Higgs selfcouplings or reconstructing the Higgs potential

Maria Krawczyk, Warsaw U.

Standard Model and 2HDM (II)

Based on:

TESLA TDR III,

GKO hep-ph/0101208

Kanemura et. al hep-ph/0209326,

Belusevic at al. hep-ph/0403303

Higgs mechanism in SM(s)

Spontaneous electroweak symmetry breaking of SU(2)x U(1) (EWSB) via the Higgs mechanism ⇒ the Lagrangian

$$\mathcal{L} = \mathcal{L}^{SM}_{\text{gauge+fermions}} + \mathcal{L}_H + \mathcal{L}_Y \, .$$

$\mathcal{L}^{SM}_{\text{gauge+fermions}}$ - $SU(2) \times U(1)$ Standard Model interaction of gauge bosons and fermions

$\mathcal{L}_H$ - Higgs scalar Lagrangian

$\mathcal{L}_Y$ - Yukawa interactions of fermions with scalars.

Standard Model (SM) = 1HDM

Standard Model (SM) - one single scalar isodoublet SU(2)
ϕ with weak hypercharge $Y = 1$

$$Q = I_{weak,3} + \frac{Y_{weak}}{2}$$

two complex fields with charged 1, and 0.

$$\phi = \begin{pmatrix} \varphi^+ \\ \varphi^0 \end{pmatrix}, \tag{1}$$

(Note also $\tilde{\phi} = i\tau_2(\phi^\dagger)^T$ with $Y = -1$).

$$I_{weak,3} = \pm 1/2$$

Higgs potential and vacuum

Higgs Lagrangian:

$$\mathcal{L}_H = (D_\mu \phi)^\dagger D_\mu \phi - V$$

with the Higgs potential

$$V = \lambda(\phi\phi^\dagger)^2/2 - m^2\phi\phi^\dagger/2$$

(both λ and $m^2 > 0$ are real - hermiticity of $\mathcal{L}$).

Minimum (in fact only extremum):

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=\langle\phi\rangle} = 0, \qquad \phi \to = v/\sqrt{2} = \sqrt{m^2/2\lambda}$$

Physical fields (with definite masses)

The standard decomposition in physical fields

$$\phi = \begin{pmatrix} \varphi^+ \\ \frac{1}{\sqrt{2}}(v + \eta + i\chi) \end{pmatrix},$$

with the vacuum

$$\phi_{vac} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix},$$

since this spontaneous symmetry breaking $SU(2)_{weak\,I} \times U(1)_{weak\,Y}$ to $U(1)_{QED}$

$$\hat{Q}\phi_{vac} = 0, \quad \text{so}\, e^{i\alpha\hat{Q}}\phi_{vac} = \phi_{vac}$$

- invariance !

Physical content

$\varphi^{\pm}$ related to long. components of $W^{\pm}$
χ - related to long. component of Z
η - Higgs field

Put into gauge-kinetic Langrangian,

$$M_W = vg/\sqrt{2}, \quad M_Z = vg/\sqrt{2}c_W$$

or

$$\rho = M_W^2/(M_Z^2 c_W^2) = 1$$

We known value of v knowing Fermi constant G_F

$$v = 246 GeV$$

Mass of Higgs field: given by λ or m^2 (since v (known) relates them)

Couplings of Higgs boson to gauge bosons and selfcouplings

Couplings of Higgs boson to gauge bosons:
$g_W^{\mathrm{SM}} = \sqrt{2} M_W / v$, $g_Z^{\mathrm{SM}} = \sqrt{2} M_Z / v$,
Selfcouplings of Higgs bosons (related to λ term in V)
trilinear and quartic couplings between physical Higgses

Standard Models

SM = 1HDM $\Rightarrow$ one Higgs SU(2) doublet

- Basic parameter v - vacuum expectation of scalar field
- one Higgs boson
- one unknown parameter describing whole sector:

  mass or selfcoupling: $M = \sqrt{2\lambda} v$
- interaction with gauge bosons: $M_V \sim gv$, coupling $\sim M_V$

Yukawa interaction with fermions: $m_f \sim g_f$

- Direct searches: $M_{H_{SM}}$ larger than 114.4 GeV.

2HDM models without and with CP violation

I. Ginzburg, M. Krawczyk, P. Osland

hep-ph/0101208, hep-ph/0101229,hep-ph/0211371

2HDM Potential: quartic and quadratic terms separated:

$$V = \frac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \frac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2$$
$$+ \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1)$$
$$+ \frac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right]$$
$$+ \left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\}$$
$$- \frac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + \left[m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right] + m_{22}^2(\phi_2^\dagger\phi_2)\right\}$$

soft violation of Z_2 symmetry

No (ϕ_1, ϕ_2) mixing if Z_2 symmetry satisfied:
$\phi_1 \to -\phi_1$, $\phi_2 \to \phi_2$ (or vice versa) $\Rightarrow \lambda_6 = \lambda_7 = m_{12}^2 = 0$

14 parameters: $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, $\lambda_5, \lambda_6, \lambda_7$, m_{11}^2, m_{22}^2, Re m_{12}^2,Im m_{12}^2

Hard violation of Z_2 symmetry: quartic terms with λ_6, λ_7

Lee, Diaz-Cruz, Mendez, Haber, Pomarol, Barroso, Santos, Hollik, Djouadi, Illana, Branco,Gunion, Akeroyd, Arhrib,...

Rephasing invariance

Physical picture the same if

$$\phi_i \to e^{-i\rho_i}\phi_i, \quad \rho_i \text{ real} \quad (i=1,2), \quad \rho_0 = \frac{\rho_1+\rho_2}{2}, \;\; \rho = \rho_2 - \rho_1, \tag{2a}$$

accompanied by

$$\begin{gathered} \lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{11(22)}^2 \to m_{11(22)}^2\,, \\ \lambda_5 \to \lambda_5\, e^{2i\rho}\,, \;\; \lambda_{6,7} \to \lambda_{6,7}\, e^{i\rho}\,, \;\; \varkappa \to \varkappa\, e^{i\rho}\,, \;\; m_{12}^2 \to m_{12}^2 e^{i\rho}\,. \end{gathered} \tag{2b}$$

If the CP is conserved in the Higgs sector,
all parameters of Lagrangian can be made real $\to$
transforming with $\rho \neq 0$, some λ's, etc. can become complex

Vaccum and special rephasing

The minimum of the potential defines the vacuum expectation values (v.e.v) of the fields ϕ_i:

$$\left.\frac{\partial V}{\partial \phi_1}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0, \qquad \left.\frac{\partial V}{\partial \phi_2}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0. \tag{3}$$

We choose one vacuum expectation value real (Branco,Diaz-Cruz)

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_2 e^{i\xi} \end{pmatrix}, \tag{4}$$

with a relative phase ξ.
SM constraint: $v_1^2 + v_2^2 = v^2$, with $v = (\sqrt{2}G_{\mathsf{F}})^{-1/2} = 246$ GeV.

$$v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad \beta \in \left(0, \frac{\pi}{2}\right). \tag{5}$$

The phase difference ξ between the v.e.v.'s - spontaneous CP violation? Not necessarily the case (Branco).

