Model statystyczny produkcji czastek w RHIC-u

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UW, 7.11.2003

WB + Wojciech Florkowski, PRL 87 (2001) 272302; PRC 65 (2002) 064905
WB+ Anna Baran + WF, Acta Phys. Polon. B33 (2002) 4235
WB+ WF+ Brigitte Hiller (Coimbra), PRC 68 (2003) 034911
Piotr Bozek+ WB+WF, `nucl-th/0310062`

Expectations

October 6, 1999 ... report summarizes technical discussions that conclude there is no danger of a "disaster" at RHIC. ... The scenarios are:

- Creation of a black hole that would "eat" ordinary matter.
- Initiation of a transition to a new, more stable universe.
- Formation of a "strangelet" that would convert ordinary matter to a new form.

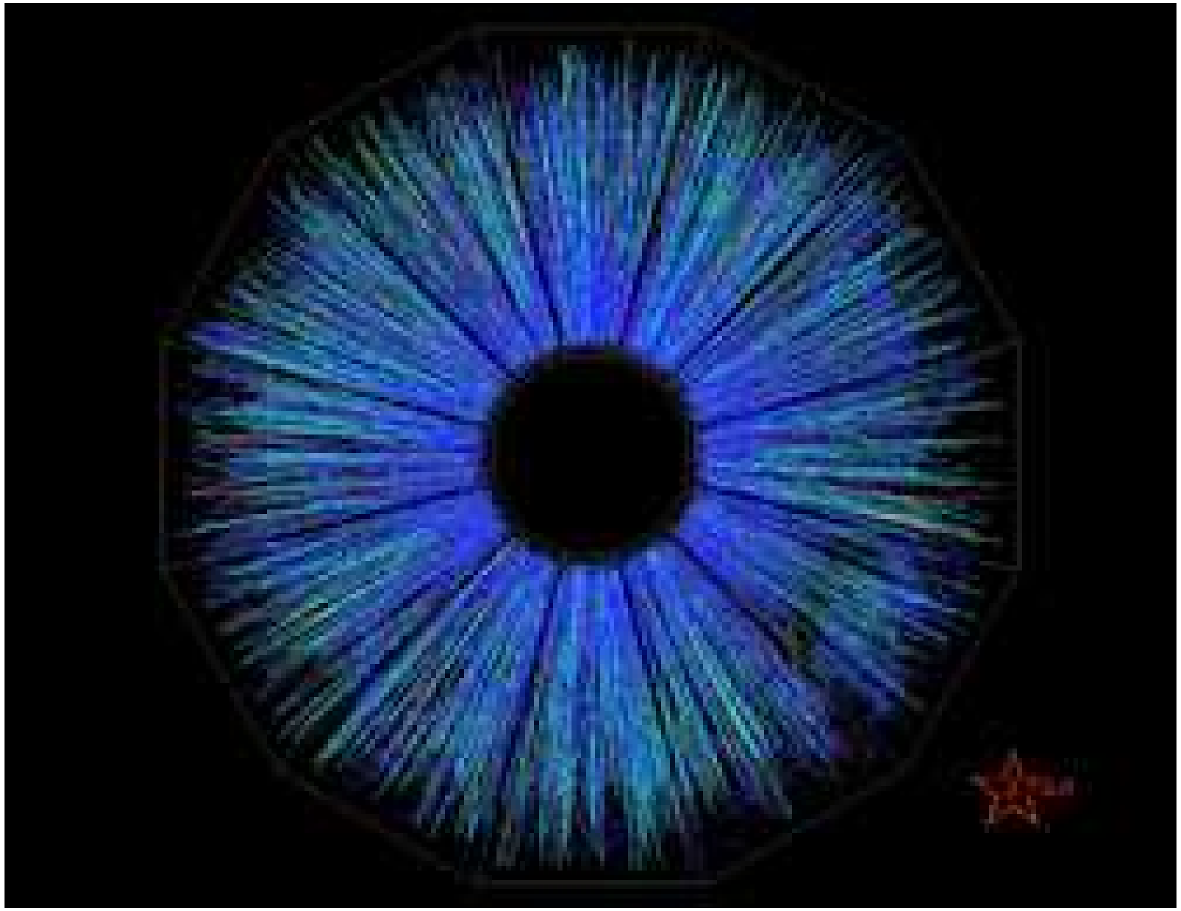
"We conclude that there are no credible mechanisms for catastrophic scenarios at RHIC," said committee chair Robert Jaffe, ...

Proclamation of quark-gluon plasma

A common assessment of the collected data leads us to conclude that we now have compelling evidence that a new state of matter has indeed been created, at energy densities which had never been reached over appreciable volumes in laboratory experiments before and which exceed by more than a factor 20 that of normal nuclear matter. The new state of matter found in heavy ion collisions at the SPS features many of the characteristics of the theoretically predicted quark-gluon plasma.

U. Heinz + M. Jacob, `nucl-th/0002042`

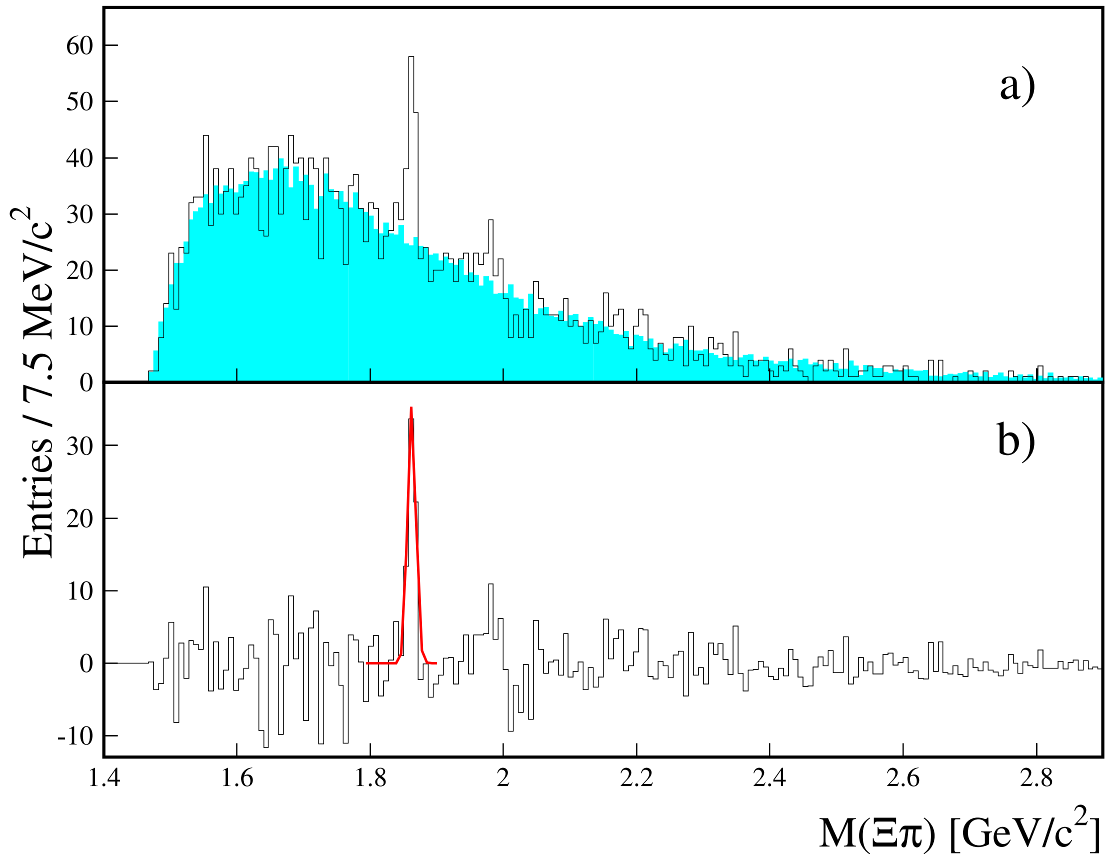
The iris of RHIC



Motivation and scope

- RHIC is a "major data provider": soft physics, hard physics, "tomography", ... A lot of detailed knowledge on identified spectra and correlations.
- Spectacular results: jet quenching, resonances seen in correlations, v_2, v_4, ...
- "Puzzles:" HBT radii, out-of-plane elongation, short life-time of the hadronic phase – problems for existing models and codes
- New spectroscopy possible: NA49 at CERN SPS found a very narrow $\Xi^{--}_{3/2}(1862)$ in $\Xi\pi$ correlations, which is a $ddss\bar{u}$ state. Possible search of $\theta^{+}(1540)$, *i.e.* $udd d\bar{s}$, ...)
- Understanding of data in simple terms desired
- Hadronic resonances are important in particle production, they appear in measurements of correlations of identified particles $(K^{*}(892), \rho, \Delta^{++}(1232), ...)$, reveal clues on the evolution of the system formed: hadronization, duration of the hadronic phase, equation of state of hot matter, size/shape at freeze-out, degree of rescattering afterwards, medium modification of particle properties, ...

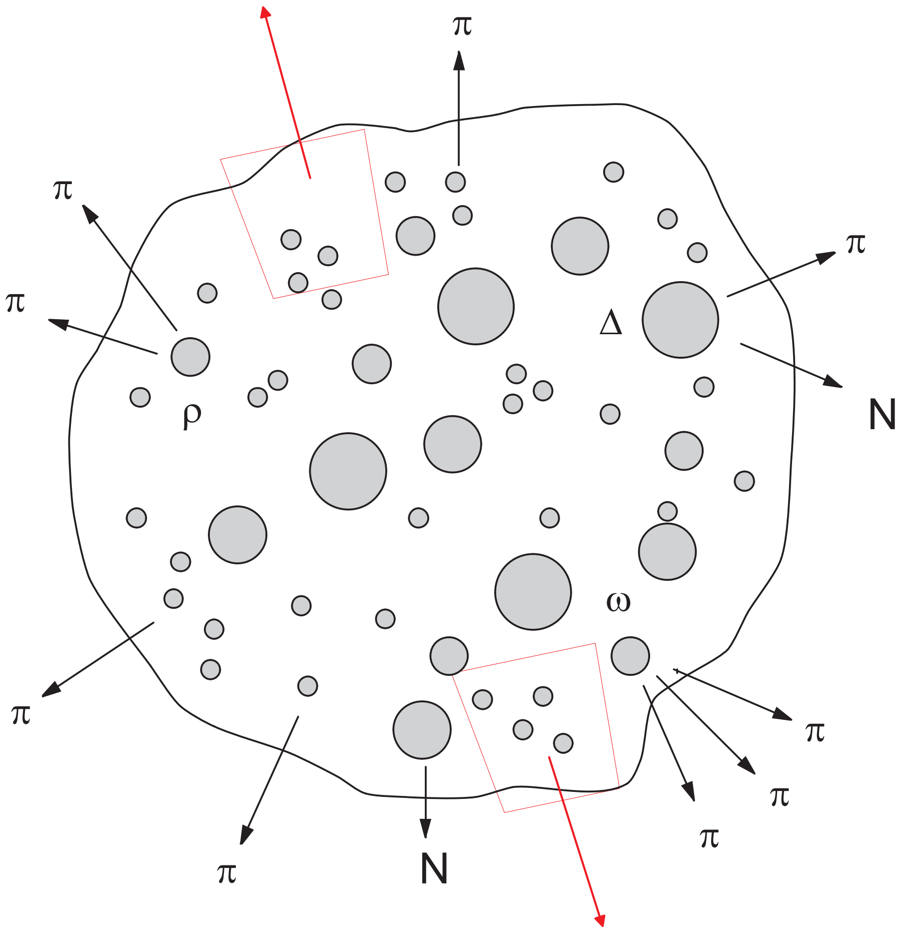
$ddss\bar{u}$



(NA49, hep-ex/0310014)

Thermal (statistical) models

Koppe (1948), Fermi (1950), Landau, Hagedorn, Bjorken, Heinz, Rafelski, Gaździcki, Mrówczyński, Redlich, Prorok, Wilk, Włodarczyk, Pluta, Kisiel, ...



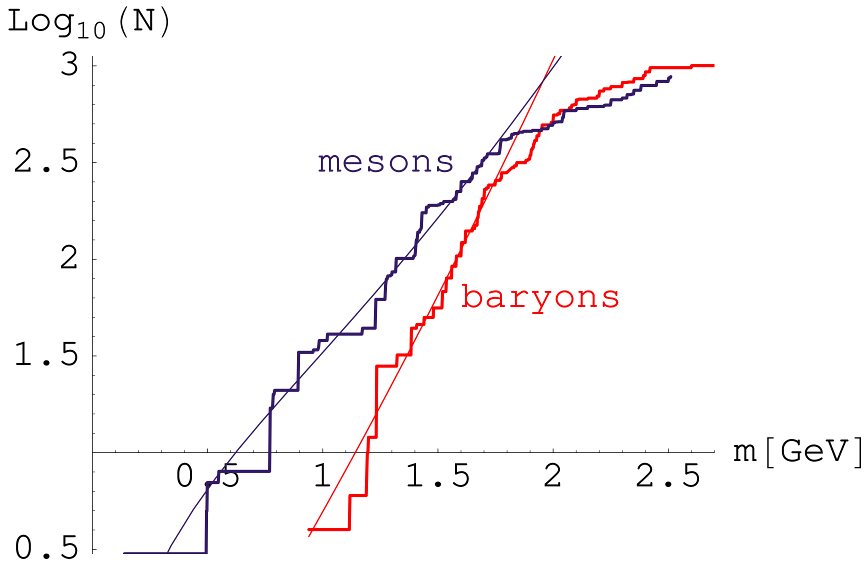
$$\sim e^{-(E-\mu)/T}$$

Our approach in a capsule

1. $T_{\rm chem} = T_{\rm kin} \equiv T$, single freeze-out (a radical simplification, supported by recent HBT results: $R_{\rm out}/R_{\rm side} \sim 1$, $R_{\rm side}(\phi)$ has out-of-plane deformation, resonances seen abundantly $\to$ short time between the freeze-outs)
2. Complete treatment of resonances (important due to the Hagedorn-like exponential growth of the number of states)
3. Assumed simple freezeout hypersurface (longitudinal and transverse flow)
4. 4 parameters: T,μ_B (fixed by the ratios of the particle abundances), invariant time at freeze-out τ (controls the overall normalization), transverse size $\rho_{\rm max}$ ($\rho_{\rm max}/\tau$ controls the slopes of the $p_\perp$ spectra)
5. Hubble-like flow, $u^\mu = x^\mu/\tau$ (supported by the so-called *scaling* solution to hydrodynamics)

... and it works remarkably well for soft physics, $p_\perp <\sim 2$ GeV

Hagedorn



(from WB+WF, PLB 490 (2000) 223)

75% of pions and protons come from decays of higher states!

