Higgs mechanism with two doublets -

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CP, FCNC, Z_2-symmetry, rephasing invariance.. - A pedagogical lecture

- Standard Model
- 2 Higgs Doublet Model

Higgs mechanism in SM(s)

Spontaneous electroweak symmetry breaking of SU(2)x U(1) (EWSB) via the Higgs mechanism ⇒ the Lagrangian

$$\mathcal{L} = \mathcal{L}^{SM}_{\text{gauge+fermions}} + \mathcal{L}_H + \mathcal{L}_Y \,.$$

$\mathcal{L}^{SM}_{\text{gauge+fermions}}$ - $SU(2) \times U(1)$ Standard Model interaction of gauge bosons and fermions

$\mathcal{L}_H$ - Higgs scalar Lagrangian

$\mathcal{L}_Y$ - Yukawa interactions of fermions with scalars.

Standard Model (SM) = 1HDM

Standard Model (SM) - one single scalar isodoublet SU(2) ϕ with weak hypercharge $Y = 1$

$$Q = I_{weak,3} + \frac{Y_{weak}}{2}$$

two complex fields with charged 1, and 0.

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \tag{1}$$

(Note also $\tilde{\phi} = i\tau_2(\phi^\dagger)^T$ with $Y = -1$).

$$I_{weak,3} = \pm 1/2$$

Higgs potential and vacuum

Higgs Lagrangian:

$$\mathcal{L}_H = (D_\mu \phi)^\dagger D_\mu \phi - V$$

with the Higgs potential

$$V = \lambda(\phi\phi^\dagger)^2/2 - m^2\phi\phi^\dagger/2$$

(both λ and $m^2 > 0$ are real - hermiticity of $\mathcal{L}$).

Minimum (in fact only extremum):

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=\langle\phi\rangle} = 0, \qquad \langle\phi\rangle = v/\sqrt{2} = \sqrt{m^2/2\lambda}$$

Physical fields (with definite masses)

The standard decomposition in physical fields

$$\phi = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}(v+\eta+i\chi) \end{pmatrix},$$

with the vacuum

$$\phi_{vac} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix},$$

since this spontaneous symmetry breaking $SU(2)_{weak\,I} \times U(1)_{weak\,Y}$ to $U(1)_{QED}$

$$\hat{Q}\phi_{vac} = 0, \quad \text{so } e^{i\alpha\hat{Q}}\phi_{vac} = \phi_{vac}$$

- invariance !

Physical content

$\varphi^{\pm}$ related to long. components of $W^{\pm}$
χ - related to long. component of Z
η - Higgs field

Put into gauge-kinetic Langrangian,

$$M_W = vg/\sqrt{2}, \qquad M_Z = vg/\sqrt{2}c_W$$

or

$$\rho = M_W^2/(M_Z^2 c_W^2) = 1$$

We known value of v knowing Fermi constant G_F

$$v = 246 GeV$$

Mass of Higgs field: given by λ or m^2 (since v (known) relates them)

Couplings of Higgs boson to gauge bosons and selfcouplings

Couplings of Higgs boson to gauge bosons:
$g_W^{\mathsf{SM}} = \sqrt{2} M_W / v$, $g_Z^{\mathsf{SM}} = \sqrt{2} M_Z / v$,
Selfcouplings of Higgs bosons (related to λ term in V)
trilinear and quartic couplings between physical Higgses

Couplings to fermions (Yukawa interaction)

Yukawa term has a form (one generation of quarks for simplicity):
$\mathcal{L}_Y = -(\bar{Q}_L \Gamma \phi n_R + \bar{Q}_L \Delta \tilde{\phi} p_R + h.c.)$

Γ and Δ - arbitrary complex numbers (matrix for more generations)

$$Q_L = \begin{pmatrix} p_L \\ n_L \end{pmatrix}$$

for quark doublet with chirality -1 (hypercharge - ?) and for singlets p_R, n_R - chirality 1.

