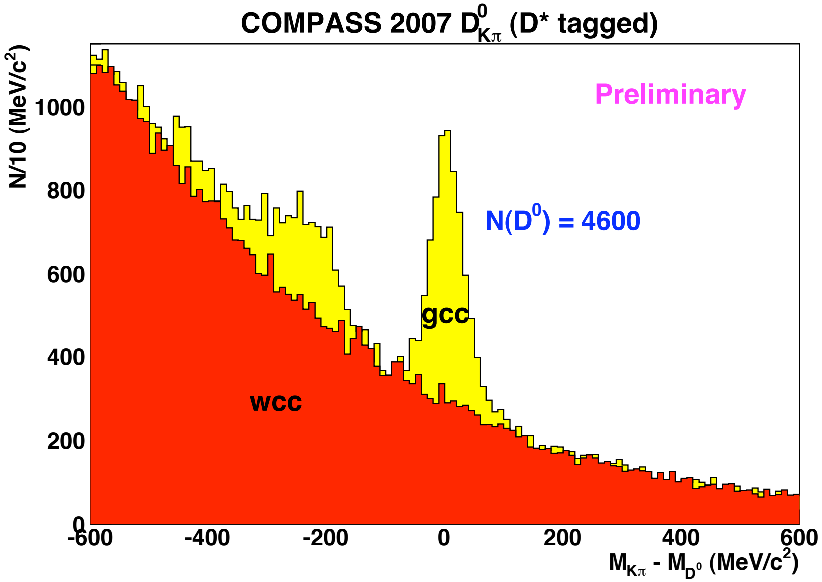


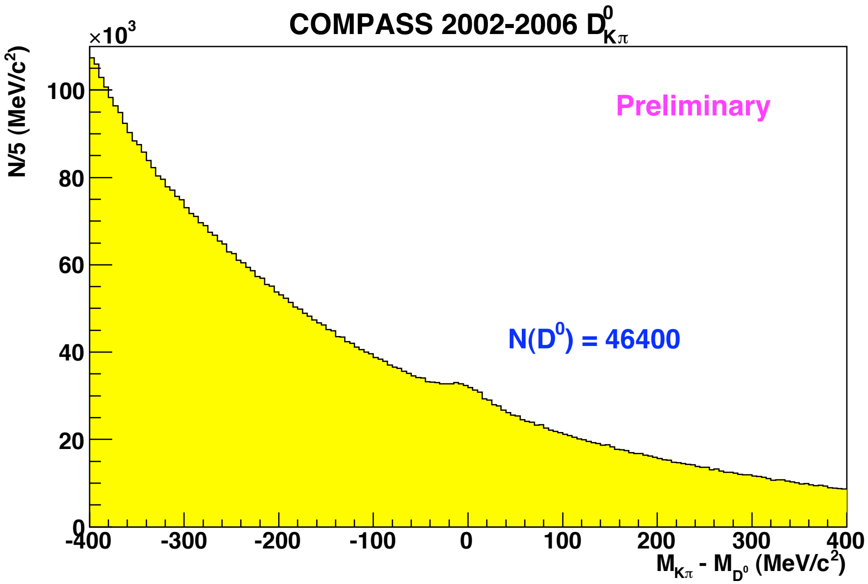
Important for Low Q^2 analysis (Q^2<1(GeV/c)^2)

D^0 meson reconstruction

Statistically weighted method based on NN approach used

- **Events considered** *(resulting from the c quarks fragmentation)*:
  - $D^0 \to K\pi$ *(BR: 4%)*
  - $D^* \to D^0\pi_s$ *(30% D^0 tagged with a D^*)*
    - $D^0 \to K\pi$
    - $D^0 \to K\pi\pi^0$ *(BR: 13%)* → **not directly reconstructed**
    - $D^0 \to K\pi\pi\pi$ *(BR: 7.5%)*
    - $D^0 \to$ sub(K)π → **no RICH ID for Kaons** *(p < 9 GeV/c)*