Equilibrium thermodynamics

$$\frac{dN_i}{d^3x d^3p} = g_i \frac{1}{(2\pi)^3} \frac{1}{\exp\left[(\sqrt{M_i^2 + p^2} - \mu_i)/T\right] \pm 1}$$

$g_i = 2J_i + 1$ is the spin degeneracy factor of the ith hadron, T is the freeze-out temperature, and μ is the chemical potential

$$\mu_i = \mu_B B_i + \mu_S S_i + \mu_I I_i$$

B_i, S_i, I_i are the baryon number, strangeness, and the third component of isospin; μ_B, μ_S and μ_I are the corresponding chemical potentials

- total (initial) strangeness $S = 0$
- $\frac{\text{total electric charge}}{\text{total baryon number}} = \frac{Z}{A} = \text{fixed } (\approx 0.4)$

μ_S and μ_I are eliminated, T and μ_B are two independent thermal parameters fitted by the mesured ratios of particles

No γ-factors for strangeness (Rafelski), excluded-volume effects (Gorenstein), canonical ensemble (Becattini, Redlich)

Particle ratios

$\sqrt{s_{NN}}$ [GeV]	130	200
T [MeV]	165 ± 7	160±5
μ_B [MeV]	41 ± 5	26±4
μ_S [MeV]	9	5
μ_I [MeV]	-1	-1
χ^2/DOF	1.0	1.5

	Model	Experiment
Ratios used in the thermal analysis for 200 GeV		
π^-/π^+	1.009 ± 0.003	$1.025 \pm 0.006 \pm 0.018$ $1.02 \pm 0.02 \pm 0.10$
K^-/K^+	0.939 ± 0.008	$0.95 \pm 0.03 \pm 0.03$ $0.92 \pm 0.03 \pm 0.10$
$\overline{p}/p$	0.74 ± 0.04	$0.73 \pm 0.02 \pm 0.03$ $0.70 \pm 0.04 \pm 0.10$ 0.78 ± 0.05
$\overline{p}/\pi^-$	0.104 ± 0.010	0.083 ± 0.015
K^-/π^-	0.174 ± 0.001	0.156 ± 0.020
$\Omega/h^- \quad \times 10^3$	0.990 ± 0.120	$0.887 \pm 0.111 \pm 0.133$
$\overline{\Omega}/h^- \quad \times 10^3$	0.900 ± 0.124	$0.935 \pm 0.105 \pm 0.140$

We also find (130 GeV)

$\varepsilon = 0.5 \text{ GeV/fm}^3$, $\quad P = 0.08 \text{ GeV/fm}^3$

$\rho_B = 0.02 \text{ fm}^{-3}$, $\quad n = 0.5 \text{ fm}^{-3}$

$\langle E \rangle / \langle N \rangle = 1$ GeV (Cleymans, Redlich)

$P = nT$, as for the ideal non-relativistic gas, classical statistics

N. Xu, Kaneta:

$T = 190 \pm 20$ MeV, $\quad \mu_B = 45 \pm 15$ MeV

Braun-Munzinger, Magestro, Redlich, Stachel:

$T = 175 \pm 7$ MeV, $\quad \mu_B = 51 \pm 6$ MeV

Karsch:

deconfinement phase transition in lattice QCD (extrapolation to the chiral limit, $\mu_B = 0$)

$T_c = 173 \pm 8$ MeV for $N_f = 2$

$T_c = 154 \pm 8$ MeV for $N_f = 3$

Expansion (flow)

Cooper-Frye formula

$$N_i = \int \frac{d^3p}{p^0} \int p^\mu d\Sigma_\mu \; f_i\,(p \cdot u)$$

$d\Sigma_\mu$ – volume element of the freeze-out hypersurface

————————————————————————————-

t = const

$d\Sigma_0 = dV, \quad d\Sigma_i = 0 \;\; (i = 1, 2, 3) \;\; u^\mu = \partial^\mu t = (1, 0, 0, 0), \;\; p \cdot u = \sqrt{p^2 + m_i^2}$

and the standard formula for particle densities is recovered

————————————————————————————-

f_i – phase-space distribution (not necessarily an equilibrium distribution)

In our model we take the shape of the hypersurface (following Bjorken and the Buda-Lund model)

$$\tau = \sqrt{t^2 - r_z^2 - r_x^2 - r_y^2} = \text{const}$$

and constrain the transverse size

$$\rho = \sqrt{r_x^2 + r_y^2} < \rho_{\max}.$$

τ and $\rho_{\max}$ are the geometric parameters, of the order of a few fm (τ^3 is the overall normalization constant, $\rho_{\max}$ controls the slopes). The hydrodynamic four-velocity is (Hubble law)

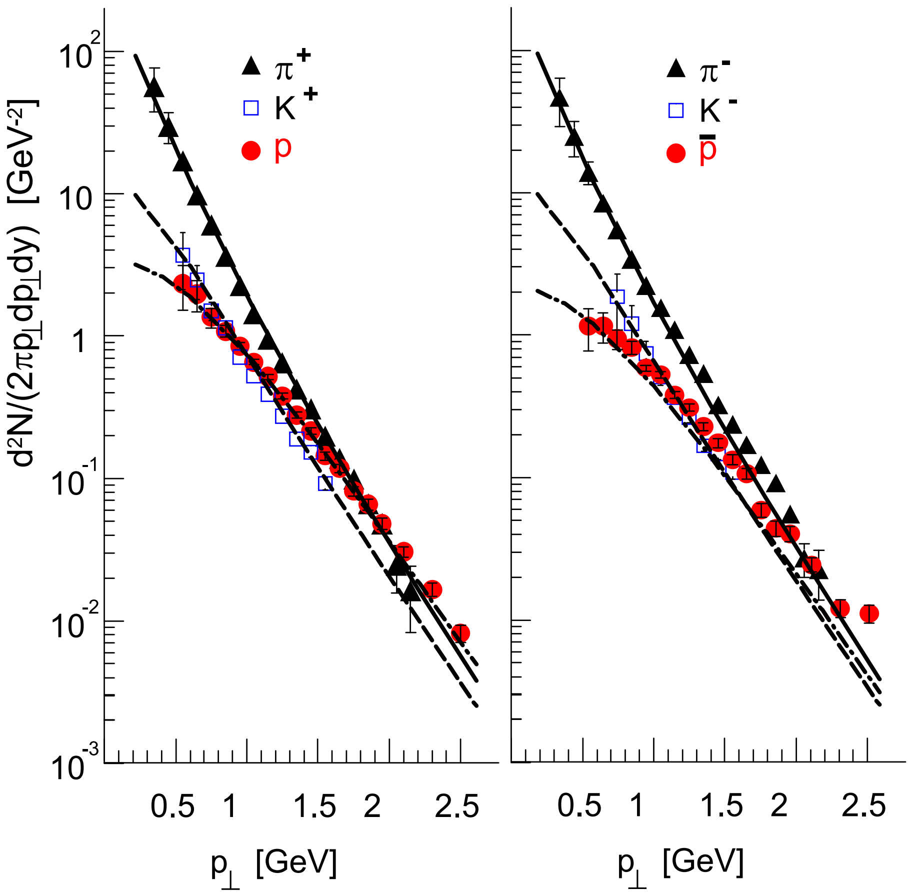
$$u^\mu = \partial^\mu \tau = \frac{x^\mu}{\tau} = \frac{t}{\tau}\left(1, \frac{r_z}{t}, \frac{r_x}{t}, \frac{r_y}{t}\right)$$

BOOST-INVARIANCE – approximate treatment of the midrapidity region

Other choices can be tested (Torrieri+Rafelski)

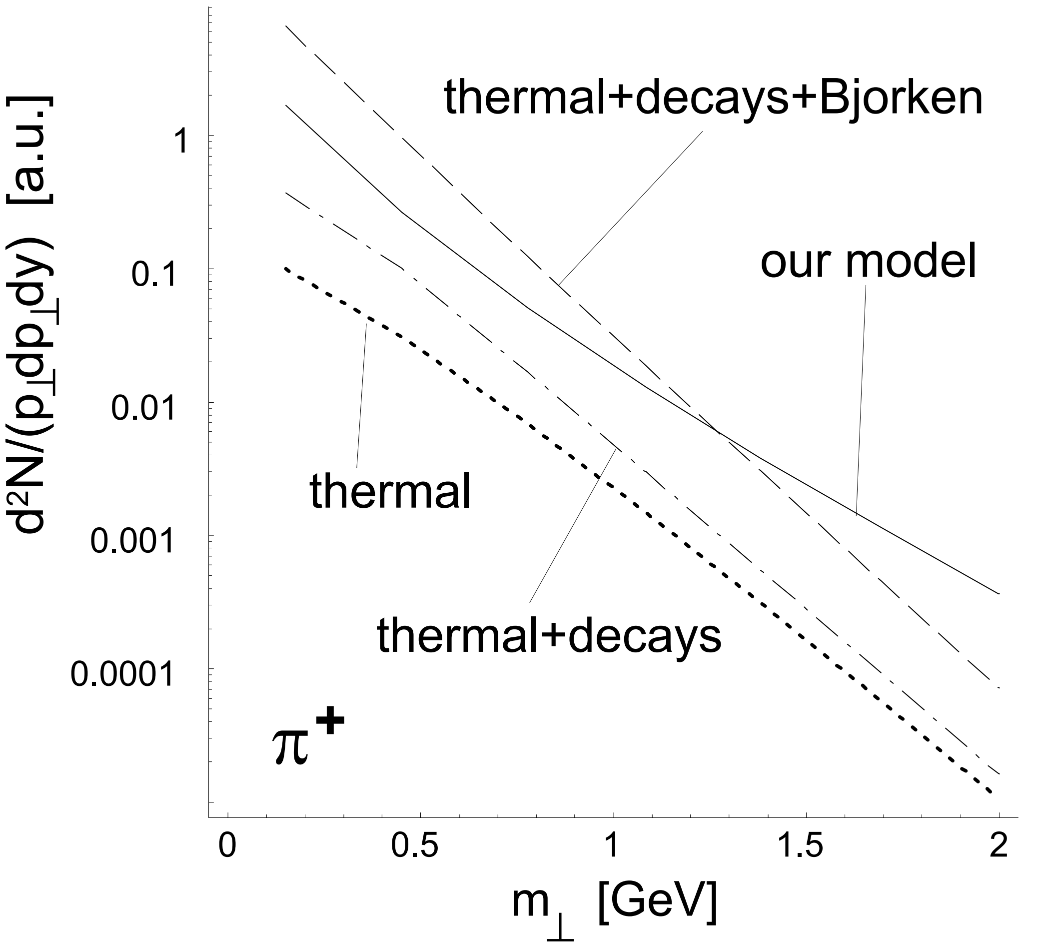
In our choice particularly simple formulas follow.

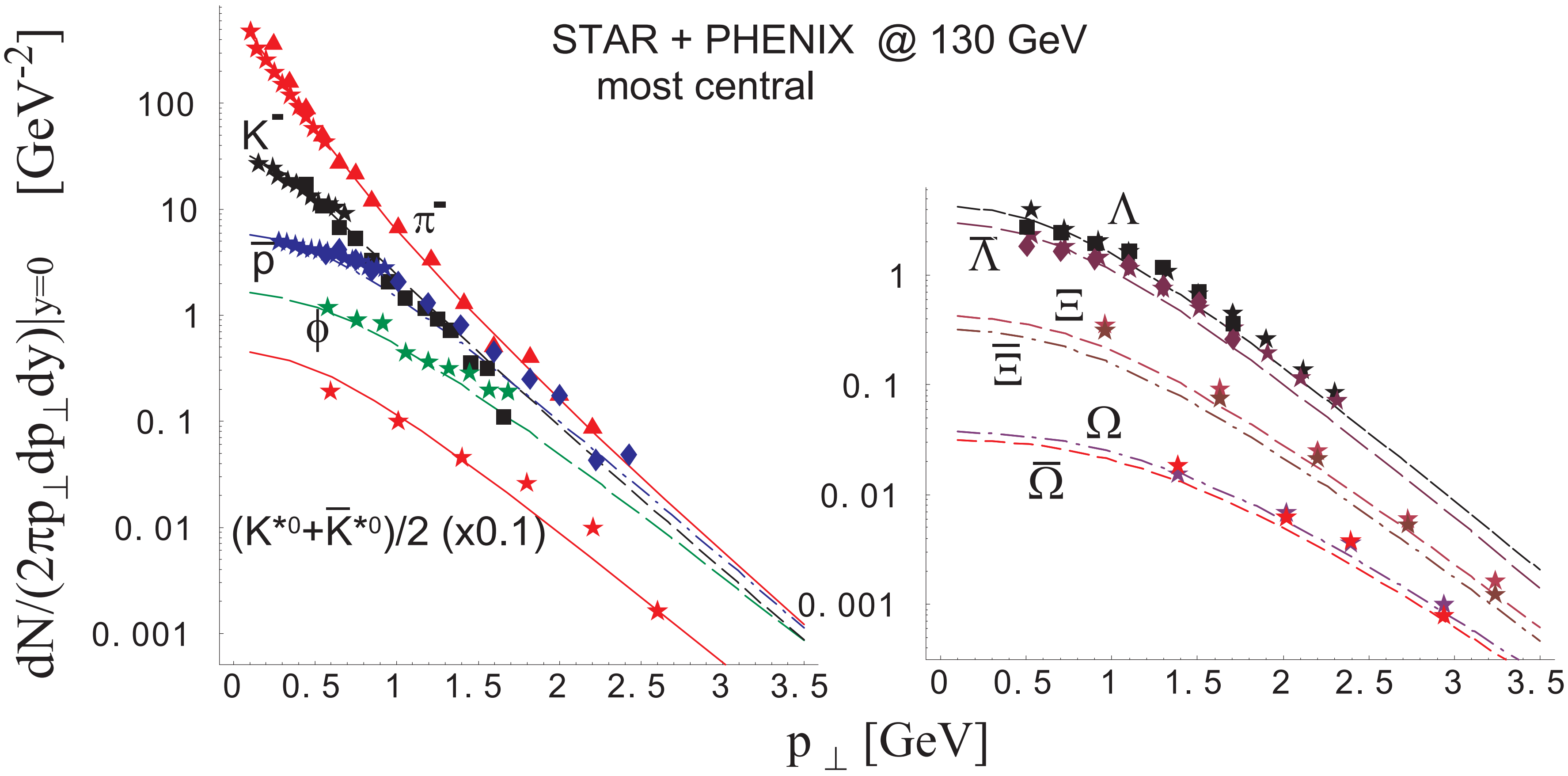
Transverse-momentum spectra



$p_\perp$ spectra of pions, kaons, protons and antiprotons as evaluated from our model with $\tau = 6$ fm, $\rho_{\max}/\tau = 0.76$, compared to the PHENIX preliminary data (Velkovska, for PHENIX, nucl-ex/0105012). Very good agreement up to $p_\perp \sim 2$ GeV. At larger values the model falls below the data, hard processes enter

