Mieszanie i łamanie symetrii CP w rozpadach cząstek powabnych w eksperymencie LHCb

Seminarium Fizyki Wielkich Energii
26 kwietnia 2013

Artur Ukleja
(Narodowe Centrum Badań Jądrowych)

Prezentowano również na Konferencji Beauty 2013
dnia 10 kwietnia 2013 roku w Bolonii

Charm mixing and CP violation at LHCb

26/04/2013

Artur Ukleja
National Centre for Nuclear Research, Warsaw

also presented on Beauty 2013, 10 April 2013, Bologna

Outline

- **Introduction:**
  - ✧ mixing D^0−anti-D^0 and CPV
    - ✓ SM predictions
    - ✓ current constraints for mixing and CPV in charm physics
    - ✓ why are we interested in charm physics?

- **Measurements of mixing and CPV in charm sector at LHCb**
  - ✧ the LHCb detector
  - ✧ observation of D^0 – anti-D^0 mixing
  - ✧ ΔA_{CP} in $D^0 \to K^+K^-$ and $D^0 \to \pi^+\pi^-$
    - ➢ pion-tagged analysis $D^{*\pm} \to D^0\pi^+_s$
    - ➢ muon-tagged analysis $B \to D^0\mu X$
  - ✧ search for direct CPV in:
    - ➢ $D^+ \to \phi\pi^+$ and $D^+_s \to K^0_s\pi^+$
    - ➢ $D^+ \to K^-K^+\pi^+$ and $D^0 \to \pi^-\pi^+\pi^-\pi^+$

- **Summary**

Introduction

Neutral mesons can oscillate between matter and anti-matter: mass eigenstates are different from flavour eigenstates

$$i\frac{d}{dt}\begin{pmatrix}|D^0\rangle\\ |\overline{D}^0\rangle\end{pmatrix} = \left[\begin{pmatrix}M_{11} & M_{12}\\ M_{12}^* & M_{22}\end{pmatrix} - \frac{i}{2}\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{12}^* & \Gamma_{22}\end{pmatrix}\right]\begin{pmatrix}|D^0\rangle\\ |\overline{D}^0\rangle\end{pmatrix}$$

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$$

theory

$$m \equiv (m_1 + m_2)/2$$

$$\Gamma \equiv (\Gamma_1 + \Gamma_2)/2$$

Two parameters describe mixing:

mass difference x:

$$x \equiv \frac{m_2 - m_1}{\Gamma} = \frac{\Delta m}{\Gamma}$$

decay width difference y:

$$y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma} = \frac{\Delta\Gamma}{2\Gamma}$$

experiment theory

$$\Delta m = M_H - M_L = 2|M_{12}|(1 + \frac{1}{8}\frac{|\Gamma_{12}|^2}{|M_{12}|^2}sin^2\phi + \ldots)$$

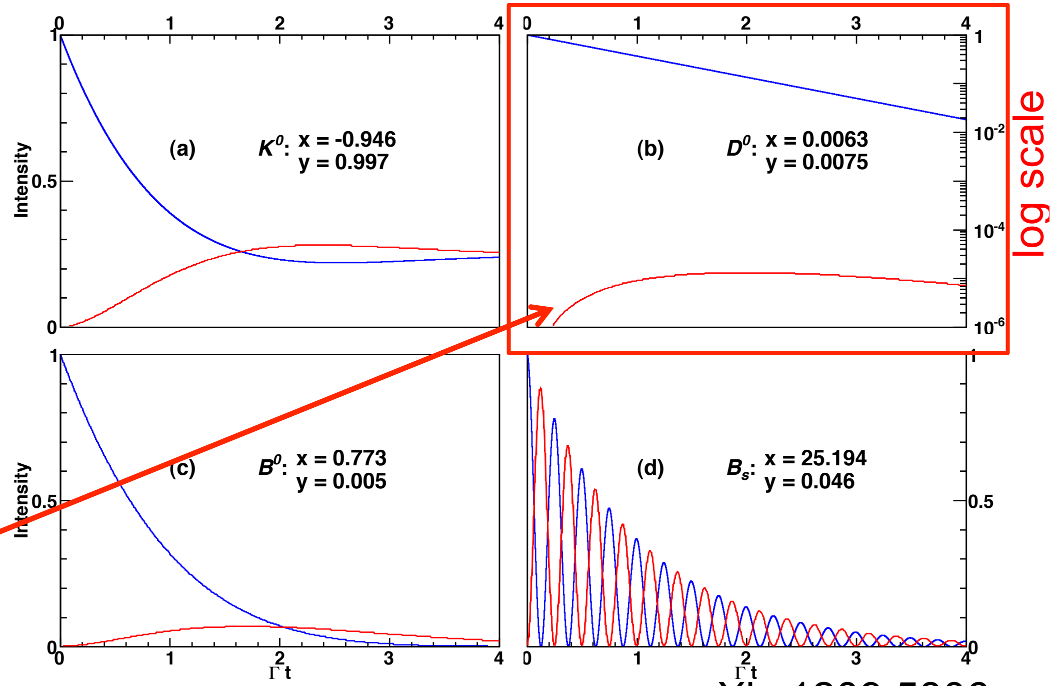
$$\Delta\Gamma = \Gamma_H - \Gamma_L = 2|\Gamma_{12}|cos\phi(1 - \frac{1}{8}\frac{|\Gamma_{12}|^2}{|M_{12}|^2}sin^2\phi + \ldots)$$

weak phase: $\phi \equiv arg(-M_{12}/\Gamma_{12})$

Δm, $\Delta\Gamma$ – measured experimentally

For charm: x = 0.0063 ; y = 0.0075

- Mixing is very slow
- Very precise measurements needed



arXiv:1209.5806

Three ways of CP violation

1. in mixing: different transition of oscillation (indirect)

$$D^0 \longrightarrow \text{anti-}D^0 \quad \neq \quad \text{anti-}D^0 \longrightarrow D^0$$

2. in decay amplitudes: decays of particles and antiparticles are not the same (direct)

$$D^0 \longrightarrow f \quad \neq \quad \text{anti-}D^0 \longrightarrow \text{anti-}f = f$$

3. interference: between CP violation in mixing and in decays (indirect)

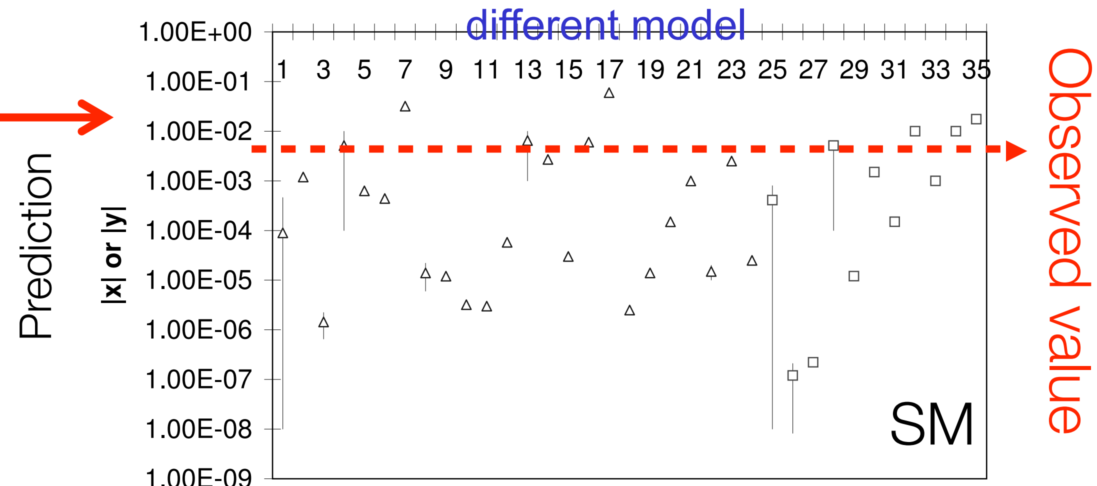
- Mixing and decay processes can be mediated via loop diagrams.
- NP is most likely to enter in loops and new particles can be exchanged

Mixing and CP violation

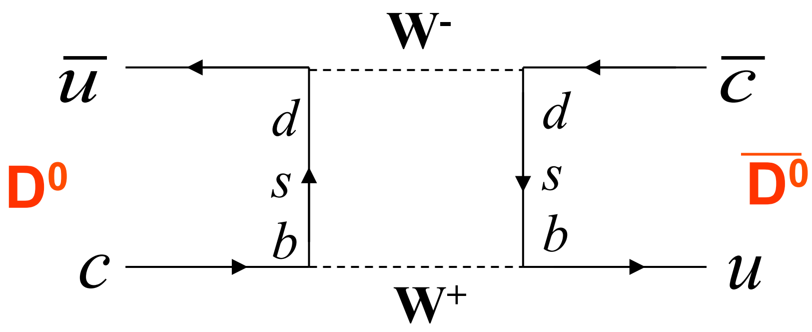
- In SM:
  - ✧ the charm mixing rate is expected to be small: $|x|,|y| \lesssim 10^{-2}$
  - ✧ expected CPV in charm sector is small $\lesssim 10^{-3}$ (much smaller than in the beauty sector) and difficult in calculation
  - ✧ SM predictions vary widely
  - ✧ New Physics contributions can enhance CPV up to 10^{-2}

Int.J.Mod.Phys.A21(2006)5381 ; Ann.Rev.Nucl.Part.Sci.58(2008)249

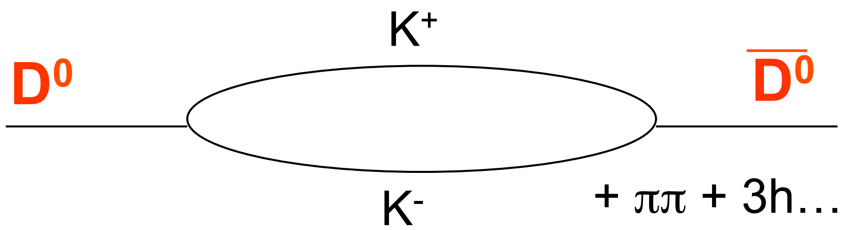
Int.J.Mod.Phys.A21(2006)5686



triangles |x| ; squares |y|



Mixing via box-diagram, short range



Mixing via hadronic intermediate states, long range (difficult to calculate)

$x \sim 1\%$ $y \sim 1\%$

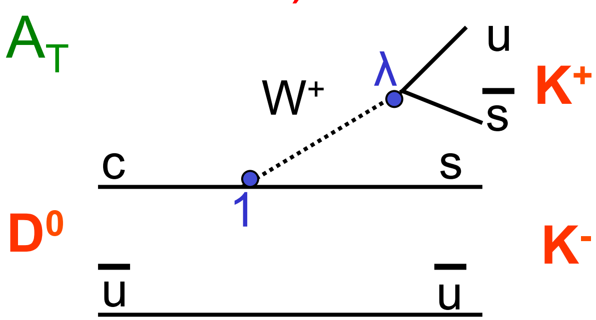
From measurements we know that **$x \sim y$**

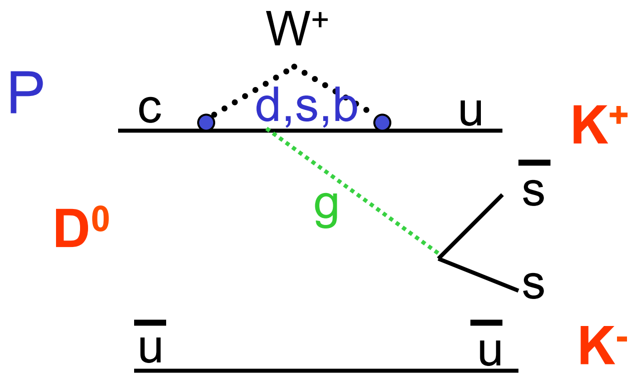
Direct decays and CP violation

If tree and penguin processes interfere with different phases then symmetry between particles and antiparticles is broken $\longrightarrow$ A ≠ anti-A

(Singly Cabibbo Suppresed decay = signal of CP ⟵ penguin diagram opens possibilities for NP searches)

$\lambda = 0.22$





$$A = \underbrace{V_{us}V^*_{cs}}_{\sim\lambda} A_T + \underbrace{V_{ud}V^*_{cd}}_{\sim\lambda} P_d + \underbrace{V_{us}V^*_{cs}}_{\sim\lambda} P_s + \underbrace{V_{ub}V^*_{cb}}_{\sim\lambda^6} P_b$$

$$Asym_{CP} \sim |A_1||A_2| sin(\phi_1 - \phi_2) sin(\delta_1 - \delta_2)$$

$=A_T$ $=P$ weak phases strong phases

- In SM CP violation in decays could be larger than in mixing (expected $\sim 10^{-3}$) and depends on final state
  → CP asymmetry should be searched elsewhere where is possible, for example: D → hh, D → hhh, D → hhhh ……

Decays without CP violation

Control decays where CP violation is negligible (no penguin contribution):

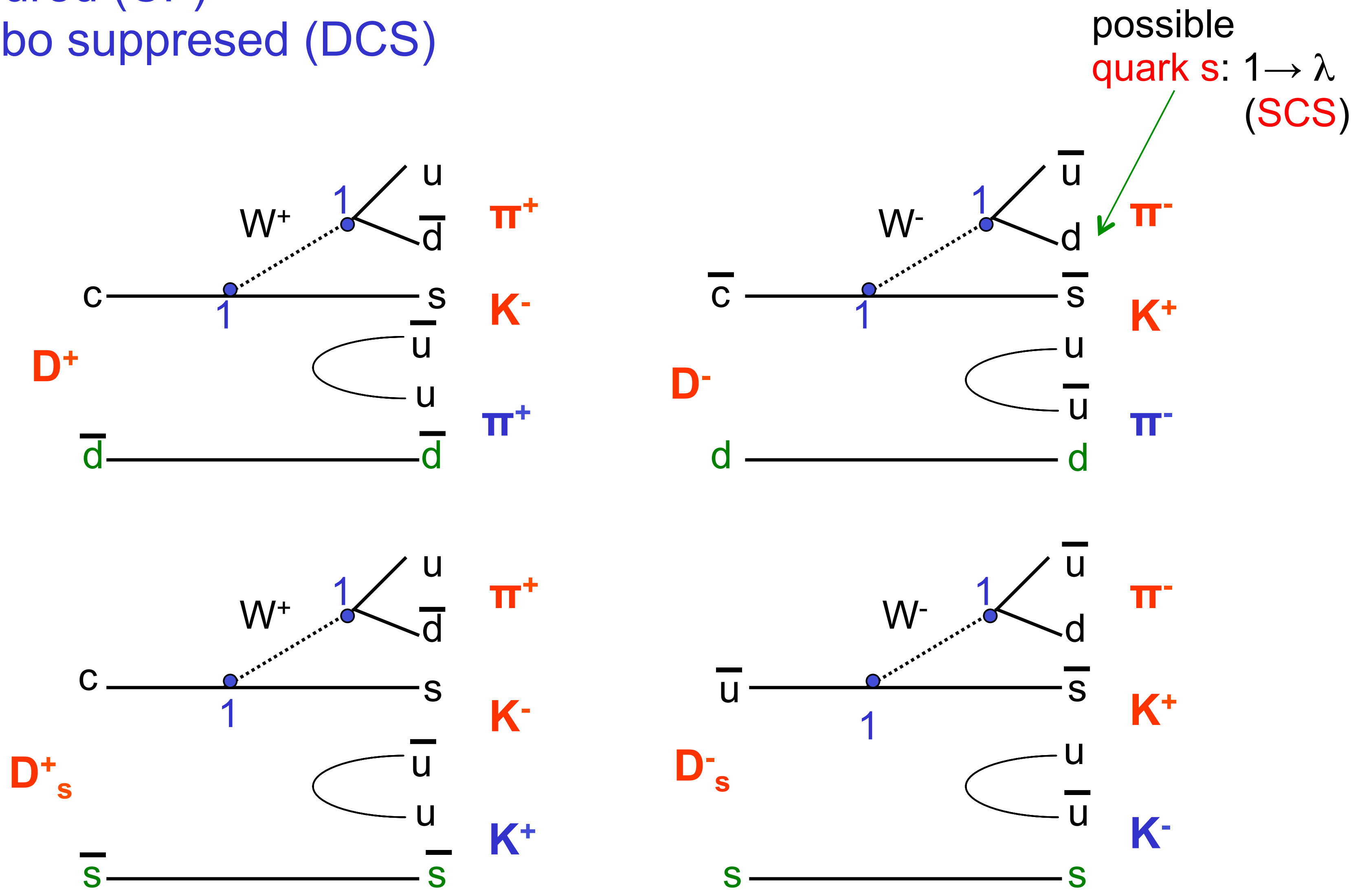
- Cabibbo favoured (CF)
- doubly Cabibbo suppresed (DCS)

$D^+ \to K^- \pi^+ \pi^+$

CF

$D^+_s \to K^- K^+ \pi^+$

CF



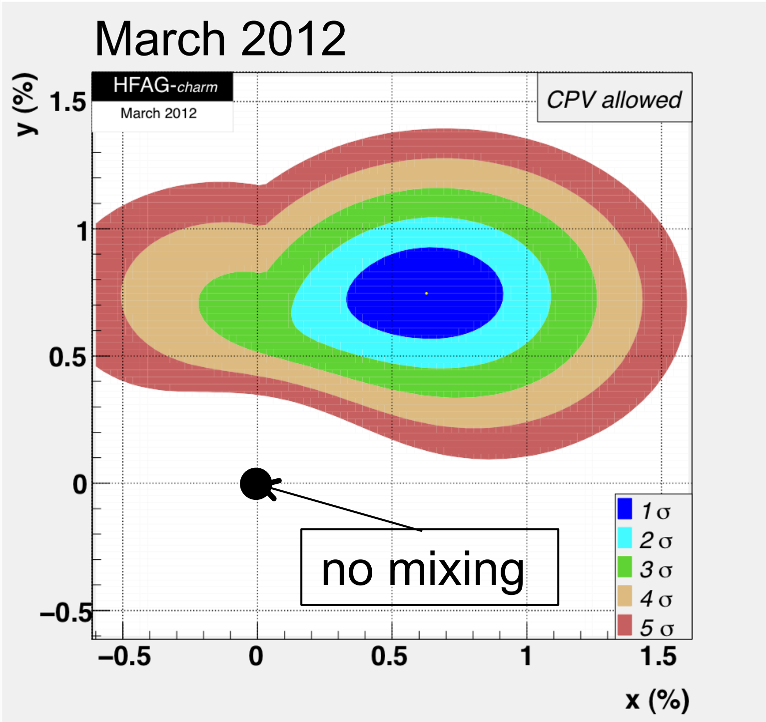
Control decays are used to check the detector effects

Current constraints

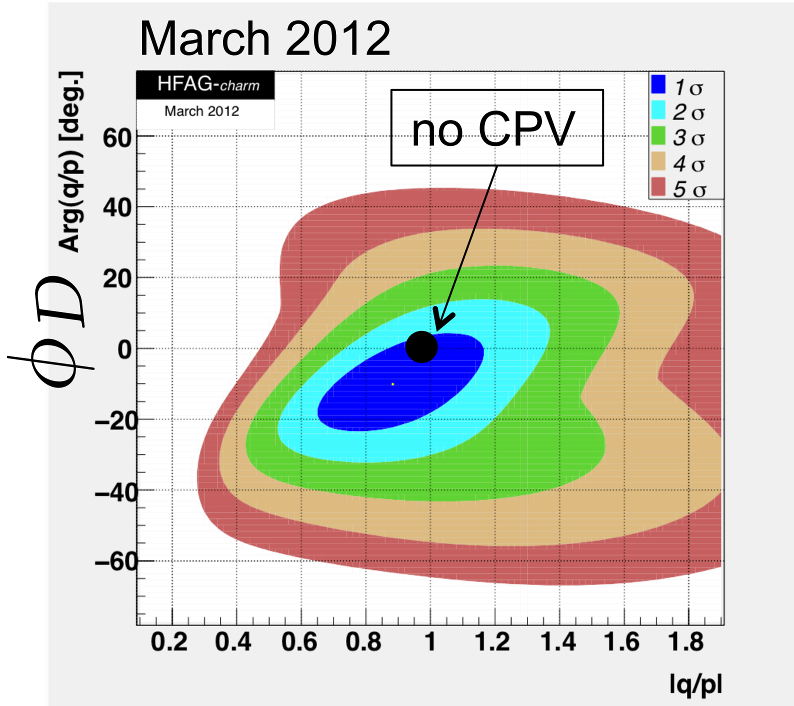
First evidence of mixing D^0-anti-D^0: BaBar, Belle (2007), CDF (2008)

- open possibilities of rich structure of CP violation in charm sector

$$y = \frac{\Delta\Gamma}{2\Gamma}$$



$$x = \frac{\Delta m}{\Gamma}$$



$$\phi_D \equiv arg(-M_{12}/\Gamma_{12})$$

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle$$

CPV in mixing: if $\phi_D \neq 0$ or $|q/p| \neq 1$

- Only the combination of all measurements provides confirmation of D^0-anti-D^0 mixing
- Before LHCb there was no observation of the phenomenon in a single measurement

Why are we interested in charm sector?