Rephasing: the ξ changes to:

$$\xi \to \xi - \rho \tag{6}$$

and rephasing invariant quantities

$$\overline{\lambda}_{1-4} = \lambda_{1-4}, \quad \overline{\lambda}_5 \equiv \lambda_5 e^{2i\xi}, \quad \overline{\lambda}_6 \equiv \lambda_6 e^{i\xi}, \quad \overline{\lambda}_7 \equiv \lambda_7 e^{i\xi}, \quad \overline{\varkappa} \equiv \varkappa e^{i\xi}, \quad \overline{m}_{12}^2 \equiv m \tag{7a}$$

(Pilaftsis, Wagner)

Rephasing invariant combination of

$$\overline{\lambda}_{345} = \lambda_3 + \lambda_4 + Re(\overline{\lambda}_5),\ \ \overline{\lambda}_{67} = \frac{v_1}{v_2}\overline{\lambda}_6 + \frac{v_2}{v_1}\overline{\lambda}_7,\ \ \overline{\lambda}_{67} = \frac{1}{2} Im\left(\frac{v_1}{v_2}\overline{\lambda}_6 - \frac{v_2}{v_1}\overline{\lambda}_7\right), \tag{7b}$$

The minimum condition $\delta = Im\left(\overline{m}_{12}^2/(2v_1v_2)\right)$ is expressed by $Im(\overline{\lambda}_{5-7})$:

$$\overline{m}_{12}^2 = 2v_1v_2(\nu + i\delta)\,, \quad \delta = \underbrace{0}_{Z_2\ sym} + \underbrace{\frac{\overline{\lambda}_5}{2}}_{soft} + \underbrace{\frac{\overline{\lambda}_{67}}{2}}_{hard} \tag{8}$$

Expressing m_{ii}^2 via λ_i, v_i and parameter ν

For rephasing with $\rho = \xi \to$ new form of Higgs potential (*zero rephasing gauge*) and rephasing invariant forms of parameters

Important

Soft Z_2 violation and special rephasing with real vaccum expectations values:

•- CPV governed by a single parameter $\sim Imm_{12}^2$

•- naturally small CPV and FCNC effects:

$\nu = Rem_{12}^2/2v_1v_2$ and $\delta = Imm_{12}^2/2v_1v_2$ both small.

Decoupling and heavy Higgs bosons

Decoupling property and masses of heavy Higgs bosons depends on ν ($\nu = Re\left(\overline{m}^2_{12}/(2v_1v_2)\right)$, $\mu^2 = \nu v^2$)

•- large $\nu \to$ decoupling, h SM-like (tree and loop couplings)

•- small $\nu \to$ non-decoupling yet h SM-like (tree) possible while deviation for loop couplings

•- unitarity constraints crucial if heavy Higgs bosons exist and ν is small

new unitarity constraints study for 2HDM with CP violation by Ginzburg, Ivanov

Large Masses

Mass of charged Higgs boson:

$$M_{H^\pm}^2 = \mu^2 - \frac{1}{2}\text{Re}(\lambda_4 + \lambda_5 + \lambda_{67})v^2$$

or

$$M_{H^\pm}^2 = v^2[\nu - \frac{1}{2}\text{Re}(\lambda_4 + \lambda_5 + \lambda_{67})]$$

To get large masses we need:

large μ^2 (leading to decoupling Haber'95..) ..
or
small μ^2 and (relatively) large Re $\lambda_{4,5,6}$ (non-decoupling)

μ-parameter crucial !

if $\mu^2 \propto \text{Re}(m_{12}) \propto \text{Re}\,\lambda_5$ small is large Im λ_5 natural?
Weak CP-violation $\rightarrow$ small $\delta \sim Im\lambda_5$

CP-consv.: masses of neutral Higgs bosons

$$M_{h,H}^2 = \frac{1}{2}\left[\mathcal{M}_{11} + \mathcal{M}_{22} \mp \sqrt{(\mathcal{M}_{11} - \mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}\right]$$
$$\equiv \frac{1}{2}\left[\mathcal{M}_{11} + \mathcal{M}_{22} \mp R\right]$$

Diagonalization gives:

$$\sin 2\alpha = \frac{2\mathcal{M}_{12}}{R}, \qquad \cos 2\alpha = \frac{\mathcal{M}_{11} - \mathcal{M}_{22}}{R}, \mathcal{M}_{11} - \mathcal{M}_{22} = R\cos 2\alpha$$

$$M_H^2 + M_h^2 = \mathcal{M}_{11} + \mathcal{M}_{22} = \mu^2 + \lambda_1 v_1^2 + \lambda_2 v_2^2 \tag{9}$$
$$M_H^2 - M_h^2 = R = \frac{2\mathcal{M}_{12}}{\sin 2\alpha} = \frac{\sin 2\beta}{\sin 2\alpha}(\lambda_{345} v^2 - \mu^2)$$
$$\mathcal{M}_{11} - \mathcal{M}_{22} = R\cos 2\alpha$$

λ_i in terms of masses, μ^2 etc.

Solving:

$$\lambda_1 v^2 = \frac{1}{\cos^2\beta}\{M_H^2 \cos^2\alpha + M_h^2 \sin^2\alpha\} - \mu^2 \tan^2\beta$$

$$\lambda_2 v^2 = \frac{1}{\sin^2\beta}\{M_H^2 \sin^2\alpha + M_h^2 \cos^2\alpha\} - \mu^2 \cot^2\beta$$

Remaining λ_i:

$$\lambda_5 = \frac{1}{v^2}[-M_{A^0}^2 + \mu^2]$$

$$\lambda_4 = \frac{1}{v^2}[M_{A^0}^2 - 2M_{H^\pm}^2 + \mu^2]$$

$$\lambda_3 = \frac{1}{v^2}\left[2M_{H^\pm}^2 + \frac{\sin 2\alpha}{\sin 2\beta}(M_H^2 - M_h^2) - \mu^2\right]$$

CP conservation: couplings

- Higgs trilinear couplings
- Higgs quartic couplings

Note, if selfcouplings are expressed in terms of masses, also μ enters!

$$g_{hhh} = -3v\Big[-\cos\beta\sin^3\alpha\,\lambda_1 + \sin\beta\cos^3\alpha\,\lambda_2 - \tfrac{1}{2}\sin 2\alpha\cos(\beta+\alpha)\lambda_{345}\Big],$$
$$= \frac{-3g}{\sin 2\beta\, M_W}\Big[(\cos\beta\cos^3\alpha - \sin\beta\sin^3\alpha)M_h^2 - \cos^2(\beta-\alpha)\cos(\beta+\alpha)\mu^2\Big],$$

Boudjema,Semenov,Dubinin,Hollik,...

