New LO & NLO Parton Parametrizations of the Real Unpolarized Photon

Paweł Jankowski   Instytut Fizyki Teoretycznej UW

Warszawa 09.01.2004

1. Motivation
2. Proton structure & DIS
3. Photon structure
4. Main problem - heavy quark contributions
5. Our parton parametrizations
6. Uncertainties of CJK parton distributions
7. Summary

Motivation

Is there a need for a new real photon parton parametrization?

- Discrepancy between theory and experiment in heavy-quark production ep, $\gamma\gamma \to b\bar{b}$
- A need of proper description of heavy-quark distributions
- Vast amount of new experimental F_2^γ data not used until '02

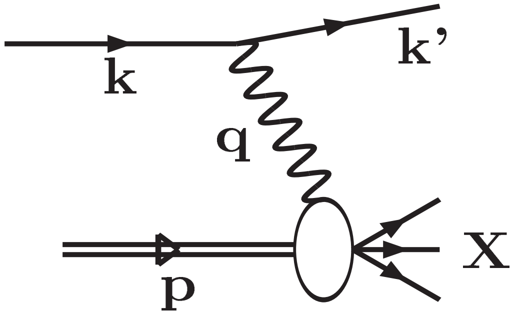
Parametrization	# of exp. F_2^γ points
GRV	$\approx$ 70
SaS	$\approx$ 80
GRSch	$\approx$ 130
NOW!	208

Recent photon parametrizations:

Parametrization	Description	Year
ALS	real	'03-'04
GRSieg	spin / virtual	'01
GRSch	real + virtual	'99
GS	real	'97
GRStr	virtual	'95
SaS	real	'95
GRV	real	'92

Albino, Klasen, Söldner, Phys. Rev. Lett. 89:122004 (2002) - fitted $\alpha_s(M_Z^2)$

Proton structure and Deep Inelastic Scattering



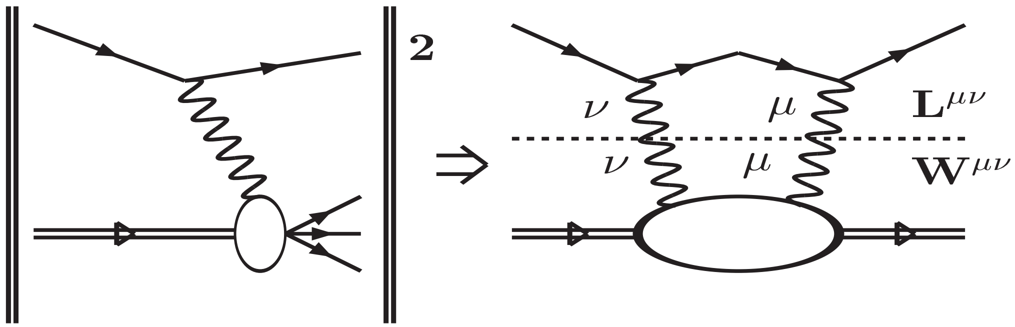
$Q^2 = -q^2$

Scaling variables:

$x = Q^2/(2p \cdot q)$ - "Bjorken x"

$y = (p \cdot q)/(p \cdot k)$

Cross-section can be divided into two parts: $|\bar{M}|^2 \sim L_{\mu\nu} W^{\mu\nu}$



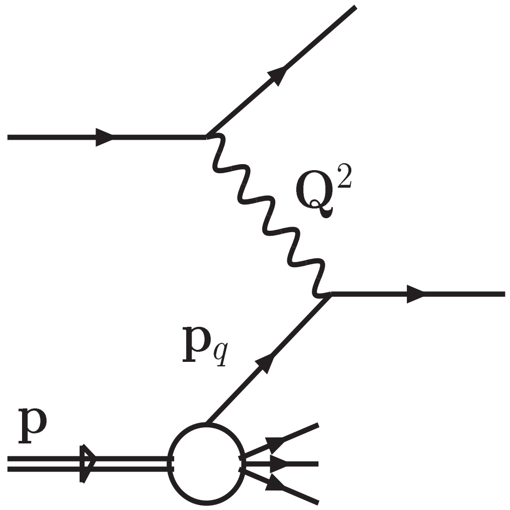
$$
\begin{aligned}
L_{\mu\nu} &= 2(k_\mu k'_\nu + k'_\mu k_\nu - k \cdot k' g_{\mu\nu}) \\
W^{\mu\nu} &= W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + \frac{W_2}{M^2} \left(p^\mu - \frac{p \cdot q}{q^2} q^\mu \right) \left(p^\nu - \frac{p \cdot q}{q^2} q^\nu \right)
\end{aligned}
$$

Unpolarized 'structure functions': $F_1^P = MW_1 \quad F_2^P = \frac{p\cdot q}{M} W_2$

$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4}\{y^2 x F_1^P + (1-y)F_2^P\} = \frac{4\pi\alpha^2}{xQ^4}\{\frac{1}{2}y^2(2xF_1^P - F_2^P) + \frac{F_2^P}{2}[1+(1-y)^2]\}$$

Quark Parton Model (QPM)

$$\frac{d^2\sigma}{dxdQ^2} = \sum_q \int_0^1 dx q(x) \frac{d^2\hat{\sigma}_{eq}}{dxdQ^2}$$



$x = p_q/p$ & $q(x)$ - quark density in proton

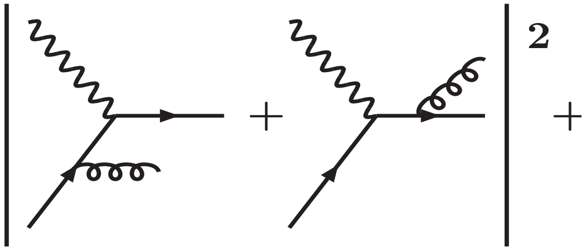
$$\frac{d^2\sigma}{dxdQ^2} = \frac{4\pi\alpha^2}{xQ^4} \sum_q \int_0^1 dx q(x) e_q^2 \frac{x}{2}[1+(1-y)^2]$$

$$F_2^P(x) = \sum_q x e_q^2 q(x)$$

QCD LO, $O(\alpha_s)$ corrections:

$$F_2^P(x,Q^2) = \sum_q xe_q^2 \left\{ q(x) + \int_x^1 \frac{dy}{y} \frac{\alpha_s(Q^2)}{2\pi} \left[P(\tfrac{x}{y}) \log \frac{Q^2}{\kappa^2} + ... \right] \right\}$$

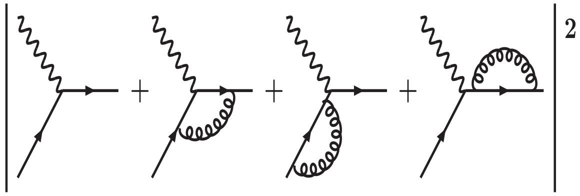
$\log \frac{Q^2}{\kappa^2}$ - collinear singularity, κ - regulator



Mass factorization: $\log \frac{Q^2}{\kappa^2} = \log \frac{Q^2}{\mu^2} + \log \frac{\mu^2}{\kappa^2}$

μ - renormalization scale

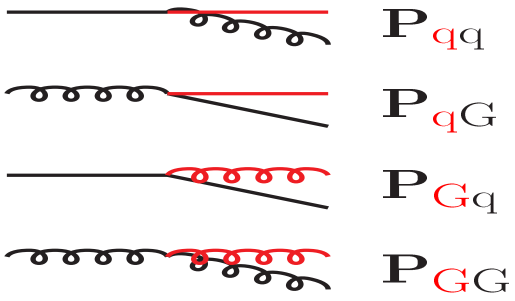
$$q(x,\mu^2) = q(x) + \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} q(y) \left[P\left(\frac{x}{y}\right) \log \frac{\mu^2}{\kappa^2} + ... \right]$$



We can choose $\mu^2 = Q^2$ and obtain Q^2 evolution:

$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) q(y,Q^2)$$

Then in LO $\qquad F_2^P(x,Q^2) = \sum_q xe_q^2 q(x,Q^2)$

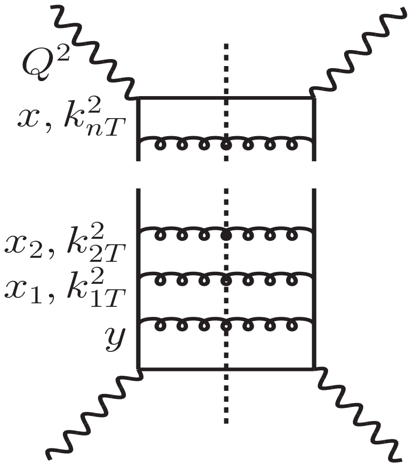


P_{ij} - 'splitting functions'

we derive to the LO DGLAP equations:

$$\frac{\partial q(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qq}\left(\frac{x}{y}\right) q(y,Q^2) + P_{qG}\left(\frac{x}{y}\right) G(y,Q^2) \right]$$
$$\frac{\partial G(x,Q^2)}{\partial \log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{Gq}\left(\frac{x}{y}\right) q(y,Q^2) + P_{GG}\left(\frac{x}{y}\right) G(y,Q^2) \right]$$

DGLAP equations resum Leading Logarithms (LL):



$\alpha_s(Q^2)^n \log^n Q^2$

nth term $\leftrightarrow$ n-rung ladder: $Q^2 \gg k^2_{nT} \gg \cdots k^2_{1T}$

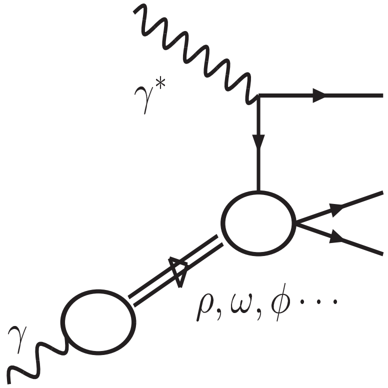
Photon structure

Photon is a gauge particle mediating electromagnetic interactions.
It couples to charged particles in a very well known (QED) way.