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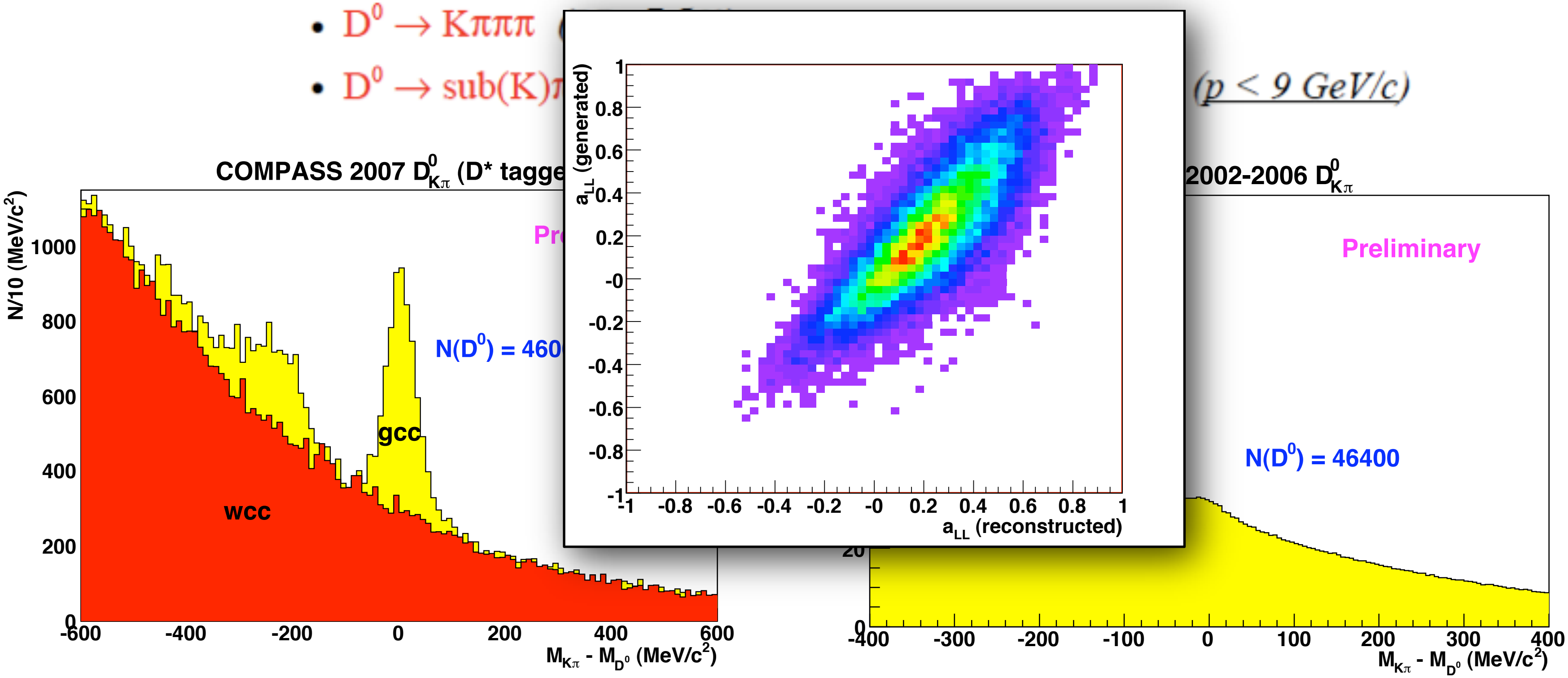
    - $D^0 \to \text{sub}(K)\pi$ *(<u>$p < 9$ GeV/c</u>)*



TMD PDFs - SIDIS cross section decomposition

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&+S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&+|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg] \\
&+|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

unpolarized
target

longitudinally
polarized target

transversely
polarized target

A.Bacchetta et al

JHEP 0702:093,2007

E-print number: hep-ph/0611265

TMD PDFs - SIDIS cross section decomposition

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&\quad + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h} + \lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}
\end{aligned}
$$

unpolarized target

Cahn & Boer-Mulders

$$
\begin{aligned}
&+ S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&+ S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]
\end{aligned}
$$

longitudinally polarized target

$$
\begin{aligned}
&+ |\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\quad + \varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)} + \varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\quad + \sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S} + \sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)} \\
&+ |\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)} + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\quad + \sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},
\end{aligned}
$$

transversely polarized target

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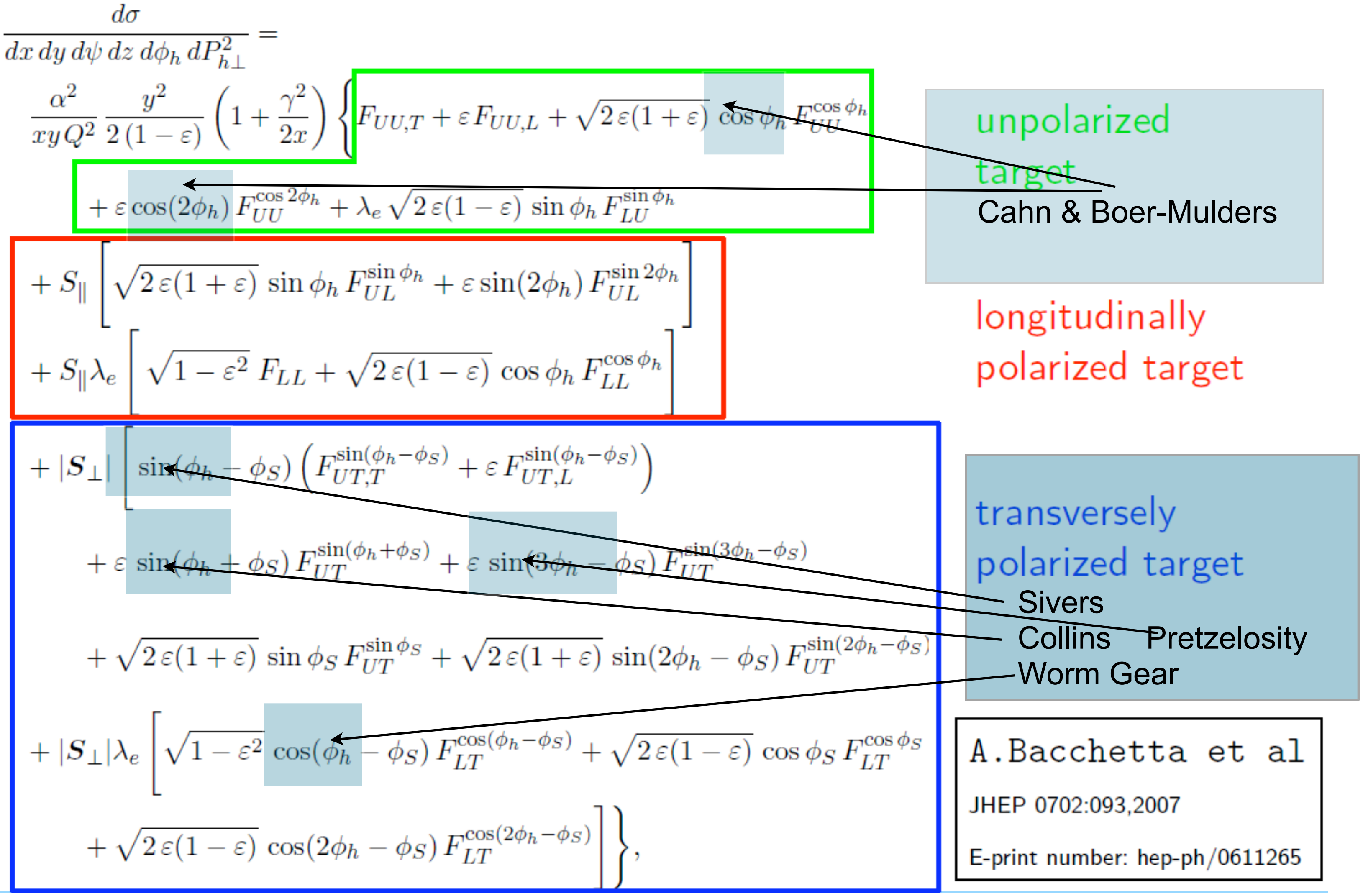
TMD PDFs - SIDIS cross section decomposition