Selfcouplings in terms of observables: masses and relative coupl.

Using relative couplings one obtains

$$g_{hhh} = \frac{-3g}{2M_W}\Big[(\cos^2\alpha\,\chi_u^h + \sin^2\alpha\,\chi_d^h)(M_h^2 - \mu^2) + \chi_V^h \mu^2\Big]$$

A relative coupling for hH^+H^-, (similarly for H):

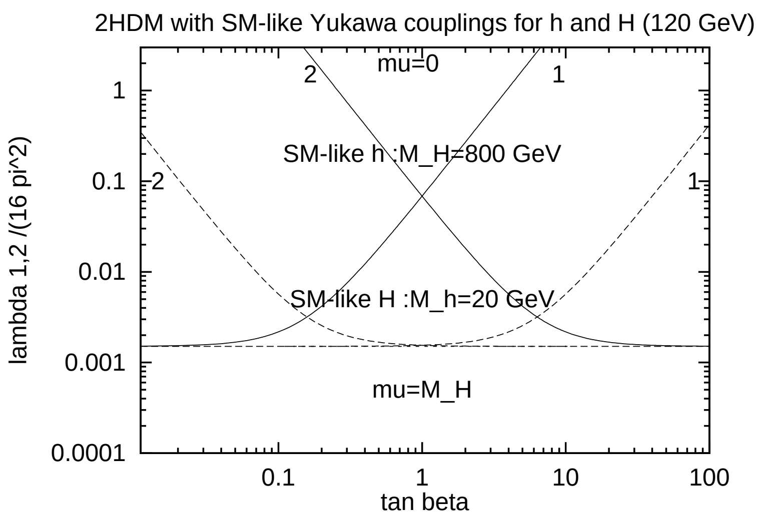
$$\chi_{H^\pm}^h \equiv -\frac{v g_{hH^+H^-}}{2iM_{H^\pm}^2} = \left(1 - \frac{M_h^2}{2M_{H^\pm}^2}\right)\chi_V^h + \frac{M_h^2 - \mu^2}{2M_{H^\pm}^2}(\chi_u^h + \chi_d^h)$$

"Finite" for "large" $M_{H^\pm}^2$ (non-decoupling for small μ^2)
important for $\gamma\gamma\phi$ loop-coupling.

Perturbativity: λ_i in terms of masses and basic couplings

$$\lambda_1 v^2 = (1+\tan\beta^2) M_H^2 - \chi_d^2 [M_H^2 - M_h^2] - \mu^2 \tan^2\beta$$
$$\lambda_2 v^2 = (1+\cot\beta^2) M_H^2 - \chi_u^2 [M_H^2 - M_h^2] - \mu^2 \cot^2\beta$$

Take $\chi_d = \chi_u = 1$ for Higgs boson with mass 120 GeV

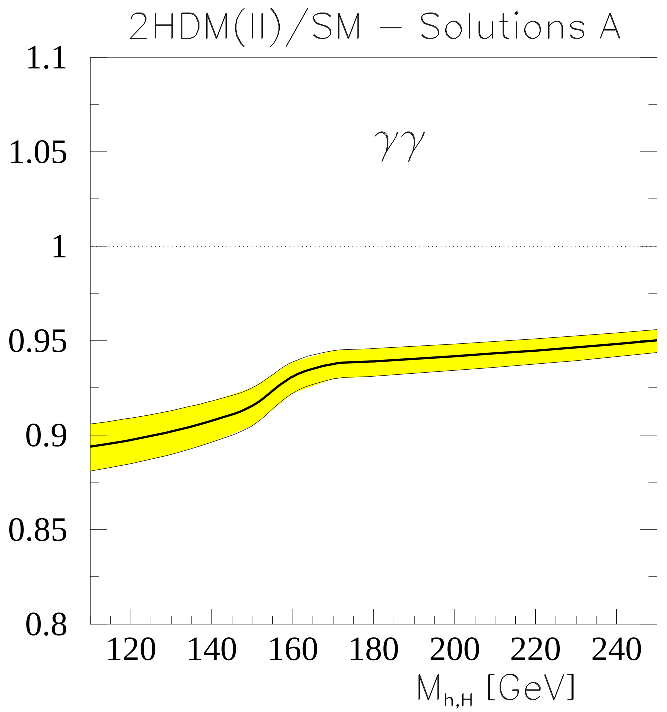


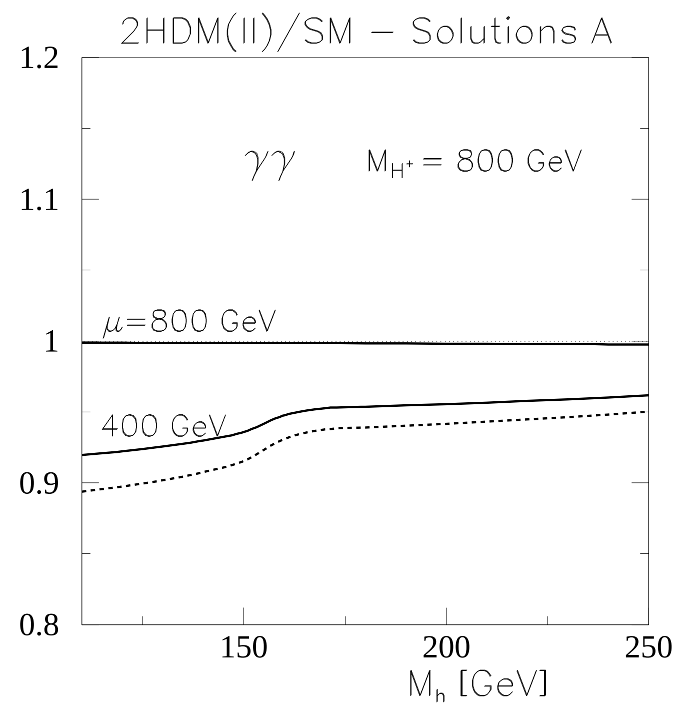
SM-like Higgs:

- h (M_H=800GeV)
- H (M_h=20GeV)

No problem...however for small μ^2 region of $\tan\beta$ maybe limited

GKO - decoupling effect due to hH+H- coupling





Kanemura et al.: Selfcoupligs - SM

$\lambda^{tree}_{hhh}(SM) = \frac{3m_h^2}{v}$.