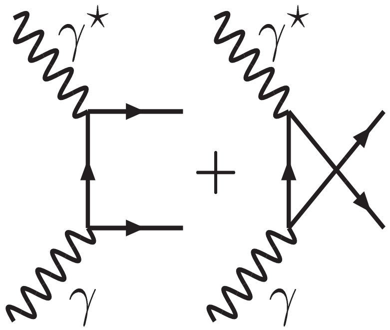
It appears that photon also exhibits hadronic structure!

Photonic structure functions F_1^γ, F_2^γ and parton distributions can be introduced by analogy to proton case: $\mathrm{DIS}_{ep} \to \mathrm{DIS}_{e\gamma}$

non-perturbative 'hadron-like' contribution can be described by Vector Dominance Model (VDM)



perturbative 'point-like' contribution $k_q^\gamma(x)$



LO inhomogenous DGLAP equations in photon case:

$$\begin{aligned}\frac{\partial q^\gamma(x,Q^2)}{\partial \log Q^2} &= \frac{\alpha}{2\pi}k_q^\gamma(x) + \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{qq}\left(\frac{x}{y}\right)q^\gamma(y,Q^2) + P_{qG}\left(\frac{x}{y}\right)G^\gamma(y,Q^2)\right]\\ \frac{\partial G^\gamma(x,Q^2)}{\partial \log Q^2} &= \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{Gq}\left(\frac{x}{y}\right)q^\gamma(y,Q^2) + P_{GG}\left(\frac{x}{y}\right)G^\gamma(y,Q^2)\right]\end{aligned}$$

Approach - Mellin moments: $q^{\gamma,n}(Q^2) = \int_0^1 x^{n-1}q^\gamma(x,Q^2)dx$

Solution of inhomogenous equations $\rightarrow$ input needed:

$$q^{\gamma,n}(Q^2) = \frac{4\pi}{\alpha_s(Q^2)}\left[1 - L^{1-2/\beta_0 P^n}\right]\frac{1}{1-\frac{2}{\beta_0}P^n}\frac{\alpha}{2\pi\beta_0}k^{\gamma,n} + L^{-2/\beta_0 P^n}q_{\text{had}}^{\gamma,n}(Q_0^2)$$

$$L = \alpha_s(Q^2)/\alpha_s(Q_0^2)$$

We make a division:

$$\begin{aligned}q_{\text{pl}}^{\gamma,n}(Q^2) &= \frac{4\pi}{\alpha_s(Q^2)}\left[1 - L^{1-2/\beta_0 P^n}\right]\frac{1}{1-\frac{2}{\beta_0}P^n}\frac{\alpha}{2\pi\beta_0}k^{\gamma,n}\\ q_{\text{had}}^{\gamma,n}(Q^2) &= L^{-2/\beta_0 P^n}q_{\text{had}}^{\gamma,n}(Q_0^2)\end{aligned}$$

Now we need hadronic input like in proton case! – Vector Meson Dominance (VMD)

Hadronic γ states approximated with vector mesons V $(J^{PC} = 1^{--})$ - ρ, ω, ϕ

$$q^{\gamma}_{\mathrm{had}}(x, Q_0^2) = \sum_V \frac{4\pi\alpha}{f_V^2} q^V(x, Q_0^2)$$

f_V^2 given by vector meson decay width into e^+e^- pair: $\Gamma(V \to e^+e^-) = \frac{\alpha^2}{3}\frac{4\pi}{f_V^2} M_V$

Often $q^{\rho}(x, Q_0^2) \sim q^{\pi}(x, Q_0^2)$

Often ρ, ω, ϕ contr. summed (non-)coherently

We fit $q^{\rho}(x, Q_0^2)$ to F_2^{γ} data

We take $V = \rho$ and calculate

$$q^{\gamma}_{\mathrm{had}}(x, Q_0^2) = \kappa \frac{4\pi\alpha}{f_\rho^2} q^{\rho}(x, Q_0^2)$$

κ accounts for other meson vector states
(Glück, Reya, Vogt, Phys. Rev. D 46, 1973 (1992))

Heavy-quark contributions

Two basic heavy-quark schemes:

- Fixed Flavour Number Scheme (FFNS)
  - photon consists only of gluons and light quarks (u, d, s)
  - $N_f = 3$ number of 'active' flavours
  - heavy quarks may appear only in final state
  - mass m_h is included in calculations
  - seems to be appropriate for $Q^2 \leq m_h^2$

- Zero-mass Variable Flavour Number Scheme (ZVFNS)
  - for $Q^2 \gg m_h^2$ heavy quark h becomes 'active'
  - nonzero heavy-quark densities exist!
  - $N_f = N_f + 1 \Rightarrow N_f = 3, 4, 5, 6$
  - $m_h = 0$ in calculations
  - seems to be appropriate for $Q^2 \gg m_h^2$

None of schemes appropriate for all Q^2 values!

Solution

General VFNS - SUM all terms emerging in FFNS and ZVFNS

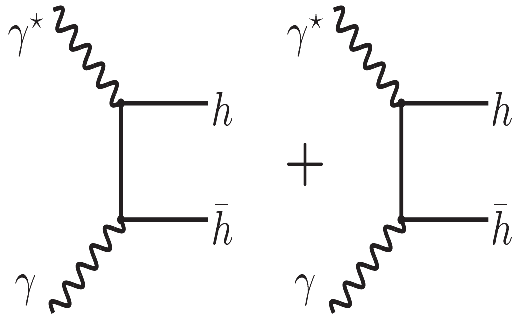
Aivazis, Collins, Olness & Tung, Phys. Rev. D50 3102 (1994)

Double counting

FFNS scheme

Only light quark densities

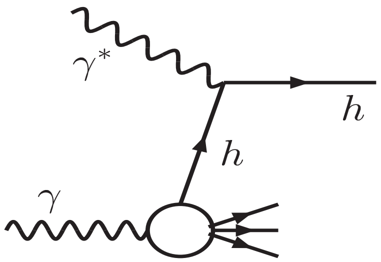
For $W > 2m_h$ $F^{\gamma}_{2,h}$ described by lowest order "direct" $\gamma^\star\gamma \to h\bar{h}$ (Bethe-Heitler) and "resolved" $\gamma^\star G \to h\bar{h}$ processes



ZVFNS scheme

Heavy-quark distribution $q^{\gamma}_h(x, Q^2)$ for $Q^2 > m_h^2$

$F^{\gamma}_{2,h} = xe_h^2 q^{\gamma}_h(x, Q^2)$

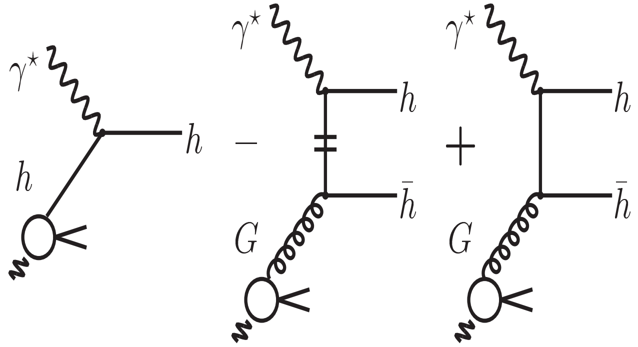


VFNS scheme

$F_{2,h}^{\gamma}$ given by heavy-quark density (ZVFNS)

plus "direct" Bethe-Heitler and "resolved"

contributions (FFNS)

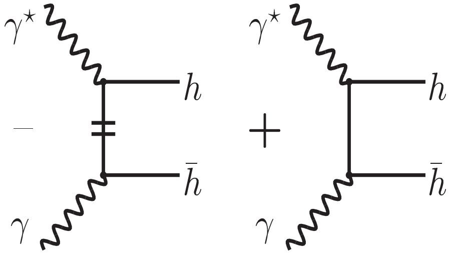


$$F_{2,h}^{\gamma}(x,Q^2) = xe_h^2 q_h^{\gamma}(x,Q^2) + F_{2,h}^{\gamma}|_{direct} + F_{2,h}^{\gamma}|_{resolved}$$

But $F_{2,h}^{\gamma}|_{direct} \quad \leftrightarrow \quad \gamma^\star\gamma \to h\bar{h} \quad (k_h^{\gamma}(x))$

and $F_{2,h}^{\gamma}|_{resolved} \quad \leftrightarrow \quad \gamma^\star G \to h\bar{h} \quad (P_{qG}(x))$

already a part of DGLAP $\Rightarrow$ need to subtract them!



$$F_{2,h}^{\gamma}(x,Q^2)|_{subtr.} = xe_h^2 \ln\frac{Q^2}{Q_0^2}\left[\frac{\alpha}{\pi}k_h^{\gamma}(x) + \frac{\alpha_s(Q^2)}{\pi}\int_x^1 \frac{dy}{y} P_{qG}\left(\frac{x}{y}\right) G^{\gamma}(y,Q^2)\right]$$

First term - exact solution of $\qquad \dfrac{dq_h^{\gamma}(x,Q^2)}{d\ln Q^2} = \dfrac{\alpha}{2\pi}k_h^{\gamma}(x)$

Second term - approximated solution of $\qquad \dfrac{dq_h^{\gamma}(x,Q^2)}{d\ln Q^2} = \dfrac{\alpha_s(Q^2)}{2\pi}\int_x^1 \dfrac{dy}{y} P_{qG}\left(\dfrac{x}{y}\right) G^{\gamma}(y,Q^2)$

Threshold

Heavy quark contributions should vanish when $W \leq 2m_h$
Difficult because scale of the process is Q^2 and $W^2 = Q^2(1/x - 1)$

Usual boundary condition at $Q^2 = m_h^2$:
$f_h(x, Q^2 \leq m_h^2) = 0$
$f_h(x, Q^2 > m_h^2) \neq 0$

ACOT: replace Q^2 with μ^2 given by
$\mu^2 = m_h^2 + cQ^2(1 - m_h^2/Q^2)^n$ for $Q^2 > m_h^2$
$\mu^2 = m_h^2$ for $Q^2 \leq m_h^2$
with $c = 0.5$ and $n = 2$

Thorne, Roberts, Phys. Lett. B421, 303 (1998) applied a very specific threshold condition
$F_2^c(x, m_h^2)|_{N_f+1} = F_2^c(x, m_h^2)|_{N_f}$
$\frac{F_2^c(x,m_h^2)|_{N_f+1}}{d\ln(Q^2/m_h^2)} = \frac{F_2^c(x,m_h^2)|_{N_f}}{d\ln(Q^2/m_h^2)}$

New approach - ACOT(χ) by Kretzer, Schmidt and Tung, J. Phys. G28, 983 (2002)
we introduce $\chi_h = x(1 + 4m_h^2/Q^2)$ and calculate $q_h^\gamma(\chi_h, Q^2)$
$\chi_h \to 1$ for $W \to 2m_h$ and $q_h^\gamma(\chi_h, Q^2) \to 0$ - proper threshold behavior!