“Cooling” via decays





$T = 165$ MeV

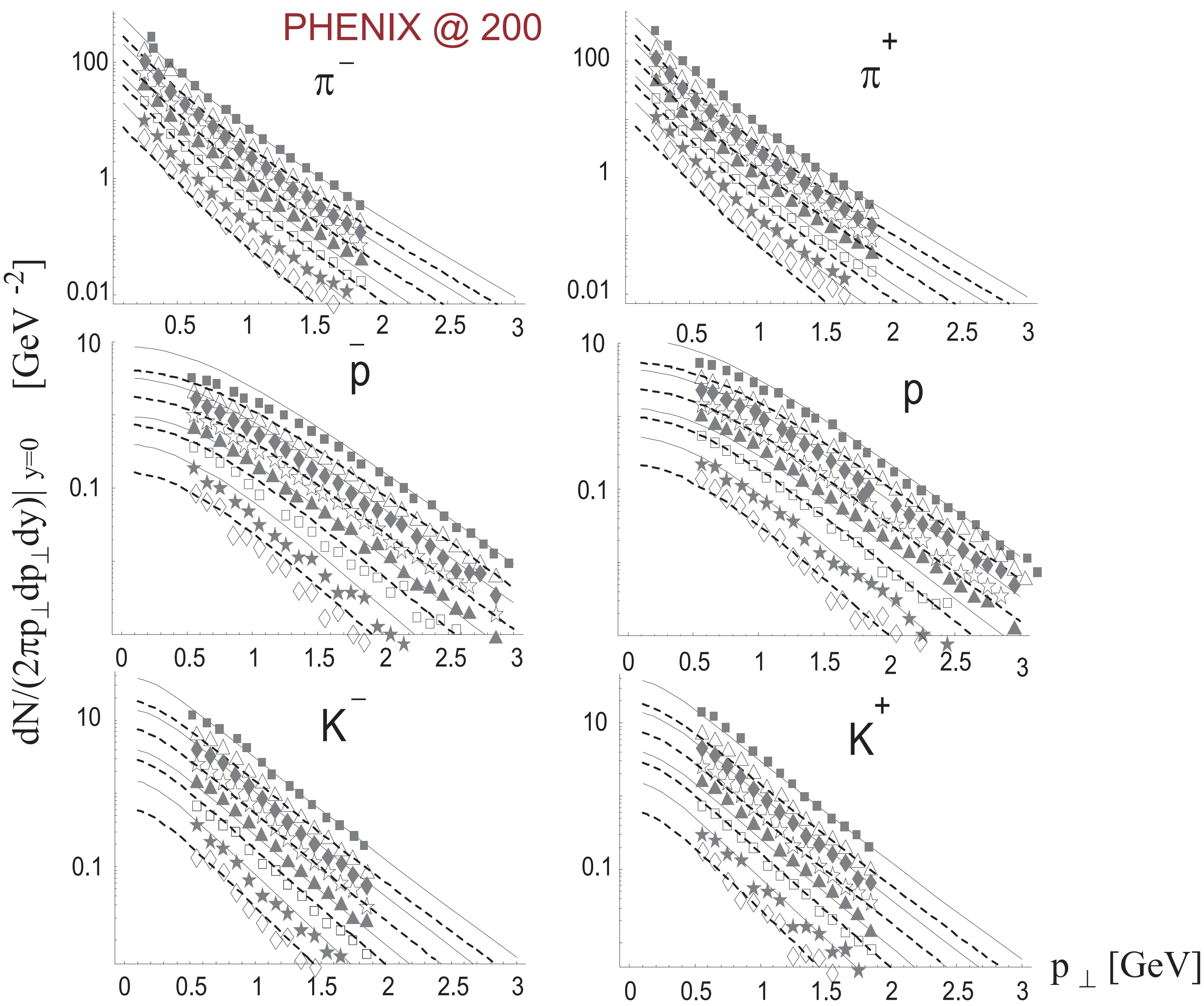
ϕ – very weak interactions, serves as a thermometer

K^* – resonance, lower T would lead to much less K^*'s

(experimental Ξ's went down by $\sim$ a factor of 2)

No special treatment of Ω's

From now on all data for 200 GeV



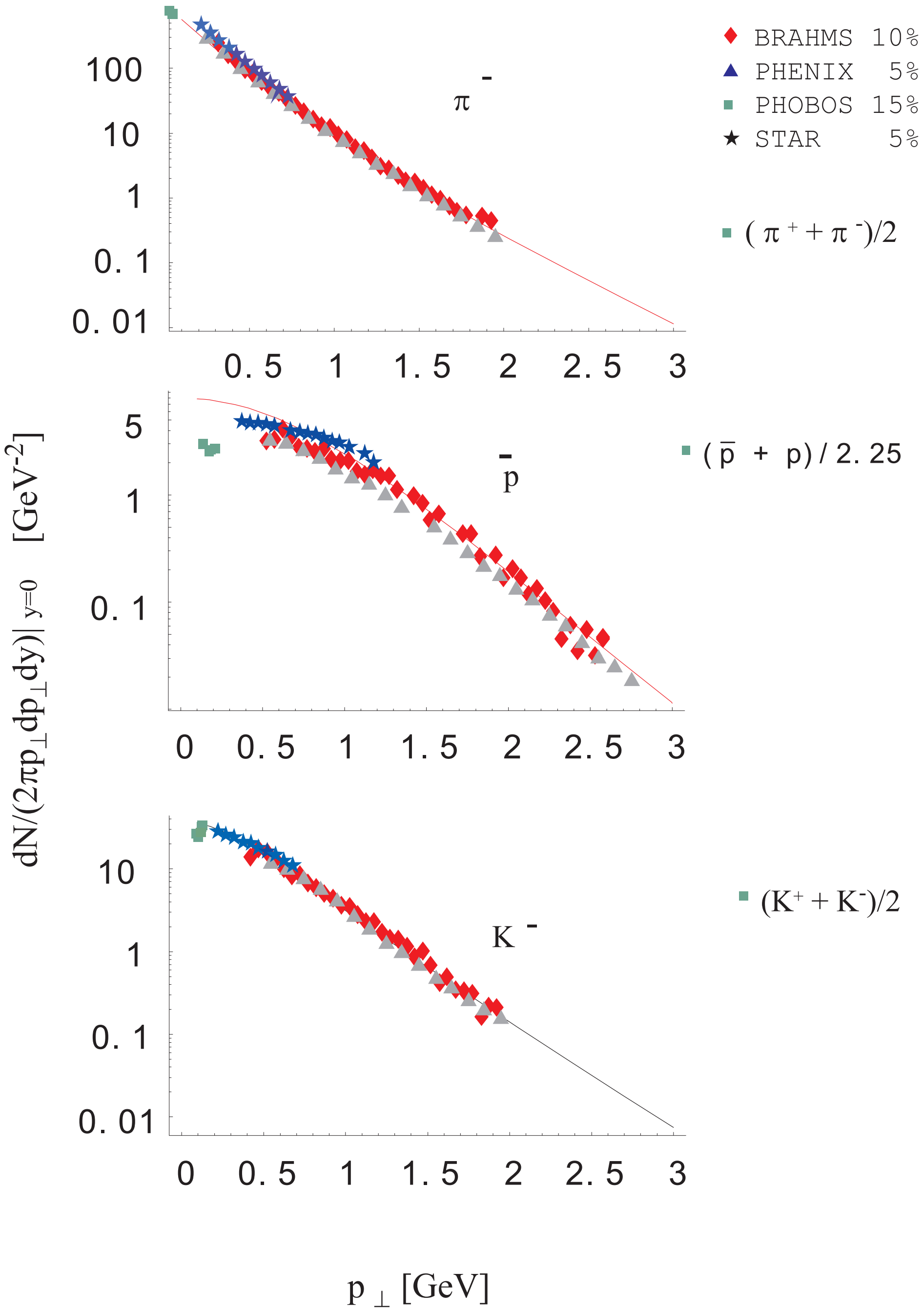
PHENIX @ 200
π^-
π^+
$\bar{p}$
p
K^-
K^+
$dN/(2\pi p_\perp dp_\perp dy)|_{y=0}$ [GeV $^{-2}$]
$p_\perp$ [GeV]

(data at different centrality, or impact parameter)

Centrality c is defined as a percentage of the most central events. To a very good accuracy

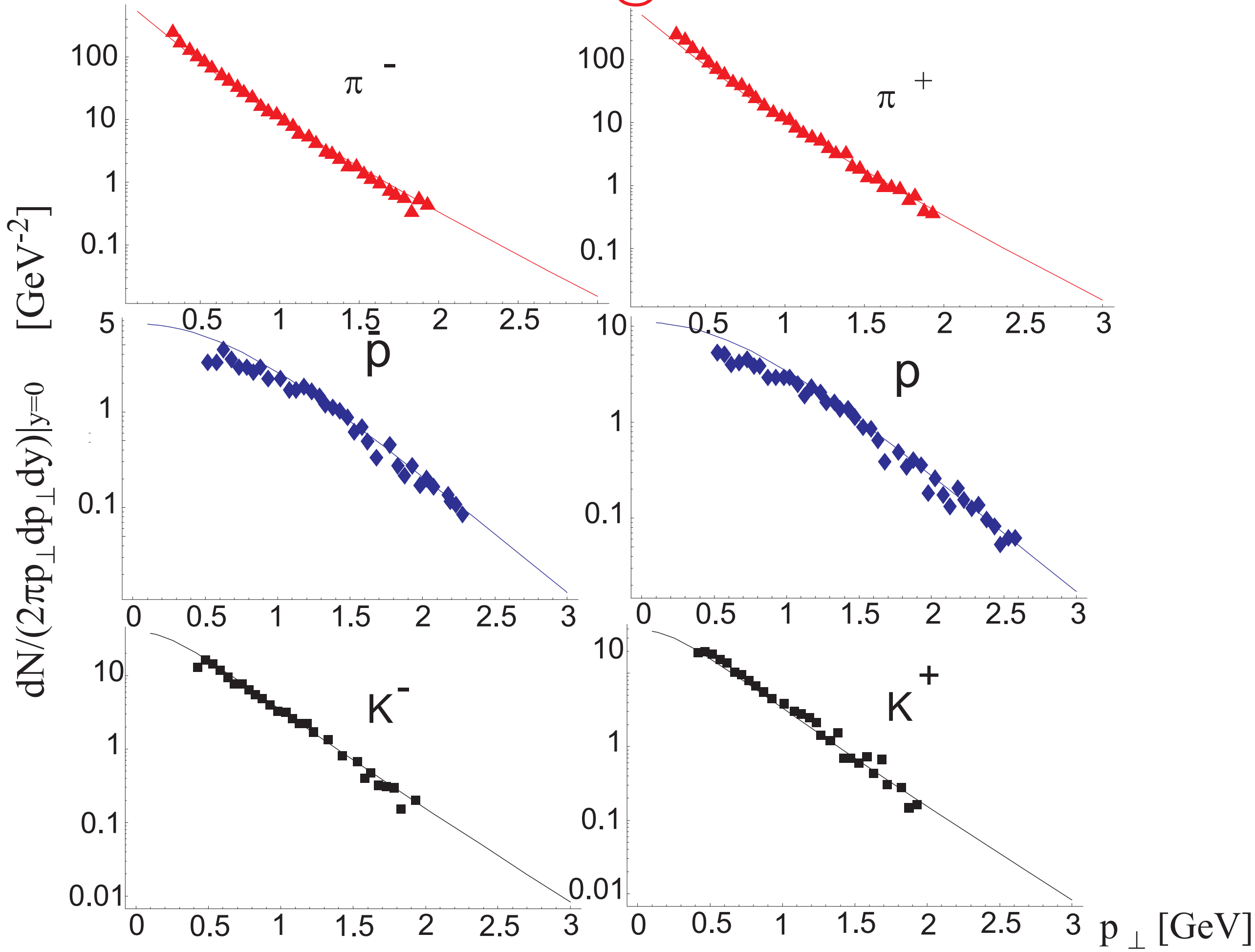
$$c \simeq \frac{\pi b^2}{\sigma_{\rm inel}^{\rm tot}} \simeq \frac{b^2}{4R^2}$$

(WB+WF, PRC 65 (2002) 024905)