The leading one-loop correction to $\lambda^{eff}_{hhh}(SM)$

$$\lambda^{eff}_{hhh}(SM) = \frac{3m_h^2}{v}\left[1 - \frac{N_c}{3\pi^2}\frac{m_t^4}{v^2 m_h^2}\left\{1 + \mathcal{O}\left(\frac{m_h^2}{m_t^2}, \frac{p_i^2}{m_t^2}\right)\right\}\right], \qquad (10)$$

(hVV ($VV = ZZ$, W^+W^-) have non-decoupling of at most $\mathcal{O}(m_t^2)$.)

•At LHC - extremely difficult for SM Higgs boson self-couplings: hhh or $hhhh$

•At LC, the trilinear coupling λ_{hhh} can be measured via the Higgs boson pair production in $e^+e^- \to Z^* \to Zhh$ and $e^+e^- \to W^{+*}\bar{\nu}W^{-*}\nu \to hh\bar{\nu}\nu$, if the Higgs boson is not too heavy.

At a 500 GeV (3 TeV) e^+e^- collider with an integrated luminosity of 1 ab^{-1} (5 ab^{-1}), λ_{hhh} can be measured to about 20% (7%) accuracy for the Higgs boson mass around 120 GeV.

Kanemura et al.: 2HDM

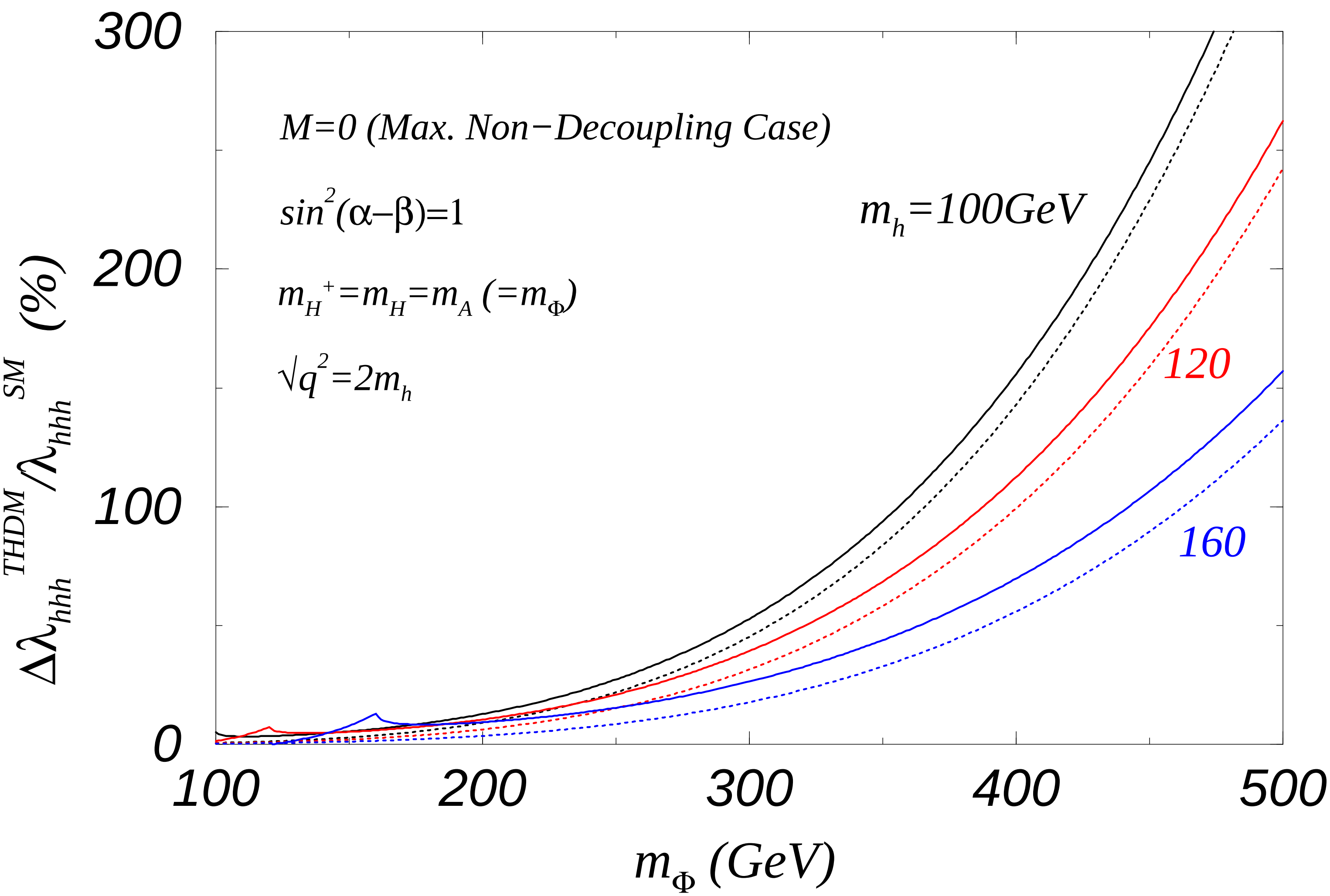
Heavy mass effects in a general THDM do not decouple.