Our parton parametrizations

Experimental F_2^γ data - e^+e^- single-tagged events $\approx$ $e\gamma$ DIS

♣ SLAC - PEP : TPC2/γ

♣ DESY - PETRA : PLUTO, JADE, CELLO, TASSO

♣ KEK - TRISTAN : TOPAZ, AMY

♣ CERN - LEP : ALEPH, DELPHI, L3, OPAL

♠ F.Cornet, P.J, M.Krawczyk, A.Lorca, Phys.Rev. D68, 014010 (2003)

♠ F.Cornet, P.J, M.Krawczyk, PHOTON 2003, hep-ph/0310029

♠ F.Cornet, P.J, M.Krawczyk, IFT-2003-30

♠ P.J, IFT-2003-31, hep-ph/0312056

♠ F.Cornet, P.J, M.Krawczyk, NLO in preparation

	MODEL	SCHEME	PARAMETR	Q_0^2 [GeV^2]	χ^2/DOF
LO	CJKL	ACOT(χ)	ANALYTIC	0.25	2.10
	FFNS$_{CJKL}$	FFNS	———	0.25	2.30
	CJK 1	ACOT(χ)	GRID	0.25	1.54
	CJK (2)	ACOT(χ)	GRID	0.25	1.54
	FFNS$_{CJK}$1	FFNS	GRID	0.25	1.75
	FFNS$_{CJK}$(2)	FFNS	GRID	0.25	1.56
NLO	CJK	ACOT(χ)	SOON GRID	0.765	1.37
	FFNS$_{CJK}$	FFNS	SOON GRID	0.765	1.29

All parton parametrizations available at
`http://www.fuw.edu.pl/~pjank/param.html`

		χ^2/DOF
AKS (only F_2^γ)	LO	0.94
	NLO	0.89
ALS ($F_2^\gamma+F_2^P$)	NLO	0.9
ALS ($F_2^\gamma+F_2^P$+dijets)	NLO	1.3

χ^2 dependence on the data

Set	TPC2$_\gamma$	DESY	KEK	ALEPH	DELPHI	L3	OPAL
χ^2_n/N_n	2.38	2.44	1.20	1.07	2.25	0.60	0.97

DESY = PLUTO + JADE + CELLO + TASSO
DELPHI = DELPHI'96 + DELPHI'98 + DELPHI'01

Set	PLUTO	JADE	CELLO	TASSO	DELPHI'96	DELPHI'98	DELPHI'01
χ^2_n/N_n	0.64	1.12	5.88	1.01	1.04	0.10	3.65

FIT	FULL	NOTPC	NOCEL	NODEL	NOTPCCEL	NOTPCDEL	NOCELDEL
χ^2	338.7	273.7	262.4	244.0	200.7	169.4	179.5
points	208	182	196	186	170	160	174
χ^2/DOF	1.66	1.54	1.37	1.34	1.21	1.08	1.06

In LO the CJK model is:

$$
\begin{aligned}
F_2^\gamma(x,Q^2) \;=\; & \sum_{i=1}^{3} x e_i^2 (q_i^\gamma + \bar{q}_i^\gamma)(x,Q^2) + \sum_{h(=c,b)}^{2} x e_h^2 (q_h^\gamma + \bar{q}_h^\gamma)(\chi_h,Q^2) \\
& + \sum_{h(=c,b)}^{2} \left[F_{2,h}^\gamma(x,Q^2)|_{direct} + F_{2,h}^\gamma(x,Q^2)|_{resolved} \right] \\
& - \sum_{h(=c,b)}^{2} x e_h^2 \ln\frac{Q^2}{Q_0^2} \left[\frac{\alpha}{\pi} k^\gamma(\chi_h) + \frac{\alpha_s(Q^2)}{\pi} \int_{\chi_h}^{1} \frac{dy}{y} P_{qG}\left(\frac{\chi_h}{y}\right) G^\gamma(y,Q^2) \right]
\end{aligned}
$$

plus conditions imposed "by hand":

♣ $k_h^\gamma(\chi_h,Q^2)=0$ for $\chi_h>1$ otherwise $k_h^\gamma(\chi_h,Q^2)=3[\chi_h^2+(1-\chi_h)^2]\to 3$ at $\chi_h\to 1$

♣ $F_{2,h}^\gamma(x,Q^2) \geq F_{2,h}^\gamma(x,Q^2)|_{direct} + F_{2,h}^\gamma(x,Q^2)|_{resolved}$
important at small- and high-x regions

α_s running:

♣ $\alpha_s^{(N_q)}(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^{(N_q)2})}$ with $\alpha_s^{(N_q)}(m_h^2) = \alpha_s^{(N_q+1)}(m_h^2)$

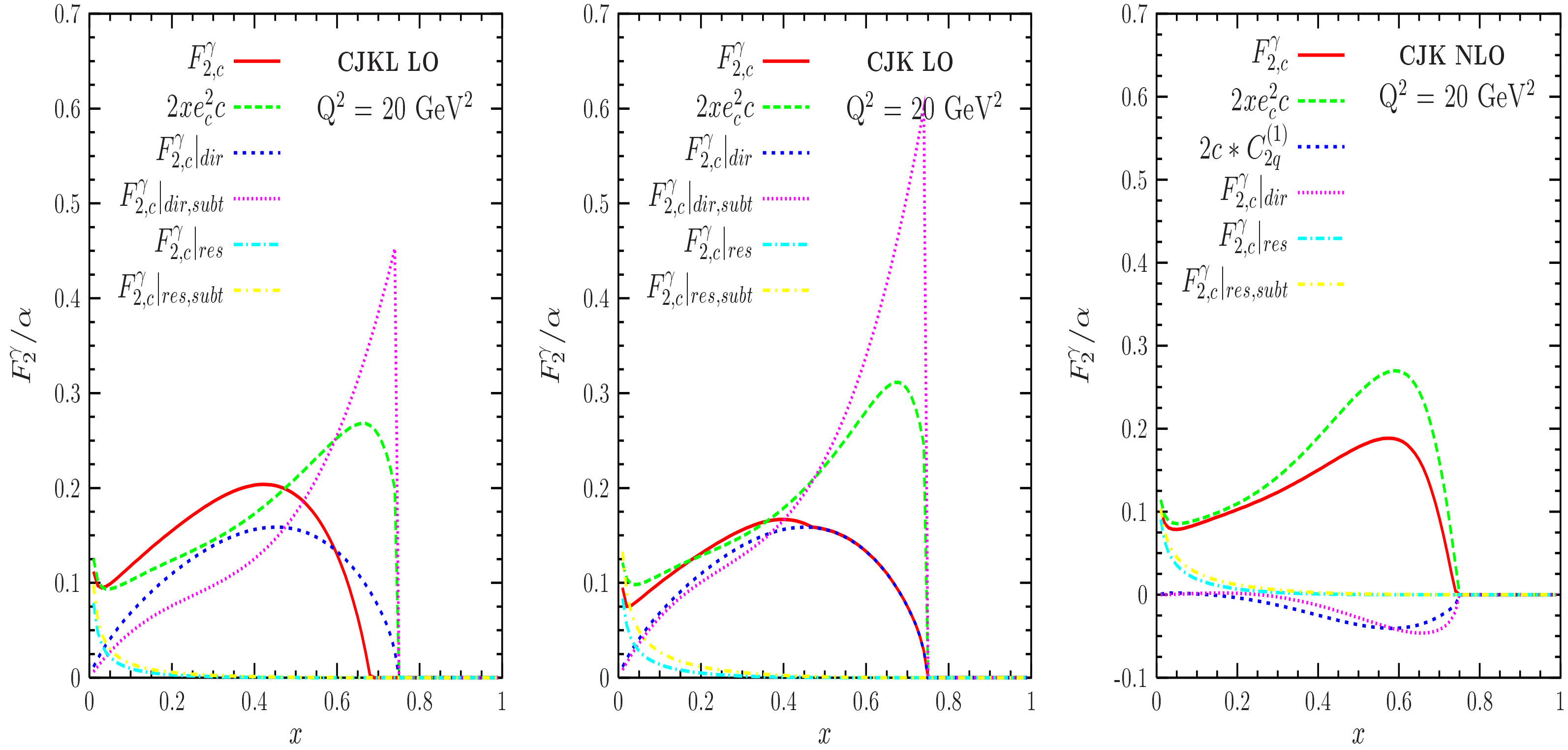
In NLO the CJK model defined in DIS$_\gamma$ factorization scheme is:

$$
\begin{aligned}
&\frac{1}{x}F_2^\gamma(x,Q^2) = \\
&\sum_{i=1}^{3} e_i^2 \left\{(q_i+\bar{q}_i)(x,Q^2) + \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[(q_i+\bar{q}_i)(y,Q^2)C_{2,q}^{(1)}(\frac{x}{y}) + G(y,Q^2)C_{2,G}^{(1)}(\frac{x}{y})\right]\right\} \\
+\quad &\sum_{h=1}^{2} e_h^2 \left\{(q_h+\bar{q}_h)(\chi_h,Q^2) + \frac{\alpha_s}{2\pi}\left[\int_x^1 \frac{dy}{y}(q_h+\bar{q}_h)(y,Q^2)C_{2,q}^{(1)}(\frac{x}{y})\right.\right. \\
+\quad &\left.\left.\int_{\chi_h}^1 \frac{dy}{y}G(y,Q^2)C_{2,G}^{h,(1)}(\frac{x}{y},m_h^2) - \ln\frac{Q^2}{Q_0^2}\cdot 2\int_{\chi_h}^1 \frac{dy}{y}G(y,Q^2)P_{qG}^{(0)}(\frac{\chi_h}{y})\right]\right\} \\
+\quad &3e_h^2\frac{\alpha}{\pi}\left[-\beta x(1-x)\frac{4m_h^2}{Q^2} + \left(x(1-3x)\frac{4m_h^2}{Q^2} - x^2\frac{8m_h^4}{Q^4}\right)\ln\frac{1+\beta}{1-\beta}\right].
\end{aligned}
$$