- So far there was no observation of CP violation in charm sector

  → next step: confirmation of CP asymmetry

- In SM expected CP asymmetry is small ($<10^{-3}$)
  - much smaller than in the beauty sector

  → perfect place for New Physics searching (small contribution from SM)

- Input to b Physics
  - a lot of B mesons decay into c particles ($b \rightarrow c$) ~50% transitions

Charm particles at LHCb

LHCb was built for b physics:

- for precise measurements of CPV in b decays and their very rare decays
- also c particle decays are reconstructed:
  - ✧ LHCb has huge charm samples
  - ✧ charm cross section ≈ 20 x b cross section within the LHCb acceptance:

$$\sigma(b\bar{b}) = 75.3 \pm 5.4 \pm 13.0\ \mu b$$

Phys.Lett.B694 (2010) 209-216

$$\sigma(c\bar{c}) = 1419 \pm 12 \pm 116\ \mu b \sim 20 \times \sigma(b\bar{b})$$

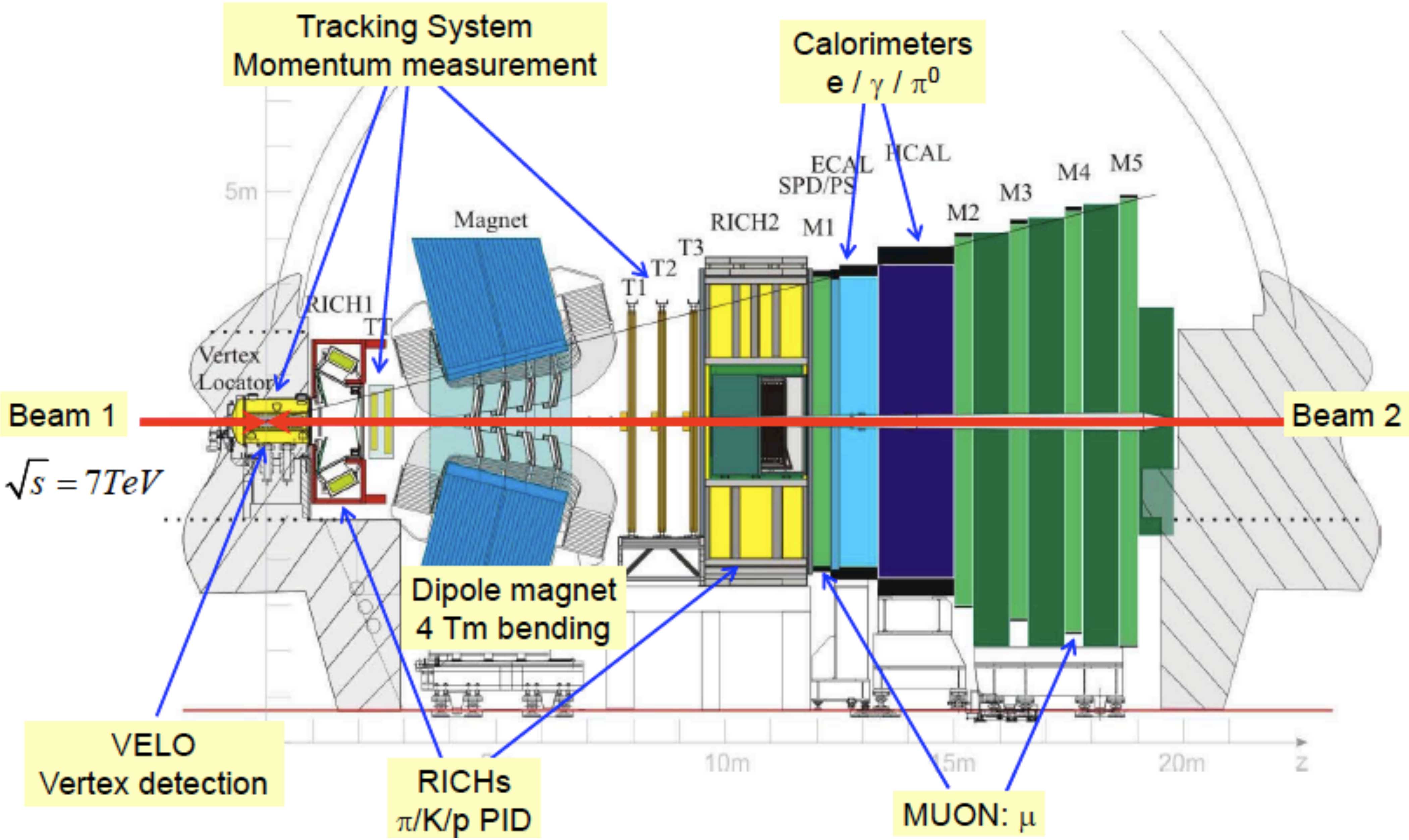
Nucl.Phys.B871 (2013) 1

✧ Largest charm samples in the world:
- ✓ 2011: 1/fb
- ✓ 2012: 2/fb

✧ for example: ~2M $D^{*\pm} \to D^0(\to K^-K^+)\pi^\pm$ reconstructed for 1/fb

LHCb – precision detector

Single-arm forward spectrometer covering range: $2<\eta<5$



LHCb – precision detector

- VELO:
  - ✓ resolution of IP: 20 μm
  - ✓ decay lifetime resolution ~ 45 fs: 0.1 $\tau(D^0)$

    (depends on the channel, for 2012 statistics ~15 fs for $D^0 \rightarrow KK$)
- Excellent tracking resolution: $\Delta p/p$ = 0.4% at 5 GeV to 0.6% at 100 GeV
- RICH:
  - ✓ very good particle identification for π and K
- Dedicated exclusive trigger lines for charm with high efficiency
  - ✓ HTL1: efficiency ~50%
  - ✓ HLT2: efficiency 50-90% for $D \rightarrow hh/3h/4h$
- The polarity of the magnet is reversed repeatedly during data taking
- LHCb has possibilities of very precise measurements of charm particles

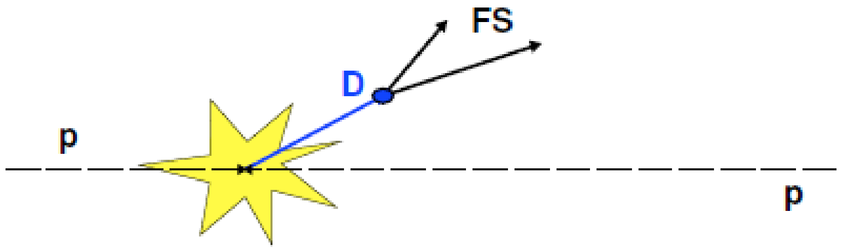
Charm production at LHCb

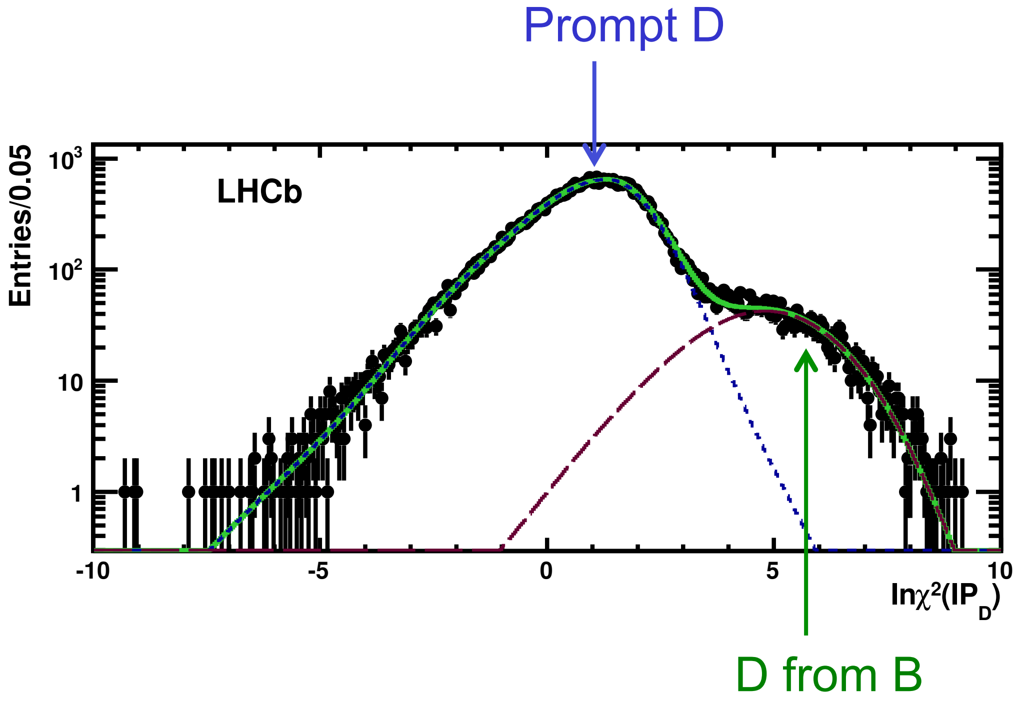
JHEP04(2012)129

Two production types of charm:

- **prompt** – produced directly in the primary vertex (PV)

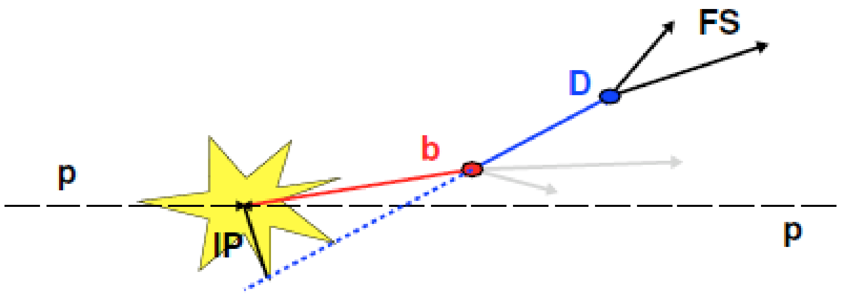
IP(D^0)~0





- **secondary** – produced in B decays (>50% of B→DX)

IP(D^0)>~0



IP – impact parameter wrt. the PV

To separate prompt charm and secondary charm decays we use the cut on χ^2(IP) parameter

The tagging of D^0 flavour

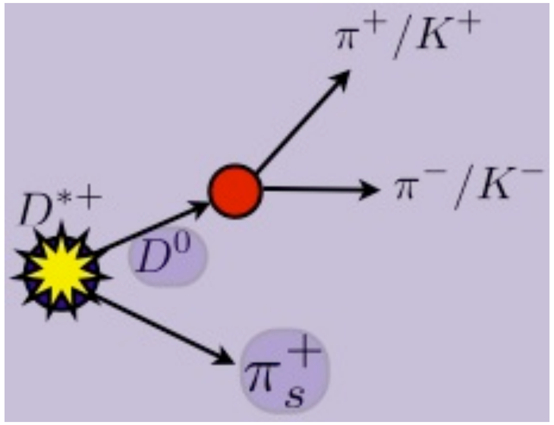
LHCb uses two methods to identify D^0 flavour at the production state

- pion-tagged method

  the sign of slow pion from D* decays is used to tag the initial D^0 flavour

  $D^{*+} \to D^0 \pi^+_s$

  $D^{*-} \to \text{anti-}D^0 \pi^-_s$



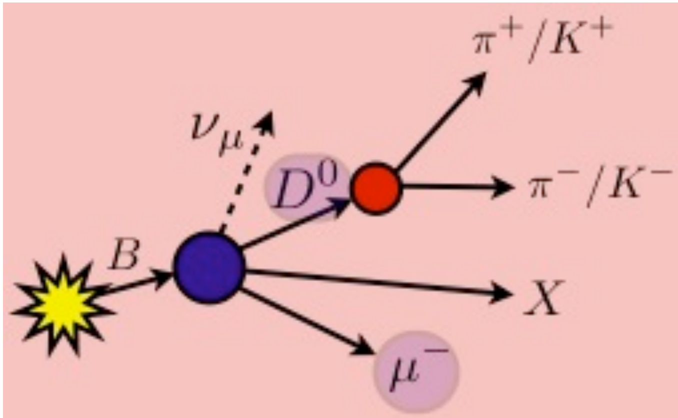
prompt D^0

- muon-tagged method

  the sign of muon from semileptonic B decays is used to tag D^0 flavour

  $B \to D^0 \mu^- \nu_\mu X$

  $B \to \text{anti-}D^0 \mu^+ \nu_\mu X$



secondary D^0

- Decays $D^0 \to h^- h^+$

  $D^0 \to K^- K^+$ (Singly Cabibbo Suppressed)

  $D^0 \to K^- \pi^+$ (Cabibbo Favoured)

  $D^0 \to K^+ \pi^-$ (Doubly Cabibbo Suppressed)

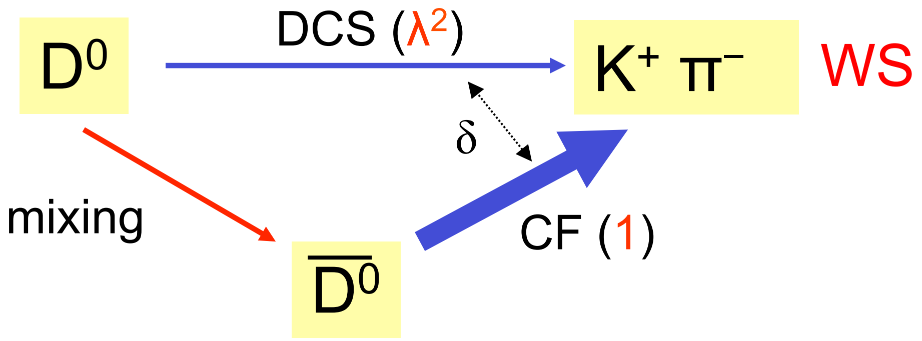
  $D^0 \to \pi^- \pi^+$ (Singly Cabibbo Suppressed)

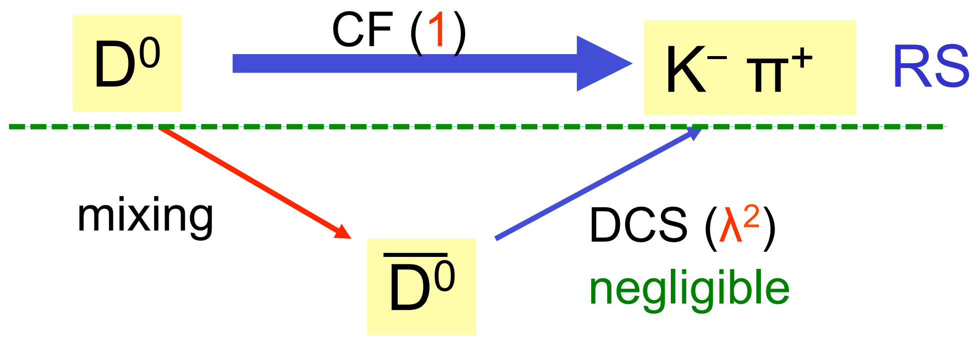
Use to measure D^0 – anti-D^0 mixing parameters

D^0 – anti-D^0 mixing

Measure the time-dependent ratio of D^0 decays with Wrong Sign to Right Sign

$$R(t) = \frac{N(D^0 \to K^+\pi^-)}{N(D^0 \to K^-\pi^+)}$$





In the limit of small mixing |x|,|y| << 1 and for no CPV:

$$R(t) = \frac{N_{WS}(t)}{N_{RS}(t)} = R_D + \sqrt{R_D}\,y' t + \frac{x'^2 + y'^2}{4} t^2$$

the ratio of DCS to CF decay rates (R_D)

the interference of the DCS and mixed decays ($\sqrt{R_D}\,y' t$)

mixing parameters ($\frac{x'^2 + y'^2}{4} t^2$)

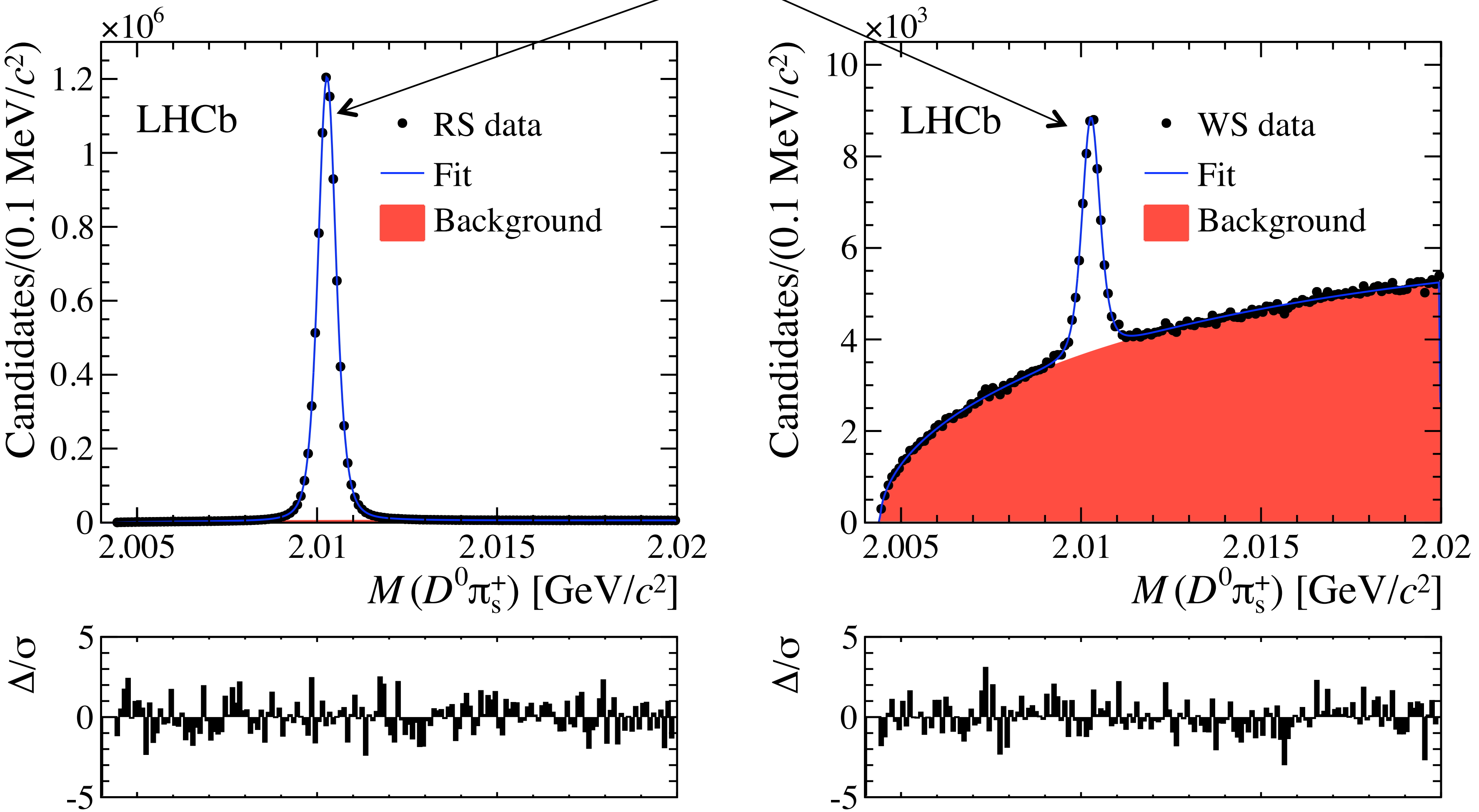
$$x' = x\cos\delta + y\sin\delta \qquad y' = y\cos\delta - x\sin\delta$$

δ is a strong phase difference between DCS and CF amplitudes

Time-integrated yields

This is NOT a Monte Carlo
This is the LHCb 2011 data, L=1/fb

Phys.Rev.Lett.
110 (2013) 101802



RS: $D^0 \to K^-\pi^+$
8.4 M decays

WS: $D^0 \to K^+\pi^-$
36 k decays

Analysis strategy

- To determine the time-dependent WS/RS ratio the data is divided into thirteen D^0 decay time bins, chosen to have a similar number of candidates in each bin

- The signal yields for the RS and WS samples are determined in each decay time bin using fits to the $M(D^0\pi^+_s)$ distribution