From this mass terms: -$\bar{n}_L M_n n_R$, etc

$$M_n = v\Gamma$$

Also Yukawa couplings: with $g_f^{\mathrm{SM}} = \sqrt{2} m_f / v$.

Generations

3 generations: Γ and Δ - 3 x 3 matrices.
n principle - arbitrary (flavour problem)

One can diagonalize M_p, M_u by transforming fields fields p, n to u, d -
physical quark fields with definite masses.

The charged-current interaction:

$$\mathcal{L}_W^q = g/\sqrt{2}(W_\mu^+ \bar{u_L} \gamma^\mu V d_L + W_\mu^- ...)$$

$$V = U_L^{p\dagger} U_L^n$$

K-M matrix (Cabibbo 63, Kobayashi-Maskawa 73)
For neutral-current - substitute p, n by u, d - no mixing matrix V !!

No FLAVOUR-CHANGING NEUTRAL CURRENT (FCNC) in SM at
three level. This is due to fact: all fermions of given charge and helicity
have the same isospin I_3, all fermion have one source of mass
- Glashow, Weinberg 77 and Paschos 77

The most general renormalizable Higgs Lagrangian:

$$\mathcal{L}_H = T - V\,, \qquad (2)$$

T is the kinetic term (with D_μ being the covariant derivative containing the EW gauge fields), V is the Higgs potential :

$$T = (D_\mu\phi_1)^\dagger(D^\mu\phi_1) + (D_\mu\phi_2)^\dagger(D^\mu\phi_2) + \varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2) + \varkappa^*(D_\mu\phi_2)^\dagger(D^\mu\phi_1)\,,$$

NEW TERMS possible !! - mixing kinetic terms?!

Mind the mixing terms and a symmetry ($\phi_1 - \phi_2$) (Z_2-symmetry)!!

Potential quartic terms

$$V = \frac{\lambda_1}{2}(\phi_1^\dagger\phi_1)^2 + \frac{\lambda_2}{2}(\phi_2^\dagger\phi_2)^2 + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$

$$+\frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] + \left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\} + \tilde{\mathcal{M}},$$

mixing terms possible of two types, Z_2-symmetry - ?

Potential -mass terms

$$\tilde{\mathcal{M}} = -\frac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + \left[m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right] + m_{22}^2(\phi_2^\dagger\phi_2)\right\} \ .$$

$\tilde{\mathcal{M}}$ is a mass term in the potential
λ_{1-4}, m_{11}^2 and m_{22}^2 are real (by hermiticity of the potential)
λ_{5-7} m_{12} and $\varkappa$ are in general complex parameters (in total there are 16 ndependent parameters)

Two spin zero doublets ϕ_1 and ϕ_2
with vacuum exp. values v_1 and v_2

(with $v^2 = v_1^2 + v_2^2$, often we use $\tan\beta = v_2/v_1$)

Counting degrees of freedom: 8 fields - 3 (for long.components of W/Z)
= 5

⇒ five physical Higgs bosons, spin 0, 3 -el. neutral, two charged !!

Rephasing invariance

The Lagrangian $\mathcal{L}_H$ invariant under global phase transformation of both fields with a common phase, $\phi_i \to \phi_i e^{-i\rho_0}$ (*an overall phase freedom*). Besides, the same physical reality (the same values of observables) if independent phase rotations of each field $\phi_i \to \phi_i e^{-i\rho_i}$ together with the compensating rotation by the phase difference $\rho = \rho_2 - \rho_1$ of *the complex* parameters of the Lagrangian.

We call this property *the rephasing invariance* in the space of Lagrangians, with coordinates given by λ's, m_{ij}^2, $\varkappa$.

