FCNC Decays of the B Mesons

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B-mesons	$B^+\ (u\bar{b})$	$B^0\ (d\bar{b})$	$\bar{B}^0\ (b\bar{d})$	$B^-\ (b\bar{u})$
		$B^0_s\ (s\bar{b})$	$\bar{B}^0_s\ (b\bar{s})$	
	$B^+_c\ (c\bar{b})$			$B^-_c\ (b\bar{c})$

Flavor Changing Neutral Current (FCNC) decays:

- **Two out of the four (anti)quarks that couple to the W boson close a loop.**
- **The size of the loop (in the B rest frame) is $\ll 1/\Lambda_{QCD}$.**

Any B decay with at least one hadron in the final state receives some contribution from the charged-current interactions.

Examples of the FCNC or FCNC-dominated decays:

- $B_{s(d)} \to l^+l^-$, $B_{s(d)} \to l^+l^-\gamma$, $B_{s(d)} \to \gamma\gamma$.
- $B \to MM$, $B \to MV$, $B \to VV$.
  $M = \pi, K, \eta, \eta', ...,\quad V = \rho, \omega, \phi, ...$
  e.g. $B \to \phi K_S^0$.
- $B \to V\gamma$, $B \to Vl^+l^-$, $B \to Ml^+l^-$,
  $B \to V\nu\bar{\nu}$, $B \to M\nu\bar{\nu}$.
- **Inclusive:** $B \to X_{s(d)}\gamma$, $\quad B \to X_{s(d)}l^+l^-$
  $B \to X_{s(d)}\nu\bar{\nu}$, $B \to X_{s(d)}$.

$B^0_{s(d)} \to l^+l^-$ branching ratios:

$$\mathrm{BR}(B^0_q \to l^+l^-) = \frac{\tau(B^0)}{\pi} M_{B^0_q} \left(\frac{G_F \alpha_{\mathrm{em}} \hat{F}_{B_q} \mathbf{m_l}}{4\pi \sin^2\theta_W} \right)^2 \sqrt{1 - \frac{4m_l^2}{M^2_{B^0_q}}} \left| V^*_{tq} V_{tb} Y(x_t) \right|^2$$

Mode	SM expectation (leading EW)	Experimental bound (90% CL)
$B_s \to \tau^+\tau^-$	7.8×10^{-7}	$\sim 5\%$ **from LEP? hep-ph/9607473**
$B_s \to \mu^+\mu^-$	3.6×10^{-9}	9.5×10^{-7} **CDF, hep-ex/0402020**
$B_s \to e^+e^-$	8.6×10^{-14}	5.4×10^{-5} **PDG 2002**
$B_d \to \tau^+\tau^-$	2.9×10^{-8}	$\sim 1.5\%$ **from LEP? hep-ph/9607473**
$B_d \to \mu^+\mu^-$	1.4×10^{-10}	1.6×10^{-7} **Belle, hep-ex/0309069**
$B_d \to e^+e^-$	3.3×10^{-15}	1.9×10^{-7} **Belle, hep-ex/0309069**

The present bound on $B_s \to \mu^+\mu^-$ already gives constrains on SUSY models with large $tan\beta$ where intermediate (pseudo)scalar contributions become relevant.

The CP asymmetry in $B \to \phi K_S^0$

$$\Upsilon(4S) \to (\bar{B}_d^0 B_d^0) \begin{array}{l} \nearrow \quad \phi K_S^0 \ \textbf{ at time } t_0 \\ \searrow \quad F_{\rm tag} \ \textbf{ at time } t_{\rm tag} \end{array}$$

The decay rate:

$$\mathcal{P}(\Delta t) = \frac{e^{-|\Delta t|/\tau_B}}{4\tau_B} \{1 + q\,[S \sin(\Delta M \Delta t) + C \cos(\Delta M \Delta t)]\},$$

where
$\Delta t = t_0 - t_{\rm tag}$
$\Delta M =$ **(mass difference between the two B^0 mass eigenstates),**
$\tau_B =$ **(B-meson lifetime)**
$q = +1(-1)$ **when $F_{\rm tag}$ looks like coming from B^0 ($\bar{B}^0$) in the absence of $B^0\bar{B}^0$ mixing.**

SM expectations for the parameters S and C:

$S = sin(2\beta) = +0.731 \pm 0.056$ **as measured in $B^0 \to \psi K_s^0$**

$C \simeq 0$ **(very small direct CP violation in the $b \to s s \bar{s}$ amplitude)**

Measurements:

$$S = \begin{cases} +0.47 \pm 0.34(stat)^{+0.08}_{-0.06}(syst) & \textbf{BaBar, hep-ex/0403026} \\ -0.96 \pm 0.50(stat)^{+0.09}_{-0.11}(syst) & \textbf{Belle, hep-ex/0308035} \end{cases}$$

$$C = \begin{cases} +0.01 \pm 0.33(stat) \pm 0.10(syst) & \textbf{BaBar, hep-ex/0403026} \\ -0.15 \pm 0.29(stat) \pm 0.07(syst) & \textbf{Belle, hep-ex/0308035} \end{cases}$$

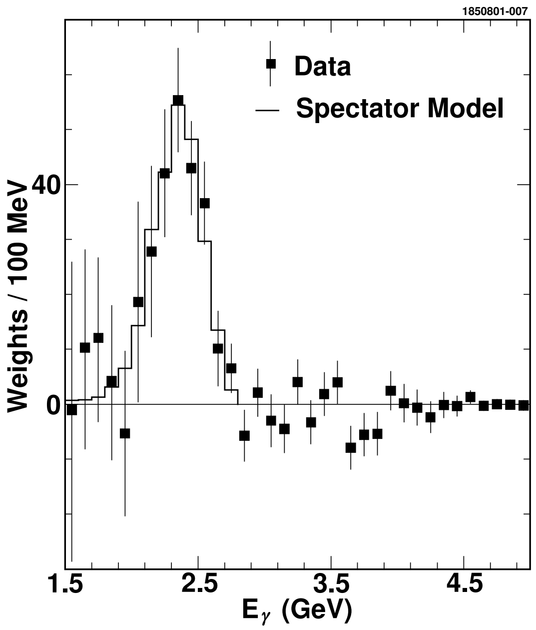
The Belle result for S is $3.5\ \sigma$ away from the SM.

