Is $\tan\beta$ a physical parameter?

Reparametrization and rephasing

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In 2HDM and MSSM parameter $\tan\beta = v_2/v_1$

at the same footing as masses of Higgs bosons
- has it the same status?

Keywords are:

Freedom of description of Higgs sector

Mixing of fields

Assumptions to simplify discussion:

no mixing in kinetic terms

no mixing in Yukawa interaction

Spontaneous electroweak symmetry breaking in SM

A spontaneous electroweak symmetry breaking of $SU(2) \times U(1)$ (EWSB) via the Higgs mechanism is described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{gf}}^{SM} + \mathcal{L}_H + \mathcal{L}_Y\,.$$

$\mathcal{L}_{\text{gf}}^{SM}$ - $SU(2) \times U(1)$ Standard Model interaction of gauge bosons and fermions

$\mathcal{L}_H$ - the Higgs scalar Lagrangian

$\mathcal{L}_Y$ - the Yukawa interactions of fermions with Higgs scalars.

In the Standard Model (SM) - the single scalar isodoublet with hypercharge $Y = 1$

$$\mathcal{L}_H = (D_\mu \phi)^\dagger D_\mu \phi - V,\ V = \lambda\phi^4/2 - m^2\phi^2/2$$

The vacuum expectation value v provides a minimum of the potential, $\langle\phi\rangle = v/\sqrt{2} = \sqrt{m^2/2\lambda}$.

2HDM $\Rightarrow$ two Higgs SU(2) doublets

Two scalar (spin zero) doublets ϕ_1 and ϕ_2

Degrees of freedom: 8 fields - 3 (for long.components of W/Z) = 5 $\Rightarrow$
five physical Higgs bosons, spin=0, 3 - neutral, two charged !!

Other charecteristics: quantum numbers - C,P...,

Number of parameters? Depends on the form of potential,
between 8 to 14

CP or not CP concervation in Higgs sector

• CP conservation: Higgs sector: h,H,A, $H^{\pm}$; $\tan\beta, \alpha$ (h,H)
h,H - CP-even, A - CP-odd

• CP violation: mixing between h_1, h_2, h_3, more mixing angles
and CP parity of Higgs bosons - not defined

more on 2HDM

Interaction with gauge bosons and fermions- Higgs bosons share obligations, eg

$$(g_W^h)^2 + (g_W^H)^2 + (g_W^A)^2 = (g_W^{H_{SM}})^2$$

Various models of Yukawa interaction with fermions:
Model II: where one scalar doublet couples to up-type quarks, other to down-type quarks and charged leptons

The potential problems related to ϕ_1, ϕ_2 mixing:

•Flavour Changing Neutral Current may be large (in nature - FCNC small)

•CP-violation may be large (in nature - small effects)

Symmetries of Two Higgs Doublet Model

Ilya F. Ginzburg, Maria Krawczyk, hep-ph/0408011

• 2HDM contains two fields, ϕ_1 and ϕ_2, with identical quantum numbers: weak isodoublets ($T = 1/2$) with hypercharges $Y = +1$ ($\rho = M_W^2/(M_Z^2 \cos^2\theta_W)$ equal to 1 at the tree level)

•Global transformations which mix these fields and change the relative phases are allowed without changing physics

• One of the earliest reason for introducing the 2HDM was to describe the phenomenon of CP violation Lee' 73
Glashow and Weinberg'77- the CP violation and the flavour changing neutral currents (FCNC) can be naturally suppressed by imposing in Lagrangian a Z_2 symmetry, that is the invariance on the Lagrangian under the interchange

$$(\phi_1 \leftrightarrow \phi_1, \phi_2 \leftrightarrow -\phi_2) \quad \text{or} \quad (\phi_1 \leftrightarrow -\phi_1, \phi_2 \leftrightarrow \phi_2).$$

