Higgs mechanism with two doublets -

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CP, FCNC, Z_2-symmetry, rephasing invariance.. - A pedagogical lecture

- Standard Model
- 2 Higgs Doublet Model

Higgs mechanism in SM(s)

Spontaneous electroweak symmetry breaking of SU(2)x U(1) (EWSB) via the Higgs mechanism $\Rightarrow$ the Lagrangian

$$\mathcal{L} = \mathcal{L}^{SM}_{\text{gauge+fermions}} + \mathcal{L}_H + \mathcal{L}_Y \,.$$

$\mathcal{L}^{SM}_{\text{gauge+fermions}}$ - $SU(2) \times U(1)$ Standard Model interaction of gauge bosons and fermions

$\mathcal{L}_H$ - Higgs scalar Lagrangian

$\mathcal{L}_Y$ - Yukawa interactions of fermions with scalars.

Standard Model (SM) = 1HDM

Standard Model (SM) - one single scalar isodoublet SU(2) ϕ with weak hypercharge $Y = 1$

$$Q = I_{weak,3} + \frac{Y_{weak}}{2}$$

two complex fields with charged 1, and 0.

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \tag{1}$$

(Note also $\tilde{\phi} = i\tau_2(\phi^\dagger)^T$ with $Y = -1$).

$$I_{weak,3} = \pm 1/2$$

Higgs potential and vacuum

Higgs Lagrangian:

$$\mathcal{L}_H = (D_\mu \phi)^\dagger D_\mu \phi - V$$

with the Higgs potential

$$V = \lambda(\phi\phi^\dagger)^2/2 - m^2\phi\phi^\dagger/2$$

(both λ and $m^2 > 0$ are real - hermiticity of $\mathcal{L}$).

Minimum (in fact only extremum):

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=\langle\phi\rangle} = 0, \qquad \langle\phi\rangle = v/\sqrt{2} = \sqrt{m^2/2\lambda}$$

Physical fields (with definite masses)

The standard decomposition in physical fields

$$\phi = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}(v + \eta + i\chi) \end{pmatrix},$$

with the vacuum

$$\phi_{vac} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix},$$

since this spontaneous symmetry breaking $SU(2)_{weak\,I} \times U(1)_{weak\,Y}$ to $U(1)_{QED}$

$$\hat{Q}\phi_{vac} = 0, \quad \text{so } e^{i\alpha\hat{Q}}\phi_{vac} = \phi_{vac}$$

- invariance !

Physical content

$\varphi^{\pm}$ related to long. components of $W^{\pm}$

χ - related to long. component of Z

η - Higgs field

Put into gauge-kinetic Langrangian,

$$M_W = vg/\sqrt{2}, \quad M_Z = vg/\sqrt{2}c_W$$

or

$$\rho = M_W^2/(M_Z^2 c_W^2) = 1$$

We known value of v knowing Fermi constant G_F

$$v = 246 GeV$$

Mass of Higgs field: given by λ or m^2 (since v (known) relates them)

Couplings of Higgs boson to gauge bosons and selfcouplings

Couplings of Higgs boson to gauge bosons:

$g_W^{\mathsf{SM}} = \sqrt{2}M_W/v$, $g_Z^{\mathsf{SM}} = \sqrt{2}M_Z/v$,

Selfcouplings of Higgs bosons (related to λ term in V)

trilinear and quartic couplings between physical Higgses

Couplings to fermions (Yukawa interaction)

Yukawa term has a form (one generation of quarks for simplicity):
$\mathcal{L}_Y = -(\bar{Q_L}\Gamma\phi n_R + \bar{Q_L}\Delta\tilde{\phi}p_R + h.c.)$

Γ and Δ - arbitrary complex numbers (matrix for more generations)

$$Q_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix}$$

for quark doublet with chirality -1 (hypercharge - ?) and for singlets p_R, n_R - chirality 1.

From this mass terms: -$\bar{n}_L M_n n_R$, etc

$$M_n = v\Gamma$$

Also Yukawa couplings: with $g_f^{\mathrm{SM}} = \sqrt{2}m_f/v$.

Generations

3 generations: Γ and Δ - 3 x 3 matrices.
n principle - arbitrary (flavour problem)

One can diagonalize M_p, M_u by transforming fields fields p, n to u, d - physical quark fields with definite masses.

The charged-current interaction:

$$\mathcal{L}_W^q = g/\sqrt{2}(W_\mu^+ \bar{u_L} \gamma^\mu V d_L + W_\mu^- ...)$$

$$V = U_L^{p\dagger} U_L^n$$

K-M matrix (Cabibbo 63, Kobayashi-Maskawa 73)
For neutral-current - substitute p, n by u, d - no mixing matrix V !!