Rephasing invariant quantities

Physical picture does not change with the change of Lagrangian under the global *rephasing transformation* of the form:

$$\phi_i \to e^{-i\rho_i}\phi_i, \quad \rho_i \text{ real} \quad (i=1,2), \quad \rho_0 = \frac{\rho_1+\rho_2}{2}, \;\; \rho = \rho_2 - \rho_1, \tag{3a}$$

$$\begin{gathered} \lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{11(22)}^2 \to m_{11(22)}^2\,, \\ \lambda_5 \to \lambda_5\, e^{2i\rho}\,,\;\; \lambda_{6,7} \to \lambda_{6,7}\, e^{i\rho}\,,\;\; \varkappa \to \varkappa\, e^{i\rho}\,,\;\; m_{12}^2 \to m_{12}^2 e^{i\rho}\,. \end{gathered} \tag{3b}$$

A set of these physically equivalent Higgs Lagrangians - *rephasing equivalent family*.

This one-parametric family is governed by the phase difference ρ - the rephasing gauge parameter.

The specific choice of this rephasing gauge parameter leads to the specific *rephasing representation*.

Rephasing invariance - cd

Rephasing invariance can be extended to whole system of scalars and fermions if the corresponding transformations for the Yukawa terms (phases of fermion fields and Yukawa couplings) supplement the transformations.

Note that if the CP is conserved in the Higgs sector, all parameters of Lagrangian can be made real.

Obviously, by transforming such Lagrangian, with $\rho \neq 0$, some λ's, etc. can become complex (however with a fixed relation of their phases).

Lagrangian and Z_2 symmetry

Z_2 symmetry.
We consider first the case with Z_2 symmetry which forbids a (ϕ_1, ϕ_2) mixing in $\mathcal{L}$. This case is described by the Lagrangian $\mathcal{L}_H$ with $\lambda_6 = \lambda_7 = \varkappa = m_{12} = 0$. Using the rephasing invariance, one can make λ_5 real. As it was mention above, it means that in such case the Higgs sector respects CP symmetry. The radiative (loop) corrections does not destroy the Z_2 invariance.

The general Lagrangian $\mathcal{L}_H$ violates Z_2 symmetry (allowing for a (ϕ_1, ϕ_2) mixing) by terms of the operator dimension 2 (with m_{12}), what is called *a soft violation of the Z_2 symmetry*, and of the operator dimension 4 (with $\lambda_{6,7}$ and $\varkappa$), called *a hard violation of the Z_2 symmetry*.

A soft violation of Z_2 symmetry.
One adds to the Z_2 symmetric Lagrangian the term $m_{12}^2(\phi_1^\dagger\phi_2) + h.c.$, with a generally complex m_{12}^2 (and λ_5) parameters.

A hard violation of Z_2.
Terms of the operator dimension 4, with generally complex parameters λ_6, λ_7 and $\varkappa$, are added to the Lagrangian with a softly broken Z_2 symmetry.

Such treatment of the case with hard violation of Z_2 symmetry is as incomplete in most of papers considering this most general 2HDM potential.

The Vacuum and Specific Choice of the Potential

The minimum of the potential:

$$\left.\frac{\partial V}{\partial \phi_1}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0, \qquad \left.\frac{\partial V}{\partial \phi_2}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0. \tag{4}$$

In order to describe the $U(1)$ symmetry of electromagnetism and using the over-all phase freedom of the Lagrangian to choose one vacuum expectation value real we take:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \tag{5}$$

with a relative phase ξ.

NOTE: THIS IS JUST ASSUMPTION.

Another parameterization:

$$v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad \beta \in \left(0, \frac{\pi}{2}\right). \tag{6}$$

The phase difference ξ between the v.e.v.'s is often interpreted as spontaneous CP violation of vacuum. However, this is not necessarily the case !!!