The measurement of $\bar{B} \to X_s \gamma$

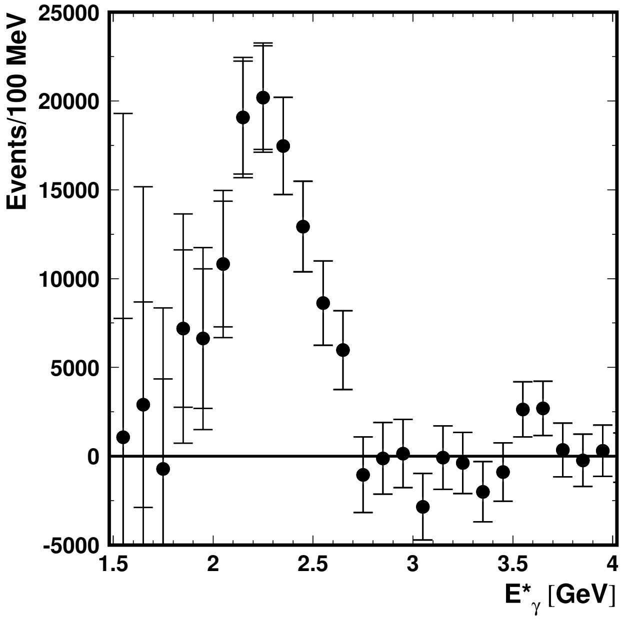
Signature:
High-energy photon in the $\bar{B}$-meson rest frame.
$E_\gamma \sim m_b/2$, **as in the two-body** $b \to s\gamma$ **decay.**
Smearing due to:

- **motion of b inside $\bar{B}$**
- **gluon Bremsstrahlung**

Measured photon energy spectra:



CLEO
hep-ex/0108032
PRL 87 (2001) 251807



BELLE
hep-ex/0403004

World average (CLEO, ALEPH, BELLE, BABAR):

$$\mathrm{BR}[\bar{B} \to X_s\gamma] = (3.42 \pm 0.30) \times 10^{-4}$$

Convention: $E_\gamma > \frac{m_b}{20}$, perturbative spectrum used for very low E_γ.

Basic approximations:

$$\Gamma[\bar{B} \to X_s \gamma] \simeq \Gamma[b \to X_s^{\rm parton} \gamma]$$
$$\equiv \Gamma[b \to s\gamma] \; + \; \Gamma[b \to s\gamma g] \; + \; \ldots$$

$$\Gamma[\bar{B} \to X_s l^+ l^-] \simeq \Gamma[b \to X_s^{\rm parton} l^+ l^-]$$
$$\equiv \Gamma[b \to s l^+ l^-] \; + \; \Gamma[b \to s l^+ l^- g] \; +$$

For appropriately chosen kinematical cuts, non-perturbative corrections to the r.h.s.'s are small ($< 5\%$). Some of them are calculable and known:

See e.g.

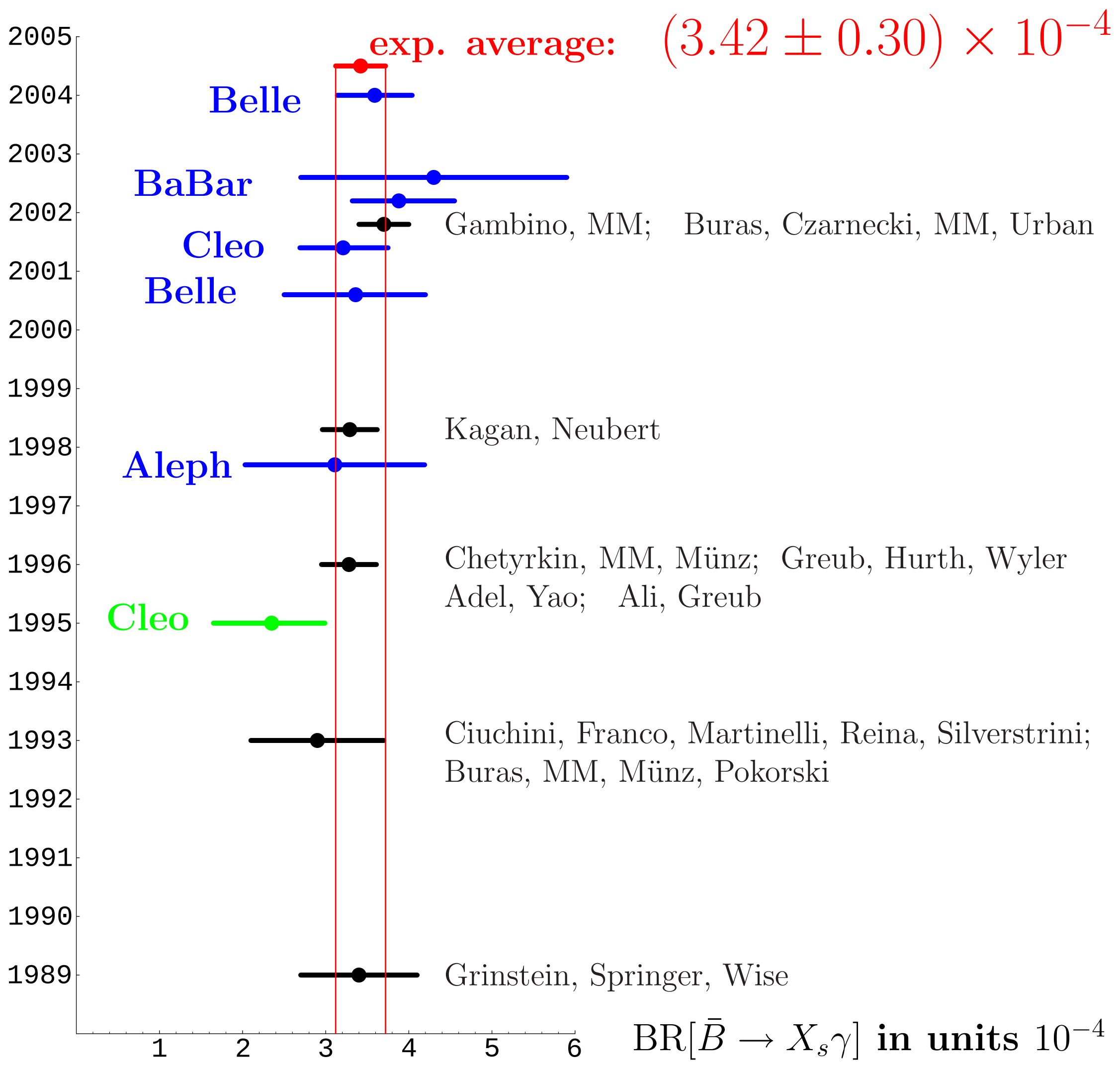
[A.F. Falk, M.E. Luke and M.J. Savage,
Phys. Rev. D 49 (1994) 3367, hep-ph/9308288].