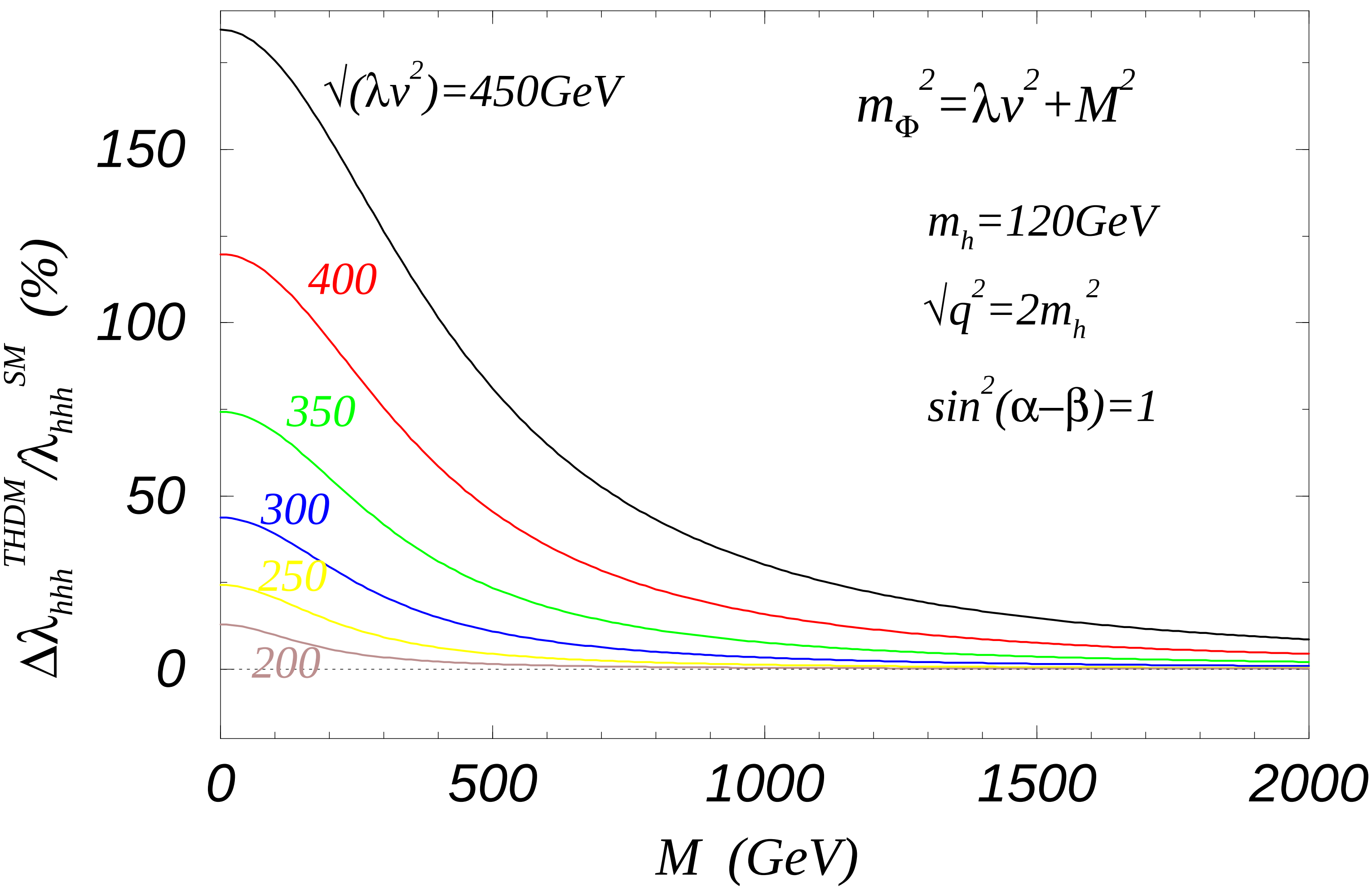
One-loop effect expressed by renormalized mass of h:
quartic dependence on the mass of the heavier Higgs bosons appears in the effective hhh coupling.

For SM-like Higgs basic couplings,
the deviation in the hhh from the SM prediction can be at the order of 100%
non-decoupling effects !.

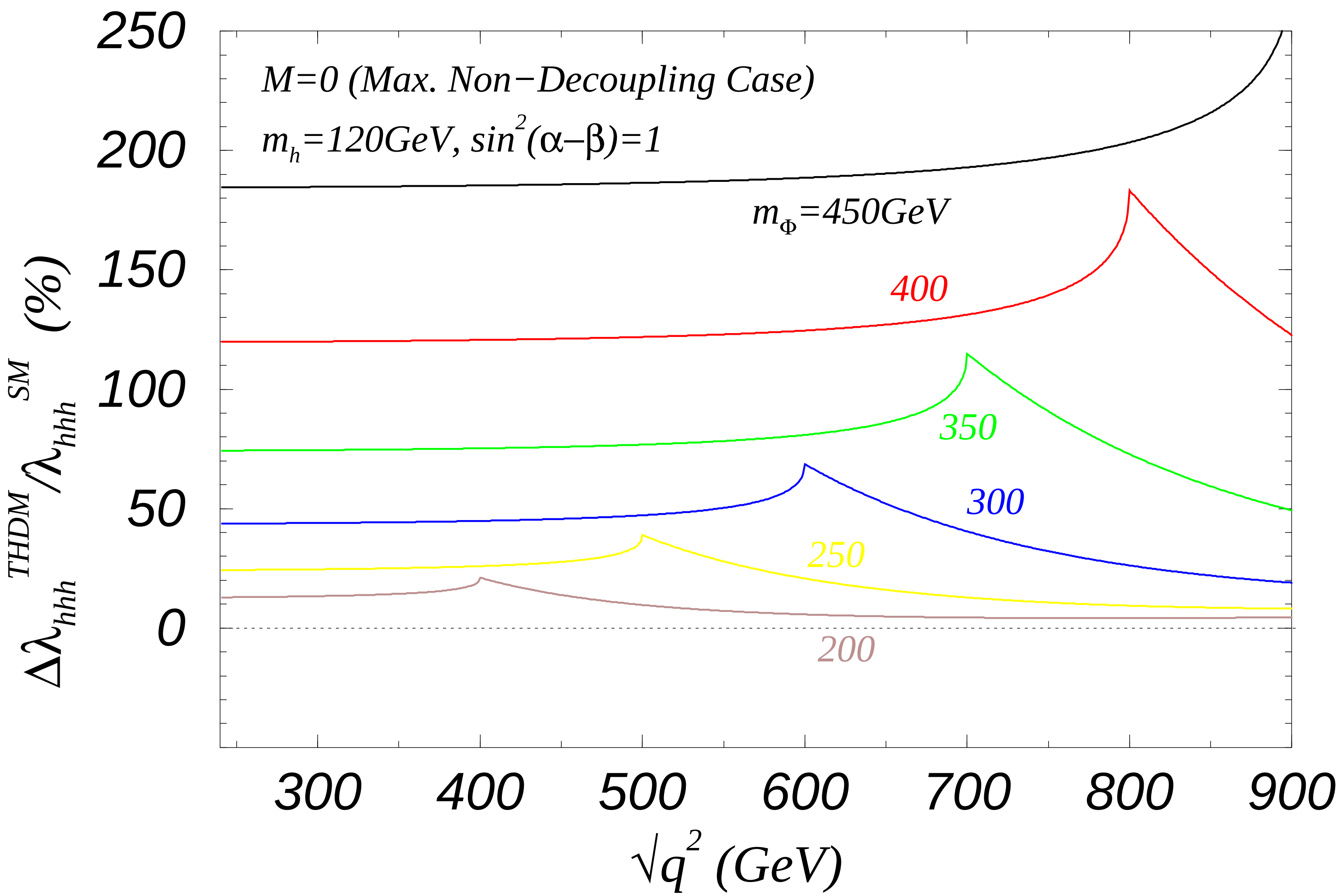
hhh in 2HDM

It depends on the mixing angles and the soft-breaking scale of the discrete symmetry μ !