Note that under the rephasing, the ξ changes to:

$$\xi \to \xi - \rho\,. \tag{7}$$

The following quantities

$$\overline{\lambda}_{1-4} = \lambda_{1-4}, \quad \overline{\lambda}_5 \equiv \lambda_5 e^{2i\xi}, \quad \overline{\lambda}_6 \equiv \lambda_6 e^{i\xi}, \quad \overline{\lambda}_7 \equiv \lambda_7 e^{i\xi}, \quad \overline{\varkappa} \equiv \varkappa e^{i\xi}, \quad \overline{m}^2_{12} \equiv m^2_{12} e \tag{8}$$

are the rephasing-invariant quantities

The standard decomposition of the fields ϕ_i in terms of physical fields is made via

$$\phi_1 = \begin{pmatrix} \varphi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \end{pmatrix}, \qquad \phi_2 = \begin{pmatrix} \varphi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \end{pmatrix}. \tag{9}$$

At $\varkappa = 0$ this decomposition leads to a diagonal form of kinetic terms for new fields φ_i^+, χ_i, η_i, while the corresponding mass matrix become off-diagonal.

The mass squared matrix can be transformed to the block diagonal form by a separation of the massless Goldstone boson fields, $G^0 = \cos\beta\,\chi_1 + \sin\beta\,\chi_2$ and $G^\pm = \cos\beta\,\varphi_1^\pm + \sin\beta\,\varphi_2^\pm$, and charged Higgs boson fields $H^\pm$, the combinations orthogonal to $G^\pm$

$$H^\pm = -\sin\beta\,\varphi_1^\pm + \cos\beta\,\varphi_2^\pm\,, \tag{10}$$

with their mass squared equal to

$$M_{H^\pm}^2 = v^2\left[\nu - \frac{1}{2}\mathrm{Re}(\lambda_4 + \bar{\lambda}_5 + \bar{\lambda}_{67})\right]. \tag{11}$$

more on 2HDM

nteraction with gauge bosons and fermions- Higgs bosons share obligations

$$(g_W^h)^2 + (g_W^H)^2 + (g_W^A)^2 = (g_W^{H_{SM}})^2$$

Various models of Yukawa interaction with fermions:
eg Model II where one scalar doublet couples to up-type quarks, other to down-type quarks and charged leptons
The potential problems:
•Flavour Changing Neutral Current may be large (in nature FCNC small)

•CP-violation may be large (in nature - small effects)

C transf.: $\phi \to exp(i\theta)\phi^*$ (θ arbitrary, put zero)
complex fields and parameters - may signal violation of C (CP)

(for spin 0 particles C conservation is equiavalent to CP conserv., when fermions are included P parity matters)

Part II - Once more on minimum

$$\begin{aligned} m_{12}^2 = 0\,, \;\; Im(\overline{\lambda}_5) = 0 \quad & \text{for no } Z_2 \text{ violation}\,, \\ Im(\overline{m}_{12}^2) = Im(\overline{\lambda}_5) v_1 v_2 \quad & \text{for soft } Z_2 \text{ violation}\,, \\ Im(\overline{m}_{12}^2) = Im(\overline{\lambda}_5 + \overline{\lambda}_{67}) v_1 v_2 \quad & \text{for hard } Z_2 \text{ violation}\,. \end{aligned} \tag{12}$$

These relations - constraints for parameters of the potential in the zero rephasing gauge.

Neutral Higgs sector. General

η_i - standard $C-$ and $P-$ even (scalar) fields
orthogonal to the Goldstone boson field G^0 is

$$A = -\sin\beta\,\chi_1 + \cos\beta\,\chi_2\,, \qquad (13)$$

C–odd (which in the interactions with fermions behaves as $P-$ odd
particle - a pseudoscalar).