[G. Buchalla, G. Isidori and S.J. Rey,
Nucl. Phys. B 511 (1998) 594, hep-ph/9705253].

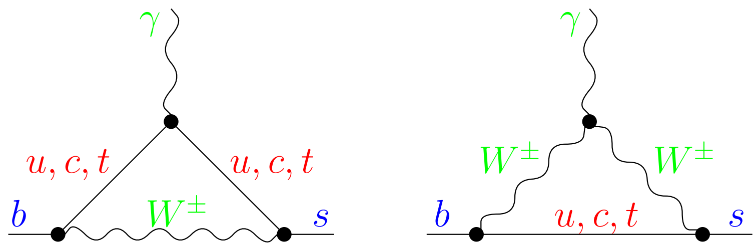
$\mathrm{BR}[\bar{B} \to X_s \gamma]$

Measurements and the SM calculations

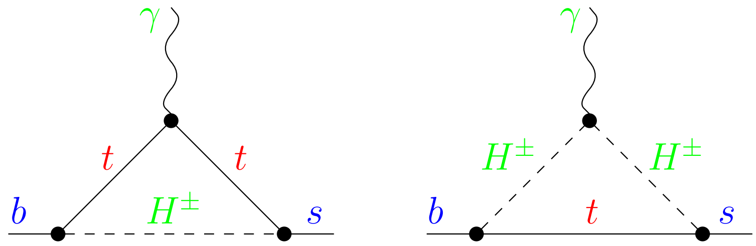
(some of them)



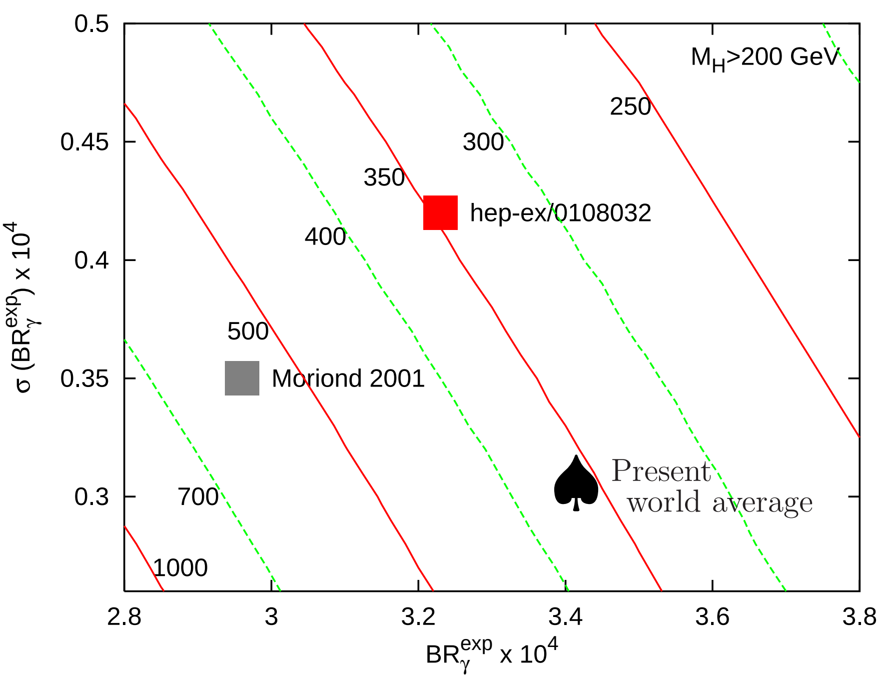
Leading SM diagrams for $b \to s\gamma$:



Example of a non-SM contribution to $b \to s\gamma$:

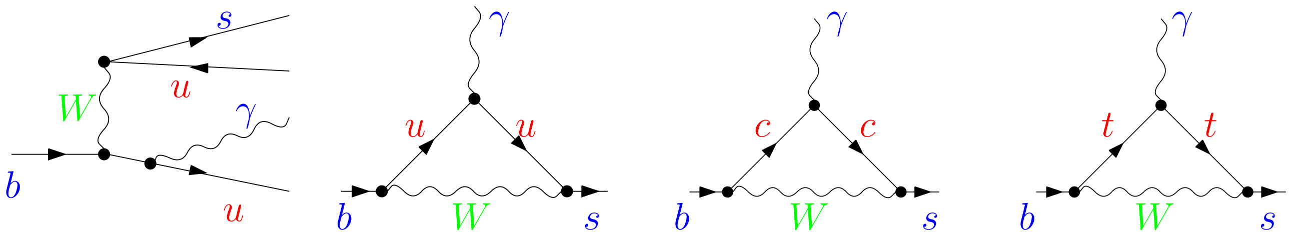


$\bar{B} \to X_s \gamma$ constraints on the charged scalar mass in the Two-Higgs-Doublet Model (II):



$$\mathrm{BR}[\bar{B} \to X_s l^+ l^-] \quad \leftrightarrow \quad \mathrm{sgn}\ \mathrm{A}[\bar{B} \to X_s \gamma]$$

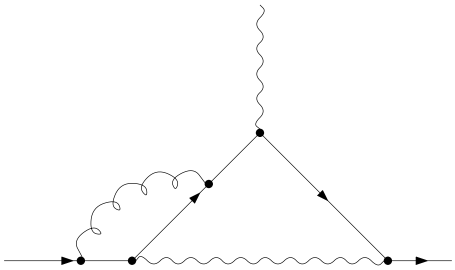
Examples of the leading electroweak diagrams for $\bar{B} \to X_s \gamma$:



$$\left|\frac{V_{ub}V_{us}}{V_{cb}}\right| \simeq \left|\frac{V_{ub}V_{us}}{V_{ts}}\right| \simeq 2\%$$

$$\underbrace{\sim +200\% \qquad\qquad \sim -100\%}$$

In the amplitude, after including LO QCD effects.



QCD logarithms $\alpha_s \ln \frac{M_W^2}{m_b^2}$ enhance BR$[\bar{B} \to X_s\gamma]$ more than twice.

Charm amplitude receives very small logarithmic corrections ($\sim 1\%$).
Top amplitude gets suppressed by around 30% $\leftrightarrow$ evolution of $m_b(\mu)$.

An effective field theory method is the most convenient for resummation of such large logarithms.

The effective Lagrangian:

$$\underset{(q=u,d,s,c,b,\;\; l=e,\mu)}{\mathcal{L} \;=\; \mathcal{L}_{QCD\times QED}(q,l)} \;+\; \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) O_i$$

$$O_i = \begin{cases} (\bar{s}\Gamma_i c)(\bar{c}\Gamma_i' b), & i=1,2, & |C_i(m_b)| \sim 1 \\ (\bar{s}\Gamma_i b)\mathbf{\Sigma}_q(\bar{q}\Gamma_i' q), & i=3,4,5,6, & |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}b_R F_{\mu\nu}, & i=7, & C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar{s}_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i=8, & C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar{s}_L\gamma_\mu b_L)(\bar{l}\gamma^\mu\gamma_5 l), & i=9,\mathbf{10} & |C_i(m_b)| \sim 4 \end{cases}$$

Three steps of the calculation:

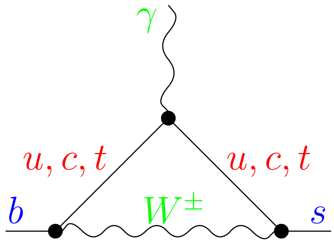
Matching: Evaluating $C_i(\mu_0)$ at $\mu_0 \sim M_W$ by requiring equality of the SM and the effective theory Green functions.

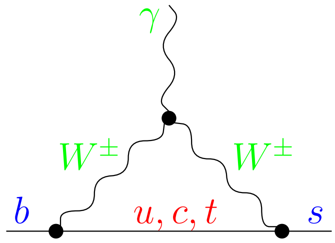
Mixing: Deriving the effective theory RGE and evolving $C_i(\mu)$ from μ_0 to $\mu_b \sim m_b$.

Matrix elements: Evaluating the on-shell amplitudes at $\mu_b \sim m_b$.

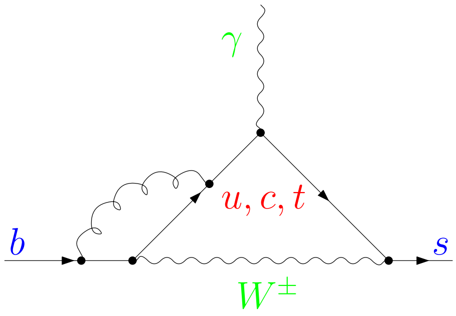
Examples of SM diagrams for the matching of $C_7(\mu_0)$

LO:
[Inami, Lim, 1981]

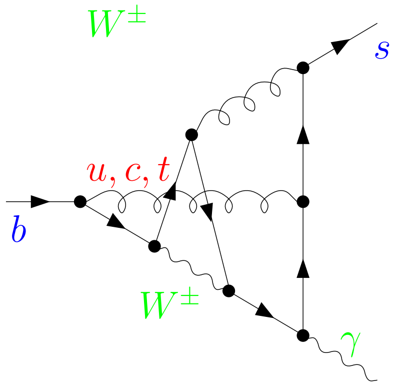




NLO:
[Adel, Yao, 1993]



NNLO:
[Steinhauser, MM, 2004]