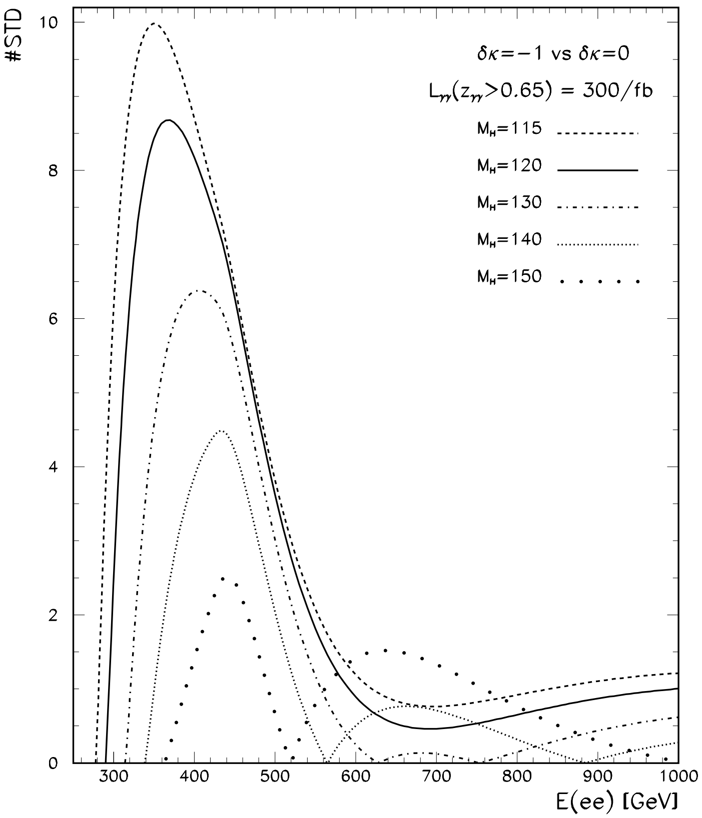

M=0 (Max. Non−Decoupling Case)
sin²(α−β)=1
m_H+=m_H=m_A (=m_Φ)
√q²=2m_h
m_h=100GeV
120
160
300
200
100
0
100
200
300
400
500
Δλ_hhh^THDM/λ_hhh^SM (%)
m_Φ (GeV)

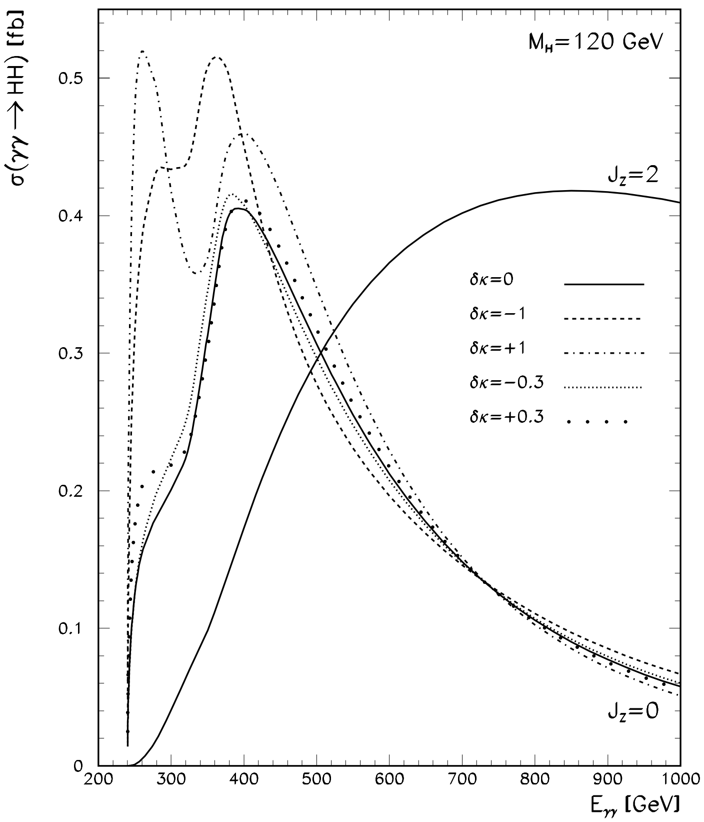


√(λv²)=450GeV
m_Φ²=λv²+M²
m_h=120GeV
√q²=2m_h²
sin²(α−β)=1
400
350
300
250
200
150
100
50
0
0
500
1000
1500
2000
M (GeV)
Δλ_hhh^THDM/λ_hhh^SM (%)