- The WS/RS ratio is calculated in each decay time bin

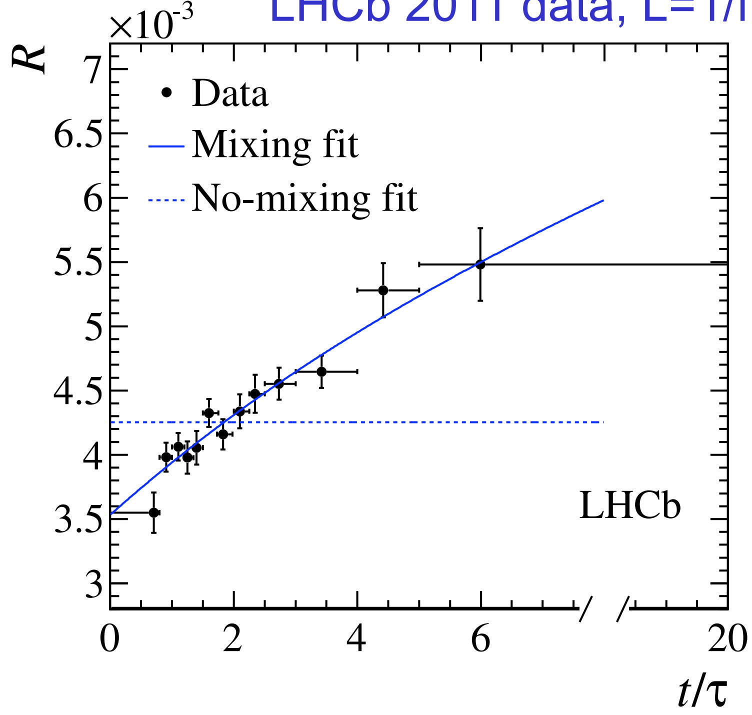
- The mixing parameters are determined in a binned χ^2 fit of the function

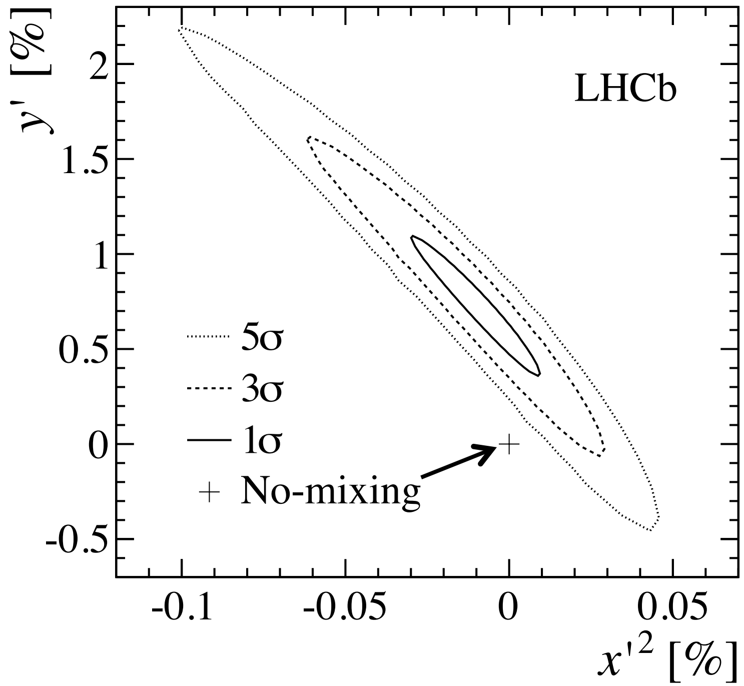
$$R(t) = \frac{N_{WS}(t)}{N_{RS}(t)} = R_D + \sqrt{R_D} y' t + \frac{x'^2 + y'^2}{4} t^2$$

to the time dependence

Results for D^0 – anti-D^0 mixing

LHCb 2011 data, L=1/fb





Phys.Rev.Lett. 110 (2013) 101802

Estimated confidence-level (CL) regions for 1-CL = 1σ,3σ,5σ

x'^2 is very small

Measurement is more sensitive to y'

Fit type (χ^2/ndf)	Parameter	Fit result (10^{-3})	Correlation coefficient R_D	y'	x'^2
Mixing (9.5/10)	R_D	3.52 ± 0.15	1	-0.954	$+0.882$
	y'	7.2 ± 2.4		1	-0.973
	x'^2	-0.09 ± 0.13			1
No mixing (98.1/12)	R_D	4.25 ± 0.04			

Uncertainties include stat. and syst. sources

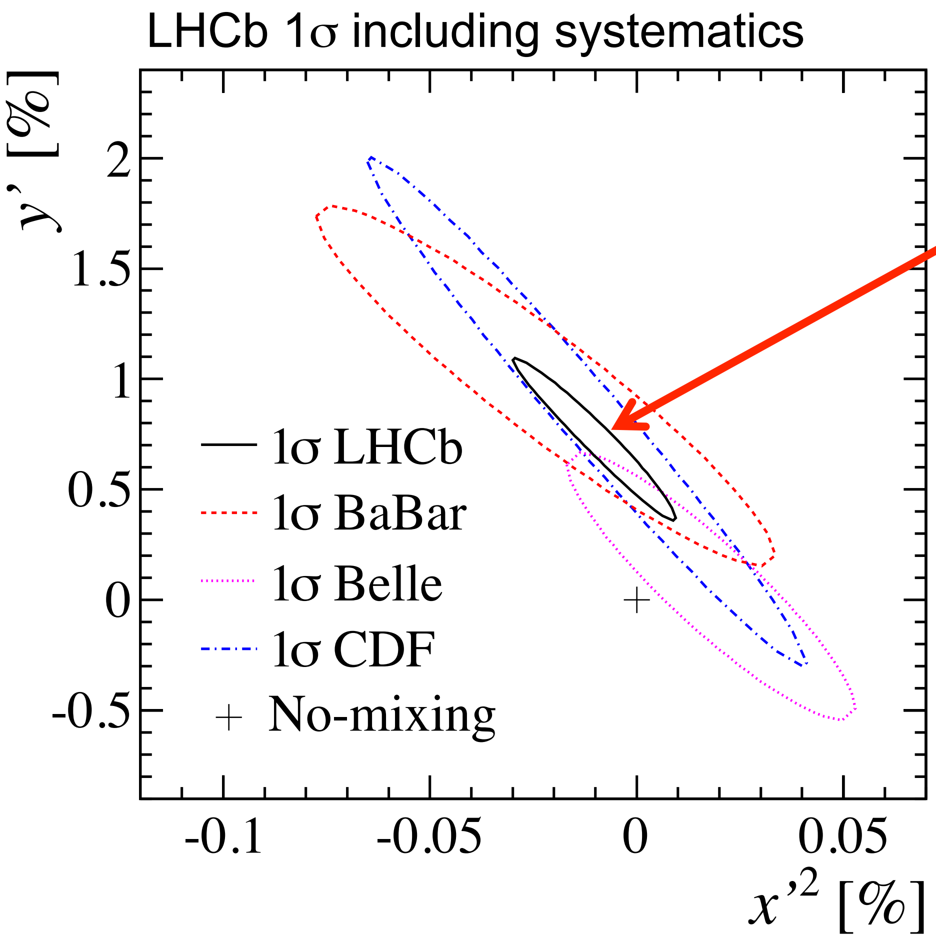
$\Delta\chi^2 = 88.6$ corresponds to p-value = 5.7×10^{-20} which excludes the no-mixing hypothesis at 9.1σ

First observation of D^0 – anti-D^0 mixing in a single measurement

Comparison with other experiments

Experiment	R_D (10^{-3})	y' (10^{-3})	x'^2 (10^{-4})
LHCb	3.52 ± 0.15	7.2 ± 2.4	-0.9 ± 1.3
BaBar	3.03 ± 0.19	9.7 ± 5.4	-2.2 ± 3.7
Belle	3.64 ± 0.17	$0.6^{+4.0}_{-3.9}$	$1.8^{+2.1}_{-2.3}$
CDF	3.04 ± 0.55	8.5 ± 7.6	-1.2 ± 3.5

LHCb: PRL 110 (2013) 101802
BaBar: PRL 98 (2007) 211802
Belle: PRL 96 (2006) 151801
CDF: PRL 100 (2008) 121802



Measured parameters at LHCb are consistent with other experiments

- 2011 data, 1/fb
- more data is on tape

Time integrated CP violation in $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ decays pion-tagged analysis

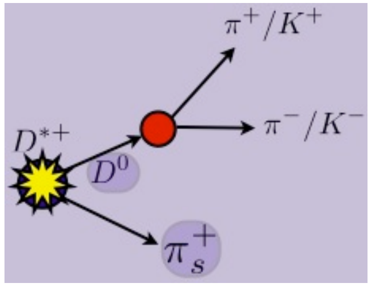
We use decays of $D^{*\pm}$:

$D^{*+} \to D^0 \pi^+_s$

$D^{*-} \to \text{anti-}D^0 \pi^-_s$

$D^0 \to K^- K^+$

$D^0 \to \pi^- \pi^+$



We want to measure asymmetry between charm particles and antiparticles:

$$A_{CP} \equiv \frac{N(D^0 \to h^- h^+) - N(\bar{D}^0 \to h^- h^+)}{N(D^0 \to h^- h^+) + N(\bar{D}^0 \to h^- h^+)}$$

Measured raw asymmetry A_{RAW} may be written as a sum of components that are physics and detector effects:

$$A_{RAW}(f)^* = A_{CP}(f) + A_D(f) + A_D(\pi_s) + A_P(D^*)$$

- $A_{CP}(f)$: CP asymmetry what we want to measure
- $A_D(f)$: detector asymmetry of D^0 reconstruction
- $A_D(\pi_s)$: detector asymmetry of π_s reconstruction
- $A_P(D^*)$: production asymmetry of D^* **in primary vertex** (different numbers of D^{*+} and D^{*-})

- A_{RAW}, A_D and A_P are defined in the same fashion as A_{CP}
- all asymmetries of order 1% or smaller

Time integrated CP violation in $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow \pi^-\pi^+$ decays pion-tagged analysis

$$A_{RAW}(f)^* = A_{CP}(f) + A_D(f) + A_D(\pi_s) + A_P(D^*)$$

Detector asymmetries for K^-K^+ and $\pi^-\pi^+$ cancel since the final states are charge symmetric

$$A_D(K^-K^+) = 0 = A_D(\pi^-\pi^+)$$

In any given kinematic region $A_D(\pi_s)$ and $A_P(D^*)$ are independent of f and thus in the first-order those terms cancel if we subtract raw asymmetries

$$A_{RAW}(K^+K^-)^* - A_{RAW}(\pi^+\pi^-)^* =$$
$$= A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \equiv \Delta A_{CP}$$

Direct and indirect CPV can contribute

ΔA_{CP} interpretation

CPV asymmetry of each final state is a sum of:

asymmetry in the decay amplitude

asymmetry due to mixing and interference between mixing and decay

Mean proper time in used sample (acceptances are functions of time and for K^-K^+ and $\pi^-\pi^+$ are slightly different)

$$A_{CP}(f) = a_{CP}^{dir}(f) + \frac{\langle t \rangle}{\tau} a_{CP}^{ind}$$

[JHEP 1106 (2011) 089]

Lifetime of D^0 (PDG)

$$\Delta A_{CP} \equiv A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$$

$$\Delta A_{CP} = [a_{CP}^{dir}(K^-K^+) - a_{CP}^{dir}(\pi^-\pi^+)] + \frac{\Delta\langle t \rangle}{\tau} a_{CP}^{ind}$$

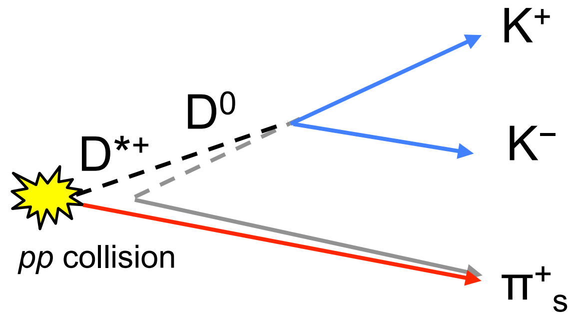
- ΔA_{CP} is equal to the difference in the direct CP asymmetry between the two decays in the limit that $\Delta\langle t \rangle$ or a^{ind} vanishes
- direct CP depends on the f
- indirect CPV is universal (up to 10^{-2} correction)
  - ✧ its contribution cancels in subtraction if lifetime acceptance same for K^-K^+ and $\pi^-\pi^+$
  - ✧ if time-acceptance is different, contribution a^{ind} remains

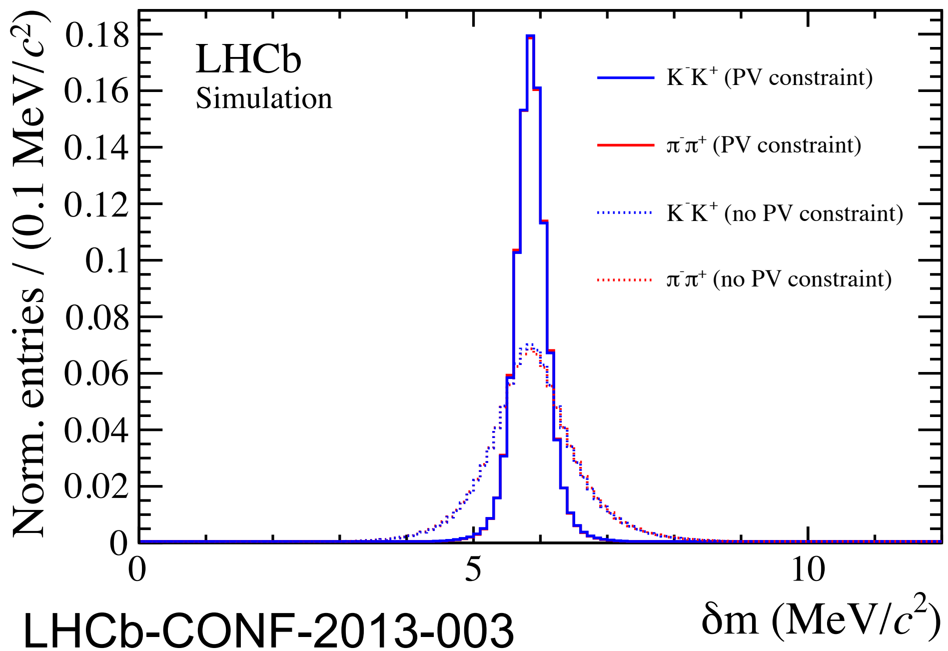
1^{st} measurement of ΔA_{CP} from D* decays

- Update of analysis from 2011 0.6/fb → 1/fb (full 2011 dataset)

- Update includes new reconstruction
  - ✧ improved tracking alignment
  - ✧ improved particle identification from RICH calibration

- New in the vertex fit
  constrain the D* vertex to the primary vertex
  - ✧ improves δm resolution by factor ~2.5
    → better background separation





LHCb-CONF-2013-003

$\delta m \equiv m(h^- h^+ \pi^+_s) - m(h^- h^+) - m(\pi^+_s)$

$D^{*+} \rightarrow D^0 \pi^+_s$

$D^0 \rightarrow K^- K^+$

$D^0 \rightarrow \pi^- \pi^+$

Signal yields

LHCb-CONF-2013-003

D^0 decays come from $D^{*+} \to D^0 \pi^+$ decays in region:

$0 < \delta m < 12$ MeV

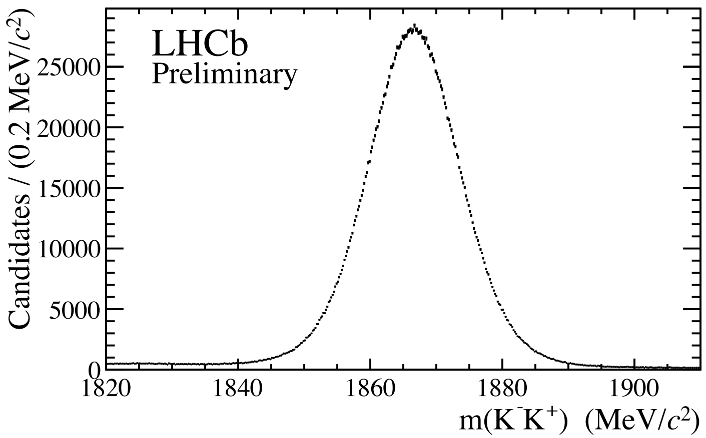
$\delta m = m(D^0\pi^+) - m(D^0) - m(\pi^+)$

For 1/fb in window mass from fit to δm:
$1844 < m(D^0) < 1884$ MeV

K^-K^+: 2.24 million events

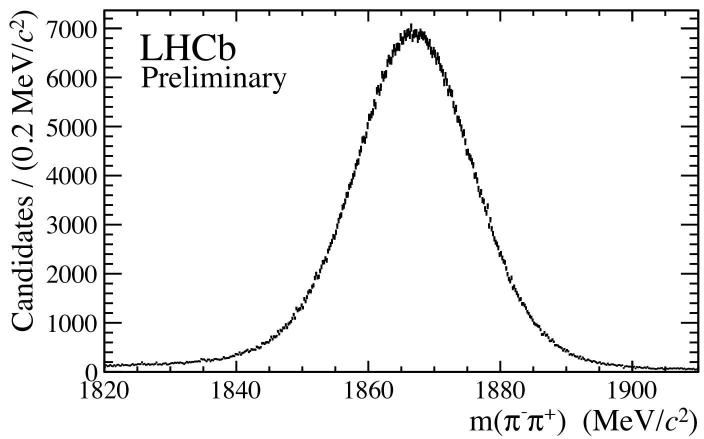
$\pi^-\pi^+$: 0.69 million events

$D^0 \to K^-K^+$

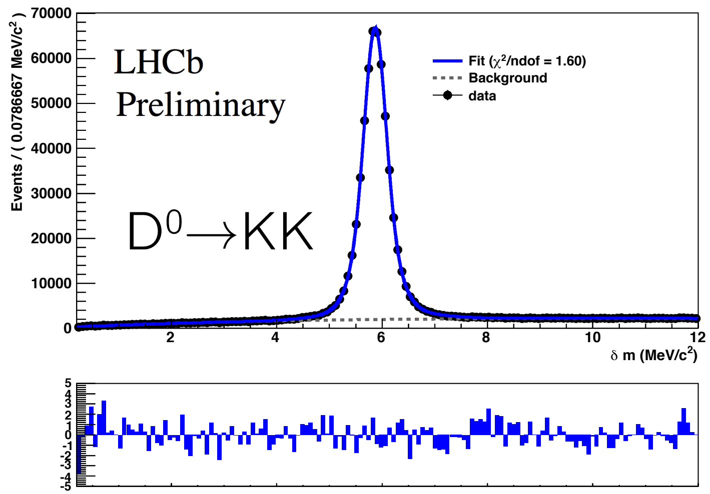


m(K^-K^+) (MeV)

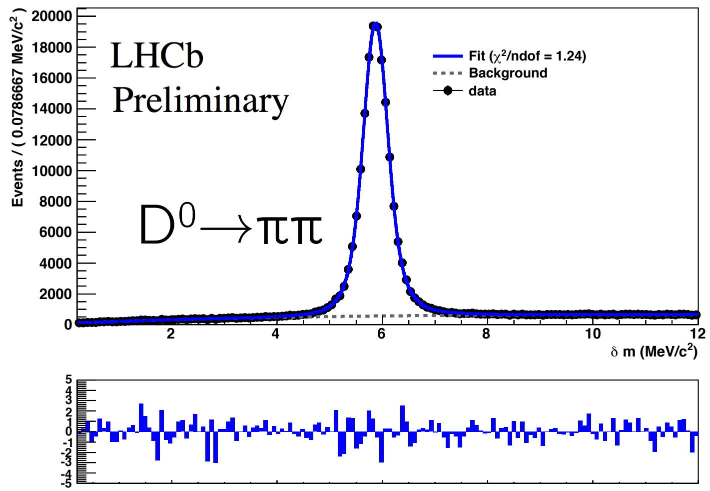
$D^0 \to \pi^-\pi^+$



m($\pi^-\pi^+$) (MeV)



δm (MeV)

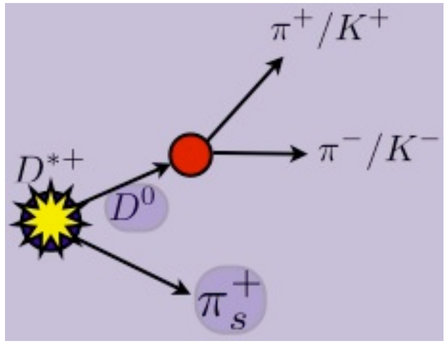


δm (MeV)

From simultaneous fits to δm for distributions of D^{*+} and D^{*-} we determine raw asymmetries $A_{RAW}(K^-K^+)$ and $A_{RAW}(\pi^-\pi^+)$ and calculate ΔA_{CP}

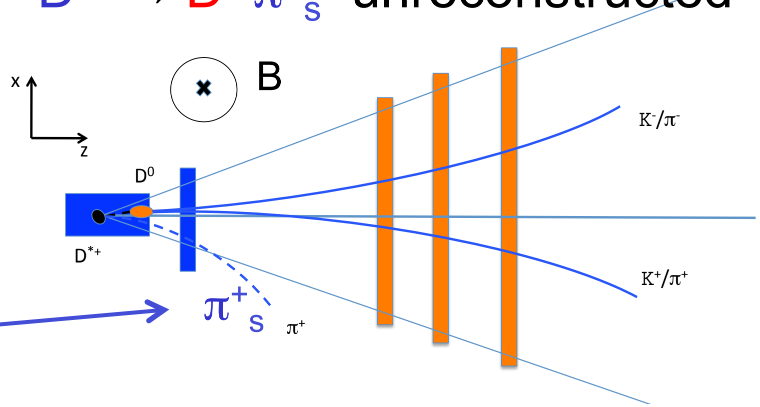
Systematic uncertainties

Systematic uncertainties with the highest contribution in change of ΔA_{CP}:

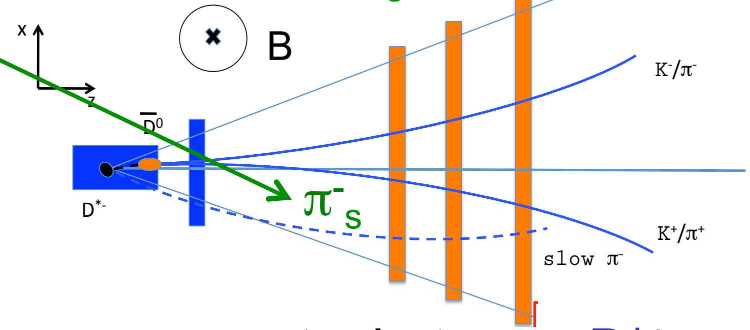


- **Imperfect reconstruction: 0.08 %**
  excluding events with imperfect reconstruction, in which π_s has a large IP w.r.t the primary vertex
- **Peaking background: 0.04 %**
  use different fits to the $m(K^-K^+)$ and $m(\pi^-\pi^+)$ spectra to test for potential peaking background contributions
- **Fit model: 0.03 %**
  sideband subtraction instead of a fit
- **Fiducial cut: 0.02 %**
  loosing fiducial requirement on π_s
- **Multiple candidates: 0.01 %**
  removing multiple candidates, keeping only one candidate per event chosen at random
- **Reweighting: 0.01%**
  due to different kinematics for K^-K^+ and $\pi^-\pi^+$

$D^{*+} \rightarrow D^0 \pi^+_s$ unreconstructed



$D^{*-} \rightarrow$ anti-$D^0 \pi^-_s$ reconstructed



Total systematic uncertainty: 0.10%
(can be reduced)

large asymmetry between D^{*+} and D^{*-} in edges of acceptance region

1st measurement of ΔA_{CP} from D* decay

Preliminary result (2011, 1/fb):

$$\Delta A_{CP} = [-0.34 \pm 0.15^{stat} \pm 0.10^{syst}]\%$$

LHCb-CONF-2013-003

Difference in decay time acceptance:

$$\Delta\langle t\rangle/\tau = [11.19 \pm 0.15^{stat} \pm 0.17^{syst}]\%$$

$$\Delta A_{CP} = [a_{CP}^{dir}(K^-K^+) - a_{CP}^{dir}(\pi^-\pi^+)] + \frac{\Delta\langle t\rangle}{\tau} a_{CP}^{ind}$$

Contributions from indirect CPV is suppressed by one order of magnitude

2^{nd} measurement of ΔA_{CP} from semileptonic B decays

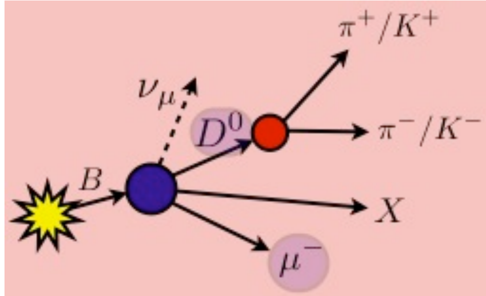
We use semileptonic B decays (independent method):

$B \to D^0 \mu^- \nu_\mu X$

$B \to \text{anti-}D^0 \mu^+ \nu_\mu X$

$D^0 \to K^- K^+$

$D^0 \to \pi^- \pi^+$



In similar way to the previous analysis

$$A_{RAW}(f)^* = A_{CP}(f) + A_D(f) + A_D(\mu^+) + A_P(B)$$

CP asymmetry what we want to measure

detector asymmetry of D^0 reconstruction cancel

detector asymmetry of μ reconstruction

production asymmetry of B

The production and muon detection asymmetries will cancel in subtraction if kinematics of μ and B meson are the same for both $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$

$$A_{RAW}(K^+K^-)^* - A_{RAW}(\pi^+\pi^-)^* =$$
$$= A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) \equiv \Delta A_{CP}$$

Signal yields

arXiv: 1303.2614

In similar way to the previous analysis ΔA_{CP} is calculated separately for two field polarities (to reduce as much as possible any residual effects of the detection asymmetry)

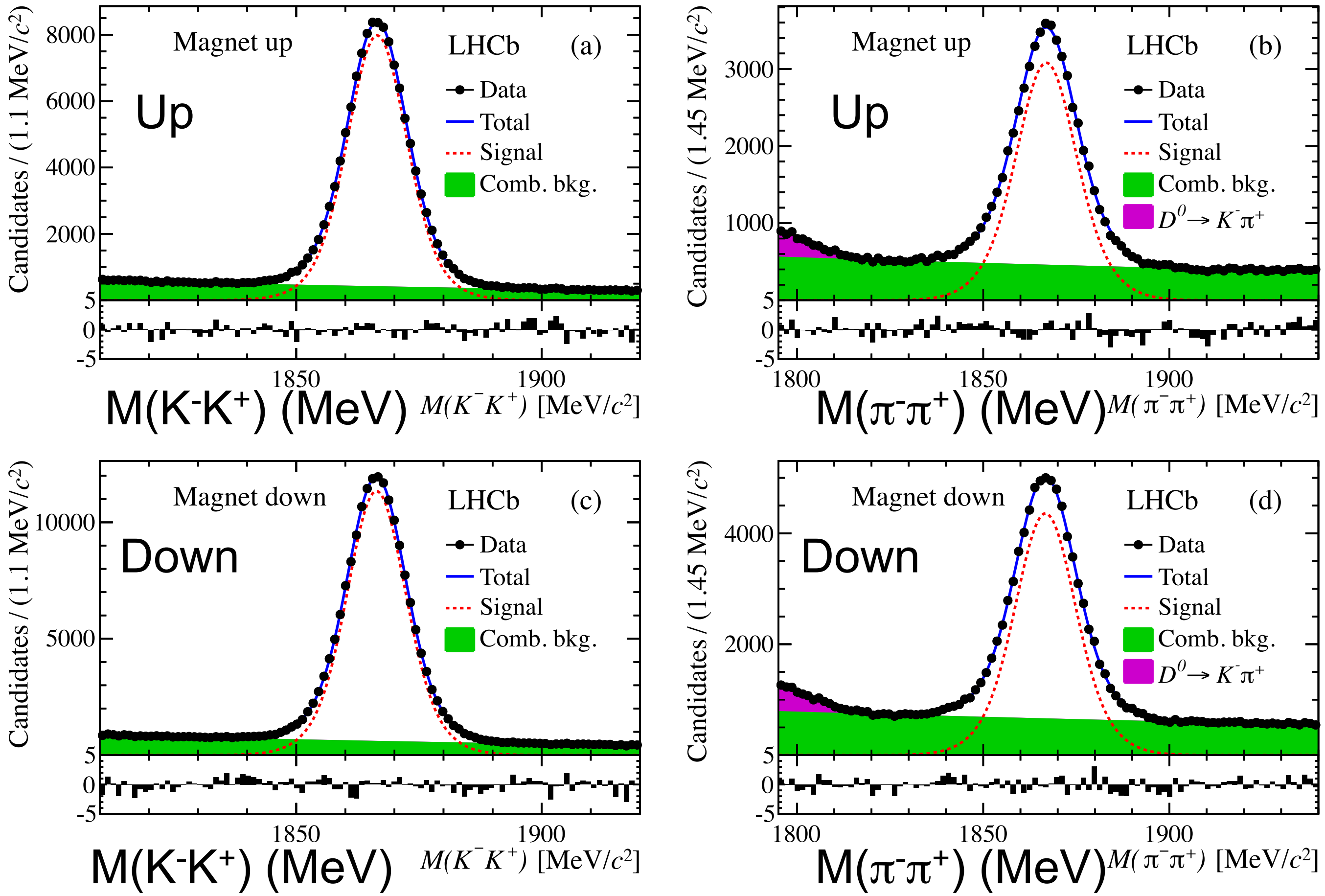
LHCb, 1/fb (full dataset 2011):
0.4/fb magnet up
0.6/fb magnet down

Clean signal
$B \rightarrow D^0 \mu^- \nu_\mu X$

559k $D^0 \rightarrow K^- K^+$

222k $D^0 \rightarrow \pi^- \pi^+$

$D^0 \rightarrow K^- K^+$ $\quad$ $D^0 \rightarrow \pi^- \pi^+$



Yields (and asymmetry) determined from fit to D^0 mass distribution (different from pion-tagged analysis where yields determined from D* mass distribution)

Measurement: ΔA_{CP}(Magnet up) = 0.86 ± 0.46 ; ΔA_{CP}(Magnet down) = 0.09 ± 0.39

(stat.only)

Systematic uncertainties

Systematic uncertainties with the highest contribution in change of ΔA_{CP}:

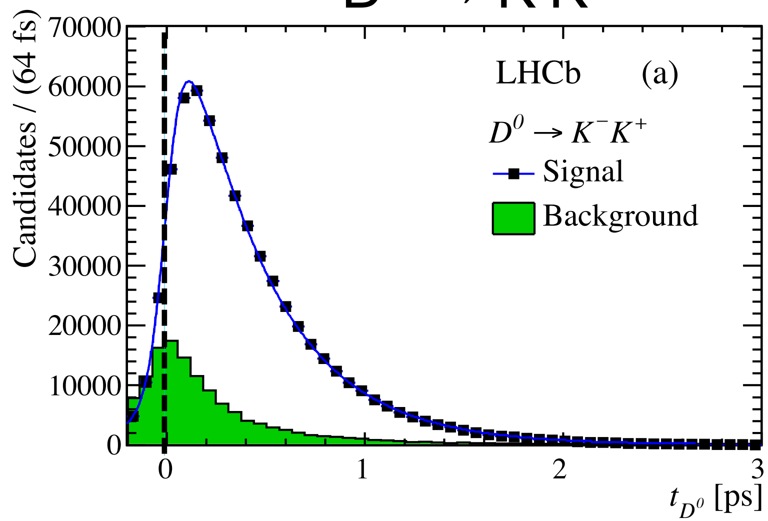
- Low-lifetime background in $D^0 \rightarrow \pi^-\pi^+$: **0.11%**
  there is more background around t=0 in $D^0 \rightarrow \pi^-\pi^+$ than in $D^0 \rightarrow K^-K^+$; evaluation of ΔA_{CP} checked when negative lifetime events were included

- Fit model: **0.05%**
  sideband subtraction instead of a fit

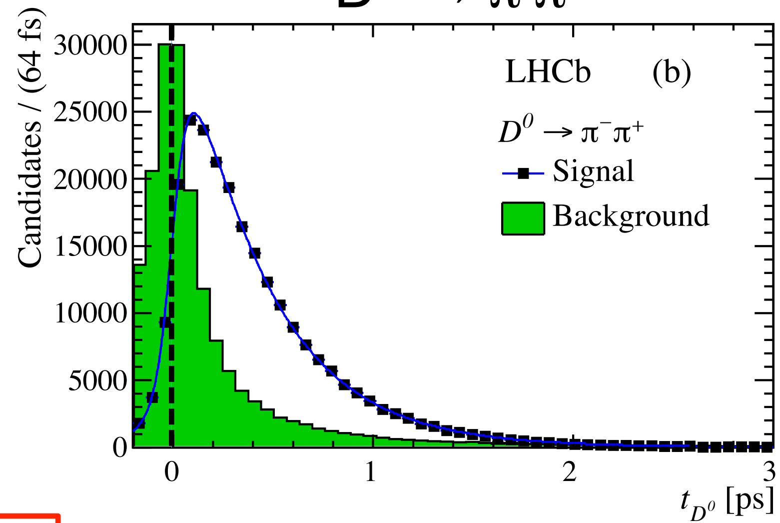
- Different weighting: **0.05%**
  after weighting the D^0 distributions in p_T and η small differences remain in muon kinematic distributions; evaluation of ΔA_{CP} checked when additional weight is applied in muon distributions p_T, η and ϕ

- Wrong muon tags: **0.02%**
  the D^0 flavour can be not tagged correctly due to muon misreconstruction; mistag probability measured using muon-tagged $D^0 \rightarrow K^-\pi^+$ (almost self-tagging) by comparison muon charge with kaon charge

$D^0 \rightarrow K^-K^+$



$D^0 \rightarrow \pi^-\pi^+$



Total systematic uncertainty: 0.14% (can be reduced)

Comparison of ΔA_{CP} measurements

1) From semileptonic B decays (arXiv: 1303.2614, Submitted to Phys.Lett.B)

$$\Delta A_{CP} = [0.49 \pm 0.30^{stat} \pm 0.14^{syst}]\%$$

Difference in decay time acceptance (small value):

$$\Delta\langle t\rangle/\tau(D^0) = 0.018 \pm 0.002^{stat} \pm 0.007^{syst}$$

Contribution from indirect CPV is negligible: $\Delta A_{CP} = \Delta a^{dir}_{CP}$

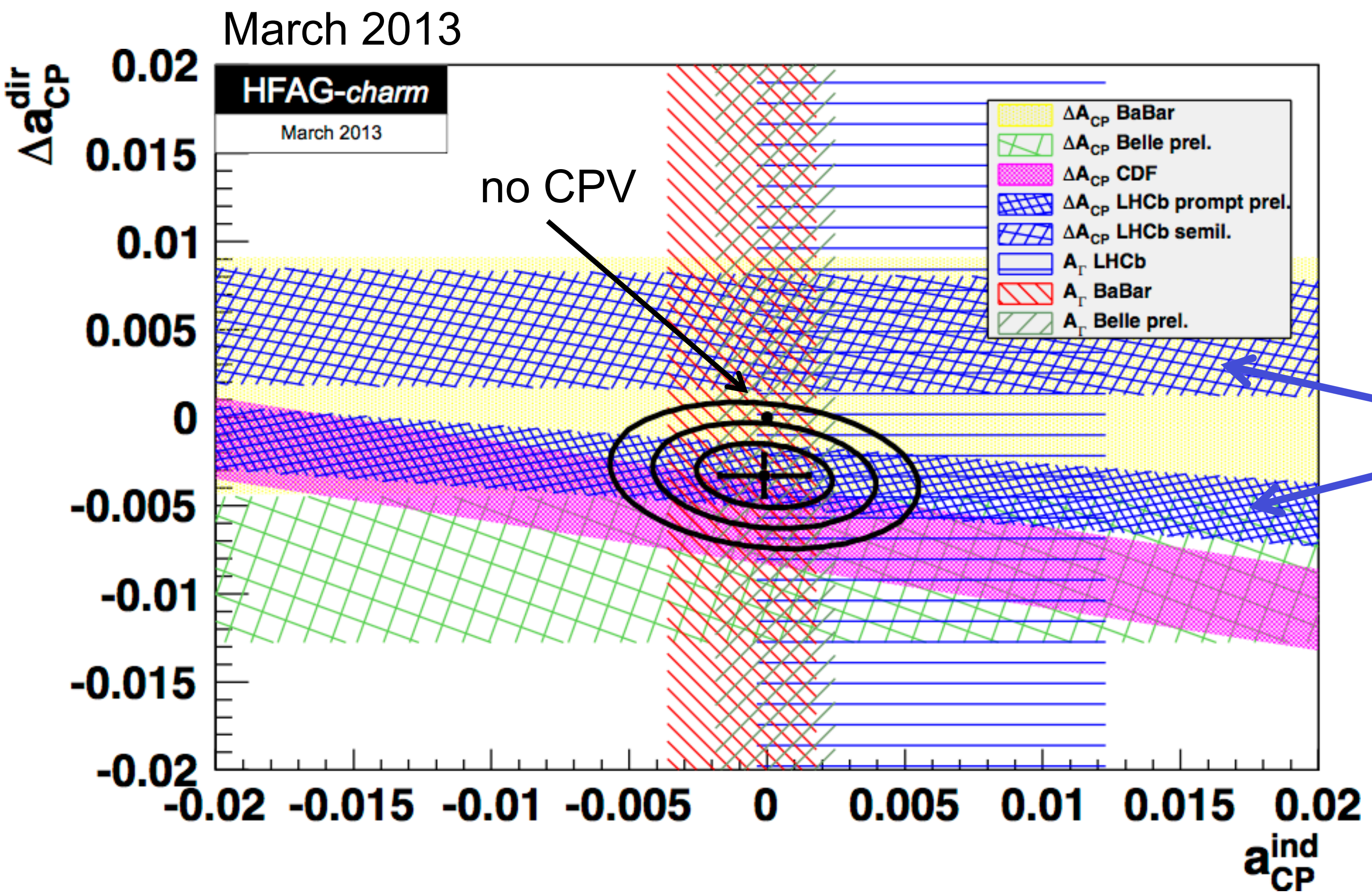
2) From pion-tagged D* decays (LHCb-CONF-2013-003)

$$\Delta A_{CP} = [-0.34 \pm 0.15^{stat} \pm 0.10^{syst}]\%$$

- Two measurements are statistically independent
- and compatible at 3% level (difference 2.2σ)

ΔA_{CP} Preliminary new world average

New average includes BaBar, CDF, Belle and new LHCb results



Naive average neglecting indirect CPV
$\Delta A_{CP} = (-0.33 \pm 0.12)\%$

Now:

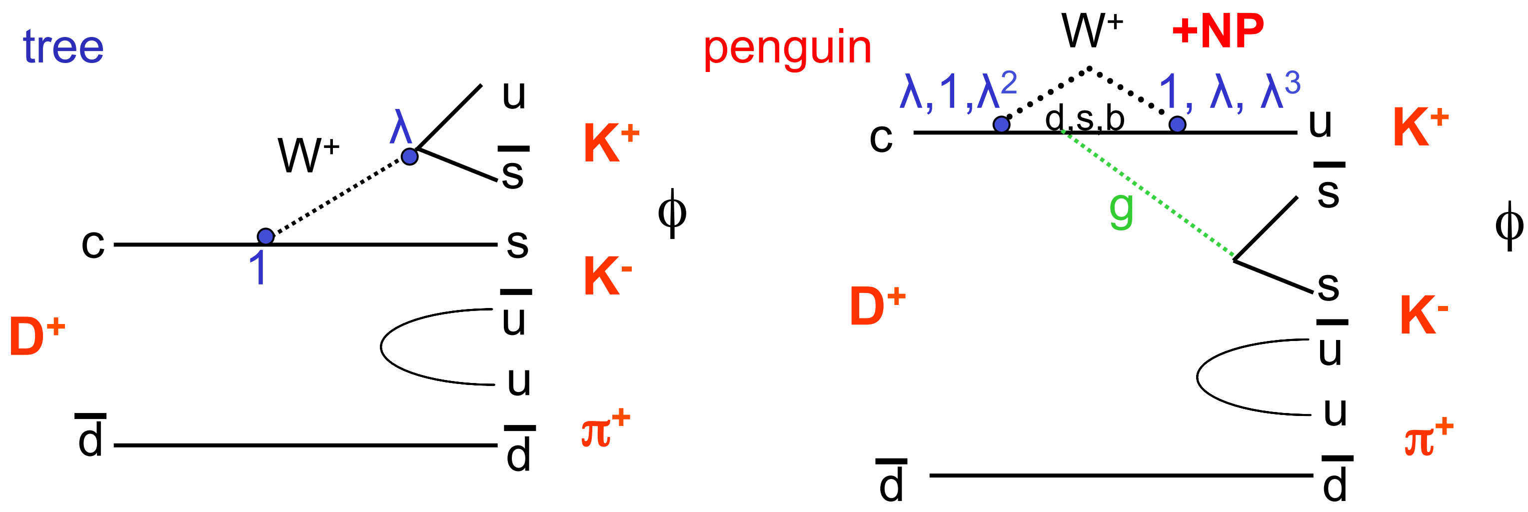
- the central value is considerably closer to zero
- result does not confirm the evidence for direct CPV in the charm sector

CP violation in $D^+ \to \phi \pi^+$ and $D^+_s \to K^0_s\pi^+$ decays

No mixing in D^+ → any CPV signal indicates direct CPV

Signal decays: $D^+ \to \phi \pi^+$ and $D^+_s \to K^0_s\pi^+$ are singly Cabibbo-suppressed decays where we expect CP asymmetry if tree and penguin processes interfere with different strong and weak phases

$\lambda = 0.22$



Control decays: $D^+ \to K^0_s\pi^+$ and $D^+_s \to \phi \pi^+$ where no CP asymmetry is expected