η_i and A are fields with opposite CP parities
Mass squared matrix $\mathcal{M}$ in the η_1, η_2, A basis

$$\mathcal{M} = \begin{pmatrix} M_{11} & M_{12} & M_{13} \\ M_{12} & M_{22} & M_{23} \\ M_{13} & M_{23} & M_{33} \end{pmatrix} \quad \text{with} \qquad (14a)$$

$$M_{11} = v^2[c_\beta^2 \lambda_1 + s_\beta^2 \nu + \frac{s_\beta}{2c_\beta} \mathrm{Re}(3c_\beta^2 \overline{\lambda}_6 - s_\beta^2 \overline{\lambda}_7)],$$

$$M_{22} = v^2[s_\beta^2 \lambda_2 + c_\beta^2 \nu + \frac{c_\beta}{2s_\beta} \mathrm{Re}(-c_\beta^2 \overline{\lambda}_6 + 3s_\beta^2 \overline{\lambda}_7)],$$

$$M_{33} = v^2[\nu - \mathrm{Re}(\overline{\lambda}_5 - \frac{1}{2}\overline{\lambda}_{67})],$$

$$M_{12} = -v^2 c_\beta s_\beta(\nu - \overline{\lambda}_{345} - \frac{3}{2}\mathrm{Re}\,\overline{\lambda}_{67}), \quad M_{13} = \delta s_\beta - v^2 c_\beta \,\mathrm{Im}\,\overline{\lambda}_6, \quad M_{23} = \delta c_\beta - v^2 s_\beta$$

where we have introduced $c_\beta = \cos\beta$, $s_\beta = \sin\beta$ and

$$\delta = -\frac{v^2}{2}\mathrm{Im}\,\overline{\lambda}_5 \tag{14c}$$

The masses squared M_i^2 of the physical neutral states h_i are eigenvalues of the matrix $\mathcal{M}$

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ A \end{pmatrix} \qquad \text{with} \quad R\mathcal{M}R^T = diag(M_1^2, M_2^2, M_3^2)\,. \tag{15}$$

In general, eigenstates h_i have no definite CP parity - they mix fields η_i and A which have opposite CP parities

Such mixing (and violation of CP) is absent in the case of real coefficients M_{13} and M_{23} (and consequently real coefficient $\overline{m}_{12}^2$,).

Case of no CP violation

This case can be realized in:
for an exact Z_2 symmetry:

• $\overline{\lambda}_5$ is real, and $Im(M_{13}) = Im(M_{23}) = 0$.

• soft violation of Z_2 symmetry the CP violation is absent only if $\overline{\lambda}_5$ ($\overline{m}_{12}^2$) is real.

• case of hard violation of Z_2 symmetry, CP violation is absent if some specific relations hold.

The mass squared matrix $\mathcal{M}$ becomes block diagonal,
the physical states become CP eigenstates: h_1, h_2, h_3 $\Rightarrow h$, H and A.

The mass of CP-odd state A is

$$M_A^2 = M_{33} \equiv v^2\,\mathrm{Re}(\nu - \overline{\lambda}_5 - \frac{1}{2}\overline{\lambda}_{67})\,. \tag{16}$$

The neutral, CP-even Higgs fields, h and H, by the rotation

$$H = \cos\alpha\,\eta_1 + \sin\alpha\,\eta_2\,,\quad h = -\sin\alpha\,\eta_1 + \cos\alpha\,\eta_2\,,\quad \alpha\in(-\pi/2,\,\pi/2) \tag{17}$$

One finds by diagonalizing the respective 2×2 matrix, that

$$M_{h,H}^2 = \tfrac{1}{2}\,(M_{11}+M_{22}\mp\mathcal{R})\,,\quad \mathcal{R} = \sqrt{(M_{11}-M_{22})^2+4M_{12}^2} \tag{18}$$

with

$$M_H^2 + M_h^2 = M_{11}+M_{22} \equiv v^2\,\mathrm{Re}\left[\nu + c_\beta^2\,\lambda_1 + s_\beta^2\,\lambda_2 + 2(\overline{\lambda}_6+\overline{\lambda}_7) - \frac{1}{2}\overline{\lambda}_{67}\right]. \tag{19}$$

It is easy to express λ's via masses of Higgs bosons, ν, $\overline{\lambda}_{345}$ and β in the case of soft violation of Z_2 symmetry...