This symmetry forbids the $\phi_1 \leftrightarrow \phi_2$ mixing

The most general renormalizable Higgs Lagrangian (2HDM)

$$\mathcal{L}_H = T - V\,,$$

T - the kinetic term, V is the Higgs potential.

$$T = (D_\mu\phi_1)^\dagger(D^\mu\phi_1)+(D_\mu\phi_2)^\dagger(D^\mu\phi_2)+\varkappa(D_\mu\phi_1)^\dagger(D^\mu\phi_2)+\varkappa^*(D_\mu\phi_2)^\dagger(D^\mu\phi_1)\,,$$

(D_μ - the covariant derivative containing the EW gauge fields)

Mixing possible... we will neglect this today

2HDM potential

2HDM Potential: quartic and quadratic terms separated:

$$
\begin{aligned}
V = & \tfrac{1}{2}\lambda_1(\phi_1^\dagger\phi_1)^2 + \tfrac{1}{2}\lambda_2(\phi_2^\dagger\phi_2)^2 \\
& + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \tfrac{1}{2}\left[\lambda_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] \\
& + \left\{\left[\lambda_6(\phi_1^\dagger\phi_1) + \lambda_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\} \\
& - \tfrac{1}{2}\left\{m_{11}^2(\phi_1^\dagger\phi_1) + \left[m_{12}^2(\phi_1^\dagger\phi_2) + \text{h.c.}\right] + m_{22}^2(\phi_2^\dagger\phi_2)\right\}
\end{aligned}
$$

soft violation of Z_2 symmetry

No (ϕ_1, ϕ_2) mixing if Z_2 symmetry satisfied (NO FCNC & NO CPV):
$\phi_1 \to -\phi_1$, $\phi_2 \to \phi_2$ (or vice versa) $\Rightarrow \lambda_6 = \lambda_7 = m_{12}^2 = 0$

14 parameters: $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, $\lambda_5, \lambda_6, \lambda_7$, m_{11}^2, m_{22}^2, Re m_{12}^2,Im m_{12}^2

Hard violation of Z_2 symmetry: quartic terms with λ_6, λ_7

Lee, Diaz-Cruz, Mendez, Haber, Pomarol, Barroso, Santos, Hollik, Djouadi, Illana, Branco,Gunion, Akeroyd, Arhrib,...

Reparametrization and rephasing

Transformation of fields

Our Lagrangian contains two fields (doublets) with identical quantum numbers: a global unitary transformations $\hat{\mathcal{F}}$ which mix these fields and change the phases:

$$\begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix} = \hat{\mathcal{F}} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \hat{\mathcal{F}} = e^{-i\rho_0} \begin{pmatrix} \cos\theta\, e^{i\rho/2} & \sin\theta\, e^{i(\tau-\rho)/2} \\ -\sin\theta\, e^{-i(\tau-\rho)/2} & \cos\theta\, e^{-i\rho/2} \end{pmatrix}. \quad (1)$$

$\boxed{\hat{\mathcal{F}} = \hat{\mathcal{F}}(\rho_0, \rho, \tau, \theta)}$ (all prameters real) ρ_0 - a overall phase

PHASE ROTATION AND MIXING OF FIELDS (changing field basis)

The particular case with $\theta = 0$

- a global transformation of fields with the independent phase rotations:

$$\phi_{1,2} \to e^{-i\rho_{1,2}}\phi_{1,2}; \quad \rho_1 = \rho_0 - \rho/2, \;\; \rho_2 = \rho_0 + \rho/2, \;\; \rho = \rho_2 - \rho_1 .$$

PHASE ROTATION FOR FIELDS (rephasing fields)

Transformation of Lagrangians

Let Λ - represents all parameters of L: $(\lambda's, m_{ij}^2)$
Let Φ -represents doublets (fields ϕ_1, ϕ_2)

Write

$$L = L(\Lambda \times \Phi)$$

Go from Φ to Φ' then $L \to L'(\Lambda \times \Phi')$ without changing physical content however if

$$L \to L'(\Lambda \times \Phi') \to L'(\Lambda \times \Phi') = L'(\Lambda' \times \Phi)$$