We measure the difference since effects of production asymmetry and of any detection asymmetry of pion cancel in subtraction

$$A_{CP}(D^+ \to \phi\pi^+) = A_{RAW}(D^+ \to \phi\pi^+) - A_{RAW}(D^+ \to K^0_s\pi^+) + A_{CP}(K^0/\bar{K}^0)$$
$$A_{CP}(D^+_s \to K^0_s\pi^+) = A_{RAW}(D^+_s \to K^0_s\pi^+) - A_{RAW}(D^+_s \to \phi\pi^+) + A_{CP}(K^0/\bar{K}^0)$$

Correction due to CPV in neutral Kaon system

Signal yields

LHCb-PAPER-2012-052

LHCb 2011, 1/fb

Very low background

Signal decays

1.6M $D^+ \to \phi\,\pi^+$

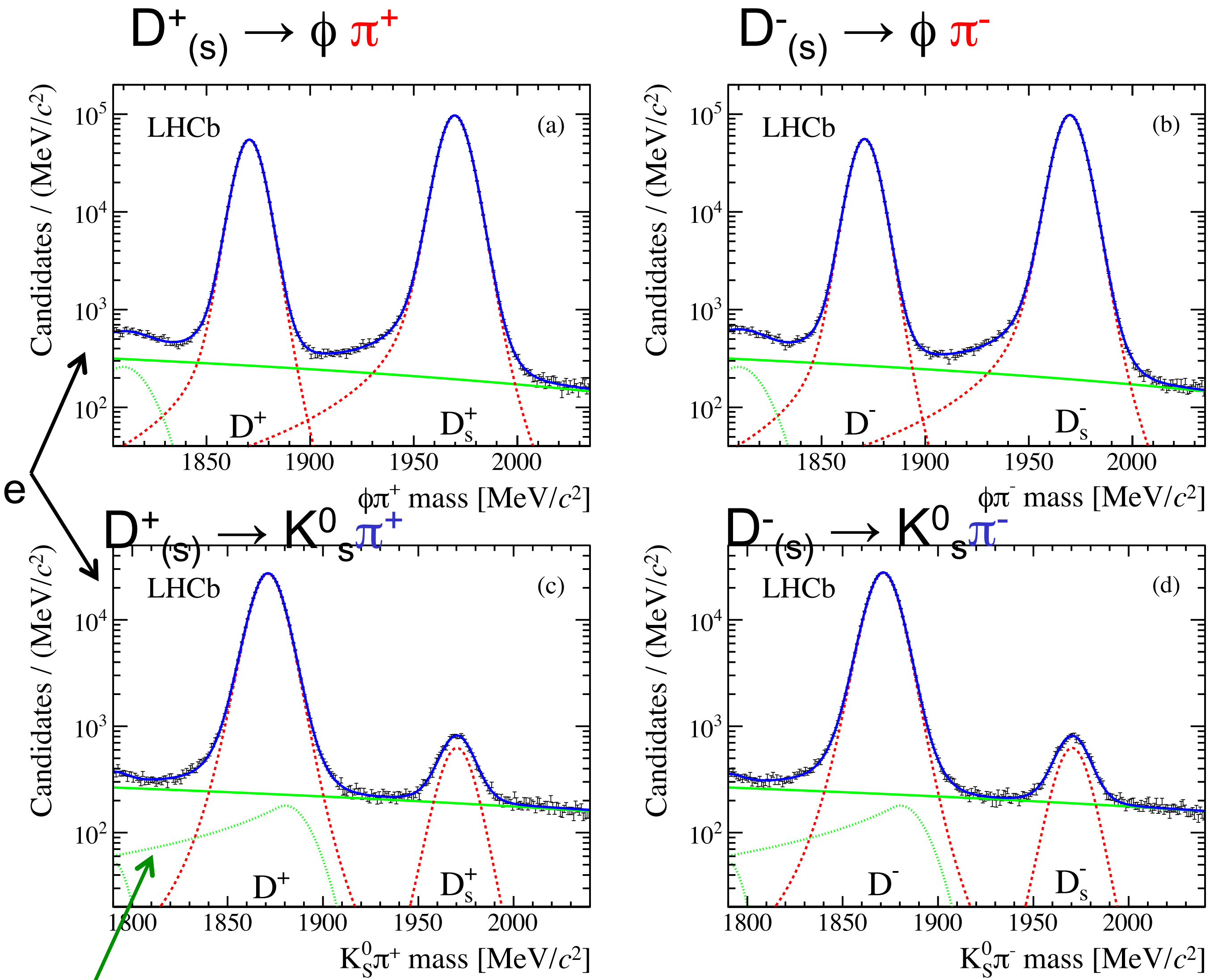
26k $D^+_s \to K^0_s\pi^+$

Control decays

1.1M $D^+_s \to \phi\,\pi^+$

3.6M $D^+ \to K^0_s\pi^+$

log scale



Background from mis-reconstructed decays:

(a) and (b) from $D^+_s \to \phi\pi^+\pi^0$

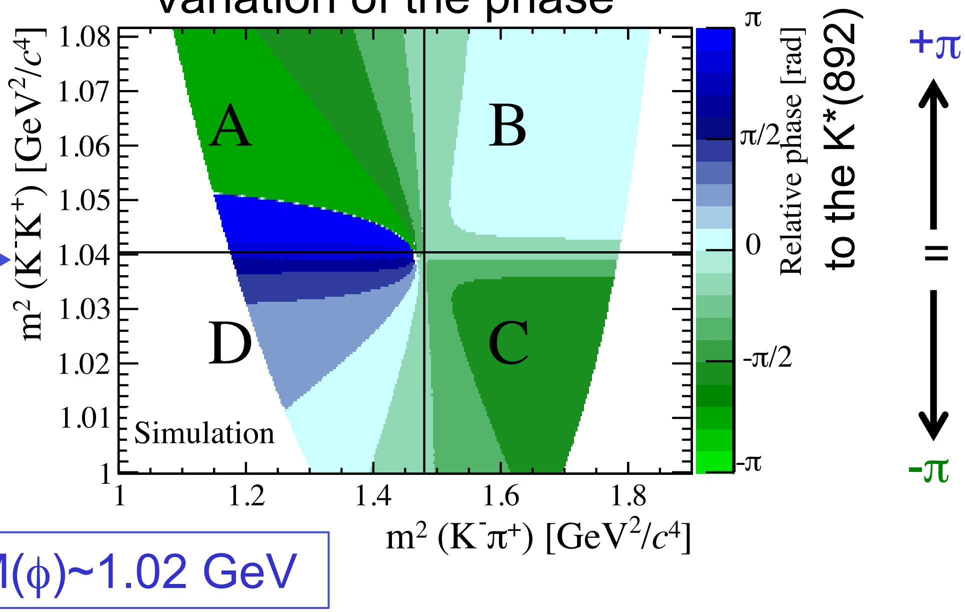
(c) and (d) from $D^+_s \to K^0_s\pi^+\pi^0$ or $D^+_s \to K^0_sK^+$

CP violation in $D^+ \to \phi \pi^+$ and $D^+_s \to K^0_s \pi^+$ decays

- To improve sensitivity to certain CPV we divide area around ϕ resonance in the Dalitz plot into four regions
- Relative strong phase varies rapidly across the ϕ region
- The division is chosen to minimize the change in phase within each region

$D^+ \to K^-K^+(\phi)\pi^+$

Variation of the phase



LHCb-PAPER-2012-052

LHCb simulation, used isobar amplitude model favoured by CLEO-c [Phys.Rev.D78 (2008) 072003]

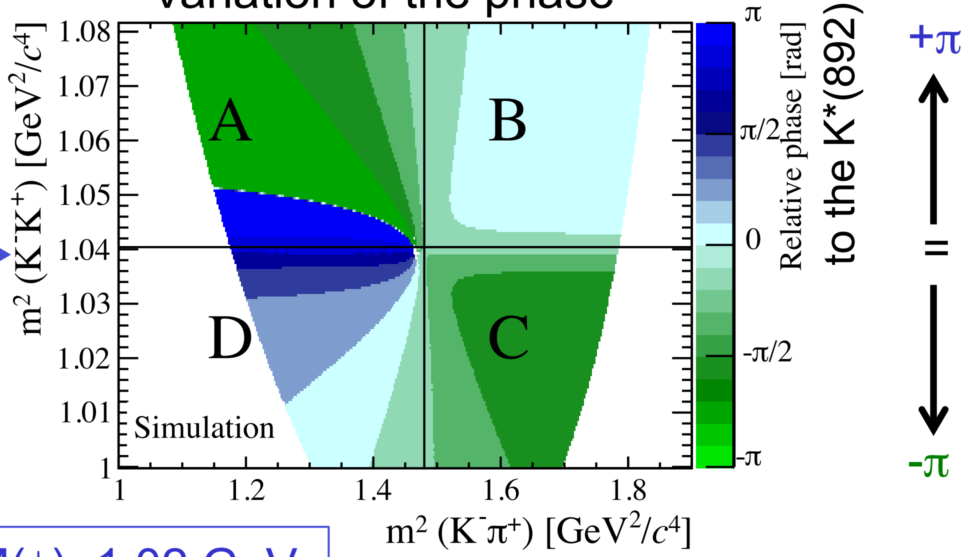
CP violation in $D^+ \to \phi \pi^+$ and $D^+_s \to K^0_s \pi^+$ decays

- To improve sensitivity to certain CPV we divide area around ϕ resonance in the Dalitz plot into four regions
- Relative strong phase varies rapidly across the ϕ region
- The division is chosen to minimize the change in phase within each region
- A difference between two diagonals with similar phases is calculated

LHCb-PAPER-2012-052

$D^+ \to K^- K^+(\phi)\pi^+$

Variation of the phase



M(φ)~1.02 GeV

LHCb simulation, used isobar amplitude model favoured by CLEO-c [Phys.Rev.D78 (2008) 072003]

$$A_{CP}|_S = \frac{1}{2}(A^A_{RAW} + A^C_{RAW} - A^B_{RAW} - A^D_{RAW})$$

Type of CPV	Mean A_{CP} (%)	Mean $A_{CP}\|_S$ (%)
$3°$ in ϕ phase	$-0.01\ (0.1\sigma)$	$-1.02\ (5.1\sigma)$
0.8% in ϕ amplitude	$-0.50\ (2.5\sigma)$	$-0.02\ (0.1\sigma)$
$4°$ in $K^*_0(1430)^0$ phase	$0.52\ (2.6\sigma)$	$-0.89\ (4.5\sigma)$
$4°$ in $K^*_0(800)$ phase	$0.70\ (3.5\sigma)$	$0.10\ (0.5\sigma)$

Simulations indicate that some types of CPV can be observed more effectively with A_{CP} and others with $A_{CP}|_S$

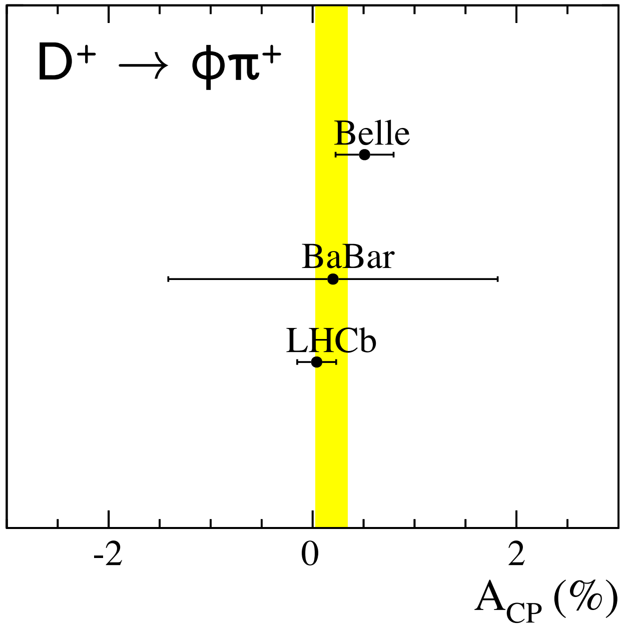
CPV in $D^+ \to \phi\pi^+$ and $D^+_s \to K^0_s\pi^+$

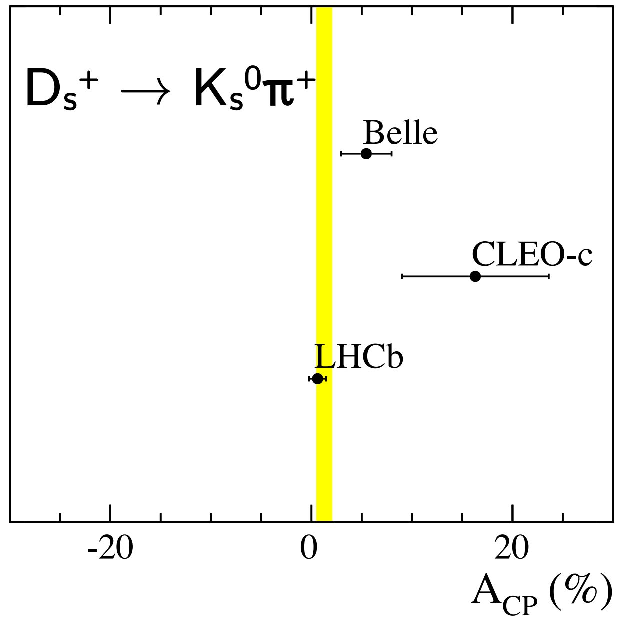
No evidence for CPV is observed

$$A_{CP}(D^+ \to \phi\pi^+) = (-0.04 \pm 0.14 \pm 0.13)\%$$
$$A_{CP}|_S(D^+ \to \phi\pi^+) = (-0.18 \pm 0.17 \pm 0.18)\%$$
$$A_{CP}(D_s^+ \to K_S^0\pi^+) = (+0.61 \pm 0.83 \pm 0.13)\%$$

errors ~1‰
1.6M events

LHCb-PAPER-2012-052





- LHCb measurements are the most precise of CP violation in ϕ region to date for both $D^+ \to \phi\pi^+$ and $D^+_s \to K^0_s\pi^+$

Searches for CPV in multi-body charm decays

We also looking for CP asymmetry in multi-body decays: in $D^{\pm} \rightarrow hhh$, $D^0 \rightarrow hhhh$

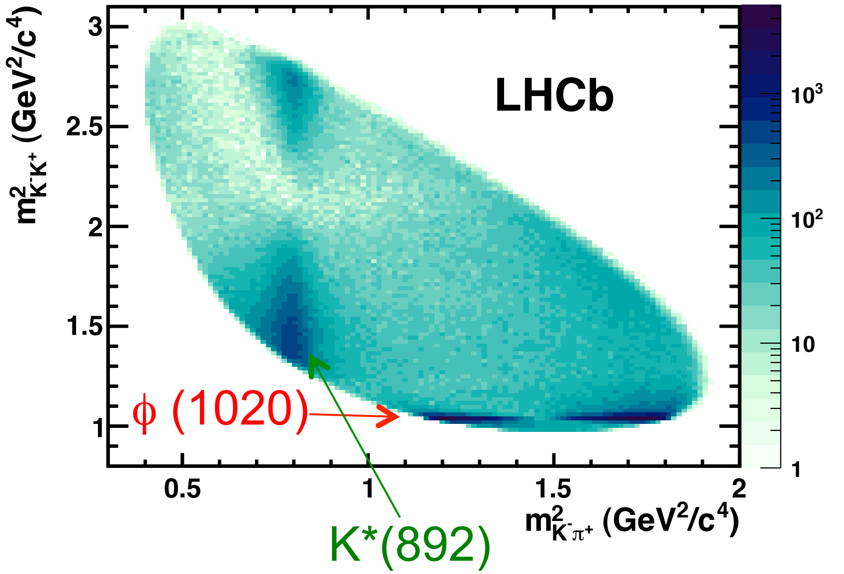
- Partition the Dalitz plot into bins
- For each bin measure **local charge asymmetry**

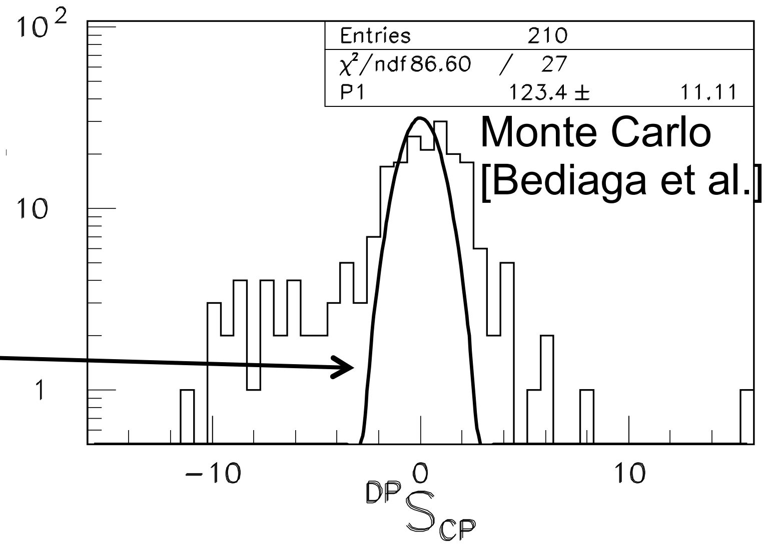
$$S^i_{CP} \equiv \frac{N^i(D^+) - \alpha N^i(D^-)}{\sqrt{N^i(D^+) + \alpha^2 N^i(D^-)}} \qquad \alpha = \frac{N(D^+)}{N(D^-)}$$

[Bediaga et al. Phys.Rev.D80(2009)096006]

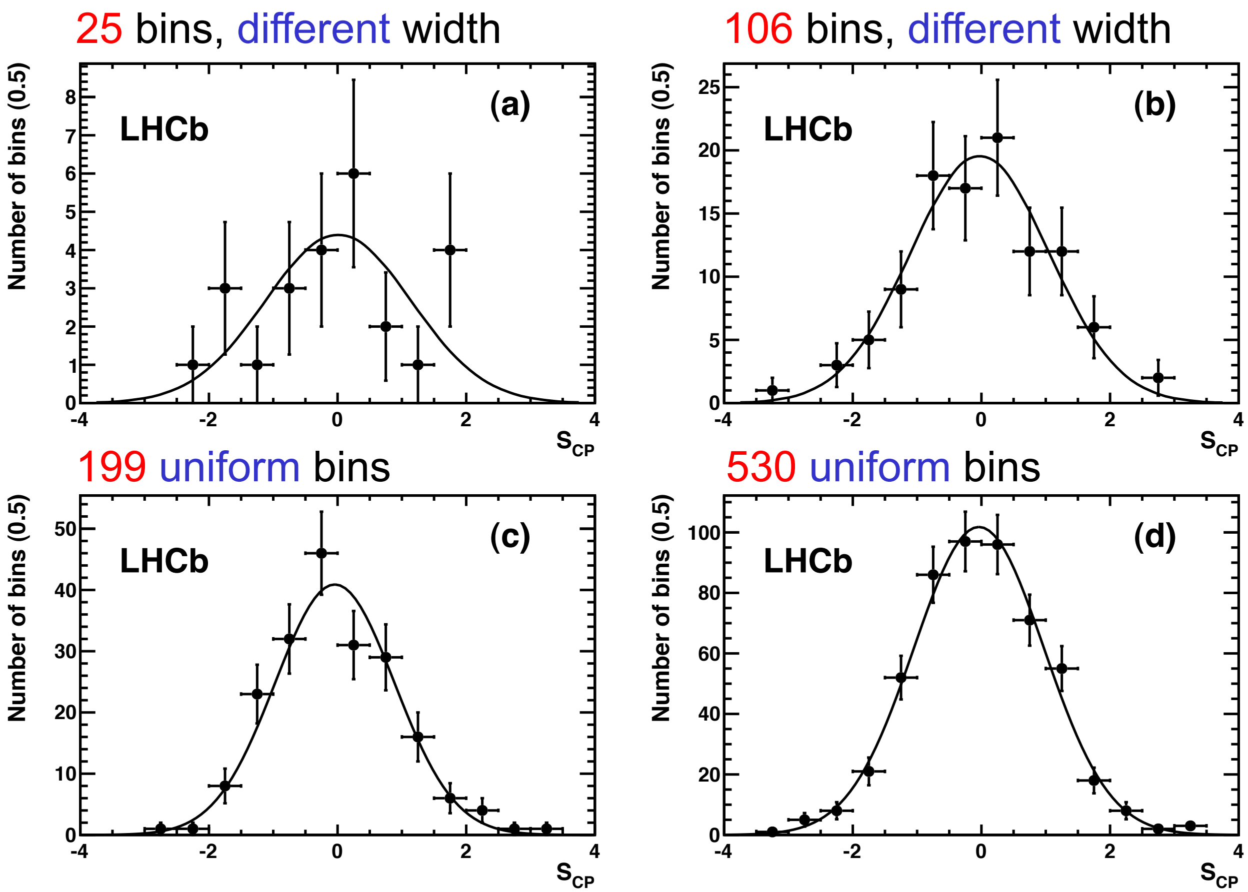
- Normalization cancels most global asymmetries (example production asymmetry)
- S_{CP} is a significance of a difference between D^+ and D^-
- Two equivalent methods:
  - ✧ If no CPV (only statistical fluctuations) then S_{CP} is Gauss distribution (μ=0, σ=1)
  - ✧ Also χ^2 test can be used: $\chi^2 = \Sigma {S^i_{CP}}^2$ → p-value