Case of CP violation

The diagonalizing matrix R - a product of three rotation matrices described by three Euler angles α_i (we define $c_i = \cos\alpha_i,\ s_i = \sin\alpha_i$)

$$R = R_3 R_2 R_1,\ R_1 = \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ R_2 = \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix},\ R_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}. \tag{20a}$$

$$R = \begin{pmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \equiv \begin{pmatrix} c_1\, c_2 & c_2\, s_1 & s_2 \\ -c_1\, s_2\, s_3 - c_3\, s_1 & c_1\, c_3 - s_1\, s_2\, s_3 & c_2\, s_3 \\ -c_1\, c_3\, s_2 + s_1\, s_3 & -c_1\, s_3 - c_3\, s_1\, s_2 & c_2\, c_3 \end{pmatrix}. \tag{20b}$$

We adopt the convention for the masses that $M_2 \geq M_1$

The rotation R_1 - diagonalizes the upper left 2×2 corner of mass-squared matrix coincides with that for the CP conserving case with two minor modifications: instead of α there is now the mixing angle $\alpha_1 = \alpha - \pi/2$,
$(-H)$ state instead of H appears

$$\begin{pmatrix} h \\ -H \\ A \end{pmatrix} = R_1 \begin{pmatrix} \eta_1 \\ \eta_2 \\ A \end{pmatrix} \quad \text{with} \quad R_1 \mathcal{M} R_1^T = \mathcal{M}_1 \equiv \begin{pmatrix} M_h^2 & 0 & M'_{13} \\ 0 & M_H^2 & M'_{23} \\ M'_{13} & M'_{23} & M_A^2 \end{pmatrix}, \qquad (21a)$$

with parameters M_h^2, M_H^2, M_A^2 and $\alpha_1 = \alpha - \pi/2$ so that one can discuss the general case in terms customary for the CP conserving case.

The full diagonalization

$$\begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix} = R_3 R_2 \begin{pmatrix} h \\ -H \\ A \end{pmatrix} \quad \text{with} \quad R\,\mathcal{M}\,R^T = R_3 R_2 \mathcal{M}_1 R_2^T R_3^T = \begin{pmatrix} M_1^2 & & \\ & M_2^2 & \\ & & M_3^2 \end{pmatrix}. \qquad (21b)$$

The angle α_2 and α_3 describe mixing between CP–even states h and H with CP–odd state A.

Soft violation of Z_2 symmetry, the CP violating effects are determined by a single parameter $\delta \propto Im(\overline{m}_{12}^2) \propto Im(\overline{\lambda}_5)$, and

$$M'_{13} = -\delta \cos(\alpha + \beta)\,, \quad M'_{13} = \delta \sin(\alpha + \beta)\,. \qquad (21c)$$

Special limits.

If $|M'_{13}/(M_A^2 - M_h^2)| \ll 1$, the Higgs boson h_1 practically coincides with h ($\alpha_2 \approx 0$). The interaction of this boson with other particles respects (with a high accuracy) CP–symmetry, while h_2, h_3 correspond to the possibly strongly mixed H and A states (with large mixing angle α_3).

Vice versa, if $|M'_{23}/(M_A^2 - M_H^2)| \ll 1$, the Higgs boson h_2 practically coincides with $-H$ ($\alpha_3 \approx 0$), and the interaction of matter with this boson does not violate CP–symmetry while h_1, h_3 can be strongly mixed states of h and A (large mixing angle α_2).

Case of weak CP violation joins above special limits: it corresponds to weak mixing between both CP–even states h, H and CP–odd state A, i.e. a small value of mixing angles α_2 and α_3 and respectively $c_2 \approx 1$, $c_3 \approx 1$.

If in addition - **a soft violation of Z_2 symmetry** even simpler..