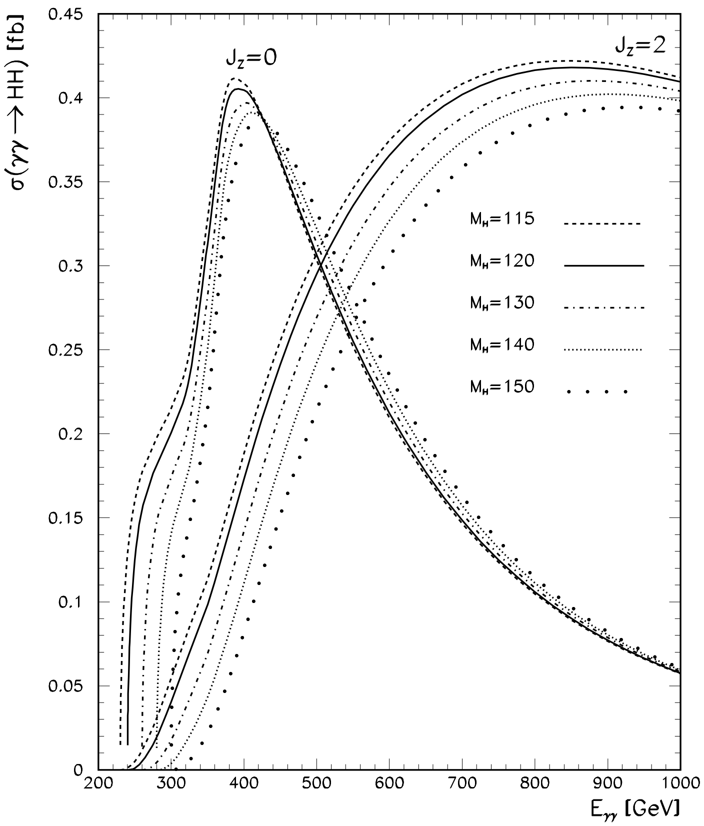

M=0 (Max. Non−Decoupling Case)
m_h=120GeV, $sin^2(\alpha-\beta)$=1
m_Φ=450GeV
400
350
300
250
200
250
200
150
100
50
0
300
400
500
600
700
800
900
$\Delta\lambda_{hhh}^{THDM}/\lambda_{hhh}^{SM}$ (%)
$\sqrt{q^2}$ (GeV)

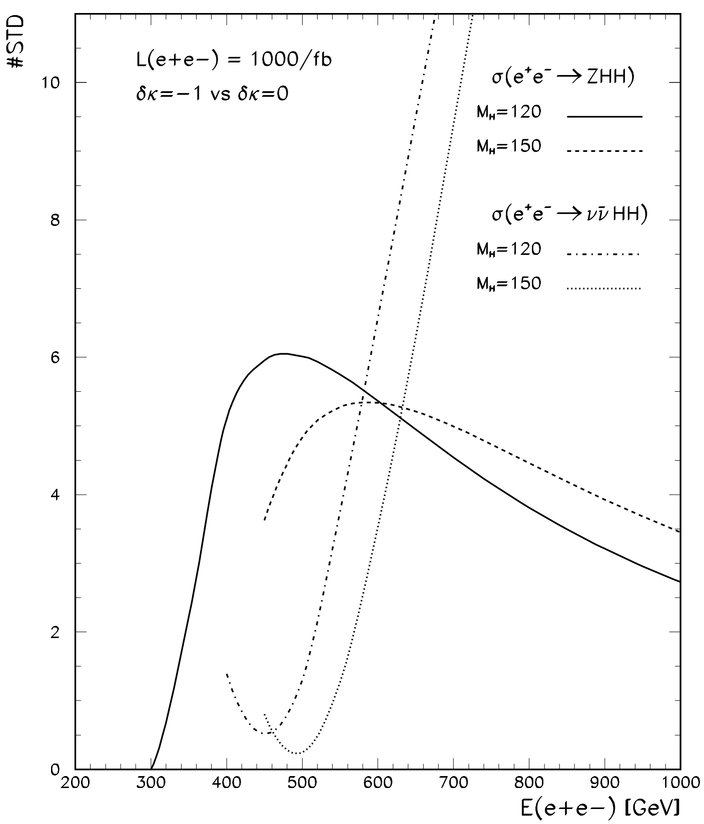
Belusevic and Jikia: LC and PLC

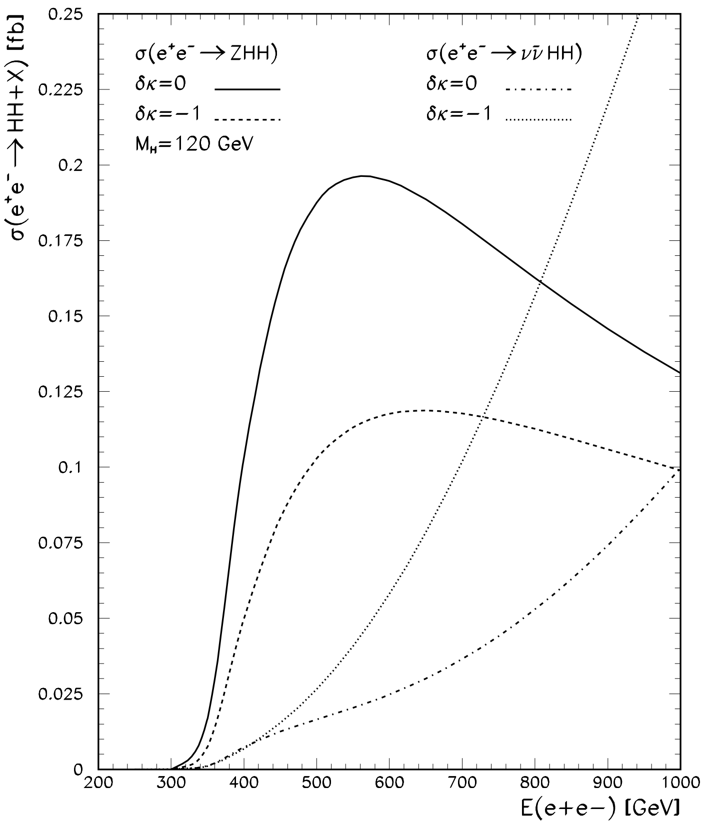




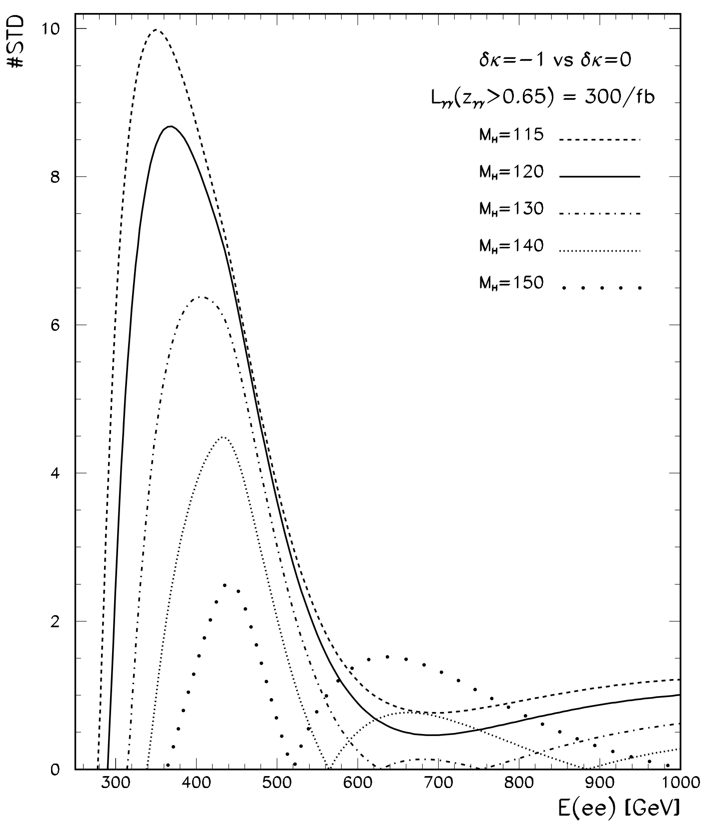
$\sigma(\gamma\gamma \to HH)$ [fb]
M_H=120 GeV
J_z=2
J_z=0
$\delta\kappa$=0
$\delta\kappa$=−1
$\delta\kappa$=+1
$\delta\kappa$=−0.3
$\delta\kappa$=+0.3
$E_{\gamma\gamma}$ [GeV]