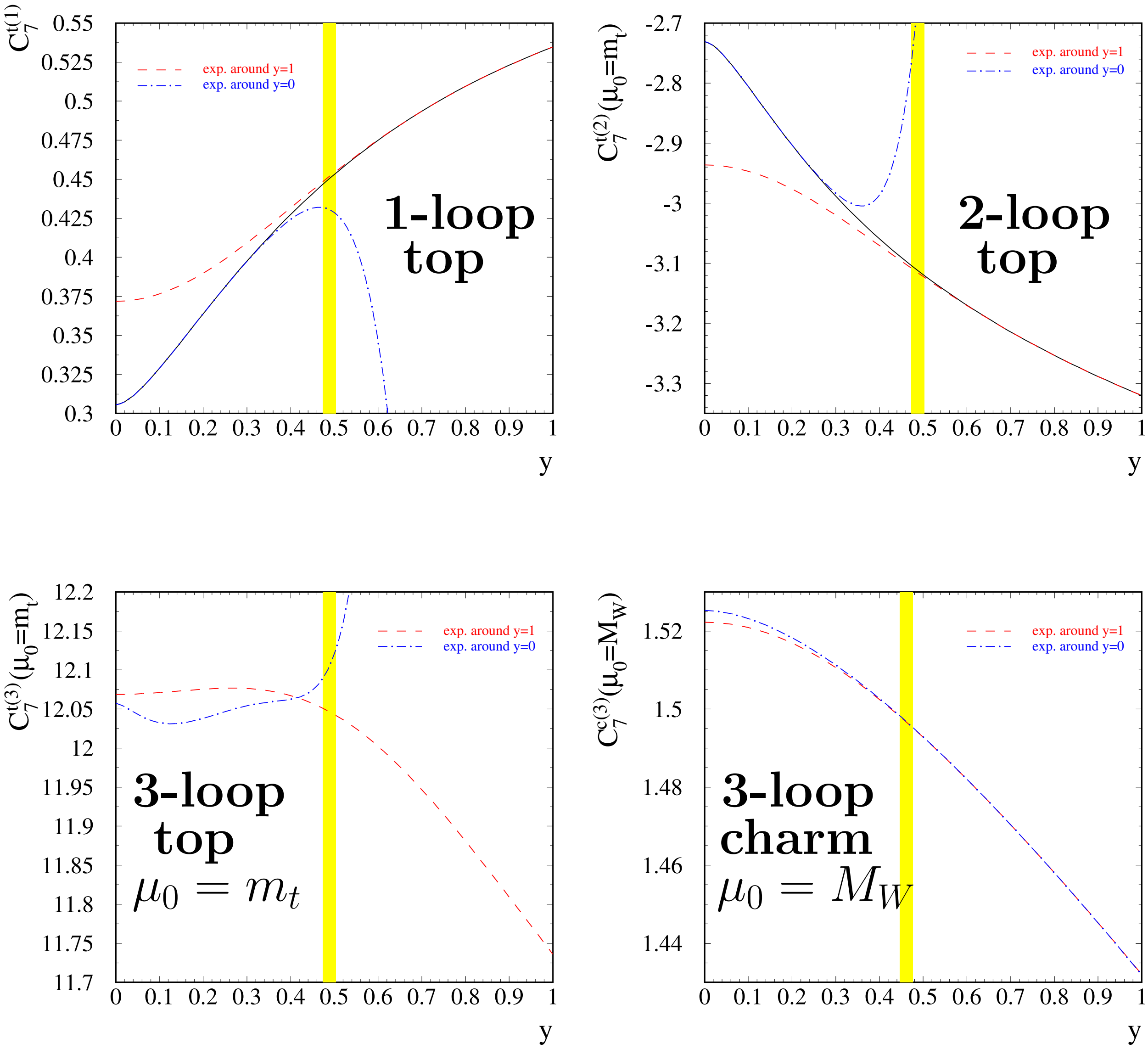
NNLO method:

- Taylor expansion in off-shell external momenta is applied before integration.
- The UV and spurious IR divergences are regulated dimensionally.
- $\Rightarrow$ In the effective theory, only tree-level diagrams survive (tree vertices and UV counterterms). The UV renormalization constants are known from former anomalous-dimension calculations.
- All the $1/\epsilon$ poles cancel in the matching equation, i.e. in the difference between the effective theory and the full SM Green functions.
- At the 3-loop level, the difference $m_t - M_W$ is taken into account with the help of expansions in y^n and $(1-y^2)^n$ up to $n = 8$, where $y = m_t/M_W$.

The “flavour-split” matching condition:

$$V_{ts}^* V_{tb} C_7(\mu_0) \equiv V_{ts}^* V_{tb} C_7^t(\mu_0) + \left(V_{us}^* V_{ub} + V_{cs}^* V_{cb}\right) C_7^c(\mu_0)$$

The coefficients $C_7^{Q(n)}(\mu_0)$ as functions of $y = \frac{M_W}{m_t(\mu_0)}$:



Resummation of large logarithms $\left(\alpha_s \ln \frac{M_W^2}{m_b^2}\right)^n$ in the $b \to s\gamma$ amplitude.