$D^+ \rightarrow K^- K^+ \pi^+$





Results for $D^+ \to K^- K^+ \pi^+$



- Several binnings in the Dalitz plot used to probe a range of CPV scenarios
- Binning shown consistent with no CPV at p=10%
- Also S_{CP} distributions consistent with standard Gauss distribution (μ~0, σ~1)
- No evidence for CP violation in the 2010 data set of 36/pb, 370k signal (SCS) $D^+ \to K^- K^+ \pi^+$

Phys.Rev.D84.112008

	μ	σ	χ^2/ndf	P-value
(a)	0.01±0.23	1.13±0.16	32.0/24	12.7%
(b)	-0.024±0.010	1.078±0.074	123.4/105	10.6%
(c)	-0.043±0.073	0.929±0.051	191.3/198	82.1%
(d)	-0.039±0.045	1.011±0.34	519.5/529	60.5%

More data is on tape:
for each 1/fb SCS signal decays:
~10 million of $D^+ \to K^- K^+ \pi^+$
~3 million of $D^+ \to \pi^- \pi^+ \pi^+$

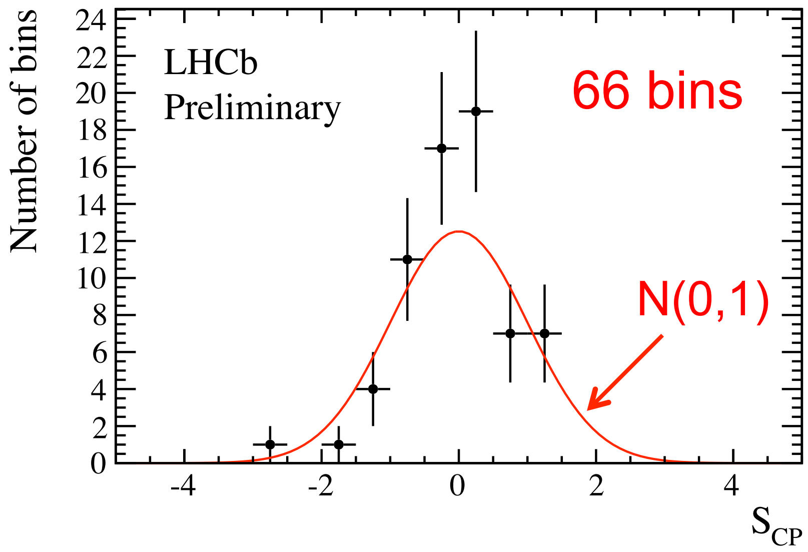
Results for $D^0 \to \pi^-\pi^+\pi^+\pi^-$

While three-body decay kinematics can be described completely in 2D Dalitz plot, a four-body decay has 5D phase space to fully describe the decay

Here we divide 5D phase space into bins and in each i^th bin we calculate S_{CP}

$$S^i_{CP} \equiv \frac{N^i(D^0) - \alpha N^i(\bar{D}^0)}{\sqrt{N^i(D^0) + \alpha^2 N^i(\bar{D}^0)}} \qquad \alpha = \frac{N(D^0)}{N(\bar{D}^0)}$$

LHCb 2011 data, L=1/fb,
180k events, 96% purity



Bins	p-values (%)
15	97.1
29	95.6
66	99.8

LHCb-CONF-2012-019

Using three different versions of binning, the results are consistent with the hypothesis of no CPV with a p-values close to 100%

Summary

- LHCb experiment has an important charm physics program and has the world's largest sample of c-hadron decays

- Using data collected in 2011 (1/fb), LHCb experiment has performed extensive studies of physics in the charm sector

- For the first time LHCb experiment has observed charm mixing in a single measurement (effect 9.1σ)

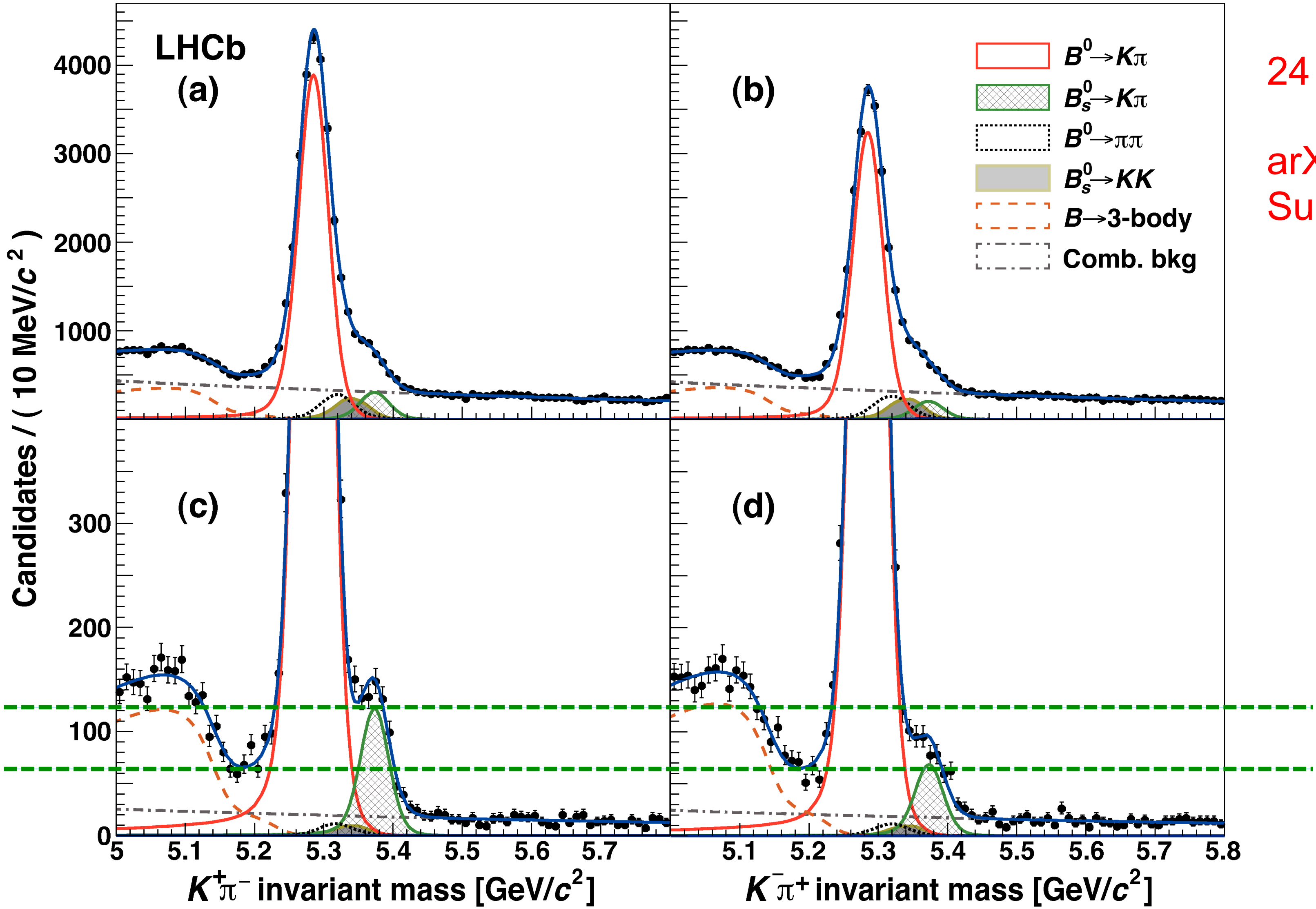
- Measured ΔA_{CP} between $D^0 \to K^- K^+$ and $D^0 \to \pi^- \pi^+$ from D* and B decays (two results statistically independent)
  - ✧ the central value is considerably closer to zero
  - ✧ result does not confirm the evidence for direct CPV in the charm sector

- No CPV observed in $D^+ \to \phi\pi^+$, $D^+_s \to K^0_s\pi^+$, $D^+ \to K^- K^+ \pi^+$, $D^0 \to \pi^- \pi^+ \pi^+ \pi^-$

- All measurements being improved with larger datasets:
  - ✧ 2011+2012: > 3/fb

- The LHCb experiment is more than beauty

First observation of CP violation in the decays of B^0_s



24 April 2013

arXiv:1304.6173
Submitted to Phys.Rev.Lett

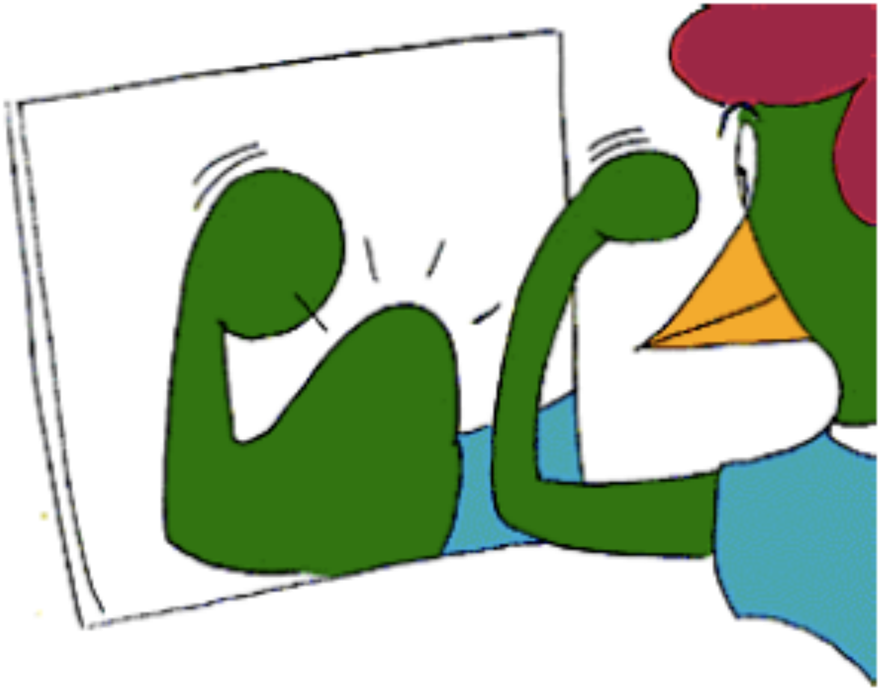
$B^0_s \to K^-\pi^+$

2011 data, L=1/fb

$$A_{CP}(B_s^0 \to K^-\pi^+) = 0.27 \pm 0.04(stat) \pm 0.01(syst)$$

$$A_{CP}(B^0 \to K^+\pi^-) = -0.080 \pm 0.007(stat) \pm 0.003(syst)$$

Backup



ΔA_{CP} from D* decay

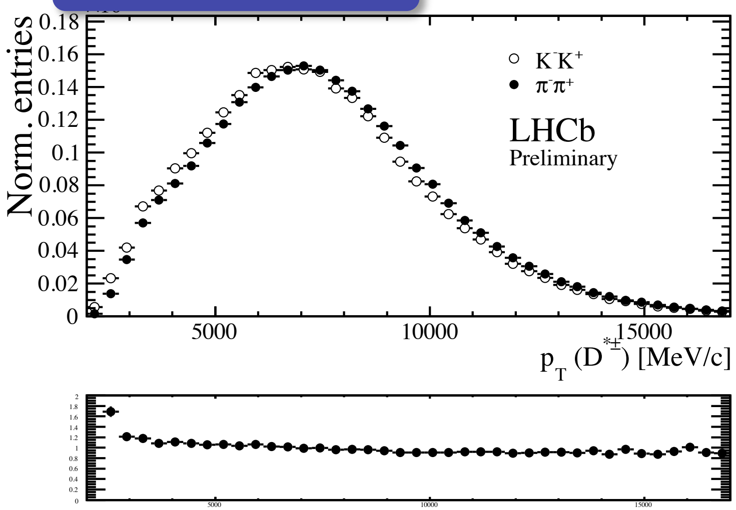
- The D^{*+} kinematic distributions are independent of the D^0 decay mode, but the selection requirements can lead to the different distributions of the K^-K^+ and $\pi^-\pi^+$ final states

- It can lead to a non-canceling second-order bias in ΔA_{CP}

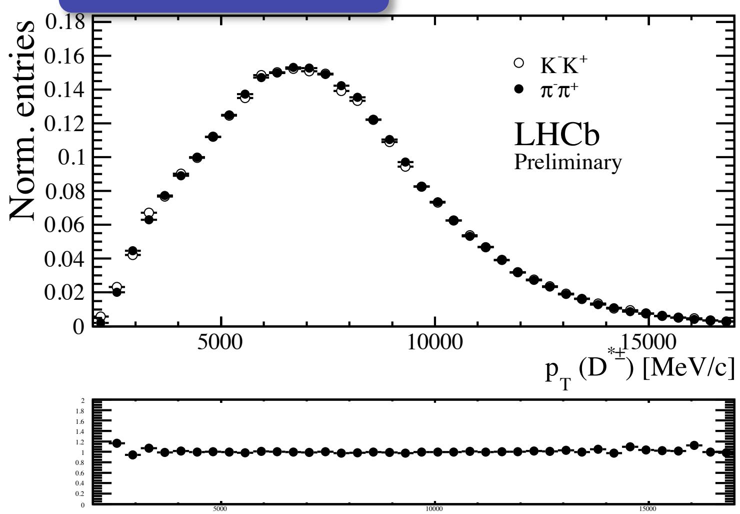
- To avoid this, we apply weighting in D* kinematic distributions of p_T, p, ϕ to ensure that $D^0 \to K^-K^+$ and $D^0 \to \pi^-\pi^+$ have the same kinematics
  - ✧ each $D^0 \to K^-K^+$ event gets a weight to match $D^0 \to \pi^-\pi^+$ kinematic distribution

Example $P_T(D^*)$

Before weighting



After weighting



1st measurement of ΔA_{CP} from D* decay

Analysis technique: split dataset into 4 subsets:

- Hardware trigger (L0) category:
  - ✧ D^0 triggered by hadronic calorimeter (Trigger On Signal)
  - ✧ event triggered on other particles from pp collision – by something else than the D* (Trigger Independent of Signal)
- Field polarity:
  - ✧ Magnet up (40%)
  - ✧ Magnet down (60%)

(stat.only)

ΔA_{CP}	Up	TOS	-0.62 ± 0.36 %
ΔA_{CP}	Down	TOS	-0.36 ± 0.30 %
ΔA_{CP}	Up	TIS	-0.30 ± 0.30 %
ΔA_{CP}	Down	TIS	-0.22 ± 0.25 %

- Weighted average of four subsets (2011, 1/fb) – Preliminary results:

$$\Delta A_{CP} = [-0.34 \pm 0.15^{stat} \pm 0.10^{syst}]\%$$

LHCb-CONF-2013-003

- Difference in decay time acceptance:

$$\Delta\langle t\rangle/\tau = [11.19 \pm 0.15^{stat} \pm 0.17^{syst}]\%$$

$$\Delta A_{CP} = [a_{CP}^{dir}(K^-K^+) - a_{CP}^{dir}(\pi^-\pi^+)] + \frac{\Delta\langle t\rangle}{\tau} a_{CP}^{ind}$$

Contribution from indirect CPV is ~10%

2nd measurement of ΔA_{CP} from semileptonic B decays

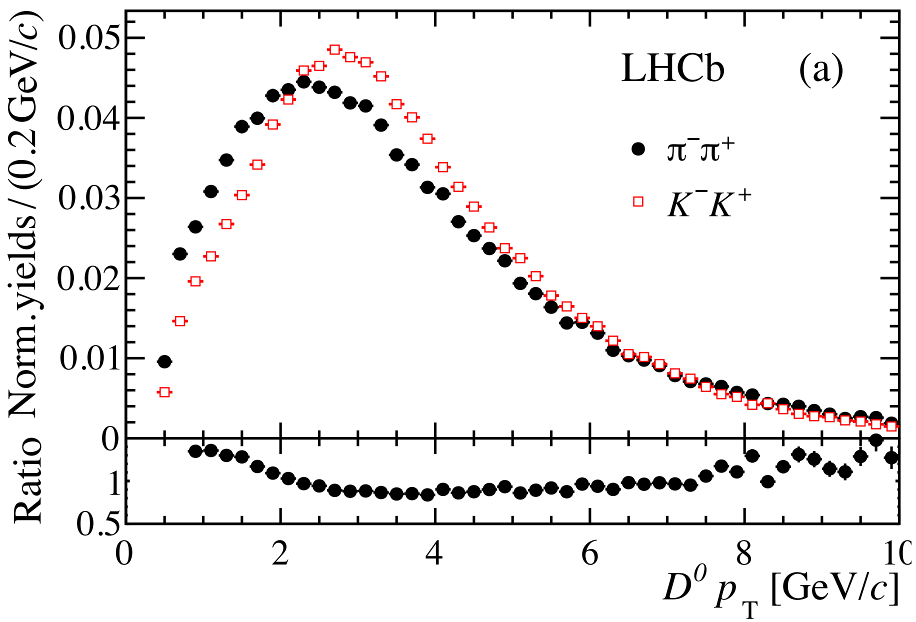
Different kinematic distributions for both decays of the K^-K^+ and $\pi^-\pi^+$ can lead to a non-canceling second-order bias in ΔA_{CP}

To obtain the same kinematic distributions for both decays we apply weighting in D^0 candidates on their p_T and η:

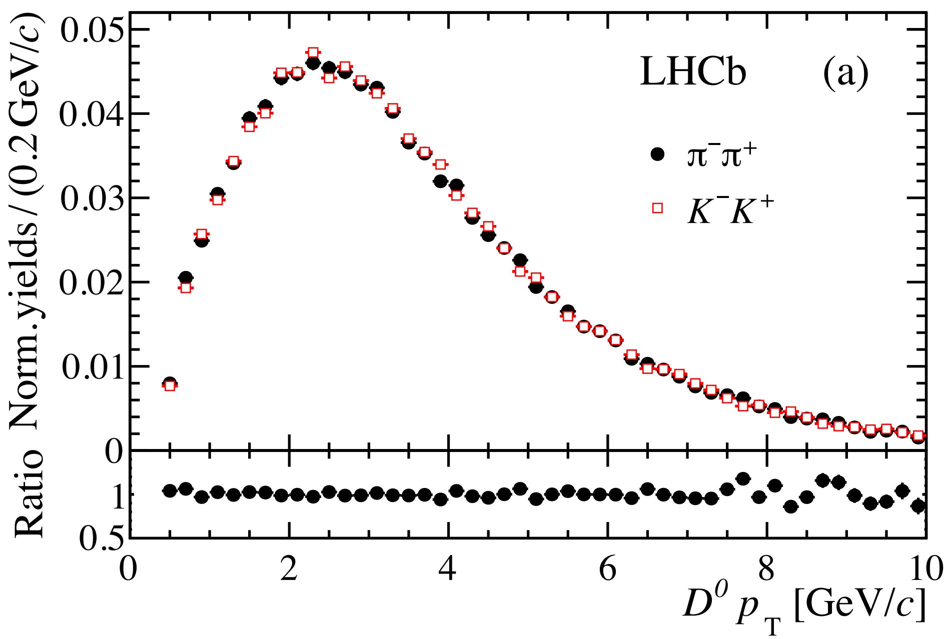
- weights are applied to either $D^0 \rightarrow K^-K^+$ and $D^0 \rightarrow \pi^-\pi^+$ candidates depending on which has most events in a given kinematic bin