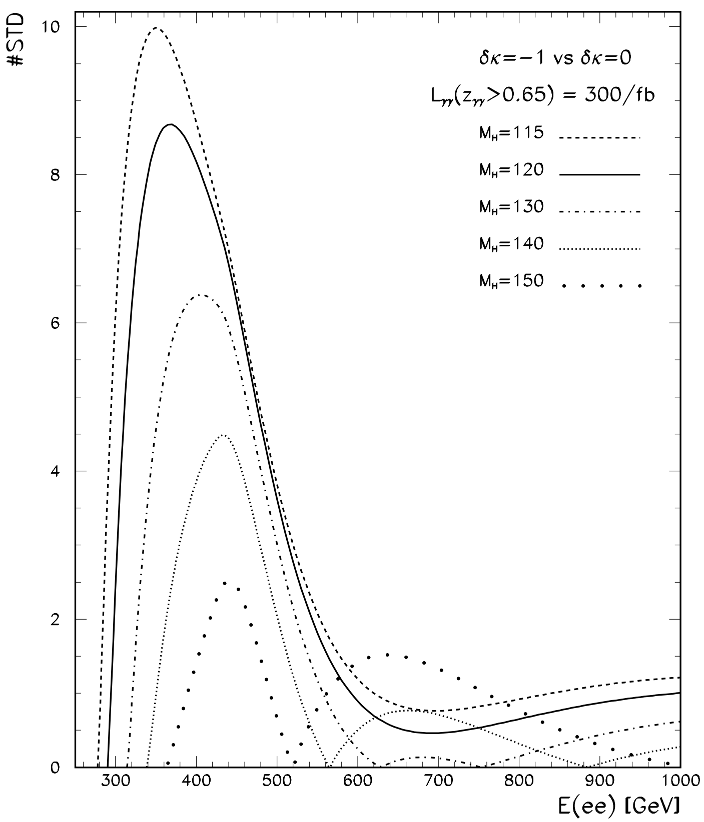

$\sigma(\gamma\gamma \to HH)$ [fb]
J_z=0
J_z=2
M_H=115
M_H=120
M_H=130
M_H=140
M_H=150
$E_{\gamma\gamma}$ [GeV]


#STD
L(e+e−) = 1000/fb
δκ=−1 vs δκ=0
σ(e+e− → ZHH)
MH=120
MH=150
σ(e+e− → νν̄ HH)
MH=120
MH=150
E(e+e−) [GeV]

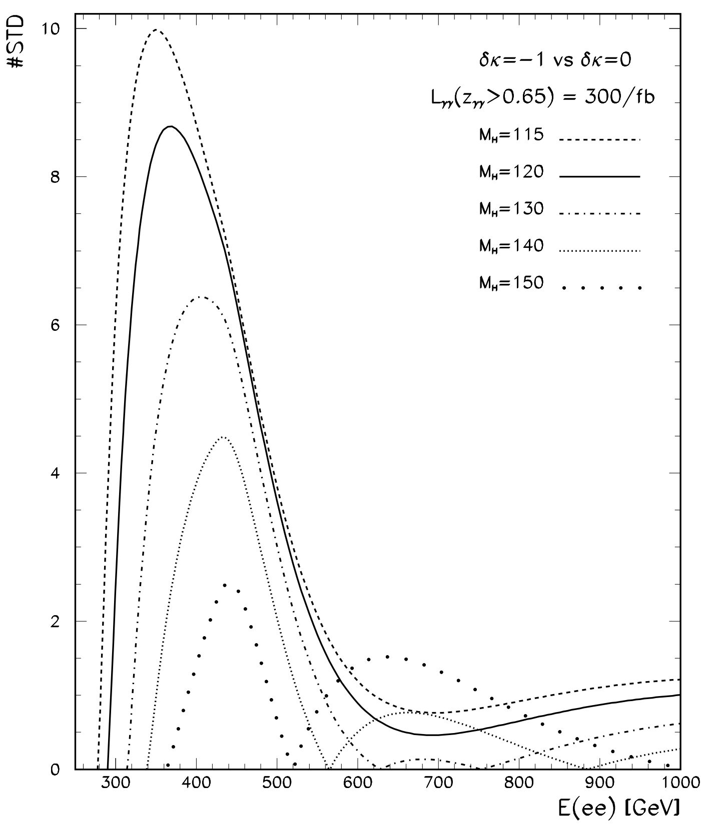


0.25
0.225
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
200
300
400
500
600
700
800
900
1000
$\sigma(e^+e^- \to HH+X)$ [fb]
E(e+e−) [GeV]
$\sigma(e^+e^- \to ZHH)$
$\delta\kappa=0$
$\delta\kappa=-1$
$M_H=120$ GeV
$\sigma(e^+e^- \to \nu\bar{\nu}\,HH)$
$\delta\kappa=0$
$\delta\kappa=-1$


#STD
10
8
6
4
2
0
$\delta\kappa$=−1 vs $\delta\kappa$=0
$L_{\gamma\gamma}(z_{\gamma\gamma}$>0.65) = 300/fb
M_H=115
M_H=120
M_H=130
M_H=140
M_H=150
300
400
500
600
700
800
900
1000
E(ee) [GeV]



#STD
δκ=−1 vs δκ=0
Lγγ(zγγ>0.65) = 300/fb
MH=115
MH=120
MH=130
MH=140
MH=150
E(ee) [GeV]


#STD
δκ=−1 vs δκ=0
Lγγ(zγγ>0.65) = 300/fb
MH=115
MH=120
MH=130
MH=140
MH=150
E(ee) [GeV]
10
8
6
4
2
0
300
400
500
600
700
800
900
1000

Higgs selfcouplings study at future collider important