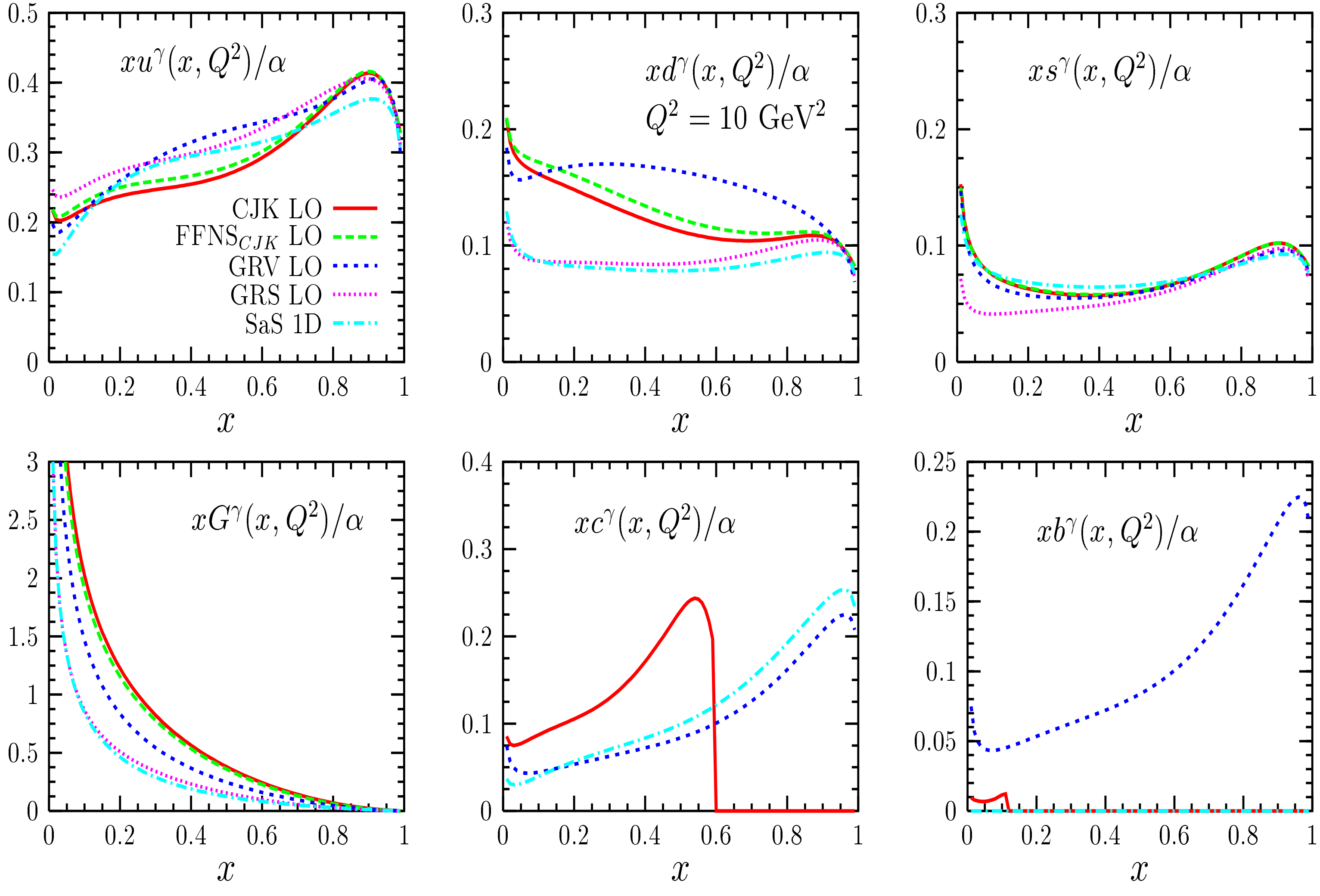
order	hard (parton) process	coefficient function
$O(\alpha_s^0)$	$\gamma^* + q(\bar{q}) \to q(\bar{q})$	$C_{2,q}^{(0)} = \delta(1-z)$
	$\gamma^* + \gamma \to q + \bar{q}$	$C_{2,\gamma}^{(0)}$
$O(\alpha_s^1)$	$\gamma^* + q(\bar{q}) \to q(\bar{q}) + G$	$C_{2,q}^{(1)}$
	$\gamma^* + G \to q + \bar{q}$	$C_{2,G}^{(1)}$

order	parton subprocess	coefficient function
$O(\alpha_s^0)$	$\gamma^* + \gamma \to h + \bar{h}$	$C_{2,\gamma}^{h,(0)}$
$O(\alpha_s^1)$	$\gamma^* + G \to h + \bar{h}$	$C_{2,G}^{h,(1)}$
	$\gamma^* + \gamma \to h + \bar{h} + G$	$C_{2,\gamma}^{h,(1)}$
$O(\alpha_s^2)$	$\gamma^* + G \to h + \bar{h} + G$	$C_{2,G}^{h,(2)}$
	$\gamma^* + q(\bar{q}) \to q(\bar{q}) + h + \bar{h}$	$C_{2,q}^{h,(2)}$, $C_{2,q}^{(2)}$

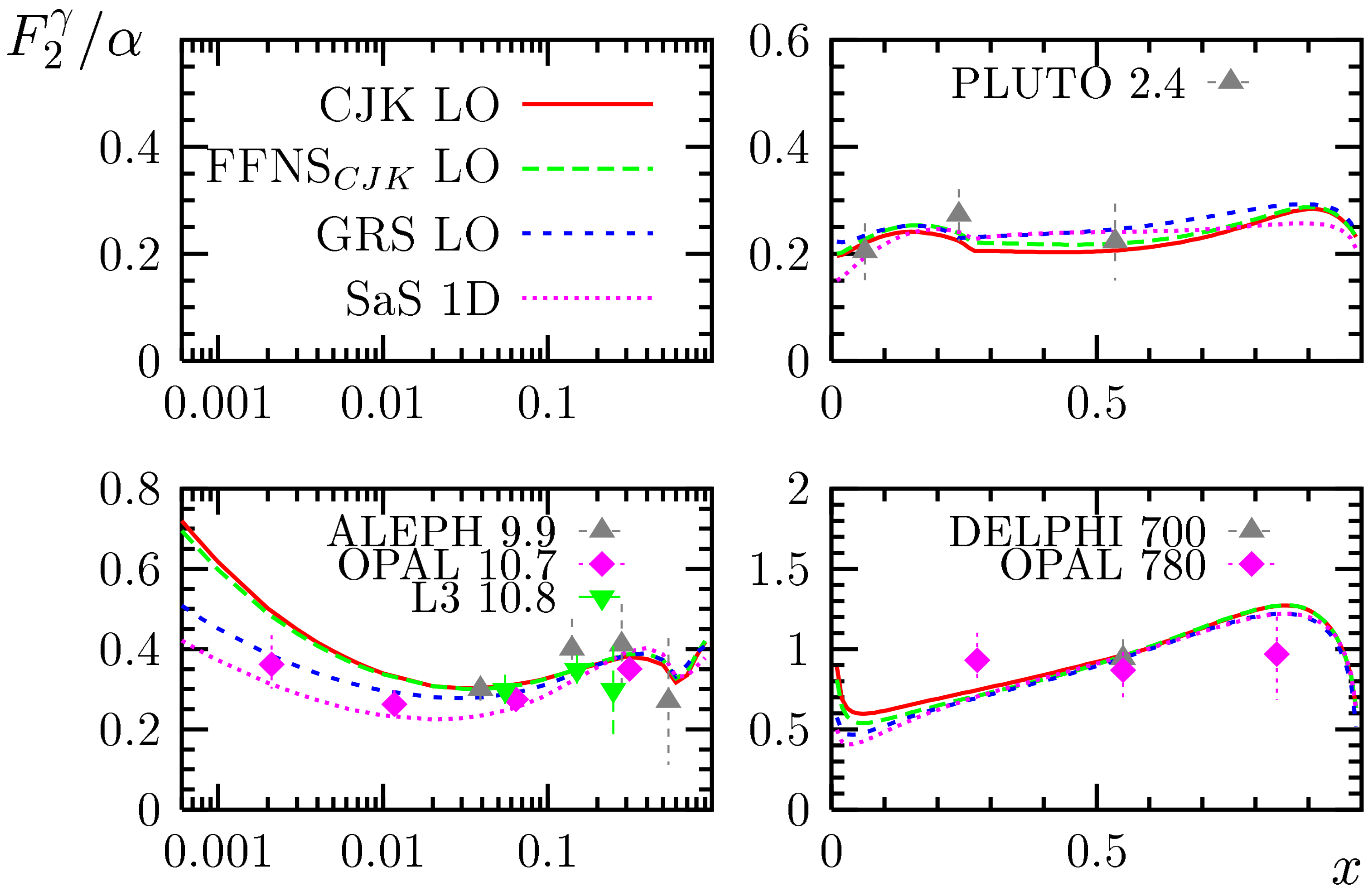
Contributions to charm structure function $F^{\gamma}_{2,c}$