No FLAVOUR-CHANGING NEUTRAL CURRENT (FCNC) in SM at three level. This is due to fact: all fermions of given charge and helicity have the same isospin I_3, all fermion have one source of mass - Glashow, Weinberg 77 and Paschos 77

2HDM $\Rightarrow$ two Higgs SU(2) doublets

To keep $\rho = M_W^2/(M_Z^2 \cos^2\theta_W)$ equal to 1 at the tree level, one assumes in 2HDM that both scalar fields (ϕ_1 and ϕ_2) are weak isodoublets ($T = 1/2$) with hypercharges $Y = \pm 1$ We use $Y = +1$ for both of them (the other choice, $Y_1 = 1$, $Y_2 = -1$, is used often in the MSSM).

The most general renormalizable Higgs Lagrangian:

$$\mathcal{L}_H = T - V\,, \tag{2}$$

T is the kinetic term (with D_μ being the covariant derivative containing the EW gauge fields), V is the Higgs potential :

$$T = (D_\mu\phi_1)^\dagger(D^\mu\phi_1) + (D_\mu\phi_2)^\dagger(D^\mu\phi_2) + \varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2) + \varkappa^*(D_\mu\phi_2)^\dagger(D^\mu\phi_1)\,,$$

NEW TERMS possible !! - mixing kinetic terms?!

Mind the mixing terms and a symmetry ($\phi_1 - \phi_2$) (Z_2-symmetry)!!

Potential quartic terms

$$V = \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

$$+\frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\} + \tilde{\mathcal{M}},$$

mixing terms possible of two types, Z_2-symmetry - ?

Potential -mass terms

$$\tilde{\mathcal{M}} = -\frac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + \left[m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right] + m_{22}^2(\phi_2^\dagger\phi_2)\right\} \; .$$

$\tilde{\mathcal{M}}$ is a mass term in the potential
λ_{1-4}, m_{11}^2 and m_{22}^2 are real (by hermiticity of the potential)
λ_{5-7} m_{12} and $\varkappa$ are in general complex parameters (in total there are 16 ndependent parameters)

Two spin zero doublets ϕ_1 and ϕ_2
with vacuum exp. values v_1 and v_2

(with $v^2 = v_1^2 + v_2^2$, often we use $\tan\beta = v_2/v_1$)

Counting degrees of freedom: 8 fields - 3 (for long.components of W/Z)
= 5

$\Rightarrow$ five physical Higgs bosons, spin 0, 3 -el. neutral, two charged !!

Rephasing invariance

The Lagrangian $\mathcal{L}_H$ invariant under global phase transformation of both fields with a common phase, $\phi_i \to \phi_i e^{-i\rho_0}$ (*an overall phase freedom*). Besides, the same physical reality (the same values of observables) if independent phase rotations of each field $\phi_i \to \phi_i e^{-i\rho_i}$ together with the compensating rotation by the phase difference $\rho = \rho_2 - \rho_1$ of *the complex* parameters of the Lagrangian.

We call this property *the rephasing invariance* in the space of Lagrangians, with coordinates given by λ's, m_{ij}^2, $\varkappa$.

Rephasing invariant quantities

Physical picture does not change with the change of Lagrangian under the global *rephasing transformation* of the form:

$$\phi_i \to e^{-i\rho_i}\phi_i, \quad \rho_i \text{ real} \quad (i=1,2), \quad \rho_0 = \frac{\rho_1+\rho_2}{2}, \ \rho = \rho_2 - \rho_1, \tag{3a}$$

accompanied by the following transformation of the parameters of the Lagrangian:

$$\begin{gathered}\lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{11(22)}^2 \to m_{11(22)}^2\,,\\ \lambda_5 \to \lambda_5\, e^{2i\rho}\,, \ \lambda_{6,7} \to \lambda_{6,7}\, e^{i\rho}\,, \ \varkappa \to \varkappa\, e^{i\rho}\,, \ m_{12}^2 \to m_{12}^2 e^{i\rho}\,.\end{gathered} \tag{3b}$$

A set of these physically equivalent Higgs Lagrangians - *rephasing equivalent family*.

This one-parametric family is governed by the phase difference ρ - the rephasing gauge parameter.

The specific choice of this rephasing gauge parameter leads to the specific *rephasing representation*.

Rephasing invariance - cd

Rephasing invariance can be extended to whole system of scalars and fermions if the corresponding transformations for the Yukawa terms (phases of fermion fields and Yukawa couplings) supplement the transformations.

Note that if the CP is conserved in the Higgs sector, all parameters of Lagrangian can be made real.

Obviously, by transforming such Lagrangian, with $\rho \neq 0$, some λ's, etc. can become complex (however with a fixed relation of their phases).