$$\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xyQ^2}\,\frac{y^2}{2\,(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h}$$
$$+\,\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\,\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h}$$
$$+\,S_\parallel\left[\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right]$$
$$+\,S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right]$$
$$+\,|\boldsymbol{S}_\perp|\Bigg[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)$$
$$+\,\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)}$$
$$+\,\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\,\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Bigg]$$
$$+\,|\boldsymbol{S}_\perp|\lambda_e\Bigg[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S}$$
$$+\,\sqrt{2\,\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Bigg]\Bigg\},$$

unpolarized target

Cahn & Boer-Mulders

longitudinally polarized target

transversely polarized target

Sivers

Collins

Pretzelosity

Worm Gear

A.Bacchetta et al

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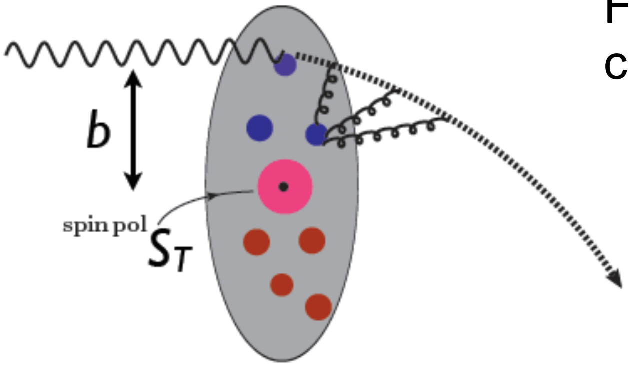
Sivers function

Chromodynamics lensing:

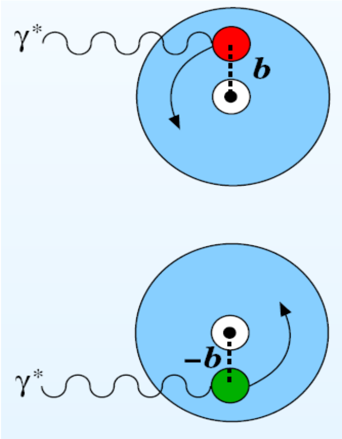


$$\vec{S} \cdot (\hat{P} \times \vec{k}_\perp)\, f_{1T}^\perp(x, \vec{k}_\perp^2)$$

M.Burhard idea:

Final State Interactions (non-trivial gauge link)
conjecture: k_T related to impact parameter b via FT?
transverse momentum always from
spacial deformation?

Spin-orbit structure



3-dim picture of the nucleon,
Connection to GPDs,

Potential measurement of angular momentum - via X.Ji sum rule - integral of GPDs

A catalog of PDFs (II)

- $f_1(x, k_T^2)$: unpolarized. Integrated in k_T^2 gives the usual $f_1(x)$.
- $g_{1L}(x, k_T^2)$: longitudinally polarized. When integrated over k_T^2 it is the helicity function $g_1(x)$. From its 1st moment one can obtain $\Delta\Sigma = \Delta u + \Delta d + \Delta s$

  $\hookrightarrow$COMPASS DIS and SIDIS results:
  PLB647(2007)8-17;
  PLB647(2007)330-340;
  PLB660(2008)458-465.
- $h_1(x, k_T^2)$: transversely polarized. When integrated over k_T^2 it survives, giving the transversity function $h_1(x)$.

  $\hookrightarrow$COMPASS SIDIS results:
  PRL94(2005)202002;
  NPB765(2007)31-70;
  PLB673(2009)127-135.

A catalog of PDFs (II)

A fly in the ointment

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  PRL94(2005)202002;
  NPB765(2007)31-70;
  PLB673(2009)127-135.

A catalog of PDFs (III)

TMD PDFs - k_T dependent

- $f_{1T}^{\perp}(x, k_T^2)$: Sivers function. It describes the distortion of the probability distribution of a non-polarized quark when it is inside a transversely polarized nucleon.