But L and L' - which differ by set of parameters Λ, describe the same physics $\rightarrow$ REPARAMETRIZATION INVARIANCE

The transformation Λ to Λ' - reparametrization transformation (or reparametrization) - given by $(\rho, \tau, c = \cos\theta, s = \sin\theta)$

$$\lambda_1 \to \lambda_1' = c^2\lambda_1 + s^2\lambda_2 - cs\Pi - 2cs\,\mathrm{Re}(\tilde\lambda_6 + \tilde\lambda_7)\,,$$
$$\lambda_2 \to \lambda_2' = s^2\lambda_1 + c^2\lambda_2 - cs\Pi + 2cs\,\mathrm{Re}(\tilde\lambda_6 + \tilde\lambda_7)\,,$$
$$\lambda_3 \to \lambda_3' = \lambda_3 + cs\Pi\,, \qquad \lambda_4 \to \lambda_4' = \lambda_4 + cs\Pi\,,$$
$$\lambda_5 \to \lambda_5' = e^{-2i\rho}\Big\{\lambda_5 + e^{i\tau}\Big[cs\Pi + 2is^2\,\mathrm{Im}\,\tilde\lambda_5 - 2ics\,\mathrm{Im}(\tilde\lambda_6 - \tilde\lambda_7)\Big]\Big\}\,,$$
$$\lambda_6 \to \lambda_6' = e^{-i\rho}\left[c^2\lambda_6 - s^2\lambda_7 + \frac{e^{i\tau/2}}{2}cs(\lambda_1 - \lambda_2) + \Psi\right]\,,$$
$$\lambda_7 \to \lambda_7' = e^{-i\rho}\left[c^2\lambda_7 - s^2\lambda_6 + \frac{e^{i\tau/2}}{2}cs(\lambda_1 - \lambda_2) - \Pi\right]\,,$$
$$m_{11}^2 \to \tilde m_{11}^2 = c^2 m_{11}^2 + s^2 m_{22}^2 - 2cs\mu_{12}^2\,, \quad m_{22}^2 \to \tilde m_{22}^2 = s^2 m_{11}^2 + c^2 m_{22}^2 + 2cs\mu_{1}^{2}$$
$$m_{12}^2 \to \tilde m_{12}^2 = e^{-i\rho}\Big\{m_{12}^2 + e^{i\tau/2}\Big[cs(m_{11}^2 - m_{22}^2) - 2s^2\mu_{12}^2\Big]\Big\}\,.$$

$$\tilde\lambda_5 = \lambda_5 e^{-i\tau}\,, \quad \tilde\lambda_{6,7} = \lambda_{6,7}e^{-i\tau/2}\,, \quad \mu_{12}^2 = \mathrm{Re}\,m_{12}^2 e^{-i\tau/2}$$

$$\Pi = cs[\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \mathrm{Re}\,\lambda_5)] + 2(c^2 - s^2)\,\mathrm{Re}(\tilde\lambda_6 - \tilde\lambda_7)$$

$$\Psi = e^{i\tau/2}\left[\frac{(c^2 - s^2)}{2}\Pi - \mathrm{Re}(\tilde\lambda_6 - \tilde\lambda_7) + ics\,\mathrm{Re}\,\tilde\lambda_5\right]$$