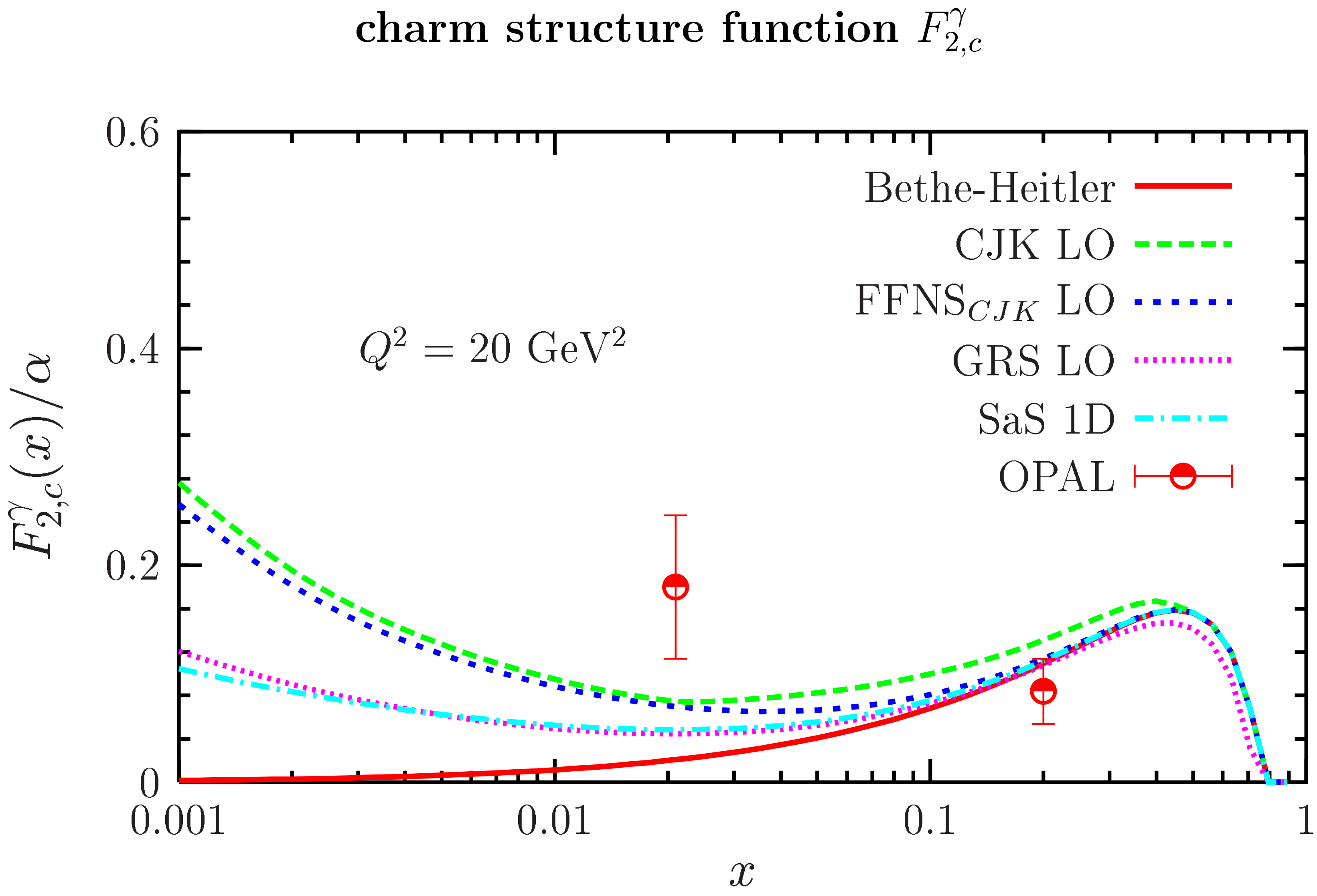


LO parton distributions



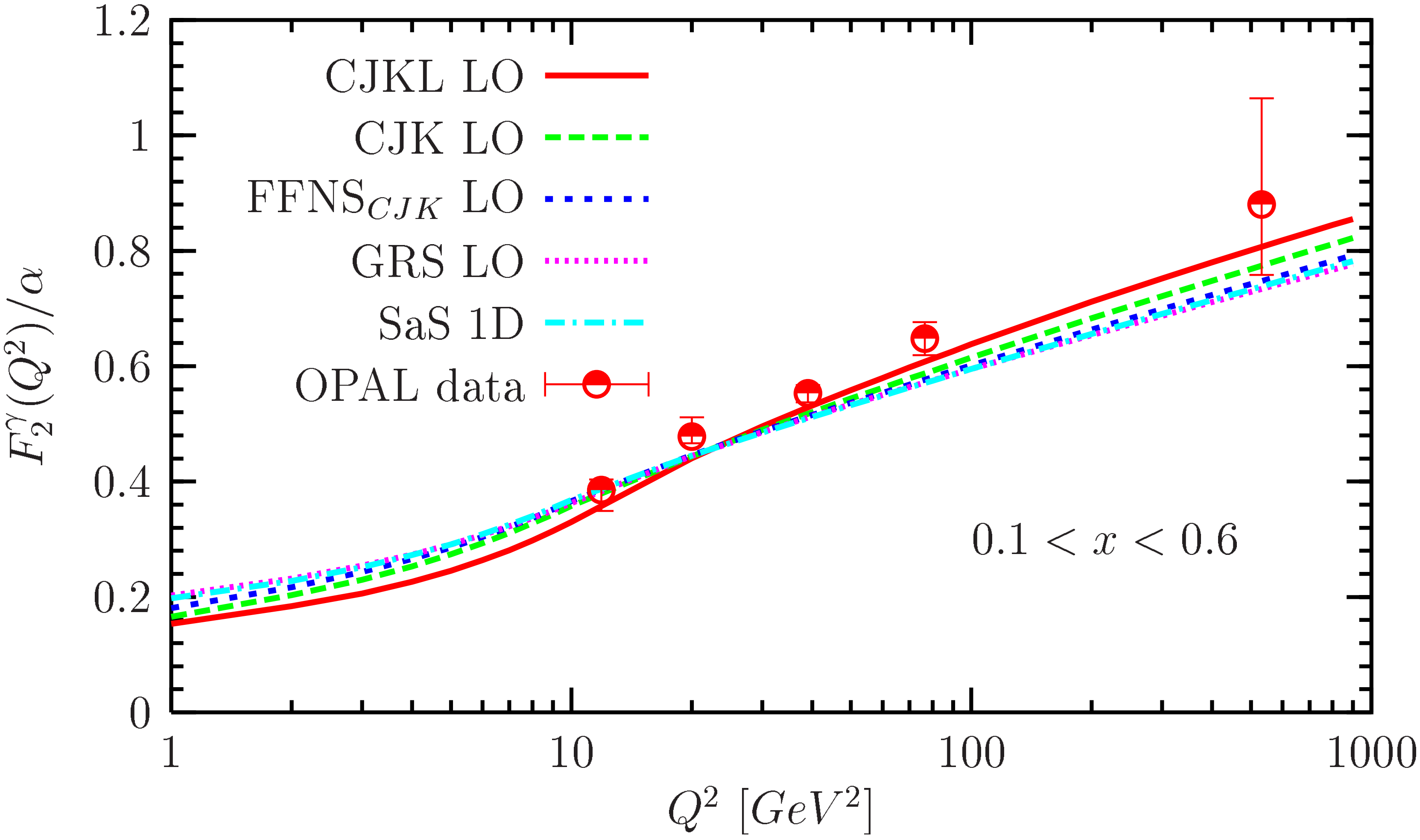
LO $F_2^\gamma(x, Q^2)$ fits



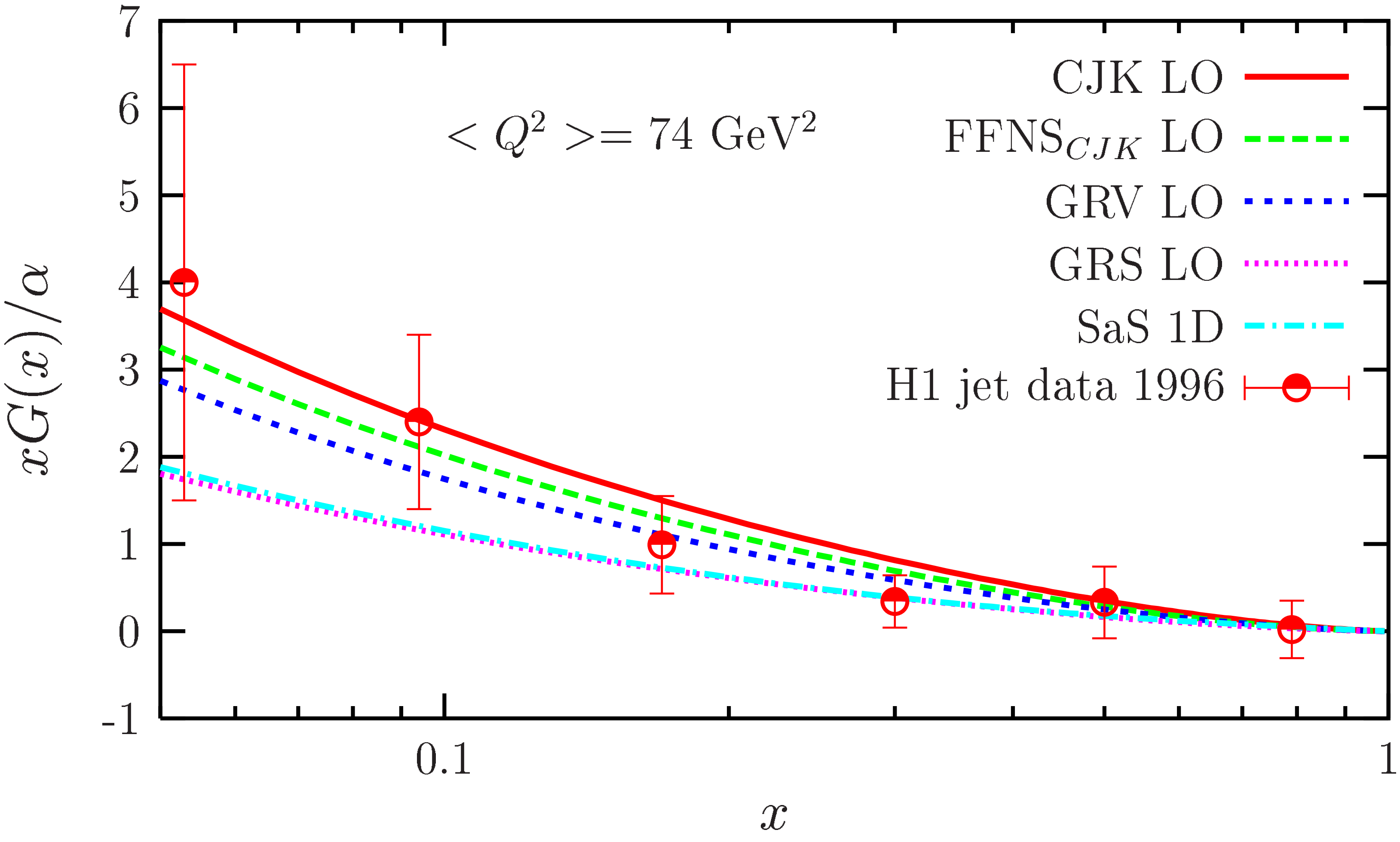


charm structure function $F_{2,c}^{\gamma}$
$F_{2,c}^{\gamma}(x)/\alpha$
$Q^2 = 20\ \mathrm{GeV}^2$
Bethe-Heitler
CJK LO
FFNS_{CJK} LO
GRS LO
SaS 1D
OPAL
0
0.2
0.4
0.6
0.001
0.01
0.1
1
x