  $\hookrightarrow$COMPASS DIS results:
  PRL94(2005)202002;
  NPB765(2007)31-70;
  PLB673(2009)127-135.

- $h_1^{\perp}(x, k_T^2)$: Boer-Mulders function. It describes the correlation between the transverse spin and the transverse momentum of a quark inside the unpolarized hadron.

- $h_{1T}^{\perp}(x, k_T^2)$: Pretzelosity function. It describes the transverse polarization of a quark, along its intrinsic k_T direction. It allows to access the orbital angular momentum information.

TMD PDFs - SIDIS cross section decomposition

$$
\begin{aligned}
\frac{d\sigma}{dx\,dy\,d\phi_S\,d\phi_h\,dz_h\,dP_{hT}^2} = {} & \frac{e^4}{32\pi^2 x Q^2}\frac{y}{1-\varepsilon}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T}+\varepsilon\,F_{UU,L}+\varepsilon\,cos(2\phi_h)F_{UU}^{cos2\phi_h} \\
& +\sqrt{2\varepsilon(1-\varepsilon)}\big(cos\phi_h F_{UU}^{cos\phi_h}+h_l sin\phi_h F_{LU}^{sin\phi_h}\big) \\
& +S_{\|}\Big[\sqrt{2\varepsilon(1+\varepsilon)}sin\phi_h F_{UL}^{sin\phi_h}+\varepsilon\,sin(2\phi_h)F_{UL}^{sin2\phi_h}\Big] \\
& +S_{\|}h_l\Big[\sqrt{1-\varepsilon^2}F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}cos\phi_h F_{LL}^{cos\phi_h}\Big] \\
& +|S_\perp|\Big[sin(\phi_h-\phi_S)\big(F_{UT,T}^{sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{sin(\phi_h-\phi_S)}\big) \\
& \qquad +\varepsilon\,sin(3\phi_h-\phi_S)F_{UT}^{sin(3\phi_h-\phi_S)}+\varepsilon\,sin(\phi_h+\phi_S)F_{UT}^{sin(\phi_h+\phi_S)} \\
& \qquad +\sqrt{2\varepsilon(1+\varepsilon)}\big(sin\phi_S F_{UT}^{sin\phi_S}+sin(2\phi_h-\phi_S)F_{UT}^{sin(2\phi_h-\phi_S)}\big)\Big] \\
& +|S_\perp|h_l\Big[\sqrt{1-\varepsilon^2}cos(\phi_h-\phi_S)F_{LT}^{cos(\phi_h-\phi_S)} \\
& \qquad +\sqrt{2\varepsilon(1-\varepsilon)}\big(cos\phi_S F_{LT}^{cos\phi_S}+cos(2\phi_h-\phi_S)F_{LT}^{cos(2\phi_h-\phi_S)}\big)\Big]\Big\},
\end{aligned}
$$

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& +|S_\perp|\Big[sin(\phi_h-\phi_S)\left(F_{UT,T}^{sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{sin(\phi_h-\phi_S)}\right) \\
& \qquad +\varepsilon\,sin(3\phi_h-\phi_S)F_{UT}^{sin(3\phi_h-\phi_S)}+\varepsilon\,sin(\phi_h+\phi_S)F_{UT}^{sin(\phi_h+\phi_S)} \\
& \qquad +\sqrt{2\varepsilon(1+\varepsilon)}\left(sin\phi_S F_{UT}^{sin\phi_S}+sin(2\phi_h-\phi_S)F_{UT}^{sin(2\phi_h-\phi_S)}\right)\Big] \\
& +|S_\perp|h_l\Big[\sqrt{1-\varepsilon^2}\,cos(\phi_h-\phi_S)F_{LT}^{cos(\phi_h-\phi_S)} \\
& \qquad +\sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_S F_{LT}^{cos\phi_S}+cos(2\phi_h-\phi_S)F_{LT}^{cos(2\phi_h-\phi_S)}\right)\Big]\Big\},
\end{aligned}
$$

Cahn & Boer-Mulders

unpolarised target

TMD PDFs - SIDIS cross section decomposition

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& + \sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_h F_{UU}^{cos\phi_h} + h_l sin\phi_h F_{LU}^{sin\phi_h}\right) \\
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& + S_{\|}h_l\left[\sqrt{1-\varepsilon^2}F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)}cos\phi_h F_{LL}^{cos\phi_h}\right] \\
& + |S_{\perp}|\Big[sin(\phi_h-\phi_S)\left(F_{UT,T}^{sin(\phi_h-\phi_S)} + \varepsilon\ F_{UT,L}^{sin(\phi_h-\phi_S)}\right) \\
& \qquad + \varepsilon\ sin(3\phi_h-\phi_S)F_{UT}^{sin(3\phi_h-\phi_S)} + \varepsilon\ sin(\phi_h+\phi_S)F_{UT}^{sin(\phi_h+\phi_S)} \\
& \qquad + \sqrt{2\varepsilon(1+\varepsilon)}\left(sin\phi_S F_{UT}^{sin\phi_S} + sin(2\phi_h-\phi_S)F_{UT}^{sin(2\phi_h-\phi_S)}\right)\Big] \\
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& \qquad + \sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_S F_{LT}^{cos\phi_S} + cos(2\phi_h-\phi_S)F_{LT}^{cos(2\phi_h-\phi_S)}\right)\Big]\Big\},
\end{aligned}
$$