Couplings to gauge bosons

The gauge bosons V (W and Z) couple only to CP − even fields η_1, η_2.
For the physical Higgs bosons h_i in terms of relative (to SM) couplings

$$\chi_V^{(i)} = \cos\beta\, R_{i1} + \sin\beta\, R_{i2}, \quad i = 1, 2, 3, \;\; V = W \;\text{ or }\; Z\,. \tag{4a}$$

For CP conservation we have

$$\chi_V^{(h)} = \sin(\beta - \alpha)\,, \quad \chi_V^{(H)} = \cos(\beta - \alpha)\,, \quad \chi_V^{(A)} = 0\,. \tag{4b}$$

A weak violation of CP symmetry:

$$\chi_V^{(1)} = \sin(\beta - \alpha)\,, \quad \chi_V^{(2)} = -\cos(\beta - \alpha)\,, \quad \chi_V^{(3)} = -s_2 \sin(\beta - \alpha) + s_3 \cos(\beta - \alpha), \tag{4c}$$

with small s_2, s_3

Higgs self-couplings

Yukawa interactions

The Yukawa Lagrangian

$$-\mathcal{L}_{\mathsf{Y}} = \bar{Q}_L[(\Gamma_1\phi_1 + \Gamma_2\phi_2)d_R + (\Delta_1\tilde{\phi}_1 + \Delta_2\tilde{\phi}_2)u_R] + \text{h.c.}, \tag{1}$$

plus similar terms for the leptons.

The rephasing invariance is extended to the full Higgs + Yukawa Lagrangian space

Soft violation of Z_2 symmetry only, one should assume that only one scalar doublet gives masses to each quark or lepton Glashow:1977,Paschos:1976.

The existence in the Yukawa couplings of off-diagonal (in family index) terms results in flavor-changing neutral-currents (FCNC). The rephasing invariance allows to consider only real diagonal elements of one matrix Γ and one matrix Δ, take matrices Γ_1, Δ_2.
Complex values of the other elements of matrices $\Gamma_{1,2}$ and $\Delta_{1,2}$ can result in the complex values of one–loop corrections to some λ's and in consequence to a CP violation in the Higgs sector discussed above (even for real m_{12}^2 and λ's in the original Higgs Lagrangian).

Therefore, in order to avoid FCNC the matrices $\Gamma_{1,2}$ and $\Delta_{1,2}$ should be diagonal.

In addition, this diagonality natural and to have only soft violation of Z_2 symmetry, one can assume that the fermions of one type (u or d or ℓ) interact with a single field (either ϕ_1 or ϕ_2)

A few models possible: $\Gamma_2 = \Delta_2 = 0$ corresponds to Model I, while $\Gamma_2 = \Delta_1 = 0$ – to Model II

Model II

$\Gamma_1 = diag(g_{d1}, g_{d2}, g_{d3})$, $\Delta_2 = diag(g_{u1}, g_{u2}, g_{u3})$ (like in MSSM)

$$-\mathcal{L}_Y^{II} = \sum_{k=1,2,3} g_{dk}\bar{Q}_{Lk}\phi_1 d_{Rk} + \sum_{k=1,2,3} g_{uk}\bar{Q}_{Lk}\tilde{\phi}_2 u_{Rk} + \sum_{k=1,2,3} g_{\ell k}\bar{\ell}_{Lk}\phi_1 e_{Rk} + \text{h.c.} \quad (2)$$

u–type and for all d–type quarks (and charged leptons).

They can be expressed via elements of the rotation matrix R

$$\chi_u^{(i)} = \frac{1}{\sin\beta}[R_{i2} - i\cos\beta\, R_{i3}]\,, \qquad \chi_d^{(i)} = \frac{1}{\cos\beta}[R_{i1} - i\sin\beta\, R_{i3}]\,. \quad (3)$$

Pattern relation and sum rules

The unitarity of the mixing matrix R allows to obtain a number of useful relations for the relative couplings of neutral Higgs particles to gauge bosons and fermions (basic relative couplings).