Reparametrization invariance

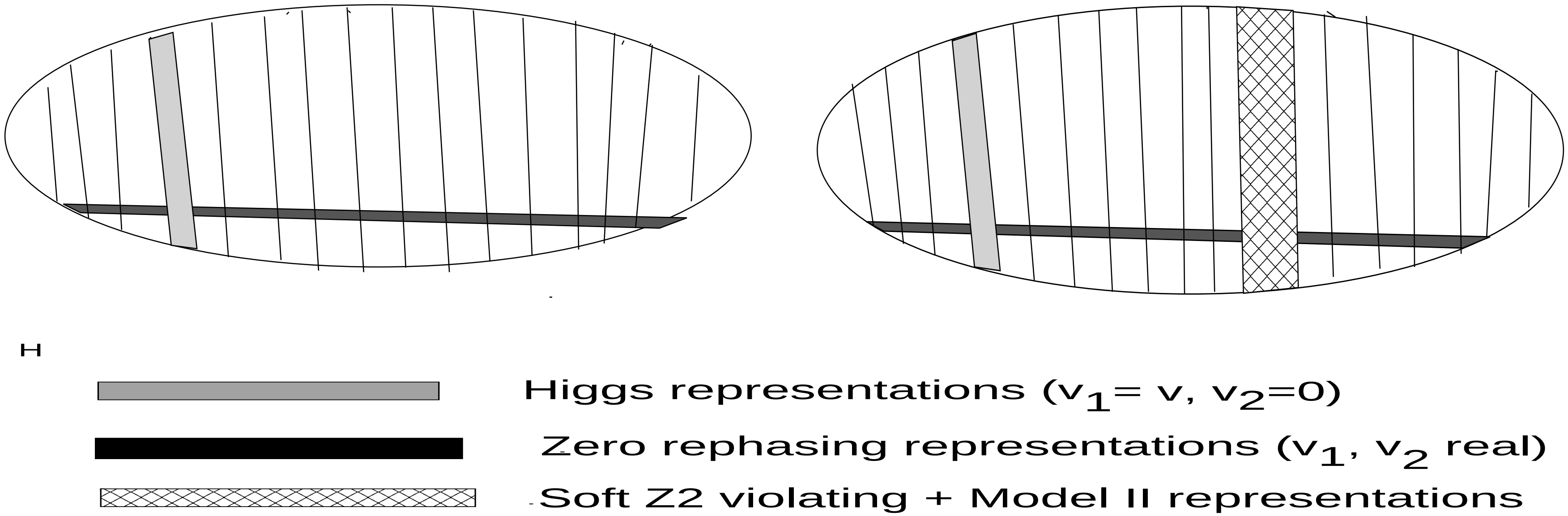
Reparametrization invariance - similar to *renormalization invariance* with set of normalization points - set of parameters $(\rho_0, \rho, \tau, \theta) \rightarrow$ set of parameters Λ

Reparametrization transformation forms the 3–parametrical **reparametrization group, operating within the space of Lagrangians** with coordinates given by λ_i, m_{ij}^2.

A set of these physically equivalent Higgs Lagrangians forms *the reparametrization equivalent space*, which is subspace of the entire space of Lagrangians. Some specific set of $\lambda' s$, m_{ij}, $\varkappa$ defines a specific *reparametrization scheme* (specific choice of normalization point in the renormalization group approach.).

In some specific reparametrization scheme some property of model are explicit In a soft Z_2 violating reparametrization scheme
Lagrangian has explicit soft violation of Z_2 symmetry (by transformation we can generate nonzero $\lambda_{6,7}$)

[h]



Schematic presentation of reparametrization equivalent Lagrangian space. *Left panel:* **hard violation of Z_2 symmetry**, *soft Z_2 violating representations cannot be realized.* *Right panel:* **(hidden) soft violation of Z_2 symmetry (or exact Z_2 symmetry)**, *soft Z_2 violating scheme*

Physical parameters

Some parameters of theory which are treated often as physical (and in principle measurable) are in fact reparametrization dependent!

A ratio of vacuum expectation values of scalar fields, $\tan\beta$ - under the transformation with $\rho = \xi$ and $\tau = 0$, angle β changes to $\beta + \theta$.