Cahn & Boer-Mulders

unpolarised target

longitudinaly polarised target

transversely
polarised target

Sivers
Collins
Pretzelosity
Worm Gear

A catalog of PDFs (IV)

Beyond collinear approximation - k_T dependence

$$\begin{aligned}
F_{UU,T} &\sim \sum_q e_q^2 \cdot f_1^q \otimes D_q^h, & F_{LT}^{\cos(\phi_h-\phi_S)} &\sim \sum_q e_q^2 \cdot g_{1T}^q \otimes D_q^h, \\
F_{LL} &\sim \sum_q e_q^2 \cdot g_{1L}^q \otimes D_q^h, & F_{UT,T}^{\sin(\phi_h-\phi_S)} &\sim \sum_q e_q^2 \cdot f_{1T}^{\perp q} \otimes D_q^h, \\
F_{UU}^{\cos 2\phi_h} &\sim \sum_q e_q^2 \cdot h_1^{\perp q} \otimes H_1^{\perp q}, & F_{UT}^{\sin(\phi_h+\phi_S)} &\sim \sum_q e_q^2 \cdot h_1^q \otimes H_1^{\perp q}, \\
F_{UL}^{\sin 2\phi_h} &\sim \sum_q e_q^2 \cdot h_{1L}^{\perp q} \otimes H_1^{\perp q}, & F_{UT}^{\sin(3\phi_h-\phi_S)} &\sim \sum_q e_q^2 \cdot h_{1T}^{\perp q} \otimes H_1^{\perp q}.
\end{aligned}$$

COMPASS data

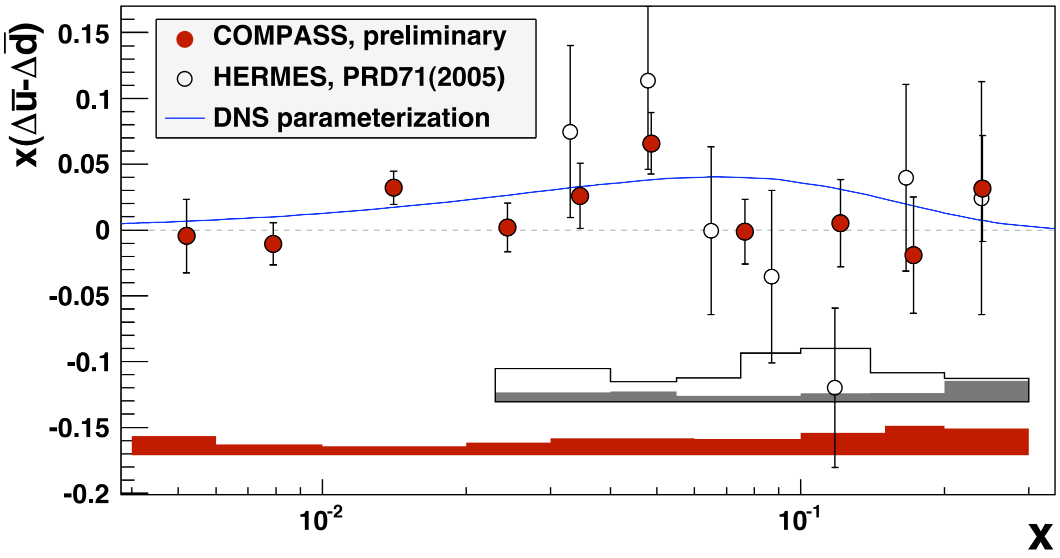
$\Delta\bar{u} \geq \Delta\bar{d}$?

Phys. Rev. Lett. 18, (1967) 1174

Pauli exclusion principle, Δ resonance etc.

NMC,E866
unpolarized asymmetry:

$$\int_0^1 \left(\bar{d} - \bar{u}\right)dx = 0.118 \pm 0.012$$



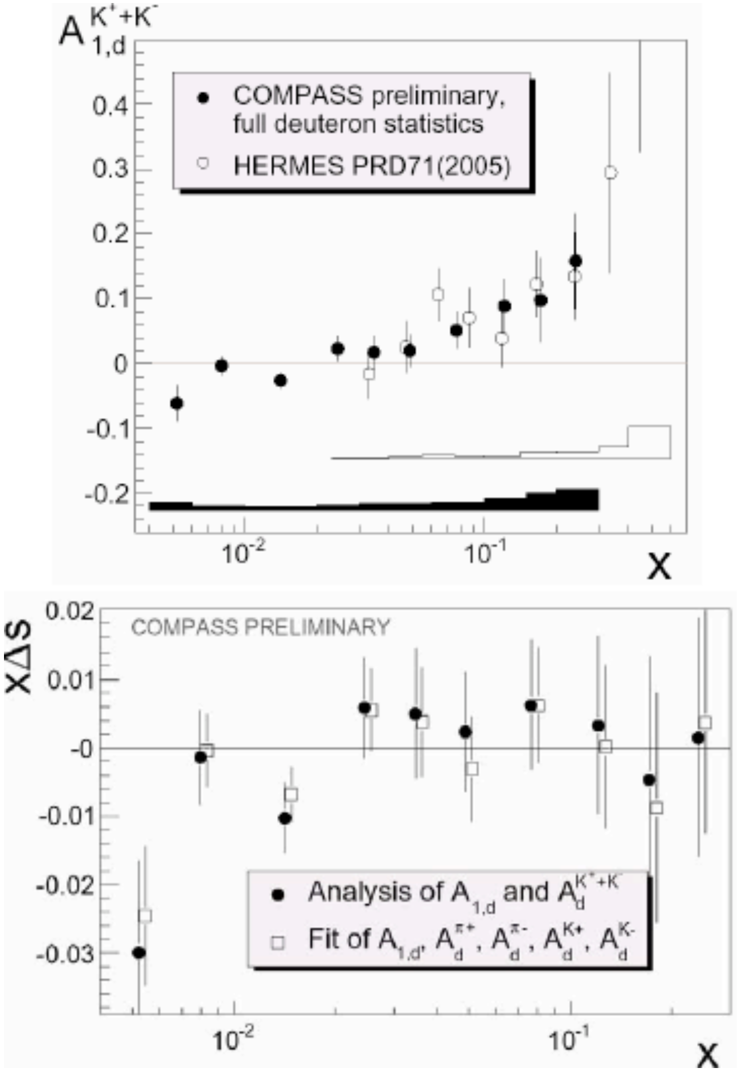
HERMES $(Q^2 = 2.5\ (\mathrm{GeV}/c)^2)$:

$$\int_{0.023}^{0.3} (\Delta\bar{u} - \Delta\bar{d})dx = 0.048 \pm 0.057(\mathrm{stat.}) \pm 0.028(\mathrm{syst.})$$

COMPASS $(Q^2 = 3\ (\mathrm{GeV}/c)^2)$:

$$\int_{0.004}^{0.3} (\Delta\bar{u} - \Delta\bar{d})dx = 0.052 \pm 0.035(\mathrm{stat.}) \pm 0.013(\mathrm{syst.})$$

COMPASS data



$$\boxed{\frac{\Delta s}{s} = A_1^d + \left(A_1^{K^+ + K^-} - A_1^d\right)\frac{Q/s + \alpha}{\alpha - 0.8}}$$

$$\alpha = \frac{2R_{UF} + 2R_{SF}}{3R_{UF} + 2} \qquad Q = u + \bar{u} + d + \bar{d}$$

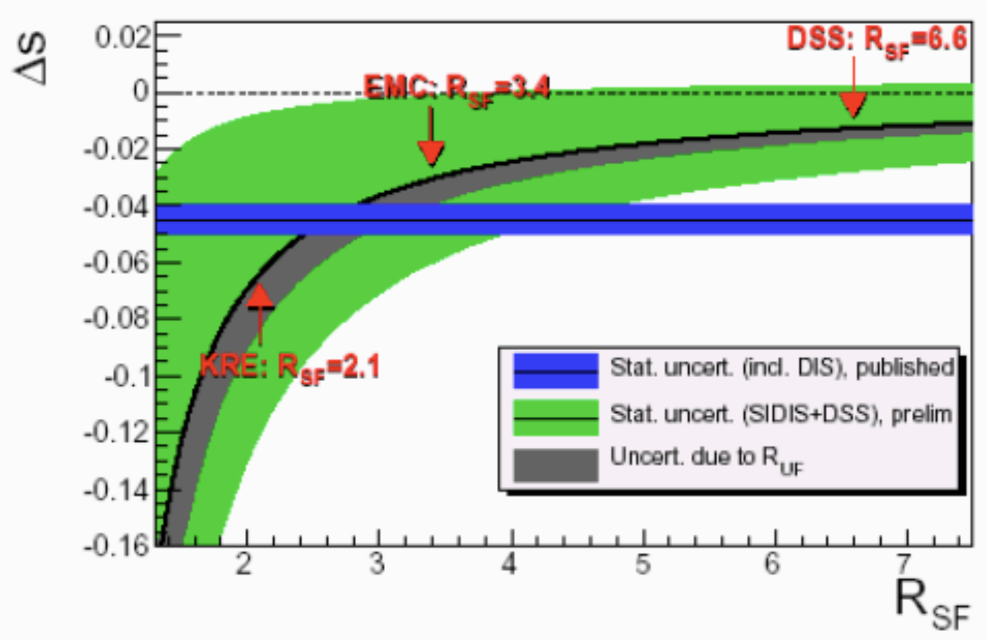
$$R_{UF} = \frac{\int D_d^{K^+}(z)dz}{\int D_u^{K^+}(z)dz} \qquad R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z)dz}{\int D_u^{K^+}(z)dz}$$

if $A_1^d = A^{K^+ + K^-} \Rightarrow \Delta s \geq 0$, insensitive to FFs

if $A^{K^+ + K^-} < 0$ (at low x) $\Rightarrow \Delta s < 0$

Δs from charged Kaon's asymmetry

COMPASS data



Phys.Lett. B 680 (2009)217-114

$$R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z)dz}{\int D_{u}^{K^+}(z)dz}$$

$$R_{UF} = \frac{\int D_{d}^{K^+}(z)dz}{\int D_{u}^{K^+}(z)dz}$$

- R_{UF} fixed at 0.14 from the DSS fragmentation functions
- Large statistical uncertainty due to R_{SF}, slight dependence on R_{UF}
- If R_{SF}>5 Δs(SIDIS)>Δs(DIS) and Δs <0 for x<0.004
- If R_{SF}<4: A^K becomes insensitive to Δs