1. The first of them is *the pattern relation* among the basic relative couplings of *each neutral Higgs particle* h_i:

$$(\chi_u^{(i)}+\chi_d^{(i)})\chi_V^{(i)} = 1+\chi_u^{(i)}\chi_d^{(i)}\,, \quad \text{or} \quad (\chi_u^{(i)}-\chi_V^{(i)})(\chi_V^{(i)}-\chi_d^{(i)}) = 1-\chi_V^{(i)2}\,, \tag{4}$$

which has the same form for each Higgs boson h_i (in particular for h, H, A).

One can also express $\tan\beta$, which is a basic parameter of the model, via the relative couplings:

$$\tan^2\beta = \frac{(\chi_V^{(i)}-\chi_d^{(i)})^\dagger}{\chi_u^{(i)}-\chi_V^{(i)}} = \frac{1-|\chi_d^{(i)}|^2}{|\chi_u^{(i)}|^2-1}\,. \tag{5}$$

The universality of these equations for each neutral Higgs boson h_i

2. These relations allow also to write for each neutral Higgs boson *sum rules*:

$$|\chi_u^{(i)}|^2 \sin^2\beta + |\chi_d^{(i)}|^2 \cos^2\beta = 1\,. \qquad (6)$$

3. The third relation provides a sum rule for each basic relative coupling to *all three neutral Higgs bosons* h_i

$$\sum_{i=V,d,u} (\chi_j^{(i)})^2 = 1\,. \qquad (7)$$

Case of no CP violation.

no CP violation $s_2 = s_3 = 0$,

[hbt] *Basic relative couplings in the CP-conserving case*

	h	H
χ_V	$\sin(\beta-\alpha)$	$\cos(\beta-\alpha)$
χ_u	$\dfrac{\cos\alpha}{\sin\beta} = \sin(\beta-\alpha) + \cot\beta\cos(\beta-\alpha)$	$\dfrac{\sin\alpha}{\sin\beta} = \cos(\beta-\alpha) - \cot\beta\sin(\beta-\alpha)$
χ_d	$-\dfrac{\sin\alpha}{\cos\beta} = \sin(\beta-\alpha) - \tan\beta\cos(\beta-\alpha)$	$\dfrac{\cos\alpha}{\cos\beta} = \cos(\beta-\alpha) + \tan\beta\sin(\beta-\alpha)$

Useful equations

$$\chi_u^{(H)} = -\chi_d^{(h)} \cot\beta, \quad \chi_d^{(H)} = \chi_u^{(h)} \tan\beta; \qquad \frac{\sin 2\alpha}{\sin 2\beta} = -\chi_u^{(h)}\chi_d^{(h)} = \chi_u^{(H)}\chi_d^{(H)}. \tag{8}$$

The relative coupling constant of neutral scalar h to the charged Higgs boson via couplings of this Higgs boson to gauge bosons and fermions:

$$\chi_{H^\pm}^{(h)} \equiv -\frac{v g_{hH^+H^-}}{2M_{H^\pm}^2} = \left(1 - \frac{M_h^2}{2M_{H^\pm}^2}\right)\chi_V^{(h)} + \frac{M_h^2 - \nu v^2}{2M_{H^\pm}^2}(\chi_u^{(h)} + \chi_d^{(h)}), \tag{9}$$

and the same expression for HH^+H^- with the change $h \to H$. This should also hold for the case with weak CP violation

Counting model parameters and possible measurements of the Higgs sector

[htb]

		measurable				in Lagrangian		
CP conservation	Yukawa interaction	Higgs masses				Z_2 violation		
			χ_V	$\chi_{u,d}$	total	no	soft	hard
yes	arbitrary	4	1 $(\beta-\alpha)$	-	5	6	7	9
	Model II	4	1 $(\beta-\alpha)$	1 (β)	6			
no	arbitrary	4	2	-	6	-	8	12
	Model II	4	4 $(\alpha_i,\, \beta)$		8			

Number of measurable parameters in different cases and number of independent parameters of Lagrangian.