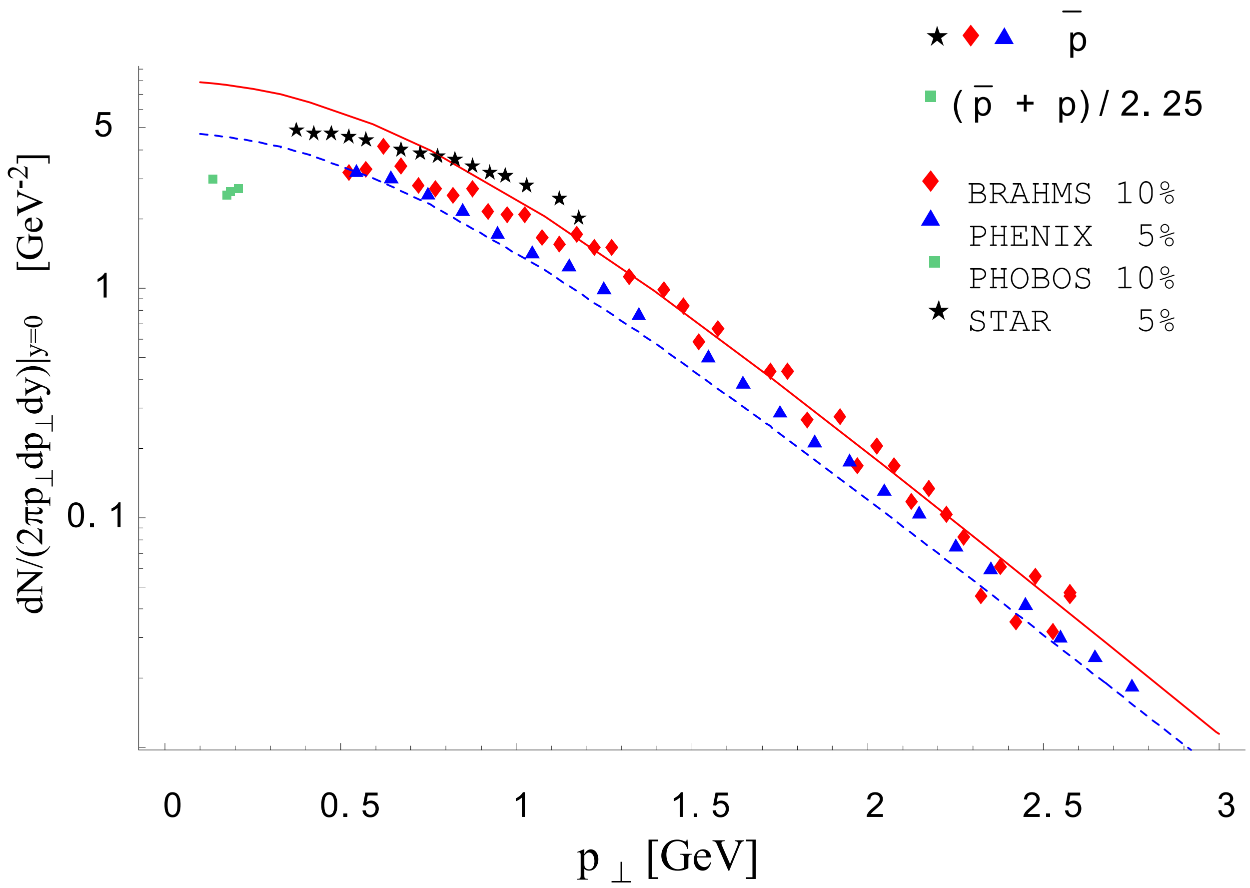


BRAHMS 10%
PHENIX 5%
PHOBOS 15%
STAR 5%
π^-
$(\pi^+ + \pi^-)/2$
$\bar{p}$
$(\bar{p} + p)/2.25$
K^-
$(K^+ + K^-)/2$
$dN/(2\pi p_\perp dp_\perp dy)|_{y=0}$ [GeV^{-2}]
$p_\perp$ [GeV]

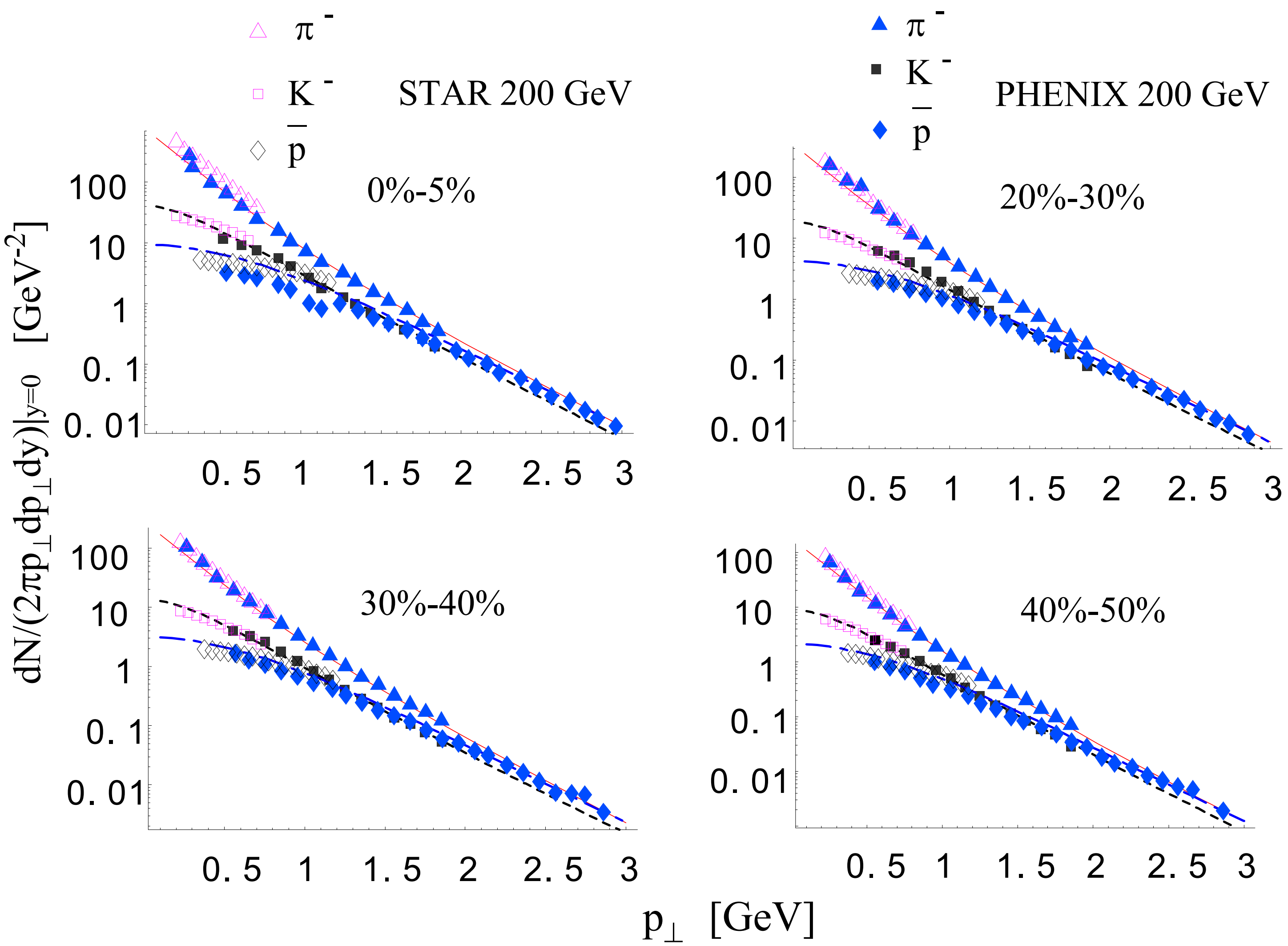
BRAHMS @ 200 GeV



BRAHMS



(solid – full feeding, dashed – no feeding from week decays)

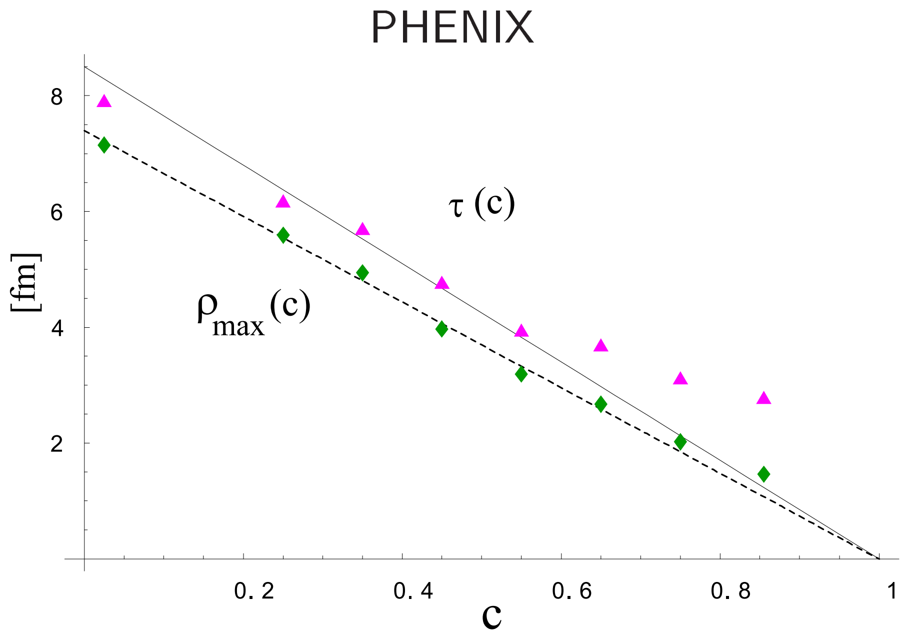


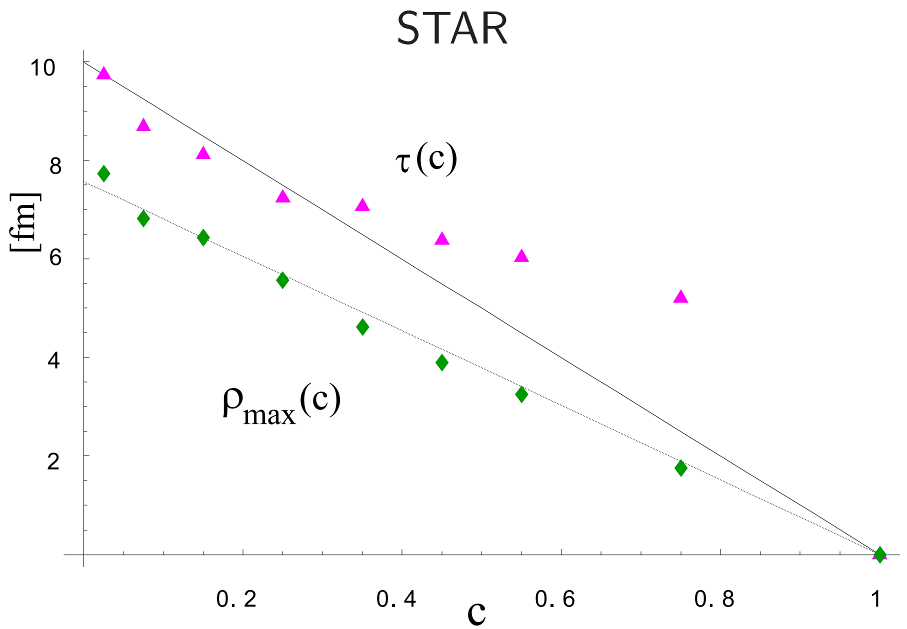
($\bar{p}$ from STAR more flat than from PHENIX)

Model works for the $p_\perp$-spectra

Compilation of geometric parameters (by A. Baran)

	c [%]	τ [fm] (norm)	$\rho_{\max}$ [fm]	$\langle\beta_\perp\rangle$ (slope)
ALL	$0-5/10$	7.58 ± 0.32	7.27 ± 0.12	0.52 ± 0.02
BRAHMS	10	7.68 ± 0.19	7.46 ± 0.05	0.52 ± 0.01
STAR	$0-5$	9.74 ± 1.57	7.74 ± 0.68	0.45 ± 0.08
	$5-10$	8.69 ± 1.39	7.18 ± 0.64	0.47 ± 0.08
	$10-20$	8.12 ± 1.31	6.44 ± 0.57	0.45 ± 0.08
	$20-30$	7.24 ± 1.18	5.57 ± 0.50	0.44 ± 0.08
	$30-40$	7.07 ± 1.17	4.63 ± 0.39	0.39 ± 0.08
	$40-50$	6.38 ± 1.02	3.91 ± 0.33	0.37 ± 0.07
	$50-60$	6.19 ± 1.09	3.25 ± 0.28	0.32 ± 0.07
	$70-80$	5.48 ± 0.81	4.03 ± 0.10	0.43 ± 0.06
PHENIX	$0-5$	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02
	$20-30$	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02
	$30-40$	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01
	$40-50$	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03
	$50-60$	3.91 ± 0.23	3.12 ± 0.07	0.45 ± 0.03
	$60-70$	3.67 ± 0.12	2.67 ± 0.03	0.42 ± 0.01
	$70-80$	3.09 ± 0.11	2.02 ± 0.02	0.39 ± 0.01
	$80-91$	2.76 ± 0.20	1.43 ± 0.03	0.32 ± 0.03