Rephasing invariance

A particular reparametrization with $\theta = 0$ is equivalent to a change of phase of the *complex* parameters of Lagrangian (sorry here -ρ):

$$\begin{gathered}\lambda_{1-4} \to \lambda_{1-4}\,, \quad m_{11}^2 \to m_{11}^2\,, \quad m_{22}^2 \to m_{22}^2\,, \\ \lambda_5 \to \lambda_5\, e^{2i\rho}\,, \;\; \lambda_{6,7} \to \lambda_{6,7}\, e^{i\rho}\,, m_{12}^2 \to m_{12}^2 e^{i\rho}, \;\; \varkappa \to \varkappa\, e^{i\rho}\,.\end{gathered} \tag{3}$$

The physical pictures described by L and L' are identical - this property is called **a rephasing invariance**.

The set of transformations - the 1–parametrical **rephasing group** with parameter ρ, *the rephasing parameter* subgroup of the reparametrization group

Rephasing equivalent space of Lagrangians

A specific choice of the of Lagrangian parameters defines a specific *rephasing scheme*.

Remarks

The rephasing (and reparametrization) invariance can be extended to the description of a whole system of scalars and fermions if the corresponding transformations for the Yukawa terms (phases of fermion fields and Yukawa couplings) supplement the transformations.

The complex values of some of these parameters in L provide *a necessary condition* for the CP violation in the Higgs sector. Obviously, this violation does not appear if the Lagrangian with complex parameters can be transformed to a form with all real parameters by means of some reparametrization. Certainly, this is possible only if parameters of the initial Lagrangian were interrelated.

The Vacuum and Special Form of the Potential

The minimum of the potential defines the vacuum expectation values (v.e.v) of the fields ϕ_i:

$$\left.\frac{\partial V}{\partial \phi_1}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0, \qquad \left.\frac{\partial V}{\partial \phi_2}\right|_{\substack{\phi_1=\langle\phi_1\rangle,\\ \phi_2=\langle\phi_2\rangle}} = 0. \tag{4}$$

In order to describe the $U(1)$ symmetry of electromagnetism and using the overall phase freedom of the Lagrangian to choose one vacuum expectation value real we take:

$$\langle\phi_1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_1\end{pmatrix} \quad \text{and} \quad \langle\phi_2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v_2 e^{i\xi}\end{pmatrix}, \tag{5}$$

with a relative phase ξ. These v_i obey SM constraint: $v_1^2 + v_2^2 = v^2$, with $v = (\sqrt{2}G_\text{F})^{-1/2} = 246$ GeV. The another parameterization of these v.e.v.'s (via parameters v and β) is also used:

$$v_1 = v\cos\beta, \quad v_2 = v\sin\beta, \quad \beta \in \left(0, \frac{\pi}{2}\right). \tag{6}$$

Rephasing invariants and vacuum condition.

The phase difference ξ between the v.e.v.'s is often interpreted as a spontaneous CP violation of vacuum. However, this is not necessarily the case..

Note that under the rephasing transformation the ξ changes to:

$$\xi \to \xi - \rho\,. \tag{7}$$

following quantities

$$\overline{\lambda}_{1-4} = \lambda_{1-4}, \quad \overline{\lambda}_5 \equiv \lambda_5 e^{2i\xi}, \quad \overline{\lambda}_6 \equiv \lambda_6 e^{i\xi}, \quad \overline{\lambda}_7 \equiv \lambda_7 e^{i\xi}, \quad \overline{m}_{12}^2 \equiv m_{12}^2 e^{i\xi} \tag{8}$$

are the rephasing-invariant quantities. Also c

$$\overline{\lambda}_{345} = \overline{\lambda}_3 + \overline{\lambda}_4 + \mathrm{Re}\,\overline{\lambda}_5, \;\; \overline{\lambda}_{67} = \frac{v_1}{v_2}\overline{\lambda}_6 + \frac{v_2}{v_1}\overline{\lambda}_7, \;\; \tilde{\lambda}_{67} = \frac{1}{2}\left(\frac{v_1}{v_2}\overline{\lambda}_6 - \frac{v_2}{v_1}\overline{\lambda}_7\right),$$