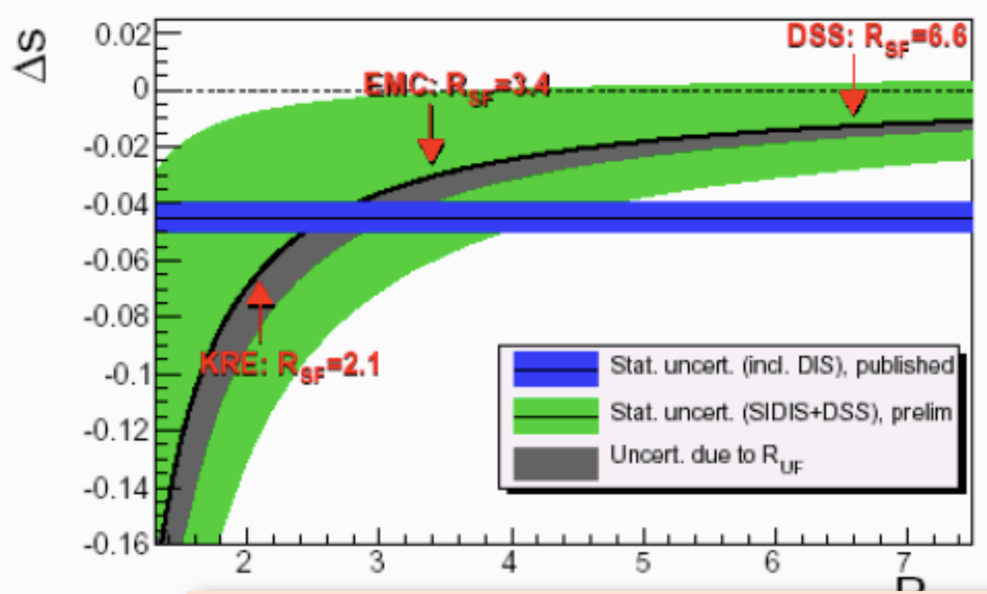
Δs (inclusive) = –0.045 ± 0.005 ± 0.010

Δs (SIDIS) = –0.01 ± 0.01 ± 0.01

in LO pQCD

Δs from charged Kaon’s asymmetry

COMPASS data



Phys.Lett. B 680 (2009)217-114

$$R_{SF} = \frac{\int D_{\bar{s}}^{K^+}(z)dz}{\int D_u^{K^+}(z)dz}$$

$$R_{UF} = \frac{\int D_d^{K^+}(z)dz}{\int D_u^{K^+}(z)dz}$$

The lesson:
Important piece of information:
precise measurement of FFs

- R_{U}
- Lar
- If R
- If R_{SF}<4: A^K becomes insensitive to Δs

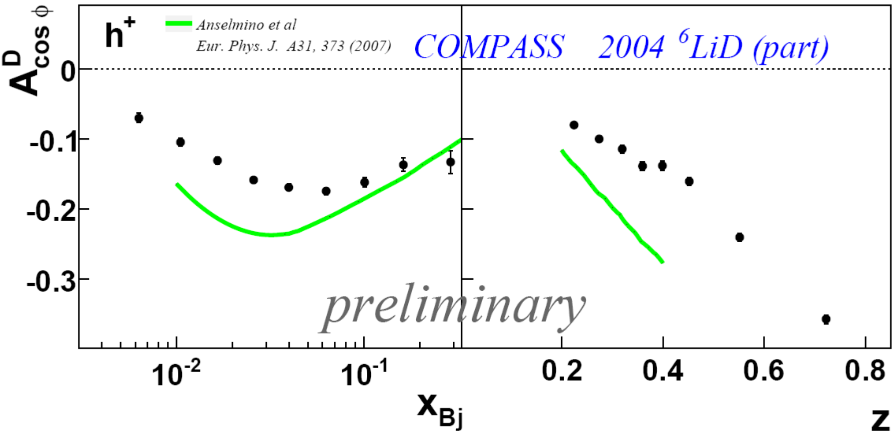
Δs (inclusive) = –0.045 ± 0.005 ± 0.010

Δs (SIDIS) = –0.01 ± 0.01 ± 0.01

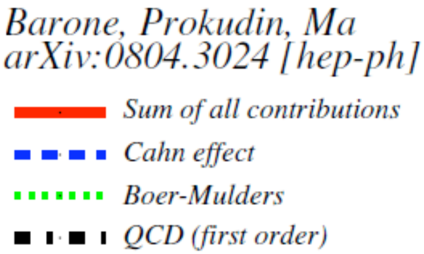
in LO pQCD

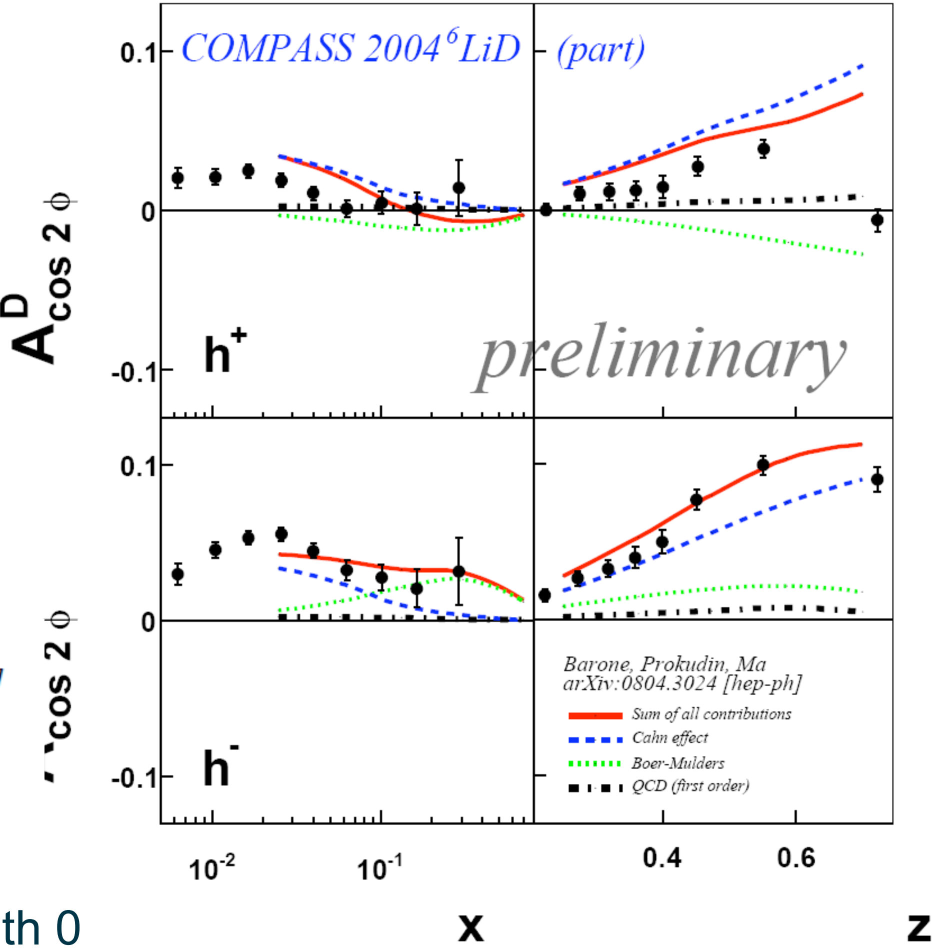
COMPASS data: cos(ϕ) & cos(2ϕ) modulation

M. Anselmino, M. Boglione, A. Prokudin, C. Türk
Eur. Phys. J. A 31, 373-381 (2007)
does not include Boer – Mulders contribution