Lagrangian and Z_2 symmetry

$\boldsymbol{Z_2}$ symmetry.
We consider first the case with Z_2 symmetry which forbids a (ϕ_1, ϕ_2) mixing in $\mathcal{L}$. This case is described by the Lagrangian $\mathcal{L}_H$ with $\lambda_6 = \lambda_7 = \varkappa = m_{12} = 0$. Using the rephasing invariance, one can make λ_5 real. As it was mention above, it means that in such case the Higgs sector respects CP symmetry. The radiative (loop) corrections does not destroy the Z_2 invariance.

The general Lagrangian $\mathcal{L}_H$ violates Z_2 symmetry (allowing for a (ϕ_1, ϕ_2) mixing) by terms of the operator dimension 2 (with m_{12}), what is called *a soft violation of the Z_2 symmetry*, and of the operator dimension 4 (with $\lambda_{6,7}$ and $\varkappa$), called *a hard violation of the Z_2 symmetry*.

A soft violation of $\boldsymbol{Z_2}$ symmetry.
One adds to the Z_2 symmetric Lagrangian the term $m_{12}^2(\phi_1^\dagger\phi_2) + h.c.$, with a generally complex m_{12}^2 (and λ_5) parameters.

A hard violation of Z_2.
Terms of the operator dimension 4, with generally complex parameters λ_6, λ_7 and $\varkappa$, are added to the Lagrangian with a softly broken Z_2 symmetry.

Such treatment of the case with hard violation of Z_2 symmetry is as incomplete in most of papers considering this most general 2HDM potential.

The Vacuum and Specific Choice of the Potential

The minimum of the potential:

$$\left.\frac{\partial V}{\partial \phi_1}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0, \qquad \left.\frac{\partial V}{\partial \phi_2}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0. \tag{4}$$

In order to describe the $U(1)$ symmetry of electromagnetism and using the over-all phase freedom of the Lagrangian to choose one vacuum expectation value real we take:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \tag{5}$$

with a relative phase ξ.

NOTE: THIS IS JUST ASSUMPTION.

Another parameterization:

$$v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad \beta \in \left(0, \frac{\pi}{2}\right). \tag{6}$$

The phase difference ξ between the v.e.v.'s is often interpreted as spontaneous CP violation of vacuum. However, this is not necessarily the case !!!

Note that under the rephasing, the ξ changes to:

$$\xi \to \xi - \rho\,. \tag{7}$$

The following quantities

$$\overline{\lambda}_{1-4} = \lambda_{1-4}, \quad \overline{\lambda}_5 \equiv \lambda_5 e^{2i\xi}, \quad \overline{\lambda}_6 \equiv \lambda_6 e^{i\xi}, \quad \overline{\lambda}_7 \equiv \lambda_7 e^{i\xi}, \quad \overline{\varkappa} \equiv \varkappa e^{i\xi}, \quad \overline{m}^2_{12} \equiv m^2_{12} e \tag{8}$$

are the rephasing-invariant quantities

The standard decomposition of the fields ϕ_i in terms of physical fields is made via

$$\phi_1 = \begin{pmatrix} \varphi_1^+ \\ \dfrac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \varphi_2^+ \\ \dfrac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix}. \qquad (9)$$

At $\varkappa = 0$ this decomposition leads to a diagonal form of kinetic terms for new fields φ_i^+, χ_i, η_i, while the corresponding mass matrix become off-diagonal.

The mass squared matrix can be transformed to the block diagonal form by a separation of the massless Goldstone boson fields, $G^0 = \cos\beta\,\chi_1 + \sin\beta\,\chi_2$ and $G^\pm = \cos\beta\,\varphi_1^\pm + \sin\beta\,\varphi_2^\pm$, and charged Higgs boson fields $H^\pm$, the combinations orthogonal to $G^\pm$

$$H^\pm = -\sin\beta\,\varphi_1^\pm + \cos\beta\,\varphi_2^\pm\,, \qquad (10)$$

with their mass squared equal to

$$M_{H^\pm}^2 = v^2\left[\nu - \frac{1}{2}\mathrm{Re}(\lambda_4 + \bar{\lambda}_5 + \bar{\lambda}_{67})\right]. \qquad (11)$$

more on 2HDM

nteraction with gauge bosons and fermions- Higgs bosons share obligations

$$(g_W^h)^2 + (g_W^H)^2 + (g_W^A)^2 = (g_W^{H_{SM}})^2$$

Various models of Yukawa interaction with fermions:
eg Model II where one scalar doublet couples to up-type quarks, other to down-type quarks and charged leptons

The potential problems:

•Flavour Changing Neutral Current may be large (in nature FCNC small)

•CP-violation may be large (in nature - small effects)

C transf.: $\phi \rightarrow exp(i\theta)\phi^*$ (θ arbitrary, put zero)
complex fields and parameters - may signal violation of C (CP)

(for spin 0 particles C conservation is equiavalent to CP conserv., when fermions are included P parity matters)