Example $P_T(D^0)$

Before weighting

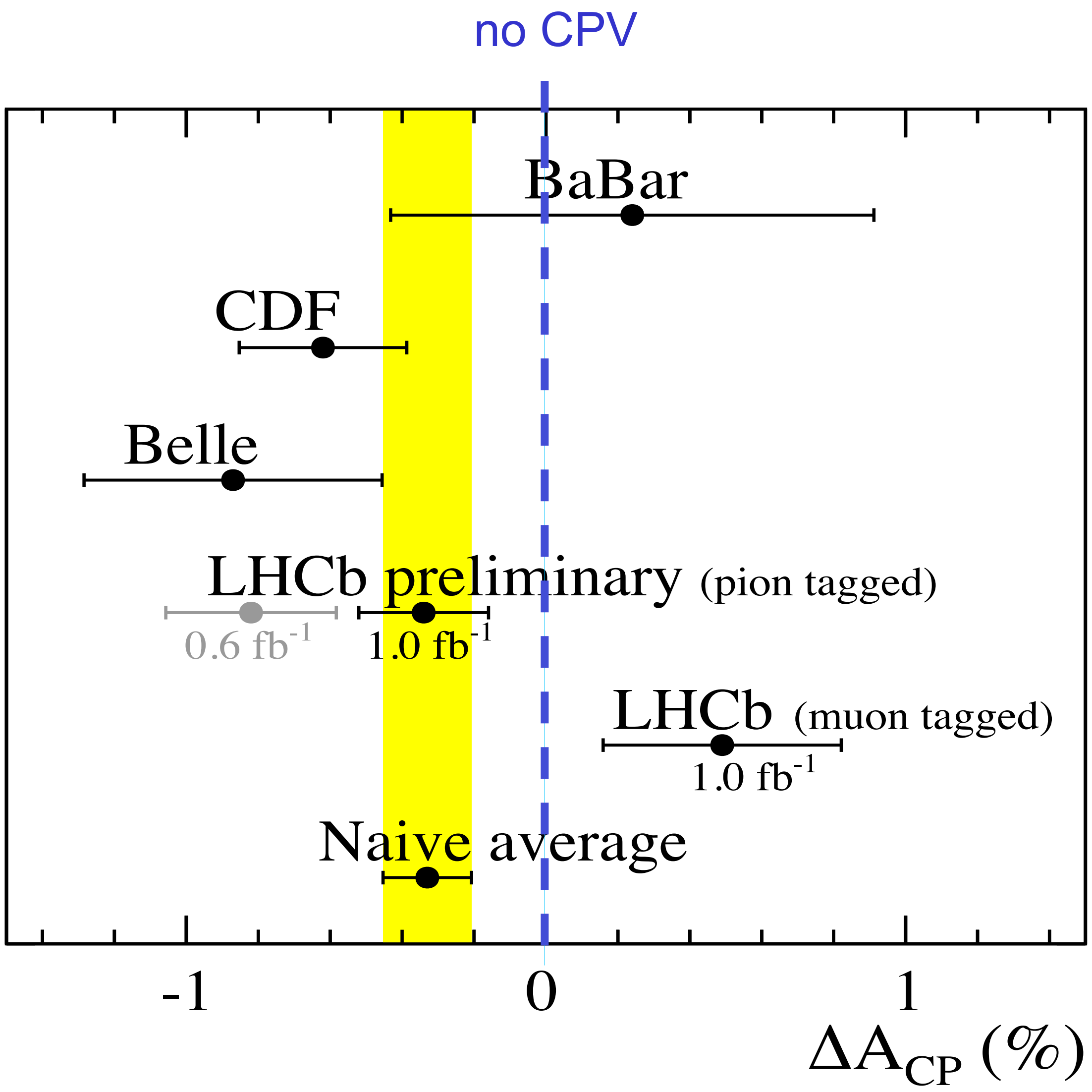


After weighting



ΔA_{CP} Preliminary new world average

New average includes BaBar, CDF, Belle and new LHCb results



Naive average neglecting indirect CPV
ΔA_{CP} = (-0.33 ± 0.12)%

Now:

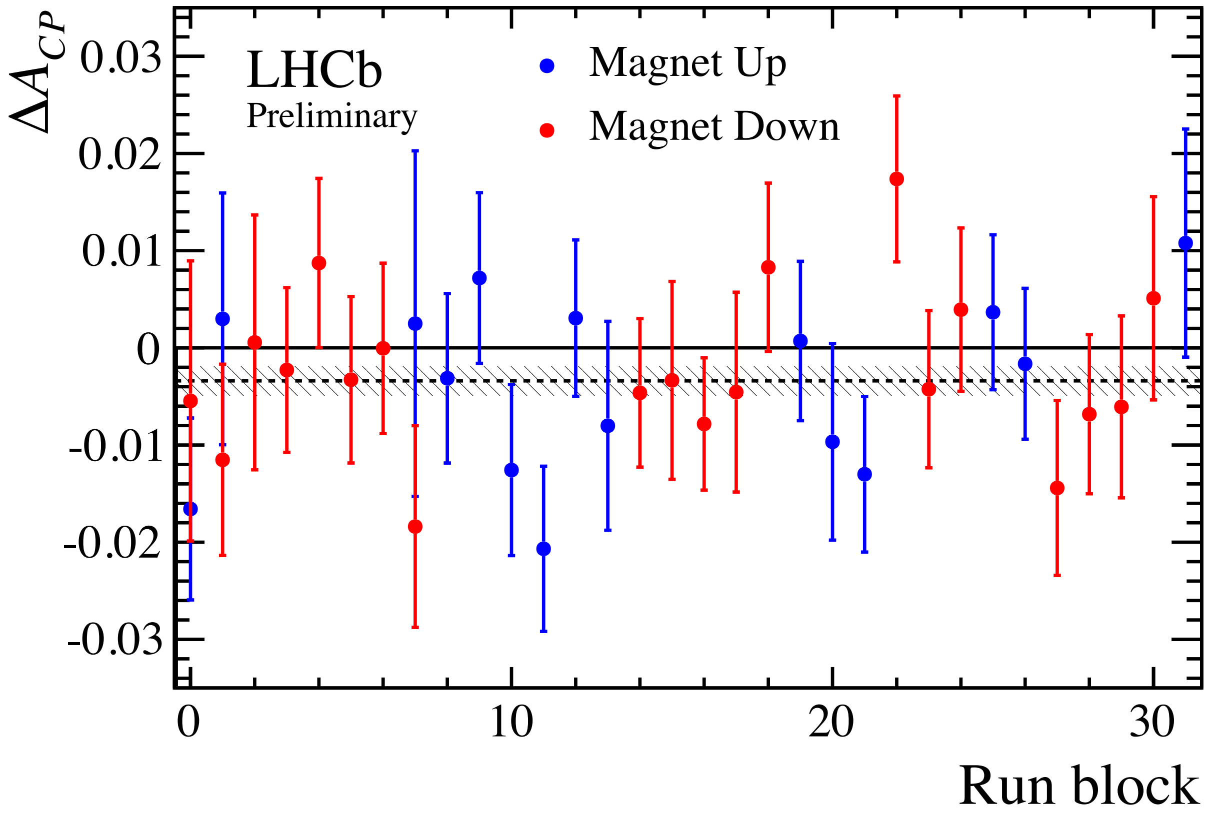
- the central value is considerably closer to zero
- result does not confirm the evidence for direct CPV in the charm sector

ΔA_{CP} stability checked

Many cross-checks performed for both methods:

- time at which data was taken
- stable versus kinematic variables: decay time, p_T, p, η, ϕ etc.
- independent cross-checks of final result by different people
- many more…
- no significant dependence is observed

Example: pion-tagged analysis



No dependence versus data taking period

Comments on ΔA_{CP}

Comments:

- The central value is considerably closer to zero the the previous result
- New result does not confirm the evidence for direct CPV in charm sector
- Several factors can contribute to the change
  - ✧ larger data sample
  - ✧ improved detector alignment and calibration
  - ✧ difference in analysis technique

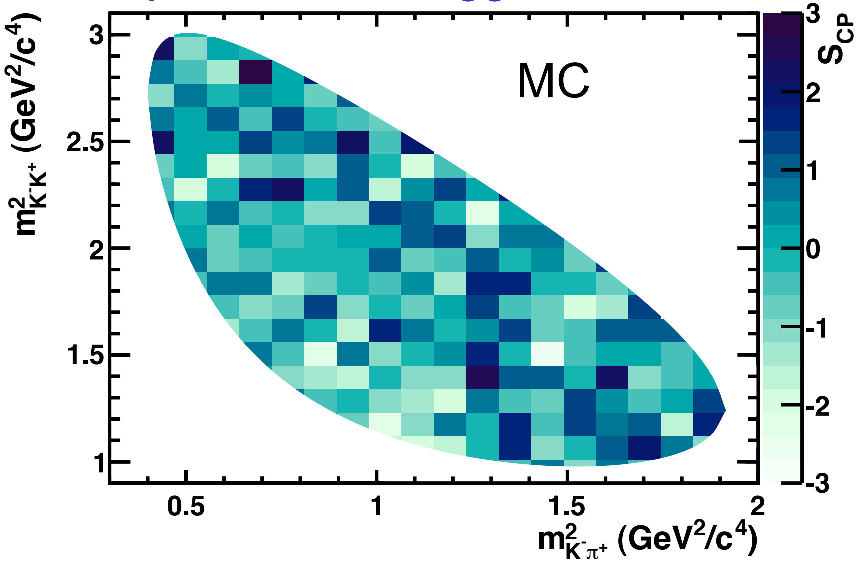
Tests of the method

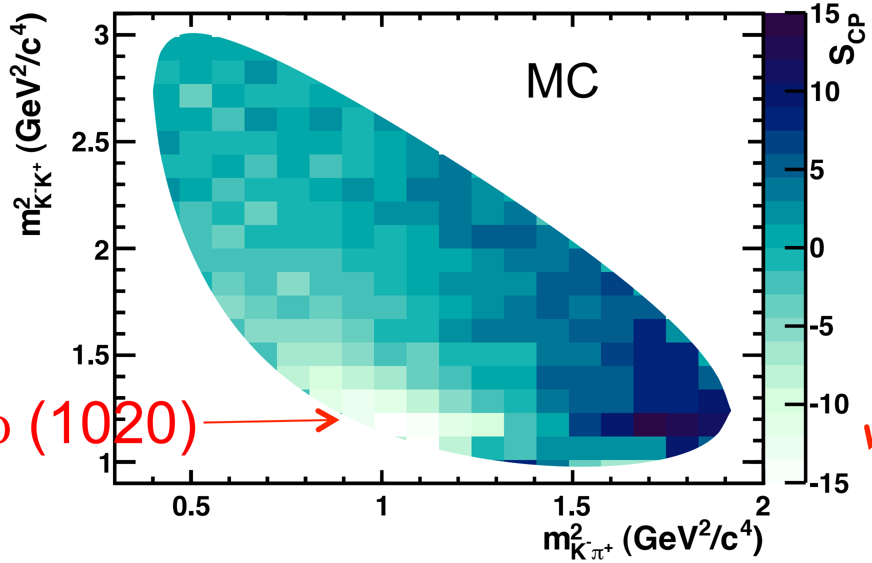
- **Check the response of the method on Monte Carlo (Dalitz models from CLEO-c, arXiv:0807.4545):**
  - should not generate signal where it is not expected
  - should give a visible signal where it is expected

$D^+ \to K^- K^+ \pi^+$

Sample 50 times bigger than 2010

5×10^7 events with 4^0 weak phase difference between amplitudes for resonance of $\phi(1020)$ from $D^+ \to \phi\pi^+$ a $D^- \to \phi\pi^-$





ϕ (1020)

The same bins
Different scale of S_{CP}

If no CPV then no signal (good)
P-value ~5%
→ no CP asymmetry

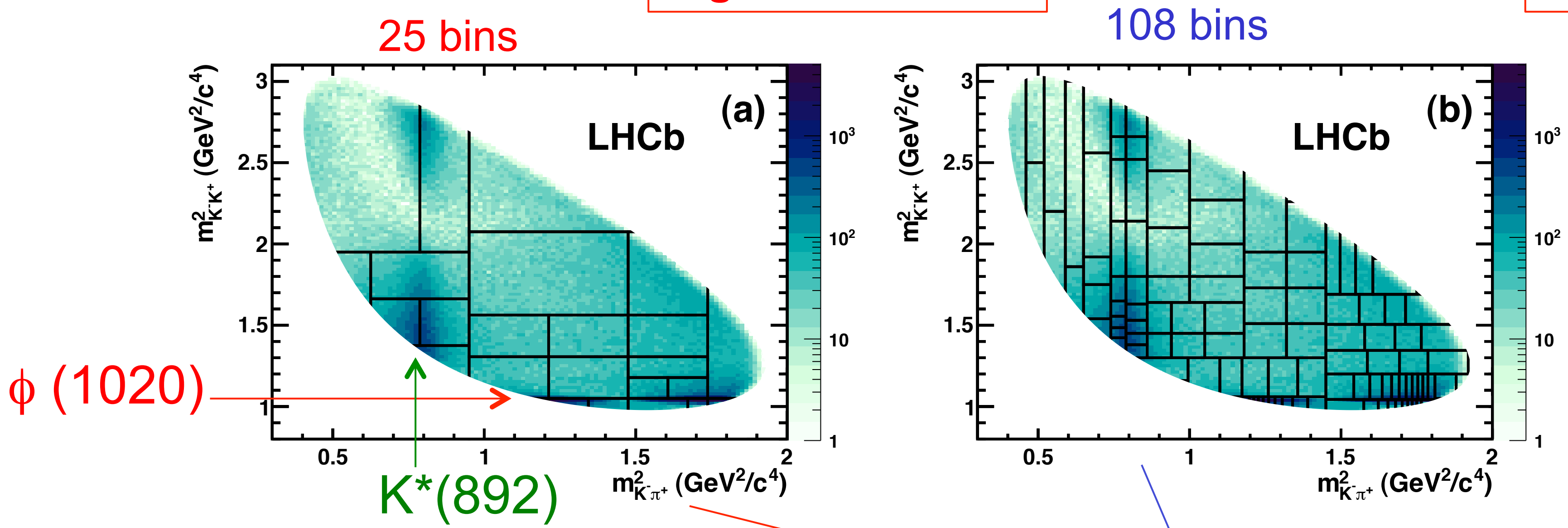
If CPV then P-value ~10^{-100}
– there is CP asymmetry
– visible sign change of S_{CP} in ϕ region

Number of bins test

Bins with different widths

Signal $D^+ \to K^- K^+ \pi^+$

Monte Carlo



	$P(3\sigma)$ (25 bins)	$P(3\sigma)$ (108 bins)
No CPV	0%	1%
6^0 weak phase difference in $\phi(1020)$	99%	98%
4^0 weak phase difference in $\phi(1020)$	76%	41%

Version with 25 bins is better

100 the same experiments and check how many times obtained 3σ

The trigger and charm physics

LHC rate ~15 MHz

hardware

L0 ~1 MHz
3 subdetectors:
ECAL, HCAL, Muon

c-anti-c ~10%

c-anti-c ~50%

After L0 **~500 kHz c-anti-c** events

No possibility of an inclusive charm trigger!

Possible only dedicated exclusive trigger lines tuned for the needs of specific analyses to deliver high signal efficiency and purity

software

Hlt1 (partial reconstruction) **~50 kHz** (efficiency ~50 %)

Hlt2 (full reconstruction) **3 kHz**
2 kHz – b physics
1 kHz – dedicated exclusive lines of **$D \to hh/3h/4h$** (efficiency 50-90%)

example: 5k $D^{*\pm} \to (D^0 \to K^{\pm} K^{-+}) \pi^{\pm}$ for 1 pb^{-1} (**2010:** 38 pb^{-1}, **2011:** 1.1 fb^{-1})

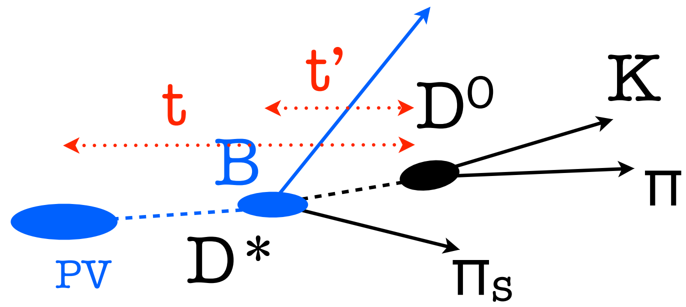
Systematics D^0 – anti-D^0 mixing

- Most of the systematic uncertainties cancel in the ratio between WS and RS events

- Two main sources of systematic uncertainties have been identified:

  (1) secondary D mesons

  ✧ D from B have wrong decay time
  ✧ such events have non-zero IP
  ✧ cut on χ^2(IP) removes most of them
  ✧ remains ~3%



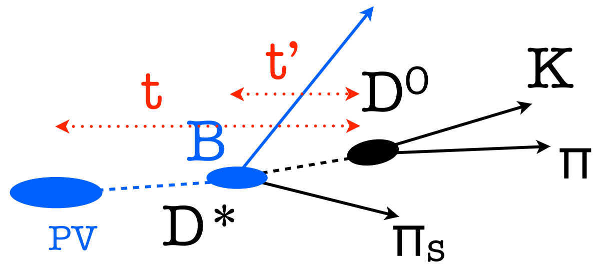
  (2) backgrounds from incorrectly reconstructed D decays – peak in $M(D^0\pi^+_s)$ (the D^0 is partially reconstructed or misidentified)

  ✧ such backgrounds are highly suppressed by tight PID cuts and two-body mass requirements
  ✧ estimated a residual (0.4±0.2)% contamination of doubly mis-identified RS events in the WS sample

- Results are dominated by statistical uncertainties

Bias from secondary D decays

$$R(t) = \frac{N_{WS}(t)}{N_{RS}(t)} = R_D + \sqrt{R_D}y't + \frac{x'^2 + y'^2}{4}t^2$$



The contamination of charm mesons produced in b-hadron decays could bias the time-dependent measurement

$$R^m(t) = \frac{N^{WS}(t) + N_B^{WS}(t)}{N^{RS}(t) + N_B^{RS}(t)} = R(t)\left\{1 - f_B^{RS}(t)\left[1 - \frac{R_B(t)}{R(t)}\right]\right\}$$

$\Delta_B(t)$ is a time-dependent bias due to the secondary contamination

where: $$f_B^{RS}(t) = \frac{N_B^{RS}(t)}{N^{RS}(t) + N_B^{RS}(t)}, \quad R_B(t) = \frac{N_B^{WS}(t)}{N_B^{RS}(t)}$$

The fraction of secondary decays in the RS sample at decay time t

Since $\Delta_B \geq 0$, it follows that the background from secondary D decays decreases the observable mixing effect. The bias in bounded by

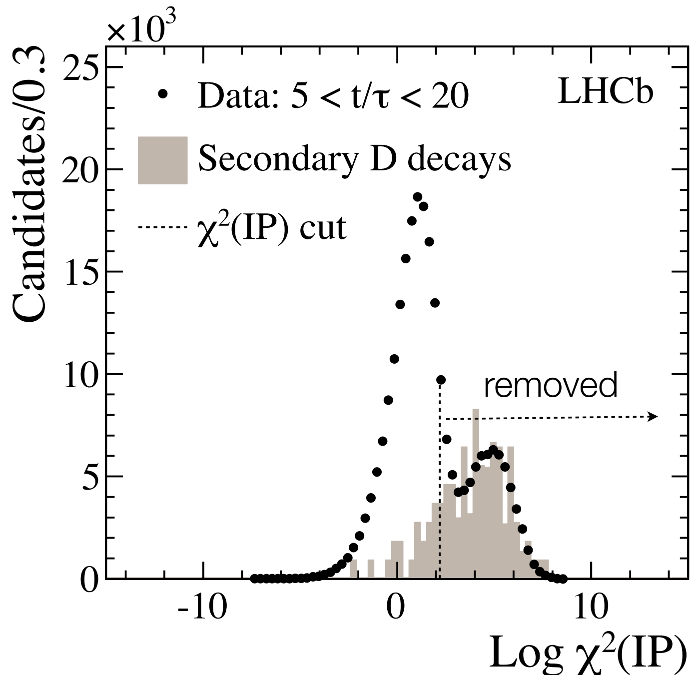
$$0 \leqslant \Delta_B(t) \leqslant f_B^{RS}(t)\left[1 - \frac{R_D}{R(t)}\right]$$

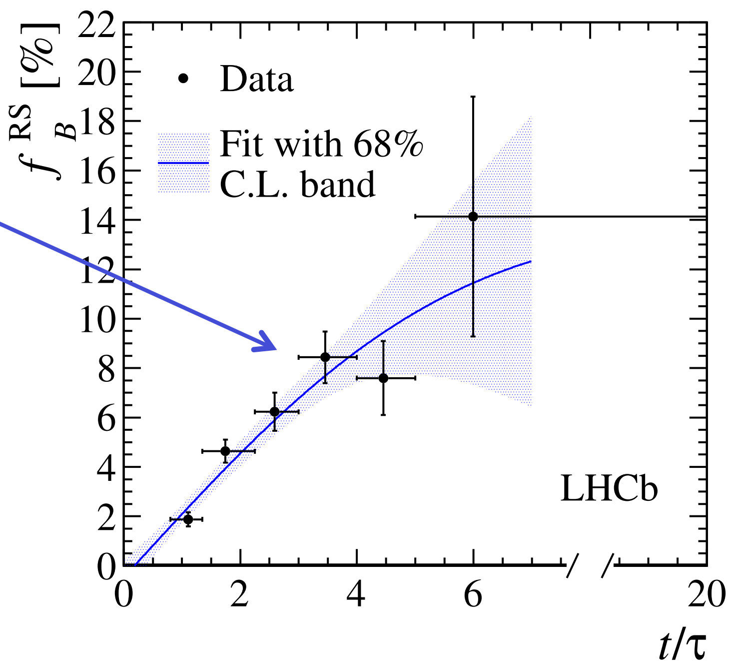
Measuring $f^{RS}_B(t)$

- A measurement of the secondary fraction is done by by fitting the χ^2(IP) distribution of the RS D^0 candidates in bins of decay time

- Secondary shape is estimated from events reconstructed also as
  $B \to D^*(3)\pi$, $B \to D^*\mu X$ or $B \to D^0\mu X$

- The value of $f^{RS}_B(t)$ is constrained in the time-dependent fit to the measured fraction