$$\overline{m}_{12}^2 = 2v_1 v_2(\nu + i\delta). \tag{9}$$

The minimum condition does not constrain $\nu = \mathrm{Re}(\overline{m}_{12}^2)/(2v_1v_2)$, while it relates the imaginary part of $\overline{m}_{12}$, $\delta = \mathrm{Im}(\overline{m}_{12}^2)/(2v_1v_2)$, to the $\mathrm{Im}(\overline{\lambda}_{5-7})$;

$$\delta = \mathrm{Im}\left\{ \underbrace{0}_{Z_2\ sym} + \underbrace{\frac{\overline{\lambda}_5}{2}}_{soft} + \underbrace{\frac{\overline{\lambda}_{67}}{2}}_{hard} \right\}. \tag{10}$$

The same minimum condition allows to express $m_{11,22}^2$ via $\overline{\lambda}$'s, v_j and the parameter ν: c

$$\mathrm{m}_{11}^2 = \underbrace{\overline{\lambda}_1 v_1^2 + \overline{\lambda}_{345} v_2^2}_{Z_2\ sym} \underbrace{-2\nu v_2^2}_{soft} + \underbrace{\frac{v_2}{v_1}\mathrm{Re}\left(3v_1^2\overline{\lambda}_6 + v_2^2\overline{\lambda}_7\right)}_{hard},$$

$$\mathrm{m}_{22}^2 = \underbrace{\overline{\lambda}_2 v_2^2 + \overline{\lambda}_{345} v_1^2}_{Z_2\ sym} \underbrace{-2\nu v_1^2}_{soft} + \underbrace{\frac{v_1}{v_2}\mathrm{Re}\left(v_1^2\overline{\lambda}_6 + 3v_2^2\overline{\lambda}_7\right)}_{hard}.$$

Special form of potential.

The rephasing transformation with $\rho = \xi$ transforms the initial Higgs potential to a new form, in which the relative phase of v.e.v.'s become equal 0 *(a real vaccum scheme)* and all rephasing dependent parameters are changed to their rephasing invariant forms

$$
\begin{aligned}
\overline{V} = {} & \frac{\overline{\lambda}_1}{2}\left[(\phi_1^\dagger\phi_1) - \frac{v_1^2}{2}\right]^2 + \frac{\overline{\lambda}_2}{2}\left[(\phi_2^\dagger\phi_2) - \frac{v_2^2}{2}\right]^2 + \overline{\lambda}_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \overline{\lambda}_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) \\
& + \frac{1}{2}\left[\overline{\lambda}_5(\phi_1^\dagger\phi_2)^2 + \text{h.c.}\right] + \left\{\left[\overline{\lambda}_6(\phi_1^\dagger\phi_1) + \overline{\lambda}_7(\phi_2^\dagger\phi_2)\right](\phi_1^\dagger\phi_2) + \text{h.c.}\right\} \\
& - \frac{1}{2}\left(\overline{\lambda}_{345} + \text{Re}\,\overline{\lambda}_{67}\right)[v_2^2(\phi_1^\dagger\phi_1) + v_1^2(\phi_2^\dagger\phi_2)] - \text{Re}[\overline{\lambda}_6(\phi_1^\dagger\phi_1) + \overline{\lambda}_7(\phi_2^\dagger\phi_2)]v_1v_2 \\
& + \nu(v_2\phi_1 - v_1\phi_2)^\dagger(v_2\phi_1 - v_1\phi_2) + \delta\, 2\,\text{Im}(\phi_1^\dagger\phi_2)v_1v_2\,.
\end{aligned}
\tag{11}
$$

The Lagrangians in a real vacuum scheme form a subspace in the entire reparametrization equivalent space. Different points of this subspace correspond to different values of $\tan\beta$.

Model II

Model II = ϕ_1 couples to d-type quarks and charged leptons ℓ, while ϕ_2 couples to u-type quarks (we take neutrinos to be massless).