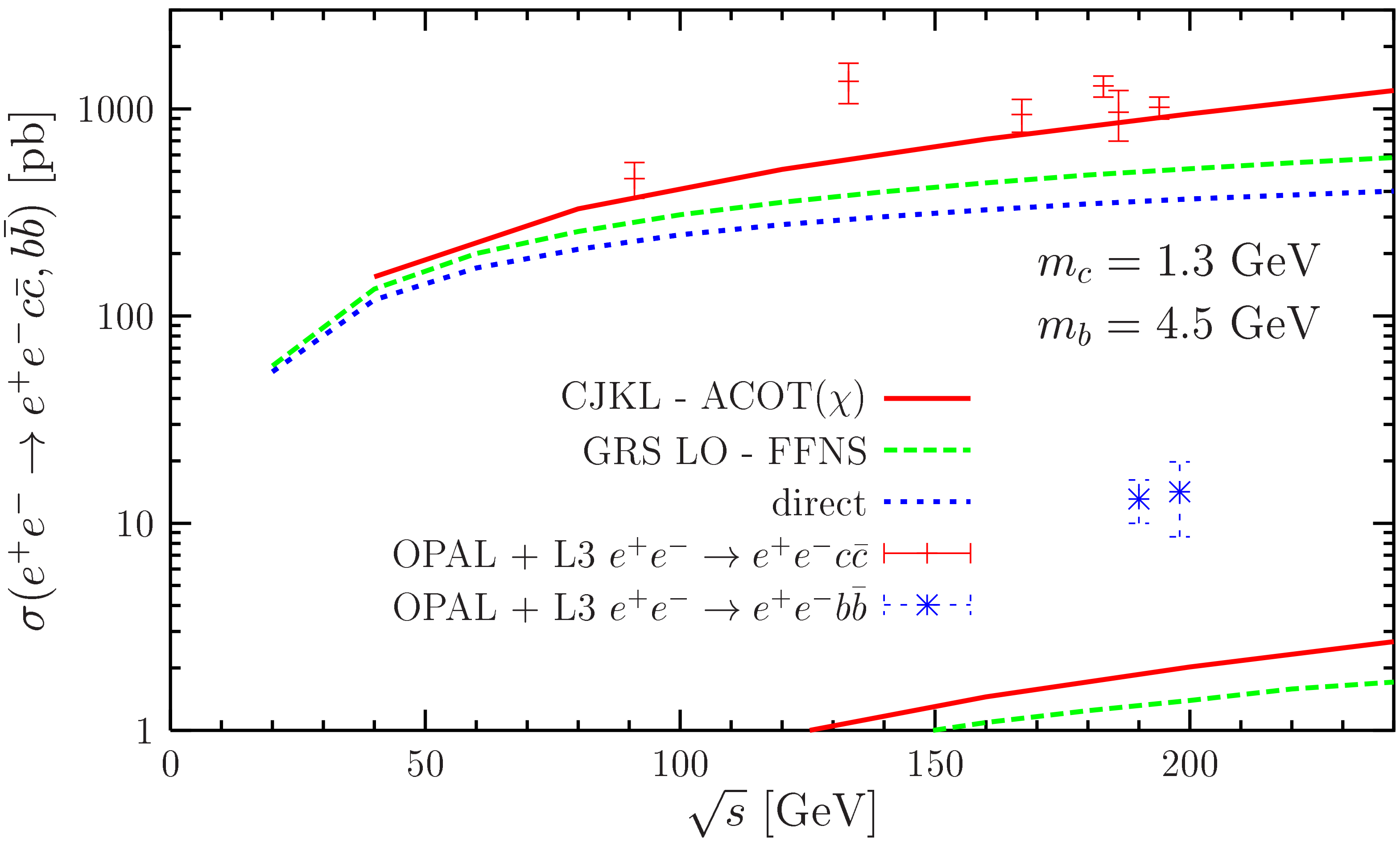
Q^2 evolution of F_2^γ



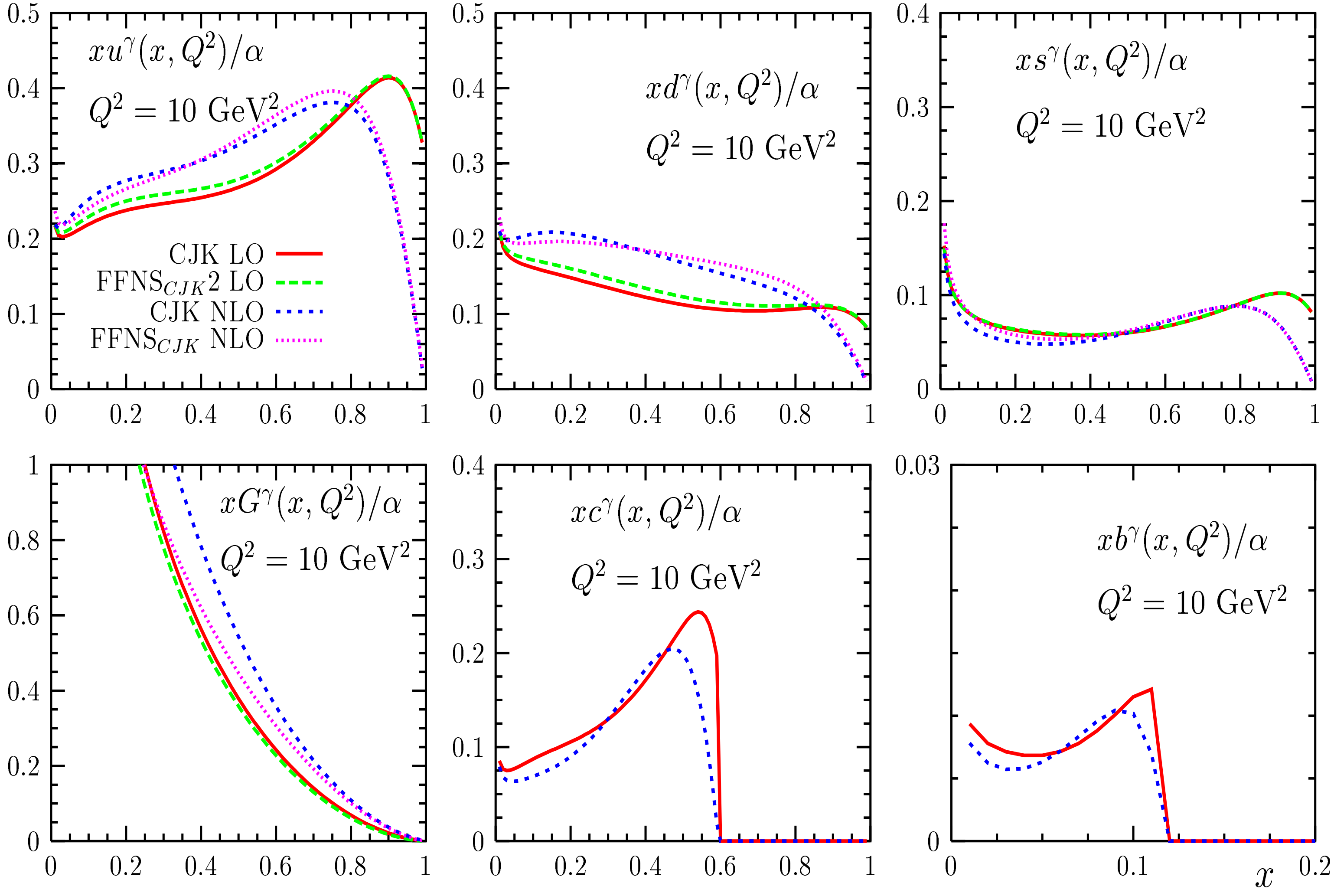
Gluon distribution at $\langle Q^2 \rangle = 74$ GeV^2