The unbinned method

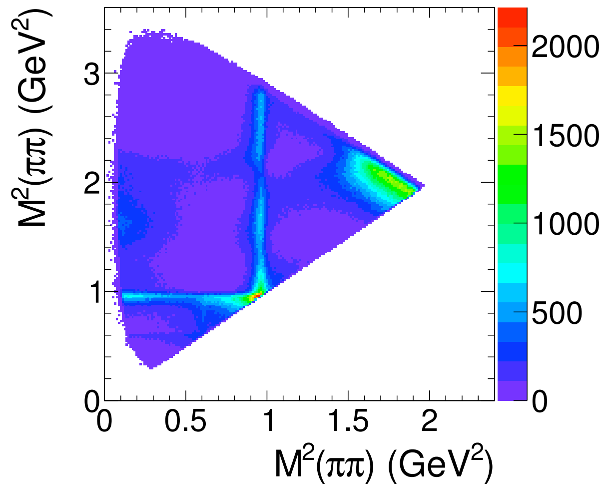
- No evidence for CP violation using the binned S_{CP} method
- The goal is to find the most sensitive method which allows us to see the differences between D^+ and D^-
- The unbinned methods could be more sensitive than the binned ones but they are more difficult in using
- There are a few unbinned method

- To analyse LHCb data Warsaw Group uses k-nearest neighbor (kNN) method:
(M.F.Schilling J.Am.Stat.Assoc.81(1986)799)

✧ used to compare the Dalitz plots for D^+ and D^- to test whether they have similar distributions or not

✧ based on the concept of counting the tag nearest neighbors (n_k):
1. in a pooled sample of particles and antiparticles we calculate distances between all event pairs
2. we find the k-nearest neighbor events to each point
3. we calculate a test statistic

$D^\pm_s \to \pi^+\pi^-\pi^\pm$ $(h_1h_2h_3)$



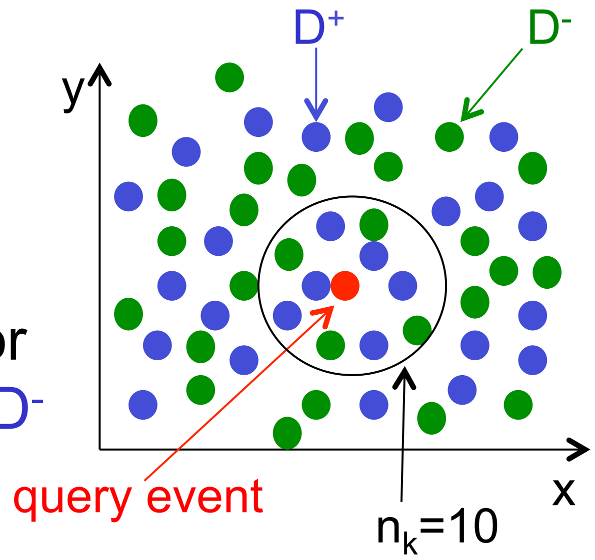
The test statistic

To test the hypothesis $f_a = f_b$ for the pooled sample of D^+ and D^- we calculate:

$$T = \frac{1}{n_k(n_a+n_b)} \sum_{i=1}^{n_a+n_b} \sum_{k=1}^{n_k} I(i,k)$$

✧ $I(i,k) = 1$ if the i^{th} query event and its k^{th} nearest neighbor belong to the same sample, like pairs: D^+—D^+ and D^-—D^-

✧ $I(i,k) = 0$ otherwise, unlike pairs: D^+—D^-



T is the mean fraction of like pairs in the pooled sample of the two data sets

Advantage:

- the expected distribution of the test statistic is known
- for the case $f_a = f_b$ the pull $(T-\mu_T)/\sigma_T$ has a limiting standard normal distribution

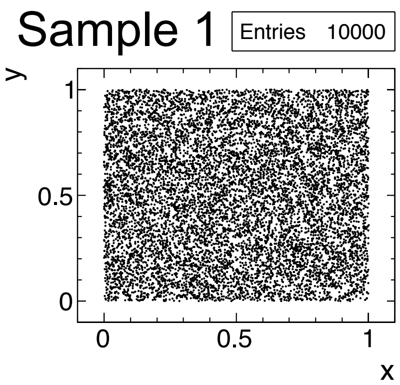
Mean: $$\mu_T = \frac{n_a(n_a-1)+n_b(n_b-1)}{n(n-1)}$$

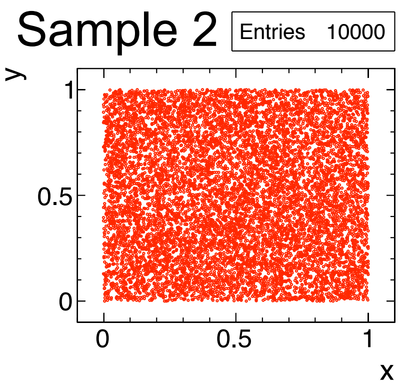
Variance: $$\lim_{n,n_k,D\to\infty} \sigma_T^2 = \frac{1}{nn_k}\left(\frac{n_a n_b}{n^2} + 4\frac{n_a^2 n_b^2}{n^4}\right)$$

with the fast convergence even for D = 2

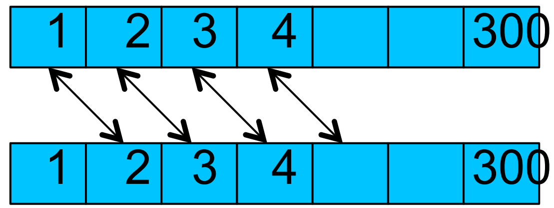
Expectation of test statistic for $n_a = n_b$ and $f_a = f_b$

300 uniform samples in two dimensions (x,y) from [0,1] are generated.
10k events in each sample.

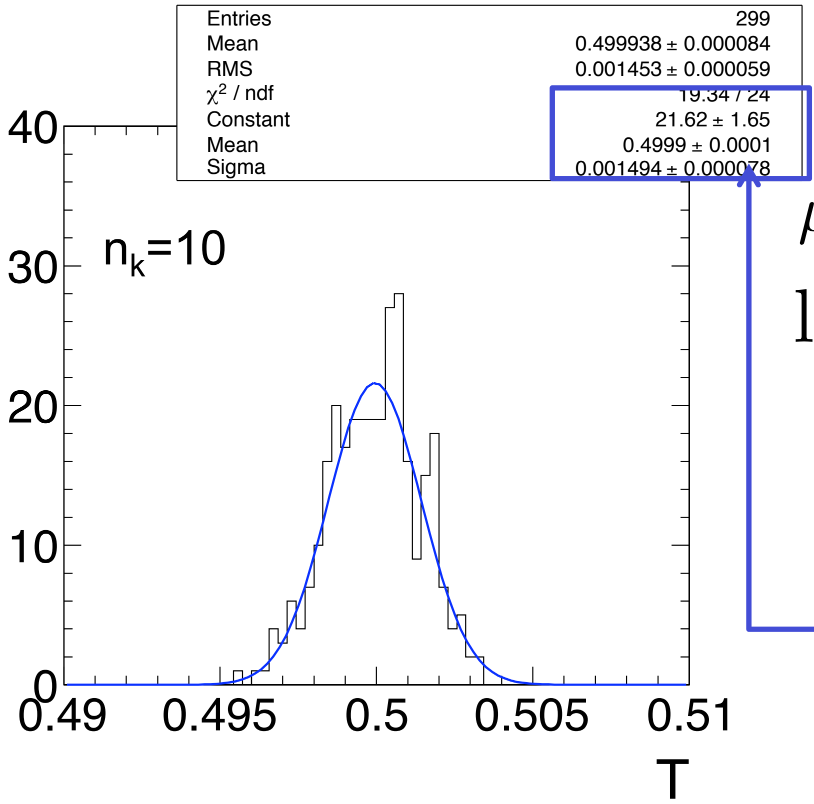




Different samples are compared.
299 combinations



$$T = \frac{1}{n_k(n_a+n_b)} \sum_{i=1}^{n_a+n_b} \sum_{k=1}^{n_k} I(i,k)$$



Expectation of μ_T and σ_T:

$$\mu_T = \frac{n_a(n_a-1)+n_b(n_b-1)}{n(n-1)} = 0.49999 \ (if \ n_a = n_b)$$

$$\lim_{n,n_k,D\to\infty} \sigma_T^2 = \frac{1}{n n_k}\left(\frac{n_a n_b}{n^2} + 4\frac{n_a^2 n_b^2}{n^4}\right)$$

for n_k = 10 expect σ_T = 0.001581

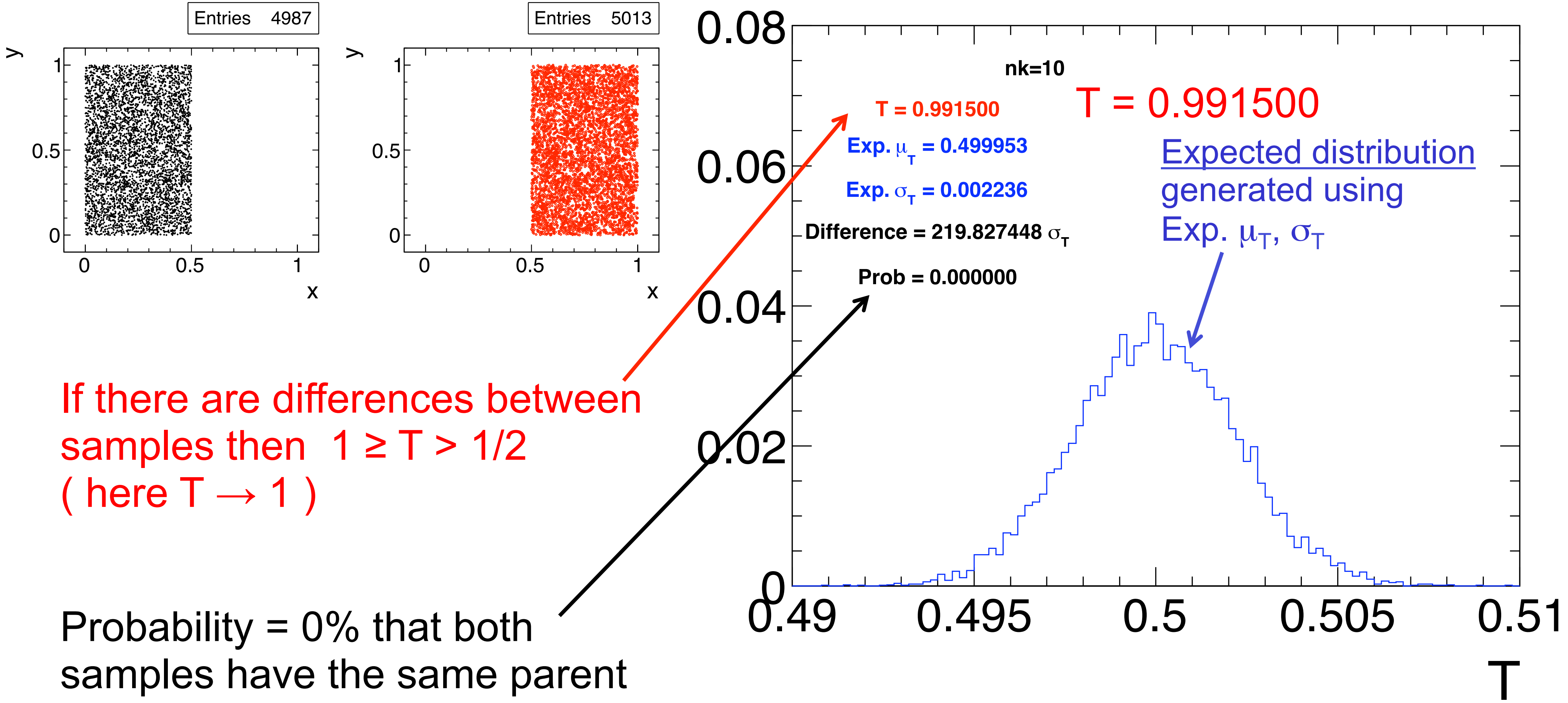
From the fit to the T distribution:

<T>=0.4999 ± 0.0001 agrees with expected μ_T

$\sigma_{T,fit}$ = 0.001494 ± 0.000078 agrees with σ_T

Expectation of test statistic for $n_a = n_b$ and $f_a \neq f_b$

Two separated samples with comparable number of events are generated



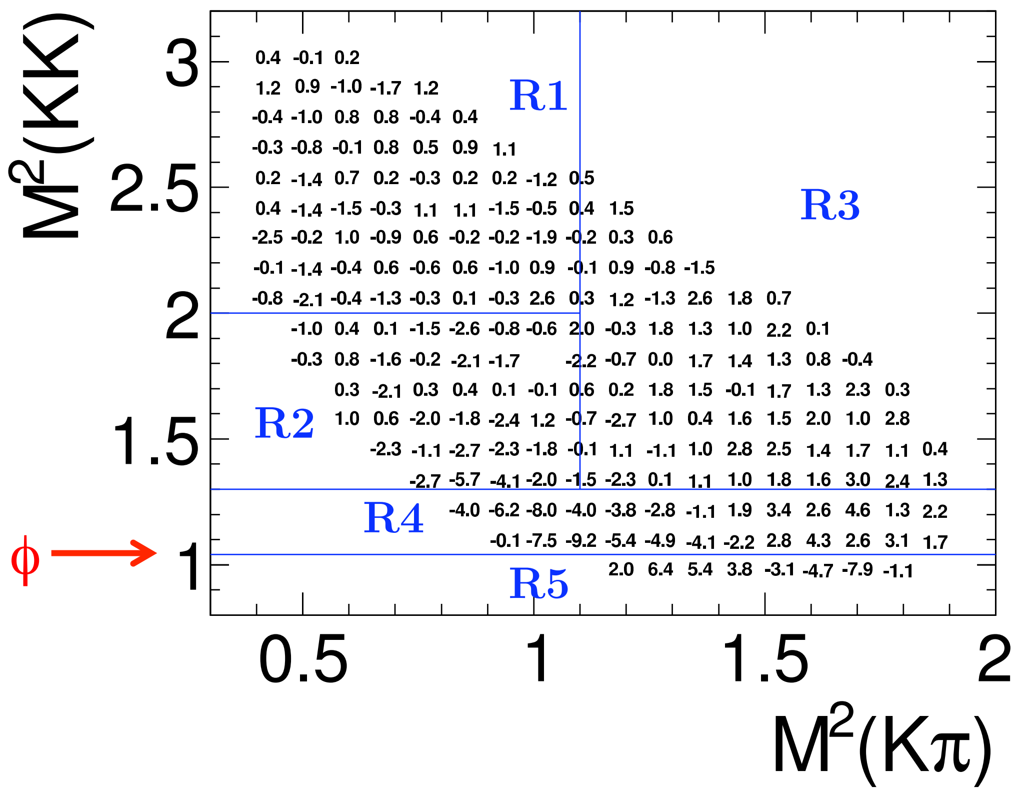
If there are differences between samples then $1 \geq T > 1/2$ (here $T \rightarrow 1$)

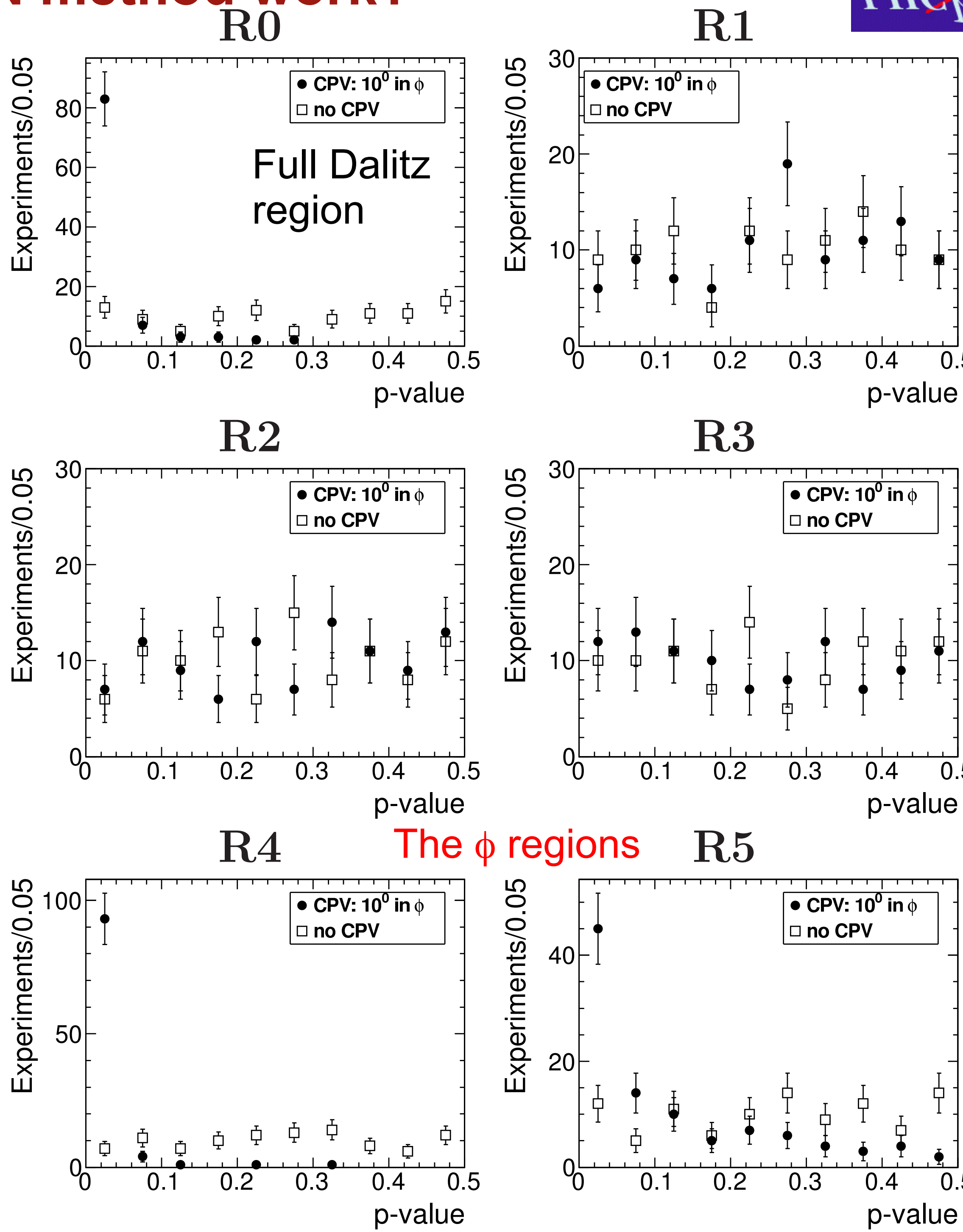
Probability = 0% that both samples have the same parent distribution

How does the KNN method work?

Monte Carlo (CLEO-c model) signal decay (SCS) $D^+ \to K^-K^+\pi^+$
100 pseudo experiments
2 million events each: with no CPV and CPV – 10^0 in ϕ, $n_k = 20$

$$S^i_{CP} \equiv \frac{N^i(D^+) - N^i(D^-)}{\sqrt{N^i(D^+) + N^i(D^-)}}$$





Clear evidence of disagreement is seen for MC CPV sample

How does the KNN method work?

Monte Carlo, signal decay (SCS) $D^+ \to K^-K^+\pi^+$
100 pseudo experiments, 2 million events each, $n_k = 20$

No CPV

Region	$\geq 1\sigma(\%)$	$\geq 2\sigma(\%)$	$\geq 3\sigma(\%)$	$\geq 4\sigma(\%)$	$\geq 5\sigma(\%)$
R0	27	7	0	0	0
R1	31	3	0	0	0
R2	28	2	0	0	0
R3	32	5	0	0	0
R4	26	2	0	0	0
R5	31	3	0	0	0

The fraction of data sets that exceed 1,2,3,4,5σ levels of significance

CPV – 10^0 in ϕ (regions R4 and R5)

Region	$\geq 1\sigma(\%)$	$\geq 2\sigma(\%)$	$\geq 3\sigma(\%)$	$\geq 4\sigma(\%)$	$\geq 5\sigma(\%)$
R0	93	69	33	9	1
R1	24	3	0	0	0
R2	28	3	0	0	0
R3	39	7	0	0	0
R4	98	87	55	19	1
R5	70	31	8	0	0

Clear evidence of disagreement is seen for MC CPV sample

Summary

- The kNN method was used to analyse LHCb data for searching local differences between D^+ and D^-

- First results for $D^+ \rightarrow \pi^-\pi^+\pi^+$ (here CP asymmetry is expected) were discussed within LHCb Group and analysis is under review (blined)

- We plan to use the kNN method for searching for CP asymmetry in different decays of:
  - ✧ charm particles,
  - ✧ beauty particles (here CP violation is larger)

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ -\lambda & 1 & \lambda^2 \\ -\lambda^3 & -\lambda^2 & 1 \end{pmatrix}$$