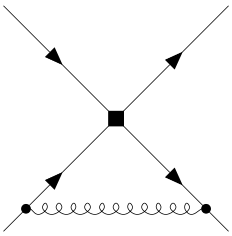
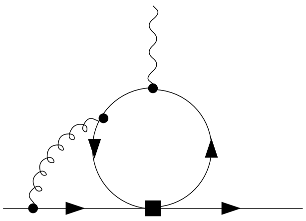
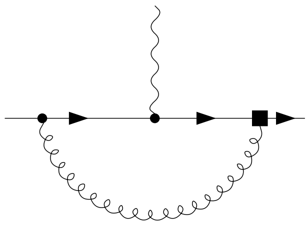
RGE for the Wilson coefficients:

$$\mu \frac{d}{d\mu} C_j(\mu) = C_i(\mu) \gamma_{ij}(\mu)$$

The anomalous dimension matrix γ_{ij} is found from eff.-theory renormalization constants, e.g.:

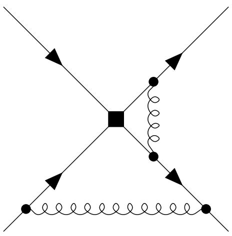
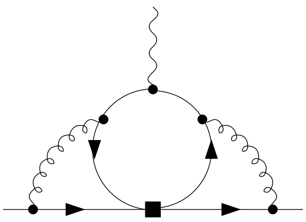
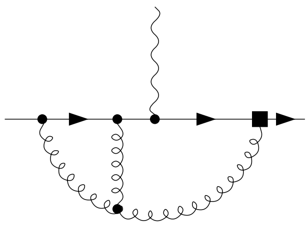
	Z_{22}	Z_{27}	Z_{87}

LO

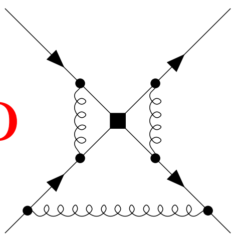
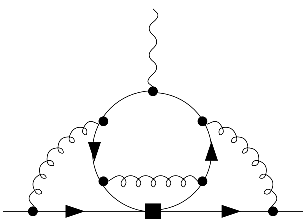
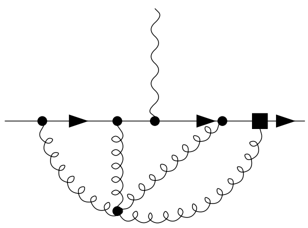
[Gaillard, Lee, 1974] [Altarelli, Maiani, 1974] — [Grinstein *et al.*, 1990] — [Shifman *et al.*, 1978] [Grigjanis *et al.*, 1988]

NLO

[Altarelli *et al.*, 1981] [Buras, Weisz, 1990] — [Chetyrkin *et al.*, 1997] — [MM, Münz, 1995]

NNLO

[Gambino *et al.*, 2004] (unpublished yet) — in progress: Czakon, Haisch, MM, Schröder. — in progress: Gambino, Gorbahn, Haisch.

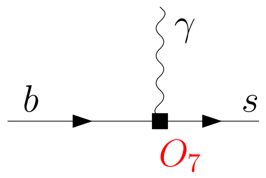
The most efficient method for the renormalization constant calculation:

- Before loop integration, one performs the Taylor expansion in the off-shell external momenta and in all the particle masses.
- A common mass m^2 is inserted into all the propagator denominators (including the gluon one) as a regulator of spurious IR divergences.
- $\Rightarrow$ Gauge invariance is broken. Unusual UV subdivergences need to be subtracted. However, after the subtraction, the MS-scheme UV counterterms are polynomial in the external momenta and masses, including the regulator mass m^2.
  The m^2-independent terms give the renormalization constants of the original theory.
- $\Rightarrow$ The only Feynman integrals to compute are single-scale massive tadpoles, for which algorithms up to 4 loops exist.

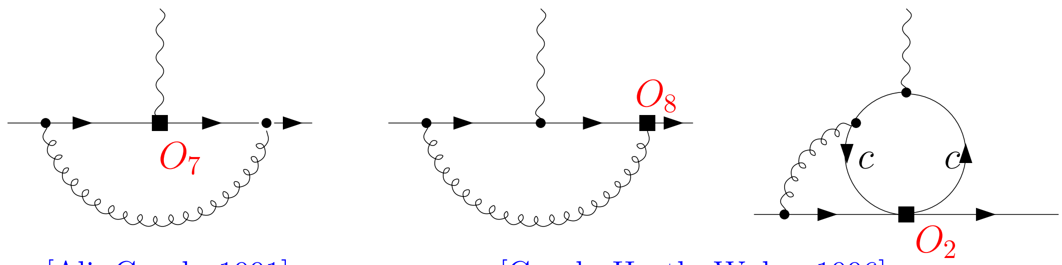
The $b \to s\gamma$ matrix elements.

Perturbative on-shell amplitudes.

LO



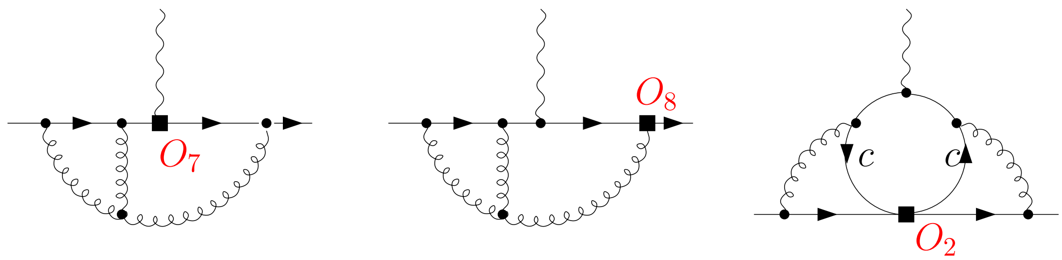
NLO



[Ali, Greub, 1991]

[Greub, Hurth, Wyler, 1996]