comparison with theory



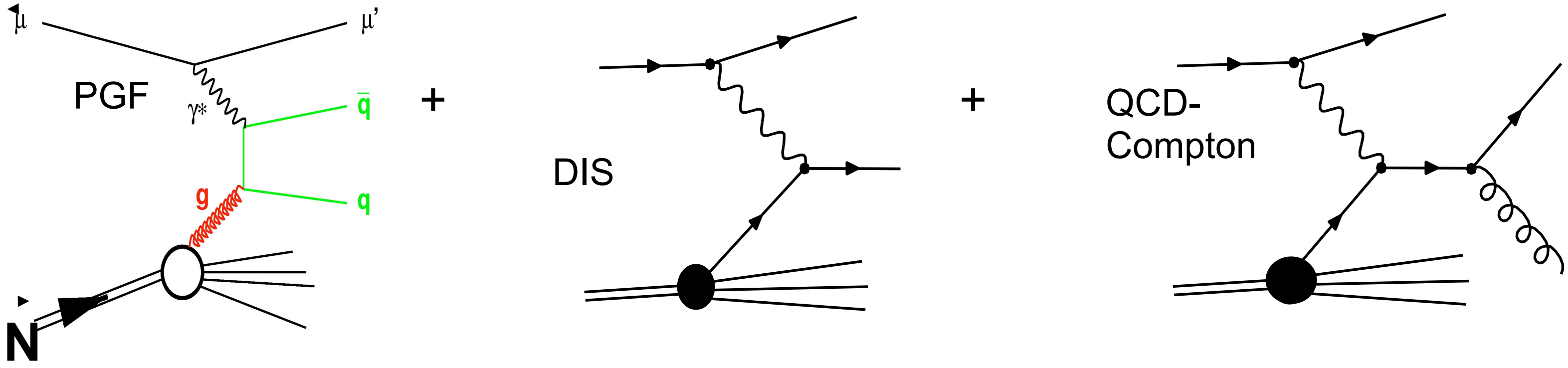


pretzelosity and worm-gear compatible with 0

Observation of the high-p_T hadron pairs

R.D.Carlitz, J.C.Collins and A.H.Mueller, Phys.Lett.B 214, 229 (1988)

Revisited by A.Bravar,D.von Harrach and A.Kotzinian, Phys.Lett.B 421, 349 (1998)
Applied by SMC, HERMES and COMPASS



The simple idea: three basic processes in LO QCD, PGF probes gluons

$$A_{LL}^{2h}(x) = \left\langle \frac{\Delta G}{G}(x_G) \right\rangle_{a_{LL}^{PGF} R_{PGF}} \langle a_{LL}^{PGF} R_{PGF} \rangle + \langle A_1^{LO}(x) \rangle_{DR_{LP}} \langle DR_{LP} \rangle + \langle A_1^{LO}(x_C) \rangle_{a_{LL}^{QCDC} R_{QCDC}} \left\langle a_{LL}^{QCDC} R_{QCDC} \right\rangle ,$$

Large statistics but Monte Carlo dependent analysis, limited so far to LO

COMPASS: statistically weighting method & Neural Network approach