PHENIX
8
6
4
2
[fm]
τ (c)
ρmax(c)
0. 2
0. 4
0. 6
0. 8
1
c

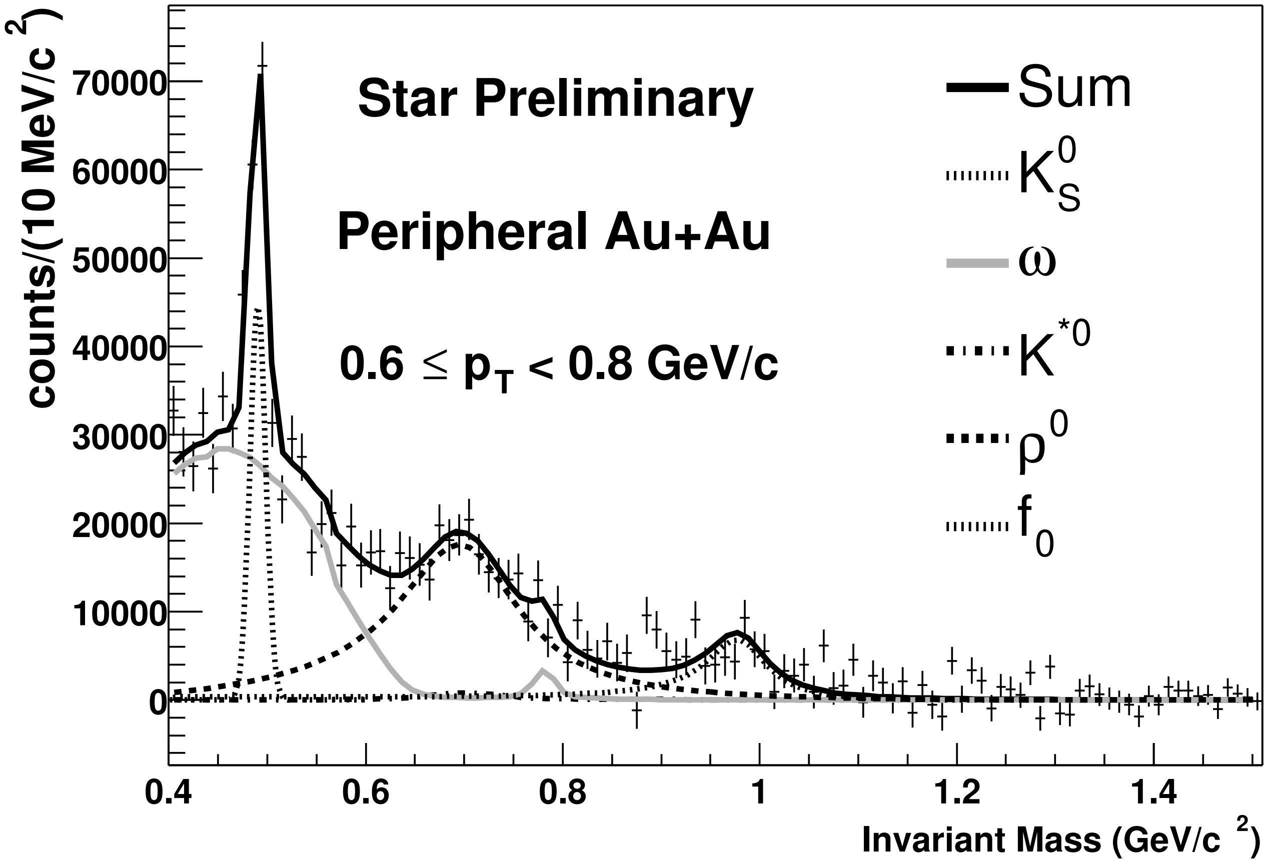

STAR
10
8
6
4
2
[fm]
τ(c)
ρmax(c)
0. 2
0. 4
0. 6
0. 8
1
c

Correlations of identified particles

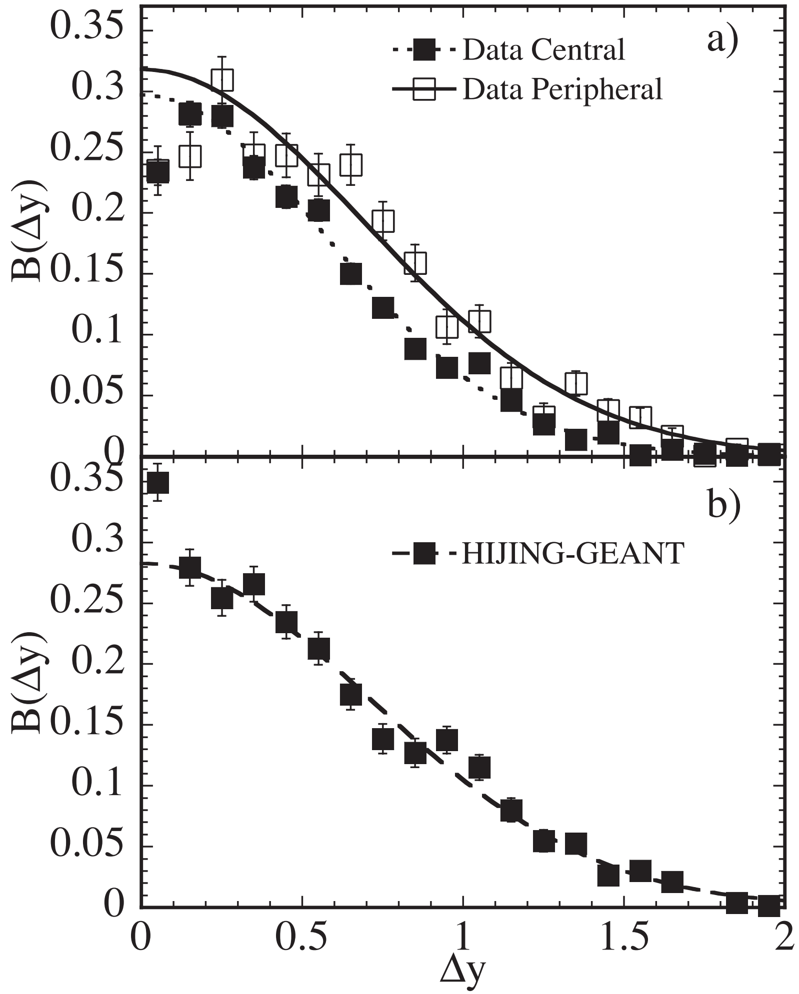
Two very clever techniques are used in order to subtract the background: mixed event ($K^*(892)$, $\Xi(1862)$) and like-sign subtraction (ρ)

- Invariant-mass spectra (K-π, π-π, to come out shortly: p-π)
- correlations in rapidity (balance functions)

$\pi^+\pi^-$ pairs from STAR



(from J. Adams et al., nucl-ex/0307023; P. Fachini, nucl-ex/0305034)



(from J. Adams et al., STAR Collaboration, Phys. Rev. Lett. **90** (2003) 172301)

Can we explain correlations in our model?

The phase-shift formula for the density of resonances

Resonances provide kinematic correlations

Beth,Uhlenbeck (1937); Dashen, Ma, Bernstein, Rajaraman (1974); **Weinhold (1998)**, Friman, Nörenberg; **WB, WF, B. Hiller**, PRC **68** (2003) 034911; Pratt, Bauer, nucl-th/0308087

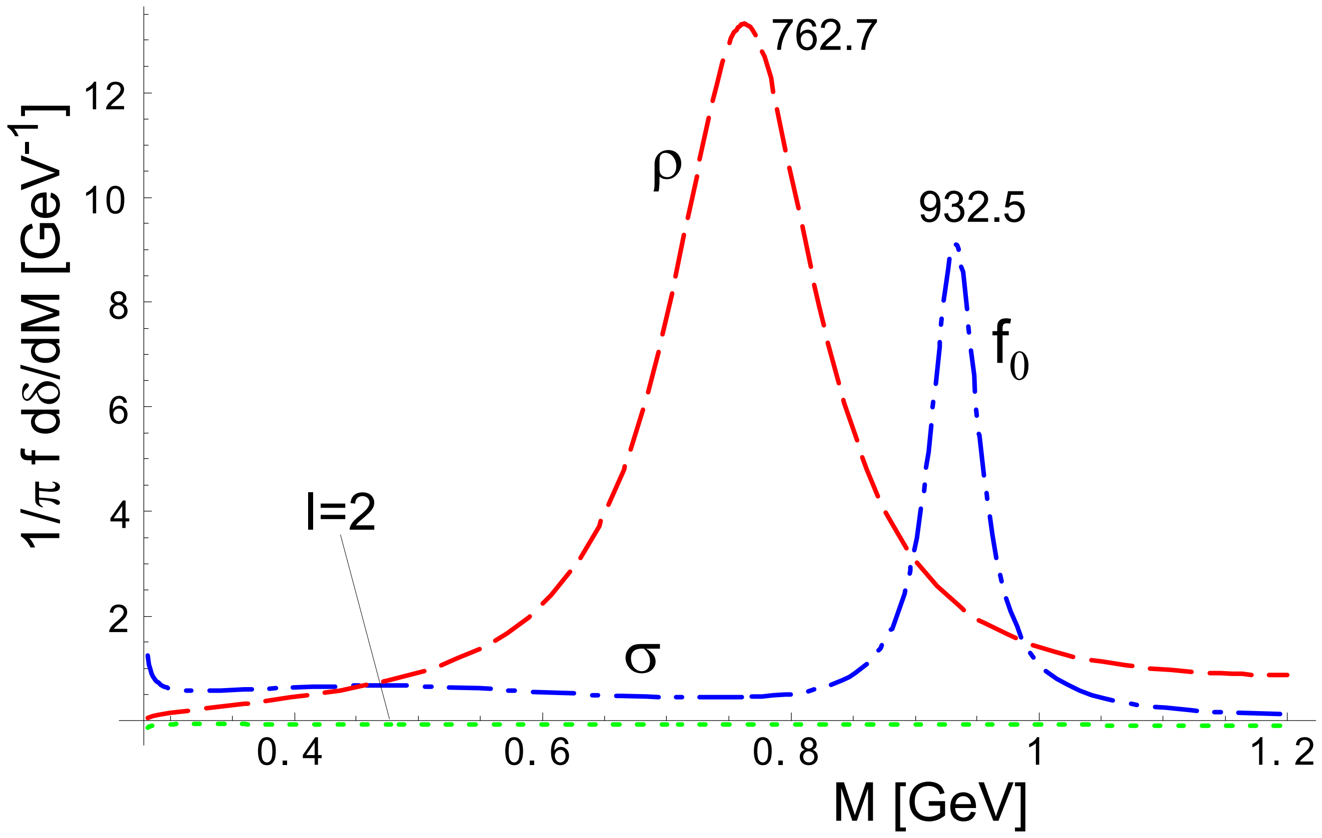
$$\frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{\pi\pi}(M)}{\pi dM} \frac{1}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

In some works the spectral function of the resonance is used *ad hoc* as the weight, instead of the derivative of the phase shift. For narrow resonances this does not make a difference, since then $d\delta(M)/dM \simeq \pi\delta(M - m_R)$, and similarly for the spectral function.

$$n^{\mathrm{narrow}} = f \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{\sqrt{m_R^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$

For wide resonances, or for effects of tails, the difference between the correct formula and the one with the spectral function is signifcant

$d\delta_{\pi\pi}(M)/dM$ from experiment

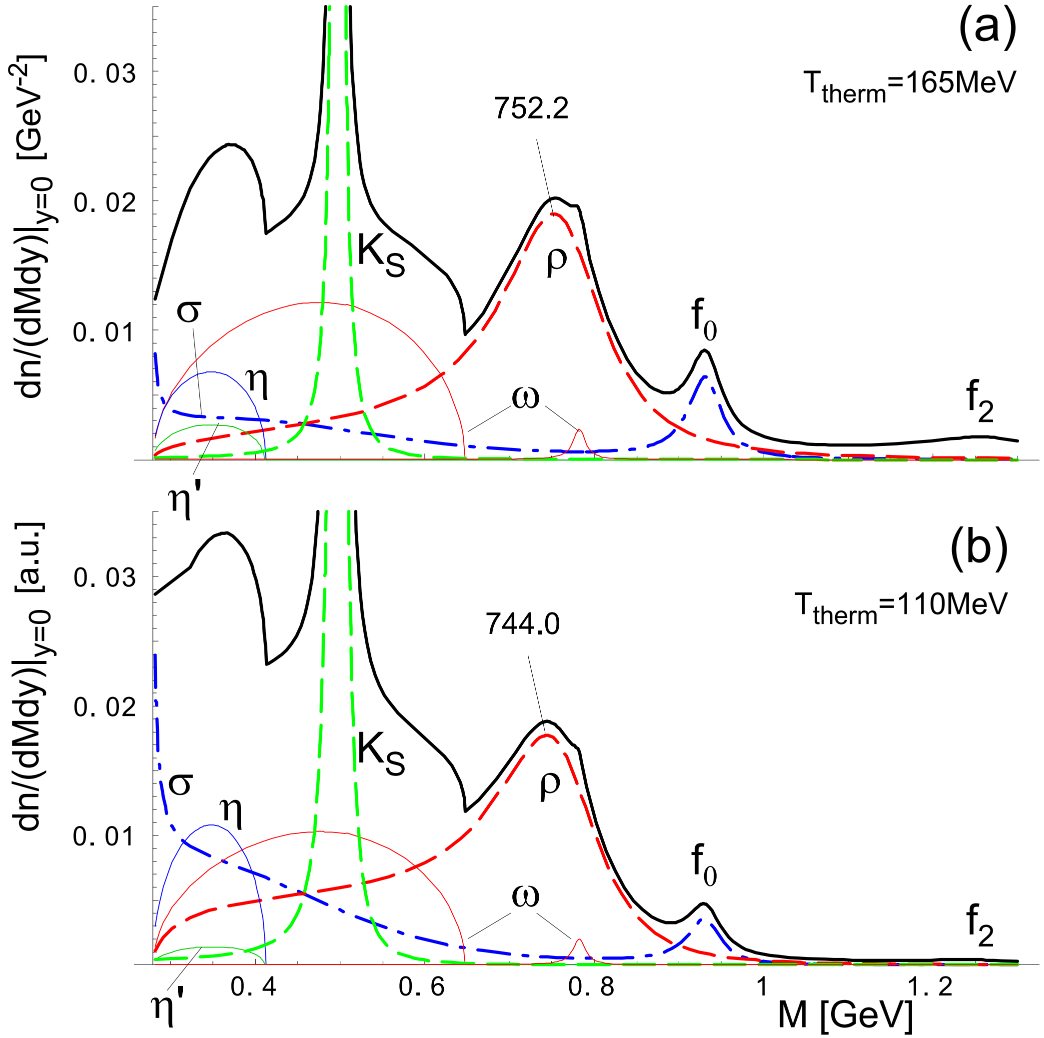


Small contribution from σ, negative and tiny contribution from $I = 2$, ρ-peak slightly shifted to lower M, $1/\sqrt{M - 4m_\pi^2}$ behavior for the σ

Warm-up calculation - static source

We compute the spectra at mid-rapidity, hence

$$\left.\frac{dn}{dMdy}\right|_{y=0} = \sum_i f_i \int_{0.2\mathrm{GeV}}^{2.2\mathrm{GeV}} \frac{p_\perp dp_\perp}{(2\pi)^2} \frac{d\delta_i(M)}{\pi dM} \frac{\sqrt{M^2+p_\perp^2}}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}_\perp{}^2}}{T}\right)-1}$$