NNLO



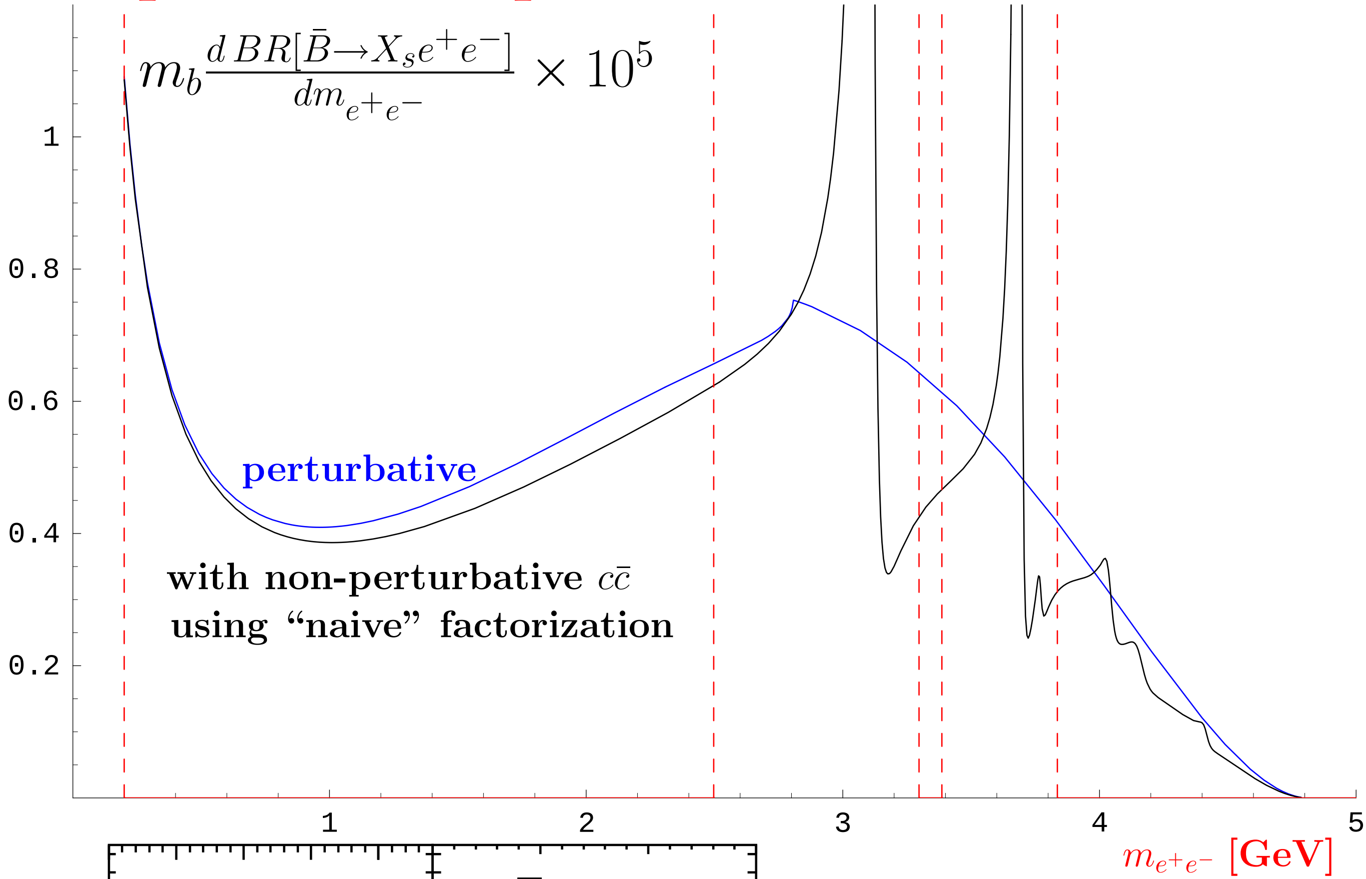
in progress: Asatrian, Greub, Hurth

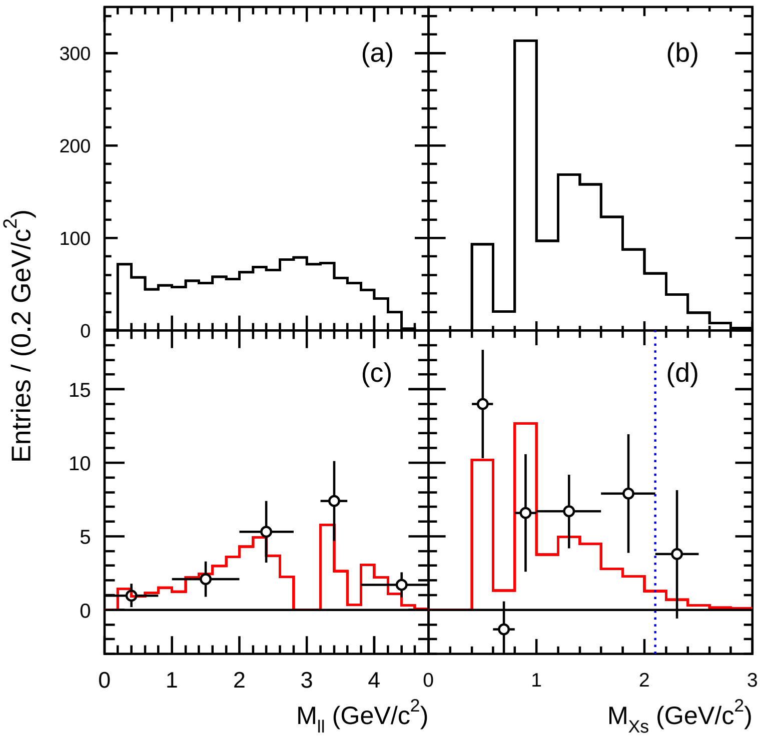
[Bieri *et al*, 2003] ($\mathcal{O}(\alpha_s^2 n_f)$)
in progress: Steinhauser, MM
(extrapolation in m_c?)