This form of Yukawa Lagrangian occurs only in some of reparametrization schemes, which we name *the Model II reparametrization schemes*.

The Lagrangian in scheme of this kind can be obtained from each other by rephasing transformation, and we use here the mentioned above zero rephasing transformation.

Pattern relation, sum rules and other useful relations in Model II

The unitarity of the mixing matrix R allows to obtain a number of useful relations between the relative couplings of neutral Higgs particles to gauge bosons and fermions (*basic relative couplings*).

1. The first of them is *the pattern relation* among the basic relative couplings of *each neutral Higgs particle* h_i :

a) $(\chi_u^{(i)}+\chi_d^{(i)})\chi_V^{(i)} = 1+\chi_u^{(i)}\chi_d^{(i)}$, or b) $(\chi_u^{(i)}-\chi_V^{(i)})(\chi_V^{(i)}-\chi_d^{(i)}) = 1-(\chi_V^{(i)})^2$, (12)

which has the same form for each Higgs boson h_i (in particular also for h, H, A in the case of CP conservation).

The universality of these equations for each neutral Higgs boson h_i can be treated as additional constraints for the couplings to be measured in future experiments.

2. The relations allow also to write for each neutral Higgs boson h_i *a horizontal sum rule*:

$$|\chi_u^{(i)}|^2 \sin^2\beta + |\chi_d^{(i)}|^2 \cos^2\beta = 1\,. \tag{13}$$

These sum rules guarantee that the cross section to produce each neutral Higgs boson h_i (or h, H, A) of the 2HDM, in the processes involving Yukawa interaction, cannot be lower than that for the SM Higgs boson with the same mass

3. The third relation provides *a vertical sum rule* for each basic relative coupling χ_j to *all three neutral Higgs bosons* h_i:

$$\sum_{i=1}^{3}(\chi_j^{(i)})^2 = 1 \qquad (j = V, d, u)\,. \tag{14}$$

For couplings to the gauge bosons this sum rule takes place independently on a particular form of the Yukawa interaction.

4. Besides, the useful *linear relation* :

$$\chi_V^{(i)} = \left\{\begin{matrix} \cos^2\beta\,\chi_d^{(i)*} + \sin^2\beta\,\chi_u^{(i)} = \\ = \cos^2\beta\,\chi_d^{(i)} + \sin^2\beta\,\chi_u^{(i)*} \end{matrix}\right\} \Rightarrow \begin{cases} \chi_V^{(i)} = \mathrm{Re}\left(\cos^2\beta\chi_d^{(i)} + \sin^2\beta\chi_u^{(i)}\right), \\ \quad \mathrm{Im}\left(\cos^2\beta\chi_d^{(i)} - \sin^2\beta\chi_u^{(i)}\right) = 0\,. \end{cases} \tag{15}$$

Some relations take place *only in the Model II reparametrization scheme*. A new relation among the couplings:

$$(1 - |\chi_d^{(i)}|^2)\,\mathrm{Im}\,\chi_u^{(i)} + (1 - |\chi_u^{(i)}|^2)\,\mathrm{Im}\,\chi_d^{(i)} = 0\,. \tag{16}$$

The relations *are reparametrization independent*.

5. One can also express $\tan\beta$, which is a basic parameter of the 2HDM defined in the Model II reparametrization scheme, via the basic relative couplings:

$$\tan^2\beta = \frac{(\chi_V^{(i)} - \chi_d^{(i)})^*}{\chi_u^{(i)} - \chi_V^{(i)}} = \frac{\mathrm{Im}\,\chi_d^{(i)}}{\mathrm{Im}\,\chi_u^{(i)}} = \frac{1 - |\chi_d^{(i)}|^2}{|\chi_u^{(i)}|^2 - 1}\,. \tag{17}$$

Summary- Higgs schemes

We should define schemes for Higgs Lagrangian
as in renormalization procedure

Higgs basis, weak basis - are already use in literature

Now we have a full picture