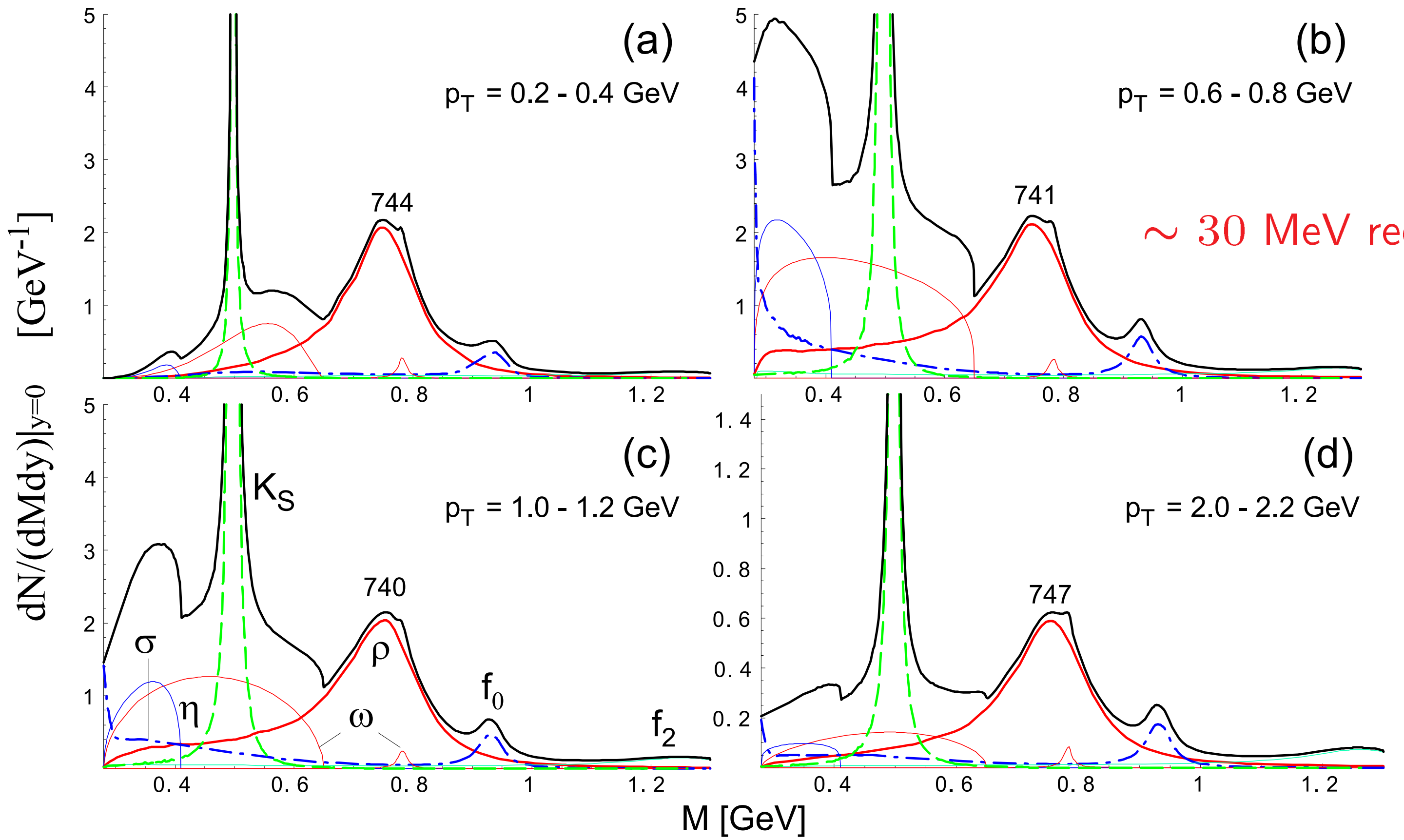


flat

steep

Cuts/flow + feeding from resonances

Flow has no effect on the invariant mass of a pair of particles produced in a resonance decay, since the quantity is Lorentz-invariant. Neverthelss, it affects the results since the kinematic cuts in an obvious manner break this invariance



~ 30 MeV reduction

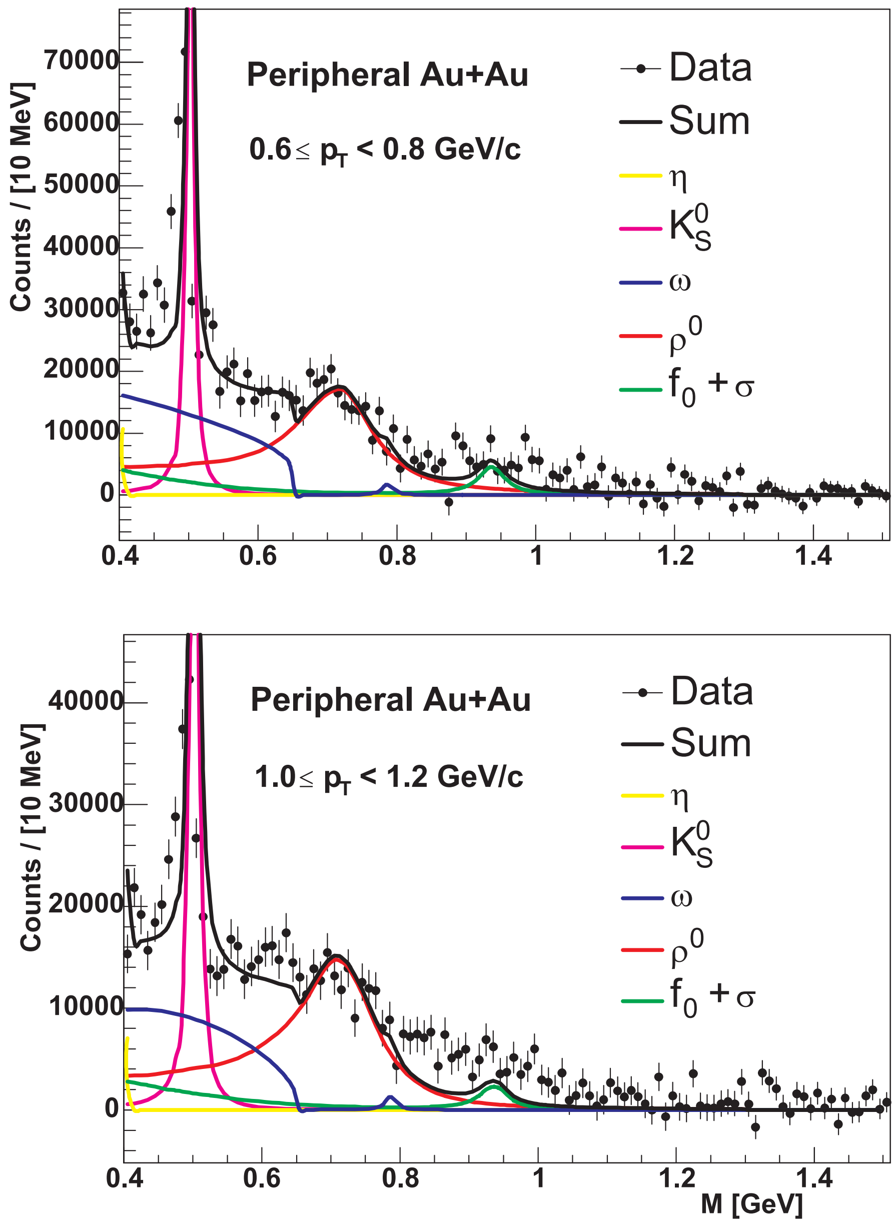
The invariant $\pi^+\pi^-$ mass spectra in the single-freeze-out model for four sample bins in the trasverse momentum of the pair, p_T, plotted as a function of M. η indicates $\eta + \eta'$. All kinematic cuts of the STAR experiment are incorporated

Resonanse decays

The higher-states decays lead to enhancement factors for low resonances: $K_S = 1.98$, $\eta = 1.74$, $\sigma = 1.13$, $\rho = 1.42$, $\omega = 1.43$, $\eta' = 1.08$, $f_0 = 1.01$, and $f_2 = 1.28$. Thus, the effects is strongest for light particles, K_S, η, ρ, and ω, while it is weaker for the heavier η' and scalar mesons.

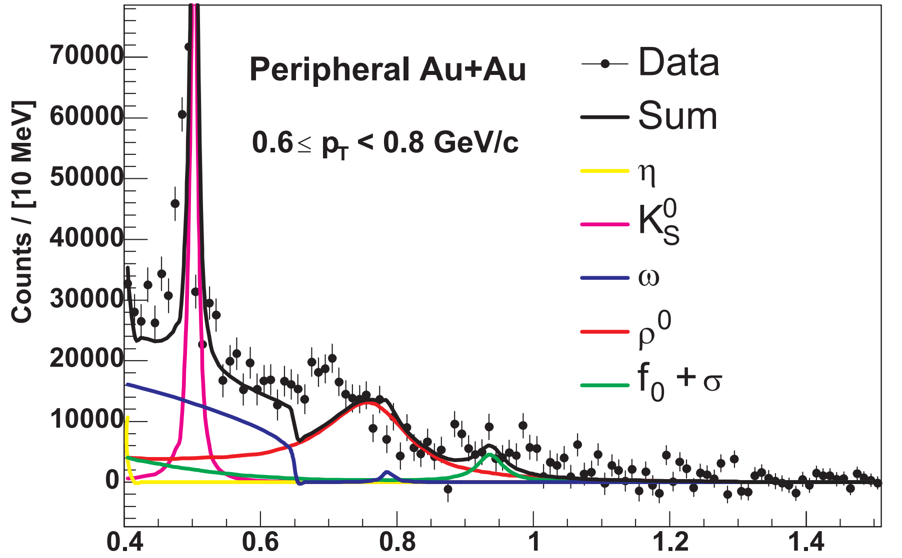
Full model, with feeding from higher resonances and flow/cuts at $T = 165$ MeV is similar to the naive model at $T = 110$ MeV !

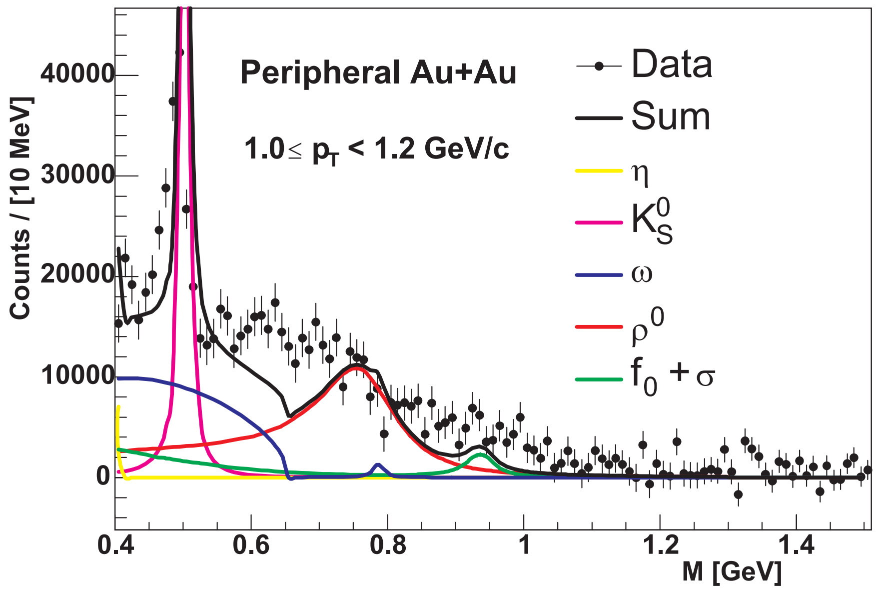
STAR vs. thermal model, lowered ρ



(prepared by P. Fachini)

vacuum ρ

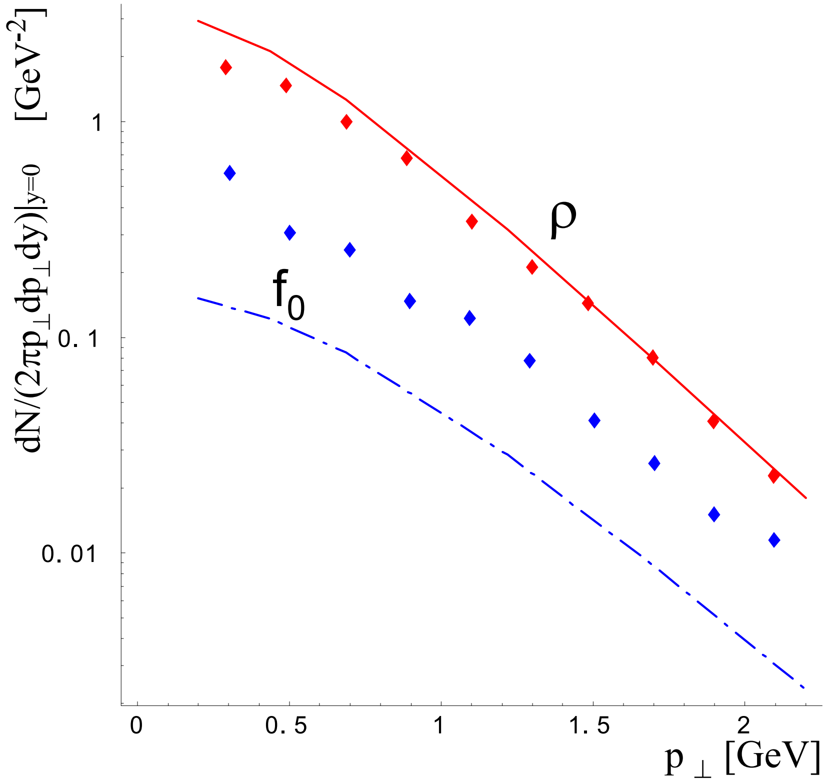




(worse agreement)

Is m_ρ lowered? (dileptons from CERES and HELIOS)