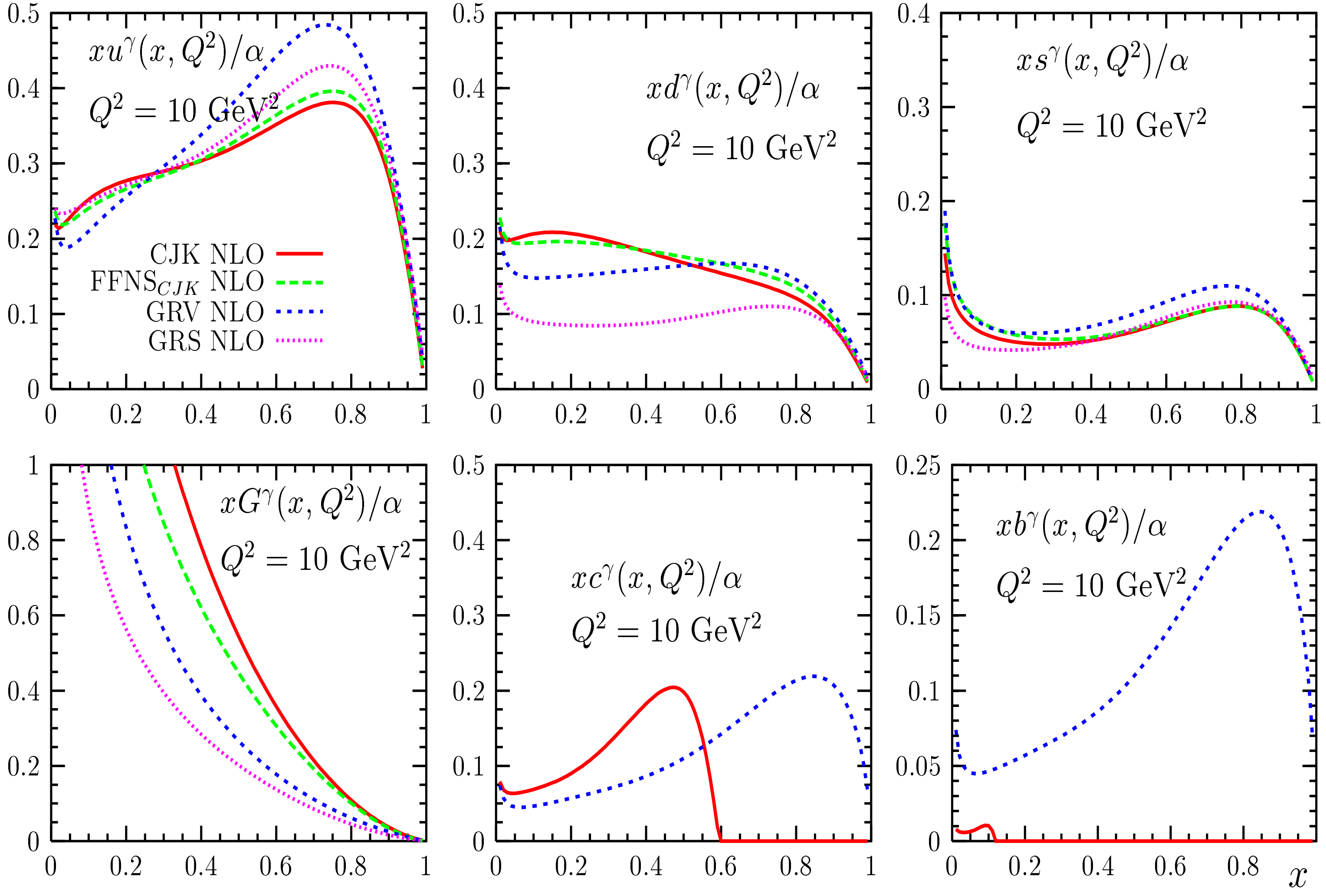
$\gamma\gamma$ heavy-quark production



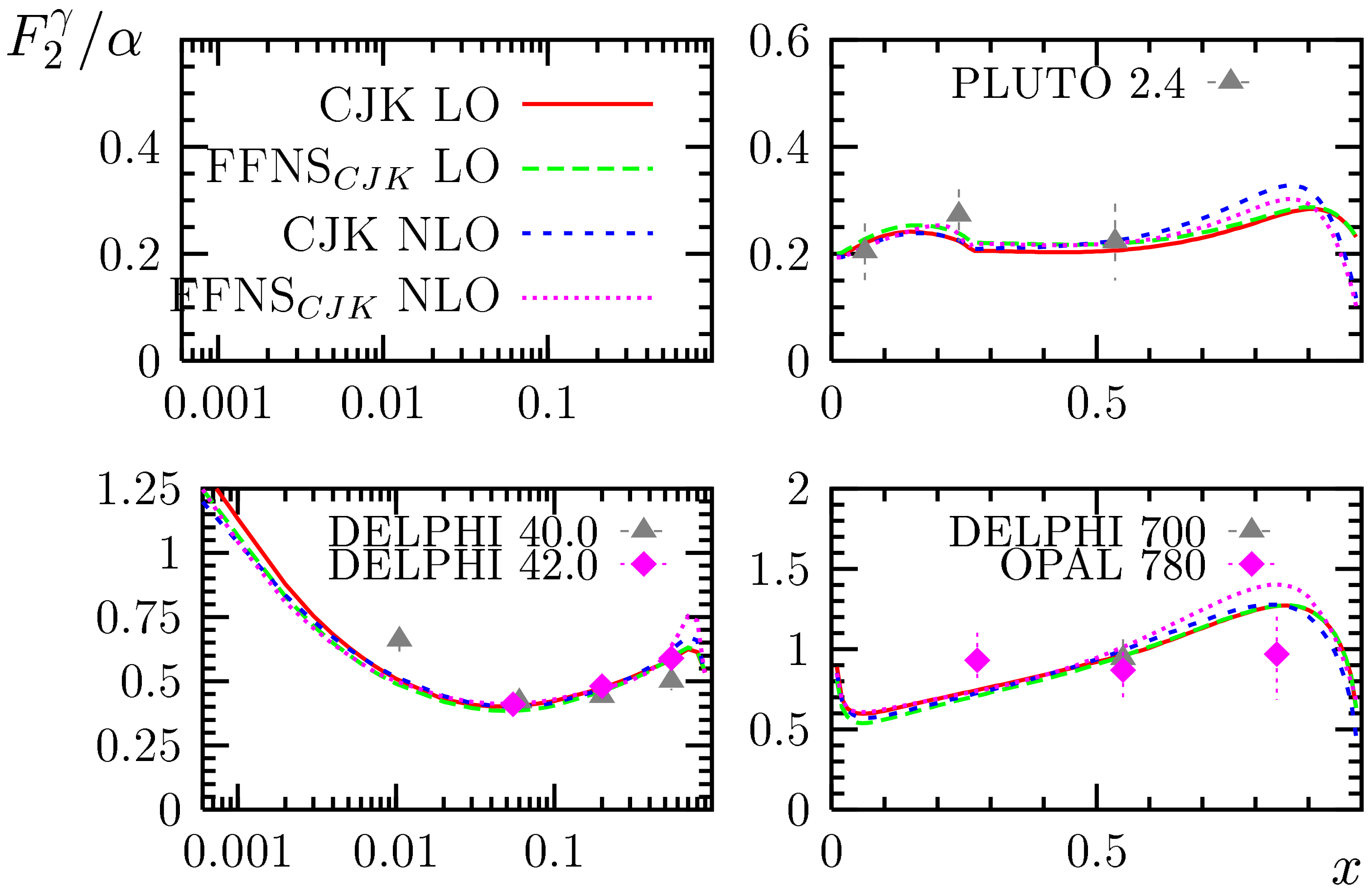
NLO vs LO parton distributions



NLO parton distributions



NLO $F_2^\gamma(x, Q^2)$ fits



Uncertainties of CJK parton densities

P.Jankowski, IFT-2003-31, hep-ph/0312056;
Following the proton analysis of CTEQ group, Phys Rev D65, 014011 (2002)...

We consider a least-square fit of a model parametrized by $\{a_i, i = 1, 2, \cdots, d\}$
We assume a quadratic expansion in the basis of parameters $\{a_i\}$

$$\Delta\chi^2 = \chi^2 - \chi_0^2 = \sum_{i=1}^{d}\sum_{j=1}^{d} H_{ij}(a_i - a_i^0)(a_j - a_j^0),$$

where $\chi_0^2 \leftrightarrow \{a_i^0\} \leftrightarrow S^0$ set - best fit and H_{ij} is an element of the symmetric Hessian matrix having a complete set of $k = 1, 2, \cdots, d$ orthonormal eigenvectors $(v_i)_k$

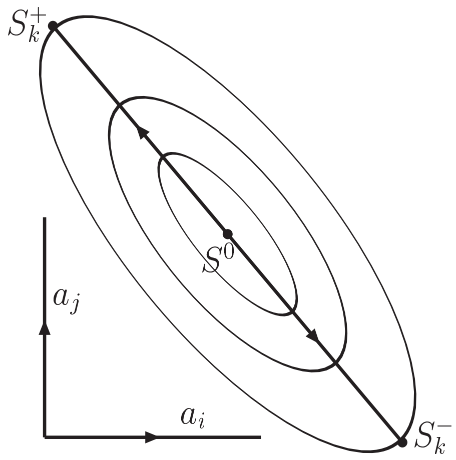
$$H_{ij} = \frac{1}{2}\left(\frac{\partial^2\chi^2}{\partial a_i \partial a_j}\right)_{a_0} \qquad \sum_{j=1}^{d} H_{ij}(v_j)_k = \epsilon_k (v_i)_k \qquad \sum_{i=1}^{d} (v_i)_j (v_i)_k = \delta_{jk}$$

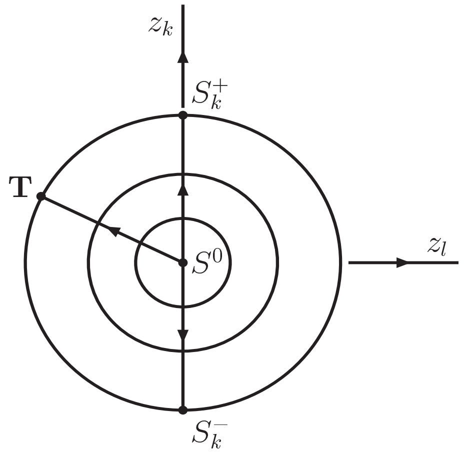
We introduce new eigenvector basis

$$a_i - a_i^0 = \sum_{k=1}^{d} s_k z_k (v_i)_k$$

$\{s_k\}$ are scale factors introduced to normalize $\{z_k\}$ so that

$$\Delta\chi^2 = \chi^2 - \chi_0^2 = \sum_{k=1}^{d} z_k^2 = T^2$$





We create a basis of test parametrizations, $\{S_k^{\pm}, k = 1, \cdots, d\}$, $S_k^{\pm} \leftrightarrow (v_i)_k$, defined by displacements of a magnitude t "up" or "down" along the corresponding eigenvector direction. $S_1^+ \leftrightarrow (z_1, z_2, z_3, \cdots) = (t, 0, 0, \cdots)$

$$a_i(S_k^{\pm}) = a_i^0 \pm t\,(v_i)_k s_k$$

Let us consider a physical observable X depending on the photon parton distributions. Its best value is given as $X(S^0)$. The uncertainty of X, for a displacement from the parton densities minimum by $\Delta\chi^2 = T^2$ can be calculated as

$$\Delta X = \frac{T}{2t}\left(\sum_{k=1}^{d}[X(S_k^+) - X(S_k^-)]^2\right)^{\frac{1}{2}}$$