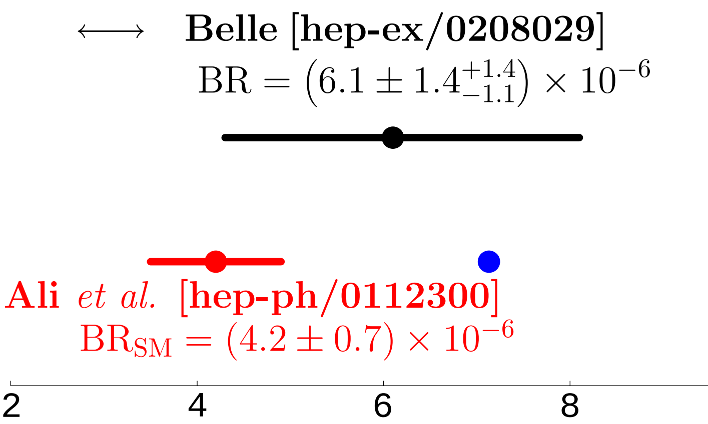
Beyond one loop:

- **Mellin-Barnes transform of Feynman-parameter integrands – expansion in m_c/m_b.**
- **"Standard" asymptotic expansions in m_c/m_b.**
- **Reduction to a (very) small number of Feynman-parameter integrals to be evaluated numerically.**
- **Reduction with the help recurrence relations (IBP).**
- **3 loops: extrapolation from large m_c ?**

Dilepton mass spectrum in $\bar{B} \to X_s e^+ e^-$.







$\bar{B} \to X_s l^+ l^-$

The (recently completed) NNLO calculation of $\bar{B} \to X_s l^+ l^-$ corresponds to the (completed) NLO calculation of $\bar{B} \to X_s \gamma$, as far as the number of loops in the diagrams is concerned.

Coefficients of the two additional operators

$$O_i = \frac{e^2}{16\pi^2}(\bar{s}_L \gamma_\mu b_L)(\bar{l}\gamma^\mu \gamma_5 l), \qquad i = 9, \mathbf{10}$$

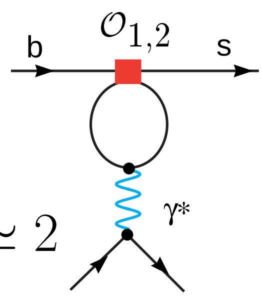
have the following perturbative expansion:

$$C_9(\mu) = \frac{4\pi}{\alpha_s(\mu)} C_9^{(-1)}(\mu) + C_9^{(0)}(\mu) + \frac{\alpha_s(\mu)}{4\pi} C_9^{(1)}(\mu) + \ldots$$

$$C_{10} = C_{10}^{(0)} + \frac{\alpha_s(M_W)}{4\pi} C_{10}^{(1)} + \ldots$$

After a formal expansion in α_s, the term $C_9^{(-1)}(\mu)$ reproduces (the dominant part of) the electroweak logarithm that originates from photonic penguins with charm quark loops:

$$\frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) = \frac{4}{9} \ln \frac{M_W^2}{m_b^2} + \mathcal{O}(\alpha_s)$$



$$C_9^{(-1)}(m_b) \simeq 0.033 \ll 1 \Rightarrow \frac{4\pi}{\alpha_s(m_b)} C_9^{(-1)}(m_b) \simeq 2$$

On the other hand: $\quad C_9^{(0)}(m_b) \simeq 2.2.$

$\Rightarrow$ **NNLO $\mathcal{O}(\alpha_s)$ corrections are necessary for $\sim 10\%$ accuracy in BR$[\bar{B} \to X_s l^+ l^-]$.**

$\Rightarrow$ **Two-loop matching, three-loop mixing and two-loop matrix elements are necessary.**

The NNLO contributions to $\bar{B} \to X_s l^+ l^-$ that had not been known from the NLO $\bar{B} \to X_s \gamma$ calculations were found by:

- **Matching:** [Bobeth, MM, Urban, 1999]
- **Mixing:** [Gambino, Gorbahn, Haisch, 2003]
- **Matrix elements:**
  [Asatrian, Asatryan, Bieri, Greub, Hovhannissyan, Walker, 2001]
  [Ghinculov, Hurth, Isidori, Yao, 2002]
  [Bobeth, Gambino, Gorbahn, Haisch, 2003]

The latter paper (hep-ph/0312090) contains the most recent phenomenological analysis, with the result:

$$BR[\bar{B} \to X_s l^+ l^-] = (1.702 \pm 0.160) \times 10^{-6}$$

for the dilepton mass $q^2 \in [1\ \mathrm{GeV}^2, 6\ \mathrm{GeV}^2]$**.**

Summary

There are two rare B decays, for which performing perturbative Next-to-Next-to-Leading-Order (NNLO) QCD calculations is necessary:

$$\bar{B} \to X_s \gamma \text{ and } \bar{B} \to X_s l^+ l^-.$$

Reasons:

- **Accurate measurements**
  $\bar{B} \to X_s \gamma$: **present** $\pm 10\%$, **future** $\sim \pm\ 5\%?$
  $\bar{B} \to X_s l^+ l^-$: **present** $\pm 30\%$, **future** $\sim \pm 10\%?$
- **Non-perturbative effects under control**
- **Sensitivity to new physics**

Status of the NNLO calculations:

- $\bar{B} \to X_s l^+ l^-$: **100% completed** $(\pm 20\% \to \pm 10\%)$
- $\bar{B} \to X_s \gamma$: $\sim \frac{1}{3}$ **way through** $(\pm 10\% \to \sim \pm 5\%$**?)**
  Until it is finished, we have at the NLO:
  $\mathrm{BR}[\bar{B} \to X_s \gamma,\ E_\gamma > 1.6\ \mathrm{GeV}] = (3.57 \pm 0.30) \times 10^{-4}$.