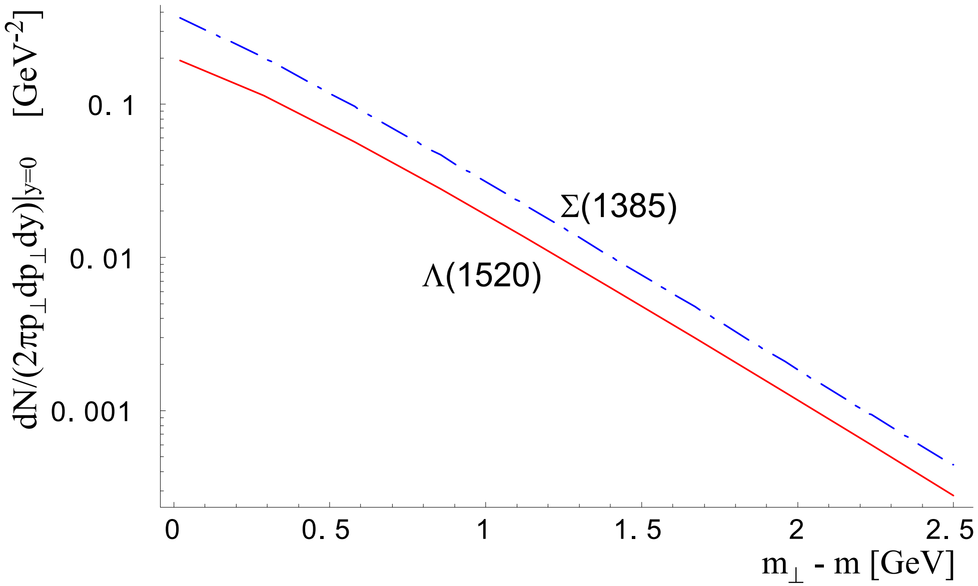
$p_\perp$ spectra of resonances



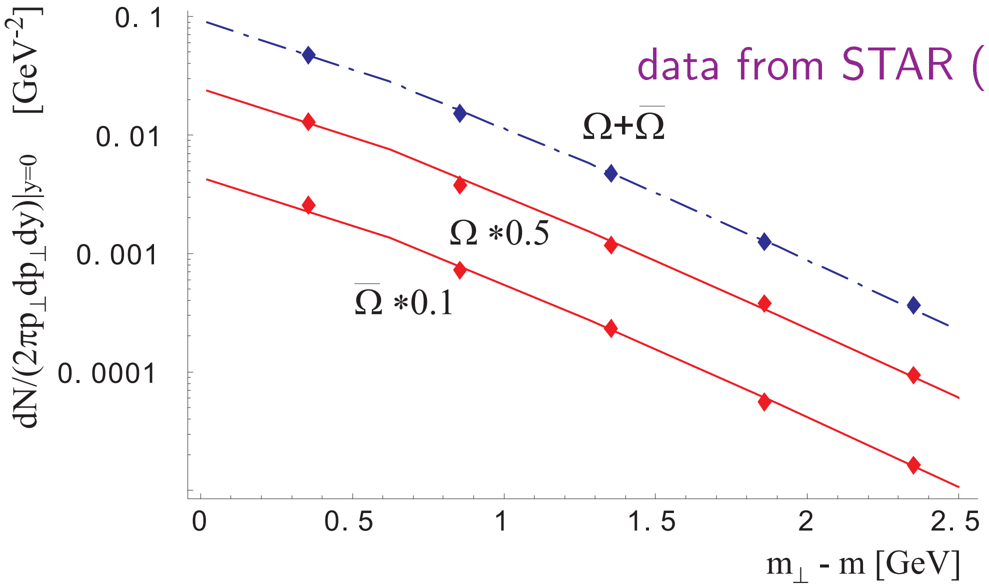
(model parameters, $\tau = 5$ fm and $\rho_{\max} = 4.2$ fm, correspond to centralities 40-80%)

For f_0 experiment $>$ thermal model!

Predictions



data from STAR (C. Suire, QM2002)



Balance functions

(based on: Piotr Bożek+WB+WF, *Balance functions in a thermal model with resonances*, `nucl-th/0310062`)

The balance functions analyzed by the STAR Collaboration at RHIC are defined as

$$B(\delta, Y) = \frac{1}{2}\left\{ \frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_{+}\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_{-}\rangle} \right\},$$

where $N_{+-}(\delta)$ counts the opposite-charge pairs when both members of the pair fall into the rapidity window Y. Their relative rapidity is $|y_2 - y_1| \equiv \delta$. N_+ is the number of positive particles in the interval Y.

For sufficiently large rapidity interval $Y \sim Y^{\max}$, the balance function of *all* charged hadrons is normalized to unity,

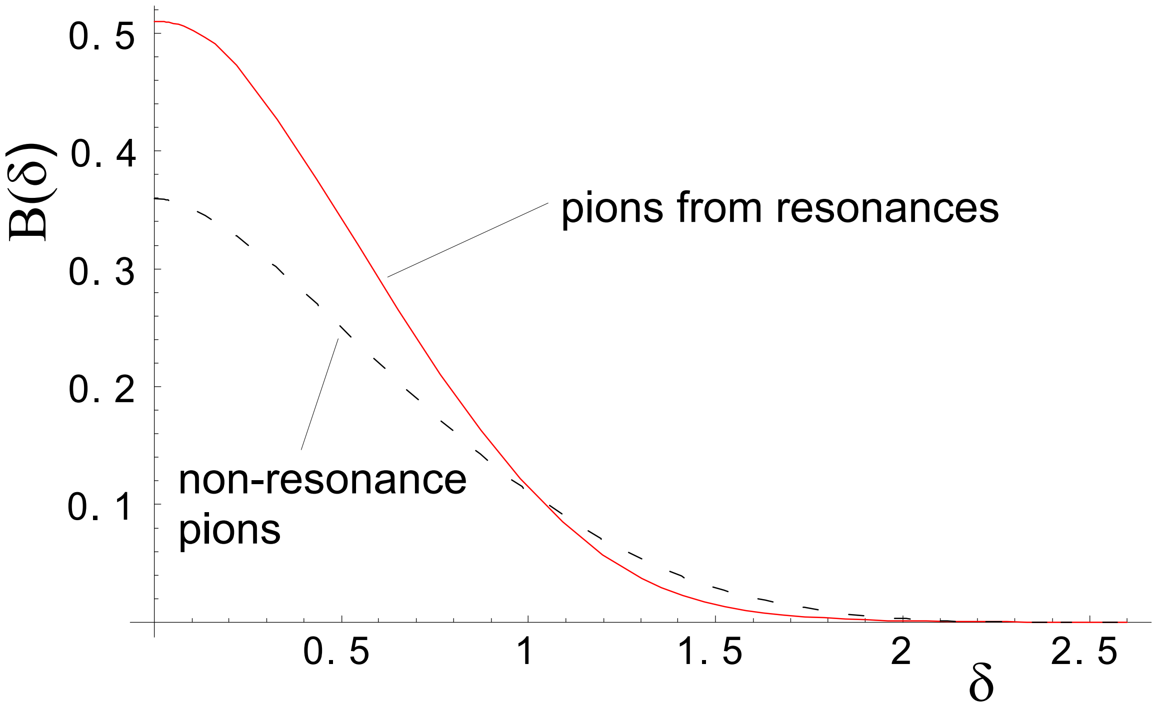
$$\int_0^{Y^{\max}} d\delta \, B(\delta, Y^{\max}) = 1,$$

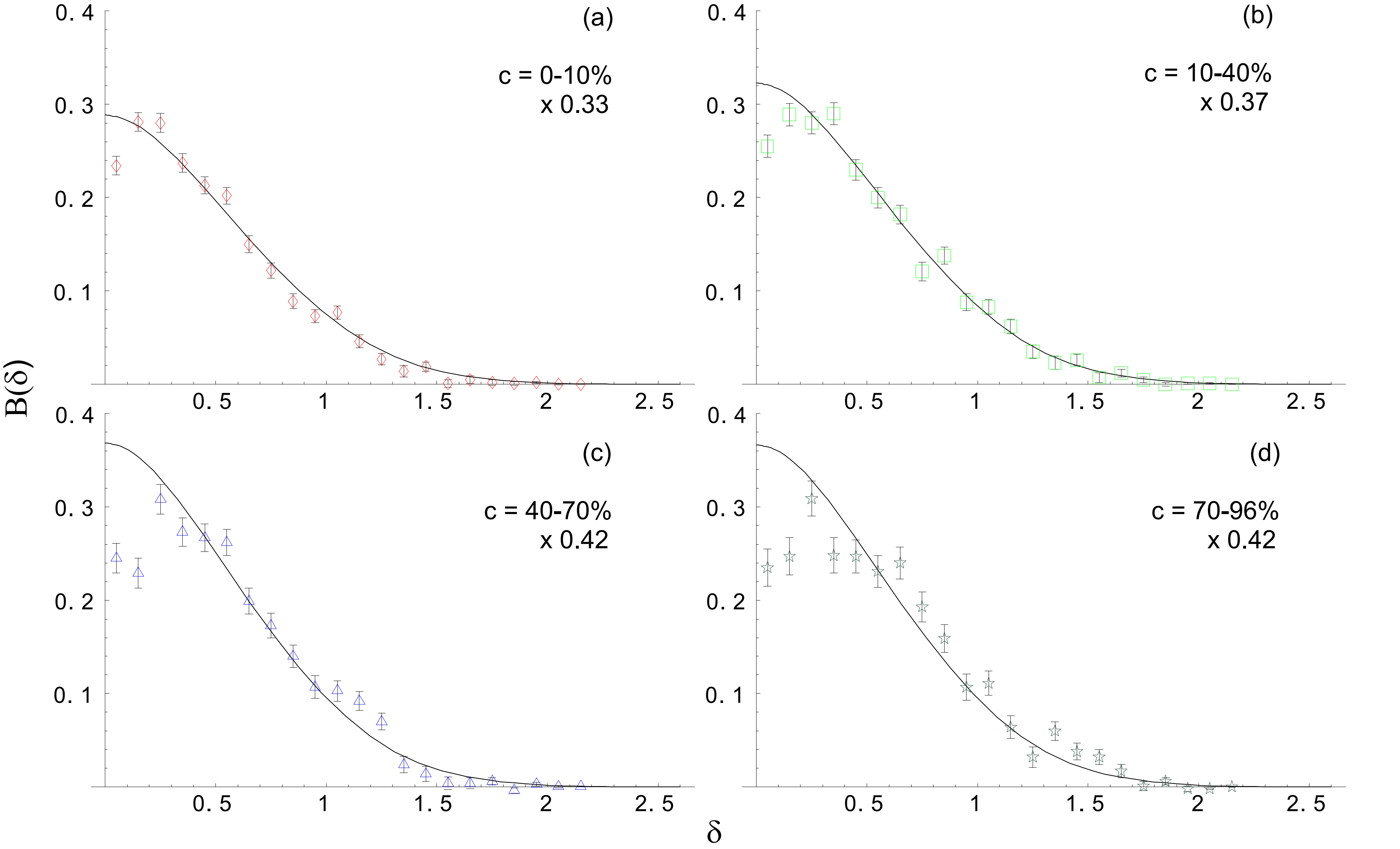
which is a condition reflecting the overall charge conservation.

Balance functions in the thermal model

Resonance and non-resonance contributions,

$$B(\delta, Y) = B_{\mathrm{R}}(\delta, Y) + B_{\mathrm{NR}}(\delta, Y)$$




B(δ)
δ
(a)
c = 0-10%
x 0.33
(b)
c = 10-40%
x 0.37
(c)
c = 40-70%
x 0.42
(d)
c = 70-96%
x 0.42

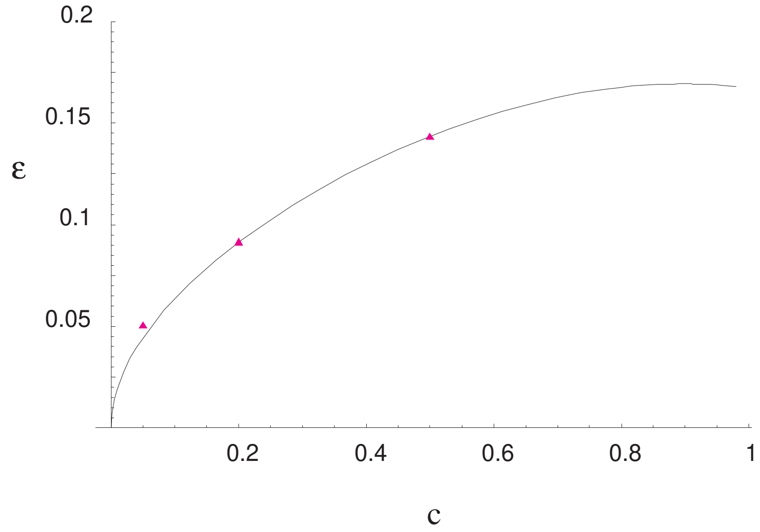
The widths of the balance functions, $\langle\delta\rangle$, are obtained (as in experiment) for the range $0.2 < \delta < 2.6$

Model				
$\rho_{\max}/\tau$	$\langle\beta_\perp\rangle$	$\langle\delta\rangle_{\mathrm{res}}$	$\langle\delta\rangle_{\mathrm{therm}}$	$\langle\delta\rangle_{\mathrm{tot}}$
0.9	0.50	0.59	0.67	0.63
Experiment				
$c = 0 - 10\%$				0.594 ± 0.019
$c = 10 - 40\%$				0.622 ± 0.020
$c = 40 - 70\%$				0.633 ± 0.024
$c = 70 - 96\%$				0.664 ± 0.029

The dependence of the width on centrality cannot be reproduced by varying the transverse flow within limits consistent the the single-particle spectra.