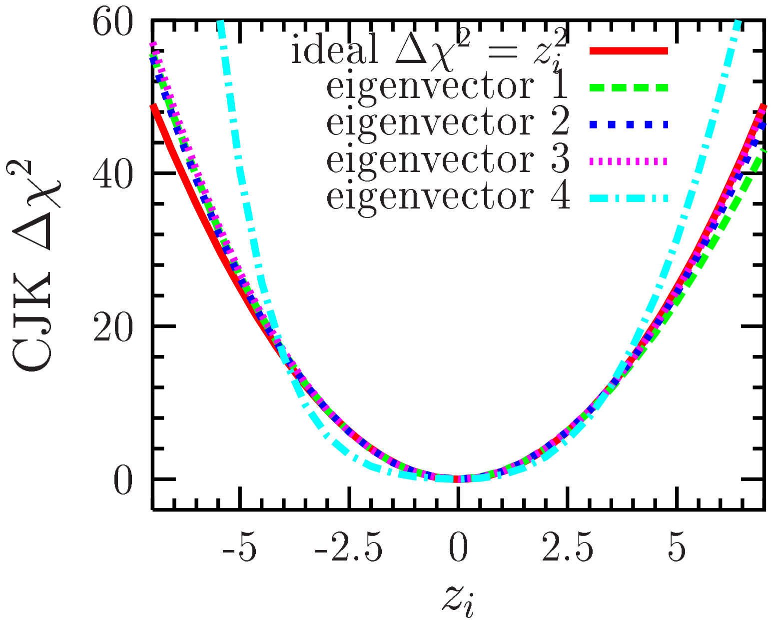
We estimate T

♣ check influence of various data

♣ compare parton distributions obtained in various models

We conclude $T \sim 5 - 10$

We test quadratic approximation



Estimation of the tolerance parameter T

n	Set	N_n	χ^2_n	χ^2_{-n}	$\Delta\chi^2_{-n}$
1	DESY	38	89.5	184.2	9.6
2	KEK	16	18.2	255.5	0.2
3	ALEPH	20	21.0	252.7	1.3
4	DELPHI	38	88.6	185.1	20.9
5	L3	28	14.9	258.8	0.6
6	OPAL	42	41.3	232.4	5.8

- χ^2_n - χ^2 for the nth set
- χ^2_{-n} - χ^2 for remaining five sets
- $\Delta\chi^2_n$ - the improvement of χ^2_{-n} gained by additional minimization

The agreement among the FFNS$_{CJK}$ and the CJK parametrizations.

	T(G^γ)	T(d^γ)	T(u^γ)	T(s^γ)	T(F_2^γ)
$1 \leq Q^2 \leq 100$ GeV^2	4.5	7.0	7.0	3.4	8
$1 \leq Q^2 \leq 1000$ GeV^2	14.0	7.0	7.0	3.4	10.5
$1 \leq Q^2 \leq 200000$ GeV^2	138.0	7.0	7.0	3.4	20.0

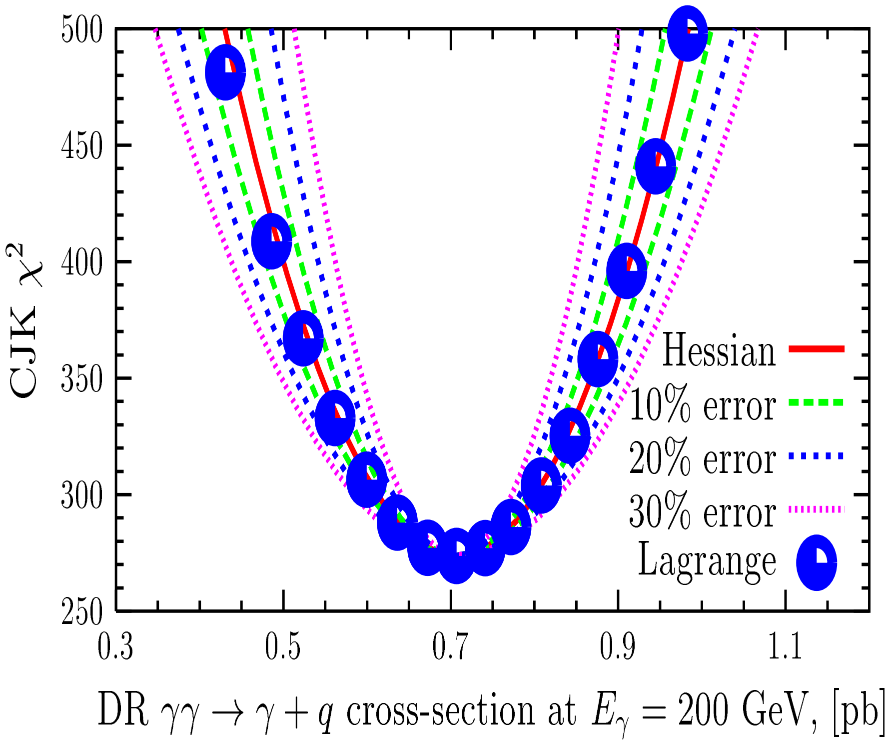
Comparison with Lagrange Multiplier Method

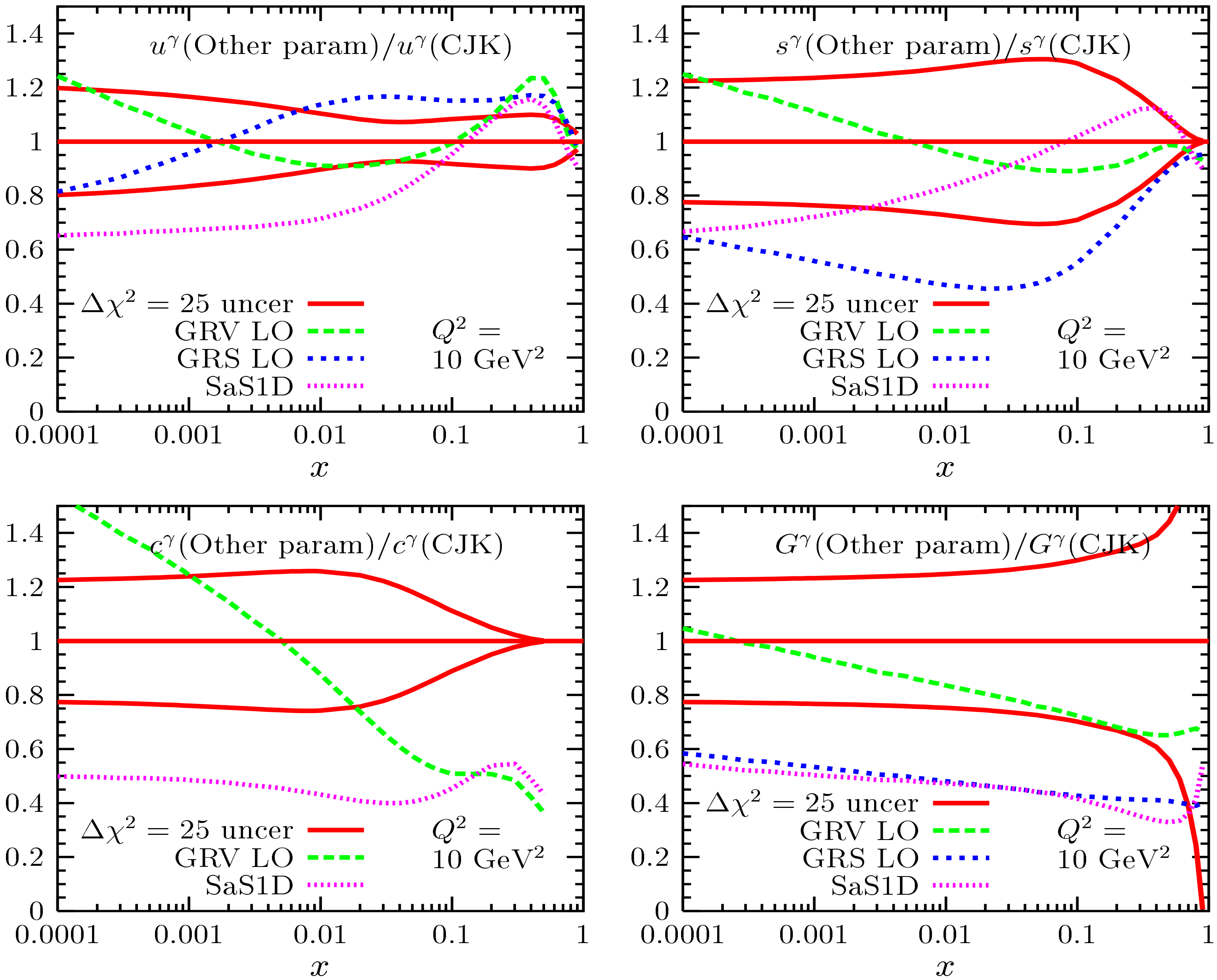
Serious of fits on

$F(\lambda, \{a_i\}) = \chi^2(\{a_i\}) + \lambda X(\{a_i\})$

As result - $(\chi^2(\lambda), X(\lambda))$ points

for various λ





u^γ(Other param)/u^γ(CJK)
s^γ(Other param)/s^γ(CJK)
c^γ(Other param)/c^γ(CJK)
G^γ(Other param)/G^γ(CJK)
$\Delta\chi^2 = 25$ uncer
GRV LO
GRS LO
SaS1D
$Q^2 =$ 10 GeV^2
x

Summary

1. New DGLAP based LO and NLO real photon parton densities were presented
2. Small $Q_0^2 \Rightarrow$ radiatively generated densities
3. New model for the heavy quarks was applied - correct c & b threshold behavior
4. All recent F_2^γ data was applied in the fits
5. No much improvement in the $e^+e^- \to b\bar{b} + X$ description
6. CJK NLO F_2^γ much higher at small-x than in ALS'03
7. Uncertainties of parton densities were examined in detail
8. All parametrizations available at `http://www.fuw.edu.pl/~pjank/param.html`