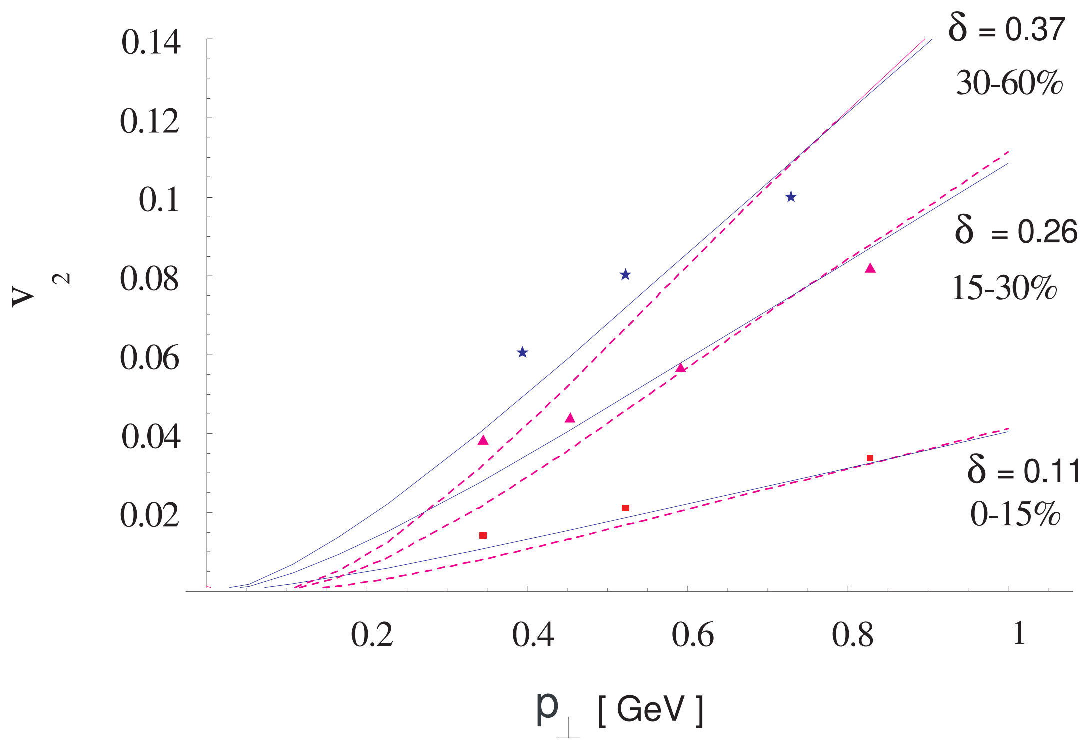
Elliptic flow

$$\varepsilon = \frac{R_y^2 - R_x^2}{R_y^2 + R_x^2}$$

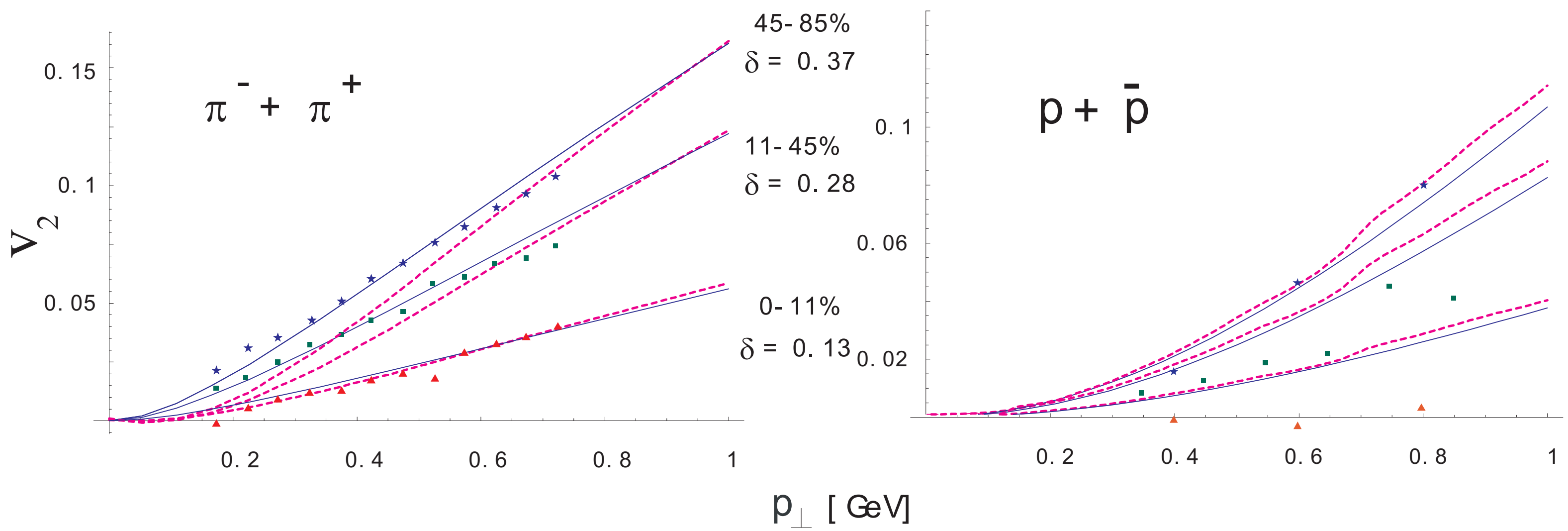


[data from STAR, M. Lisa, nucl-ex/0301005]

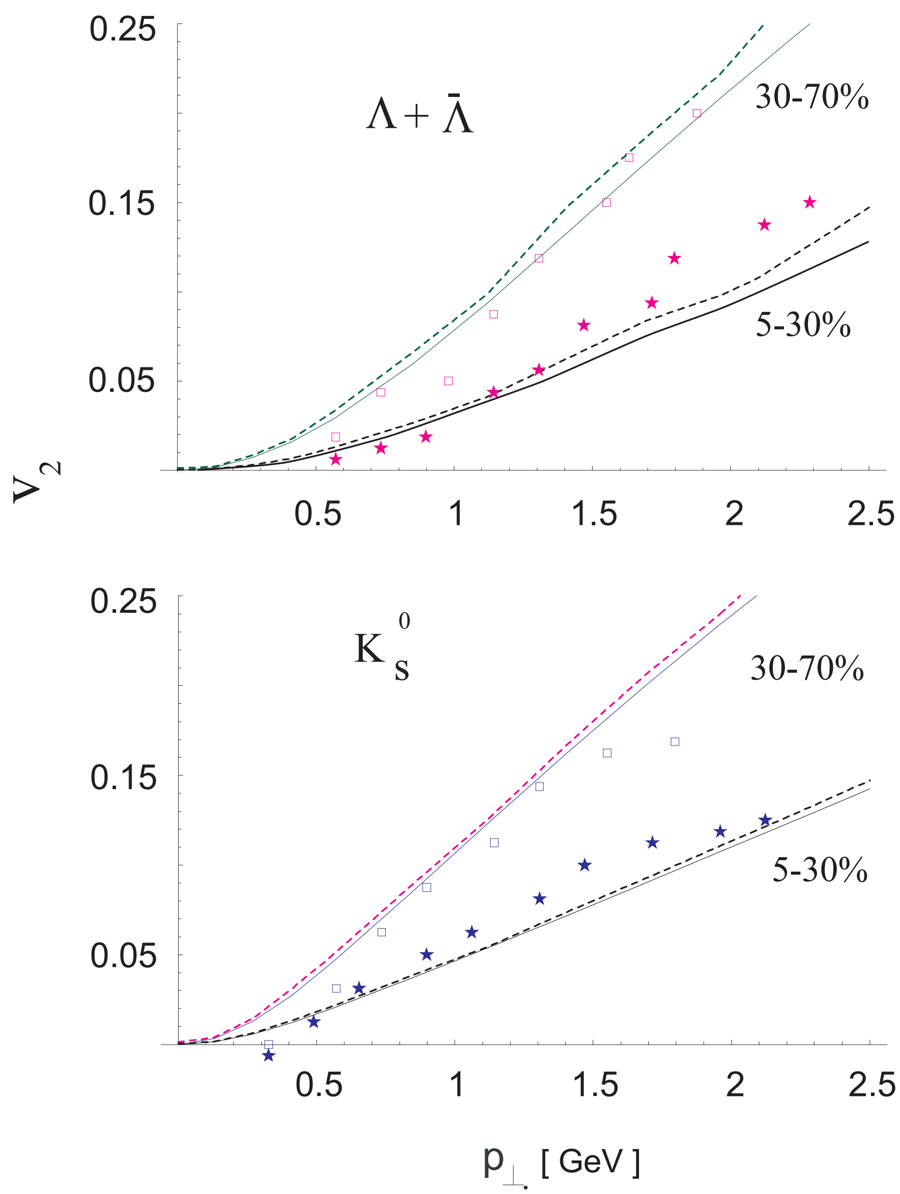
v_2, PHENIX @ 130 GeV



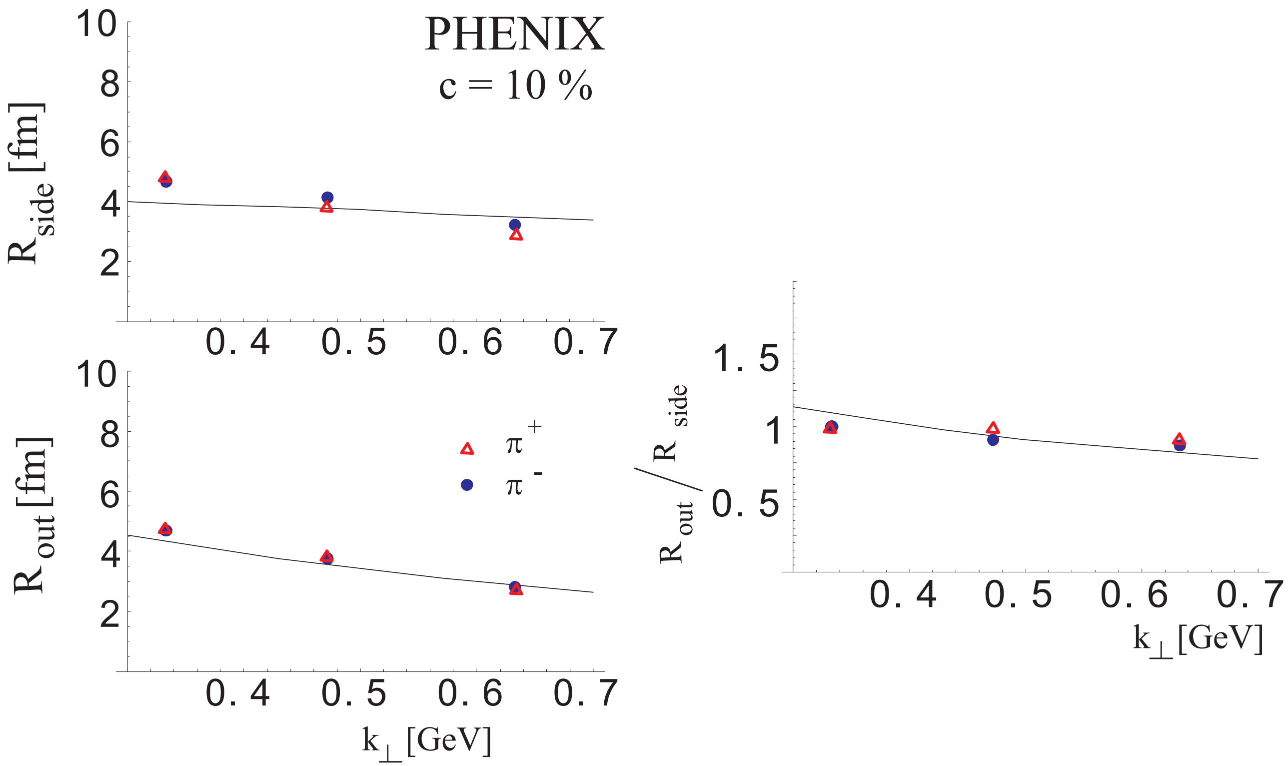
v_2, STAR @ 130 GeV



v_2 for strange particles



HBT radii



The HBT correlation radii for most-central collisions, R_{side}, R_{out}, and their ratio, as predicted by the model (solid line) and measured by the PHENIX collaboration.

Summary

1. Success for abundances, $p_\perp$-spectra, including strange particles and resonances
2. The model also works very reasonably for the HBT radii, in particular $R_{\rm out}/R_{\rm side} \sim 1$
3. ... and for the elliptic flow (A. Baran, in preparation)
4. Resonances are an important source of correlations between opposite-charge pions
5. Shape of the $\pi\pi$ "spectral line" - new thermometer, derivative of phase shifts must be used, full model gives similar results at 165 MeV to the naive calculation at 110 MeV (cooling via decays)
6. Not possible to place the ρ peak at the experimental value (medium effects - Brown-Rho scaling?, other effects?)
7. By summing up the resonance and non-resonance contributions we obtain the pion balance function with the shape similar to the data

Soft physics well desribed by the statistical (thermal) approach

W. Czyż :

Things are so complicated that

they become simple again!

Back-up slides

The STAR cuts

The cuts in the STAR analysis of the $\pi^+\pi^-$ invariant-mass spectra have the following form (Fachini):

$$\begin{aligned} &|y_\pi| \le 1, \\ &|\eta_\pi| \le 0.8, \\ &0.2 \text{ GeV} \le p_\pi^\perp \le 2.2 \text{ GeV}, \end{aligned} \tag{1}$$

while the bins in $p_T \equiv |\boldsymbol{p}_\pi^\perp + \boldsymbol{p}_\pi^\perp|$ start from the range $0.2 - 0.4$ GeV, and step up by 0.2 GeV until $2 - 2.4$ GeV.

For two-body decays, the relevant formula for the number of pairs of particles 1 and 2 has the form

$$\begin{aligned} \frac{dN_{12}}{dM} &= \frac{d\delta_{12}}{dM}\frac{bm}{p_1^*}\int_{p^\perp_{1,\text{low}}}^{p^\perp_{1,\text{high}}} dp_1^\perp \int_{y_{1,\text{low}}}^{y_{1,\text{high}}} dy_1 \int_{p^\perp_{\text{low}}}^{p^\perp_{\text{high}}} dp^\perp \int_{y_\text{low}}^{y_\text{high}} dy \\ &\times C_2^0 C_1^\eta C_2^\eta \frac{\theta(1-\cos^2\gamma_0)}{|\sin\gamma_0|} S(p^\perp), \end{aligned} \tag{2}$$

Lowering the ρ mass

In order to show how the medium modifications will show up in the $\pi^+\pi^-$ spectrum, we have scaled the $\pi\pi$ phase shift in the ρ channel, according to the simple law

$$\delta_1^1(M)_{\rm scaled} = \delta_1^1(s^{-1}M)_{\rm vacuum}, \tag{3}$$

Phase shift vs. spectral density