Metody eksperymentalne w fizyce wysokich energii

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## Wykład XIII

- Metody Monte Carlo

$\Rightarrow$ Symulacja procesów fizycznych

$\Rightarrow$ Symulacja detektorów

MC methods - introduction

- **Invented by Stanislaw Ulam, while working on Manhattan project**
- **When invented they were of purely academic interest, there were no computers in those days**
- **With the advent of computers became very useful**
- **Applied in many areas: science, financial world**
- *The Mathematics of Financial Derivatives : A Student Introduction Paul Wilmott, Sam Howison, J. Dewynne*

Simplest example: calculate area of a figure

- **Cover the figure by a grid, calculate the number of grid cells which are inside and this gives you the area**

- **Shoot at random at the figure. Count the bullets that hit it. The area of then figure is**
- S=(Nhit/Ntotal)*S(rectangle)
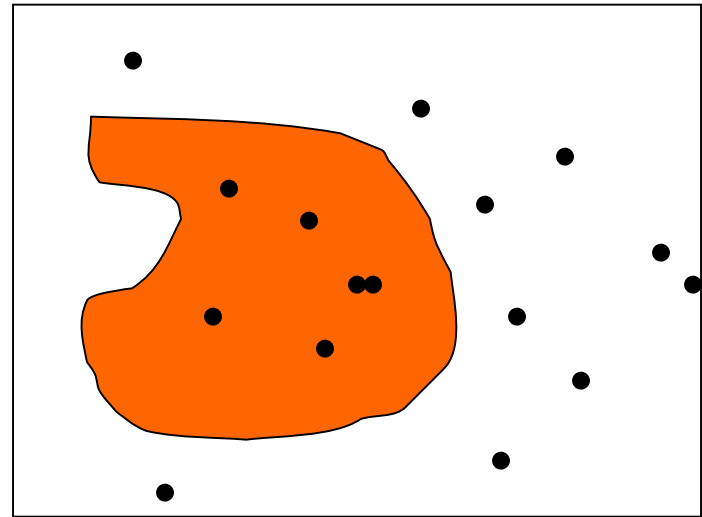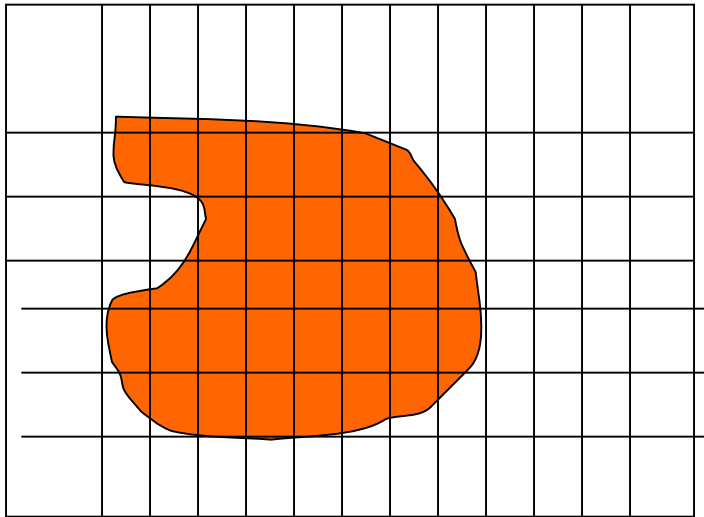
Wprowadzenie

Motywacja

Bardzo rzadko, we współczesnych eksperymentach, interesująca nas wielkość fizyczna może być zmierzona bezpośrednio w detektorze.

Wyniki pomiarów obarczone są:

- fluktuacjami w przebiegu obserwowanego zjawiska
  (np. rozwoju kaskady, wielokrotnego rozpraszania cząstek, ISR)

- wpływem tła (np. pile-up lub nakładające się przypadki)

- błędami pomiarowymi (rozdzielczość aparatury)

- wpływem nieefektywności
  (akceptacja geometryczna, nieefektywności liczników, progi układu wyzwalania)

- błędami rekonstrukcji i identyfikacji cząstek

- niepewnościami systematycznymi (kalibracji, pozycjonowania, itp.)

Wprowadzenie

Motywacja

Często też badany proces jest tak skomplikowany lub tak różnorodny, że interesująca nas wielkość (np. stała sprzężenia) nie może być wprost wyrażona przez wyniki pomiarów.

Dlatego też przeprowadza się symulację komputerową wykonanego doświadczenia, opisującą zarówno przebieg samego procesu fizycznego jak i działanie detektora.

Symulację **Monte Carlo** możemy stosować jedynie gdy znamy (ew. założymy) wszystkie szczegóły procesu fizycznego oraz działania aparatury pomiarowej.

Nie jest to sposób na uproszczenie zagadnienia, czy "zakrycie" naszej niewiedzy.

Teoretycznie, wykorzystując te same informacje moglibyśmy przedstawić oczekiwane wyniki pomiaru w postaci zbioru formuł analitycznych.

Metoda Monte Carlo jest sposobem na policzenie (przesumowanie lub przecałkowanie) takich formuły z dowolną dokładnością, praktycznie ograniczoną jedynie przez czas i szybkość działania komputera.

Przykład

Wyznaczenie akceptacji układu dwóch liczników
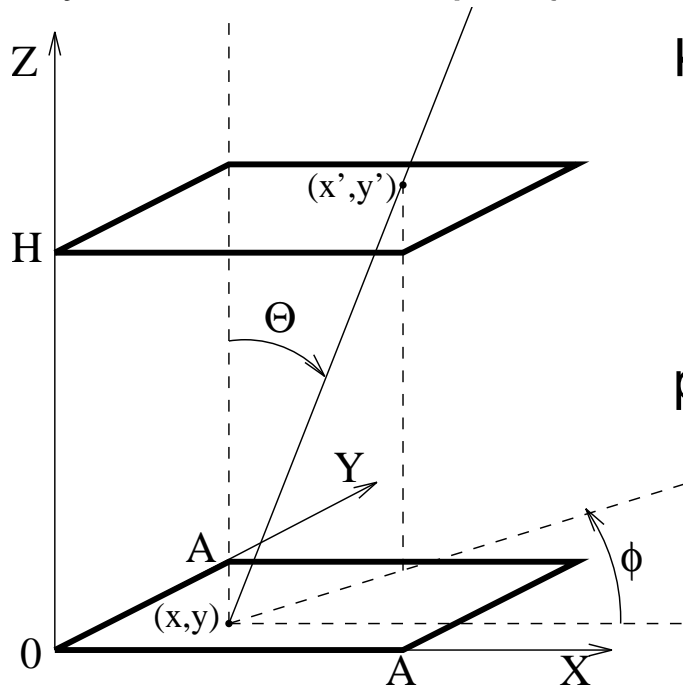
Kąt bryłowy "widziany" przez element dolnego licznika:

$$d\Omega(x,y) \;=\; \int_0^1 d(cos\Theta) \int_0^{2\pi} d\phi \cdot F(x',y')$$

Funkcja $F(x',y')$ wyraża warunek przechodzenia toru przez górny licznik ($0 < x' < A$ & $0 < y' < A$), a

$$x' \;=\; x + H \cdot tg(\theta) \cdot cos(\phi)$$
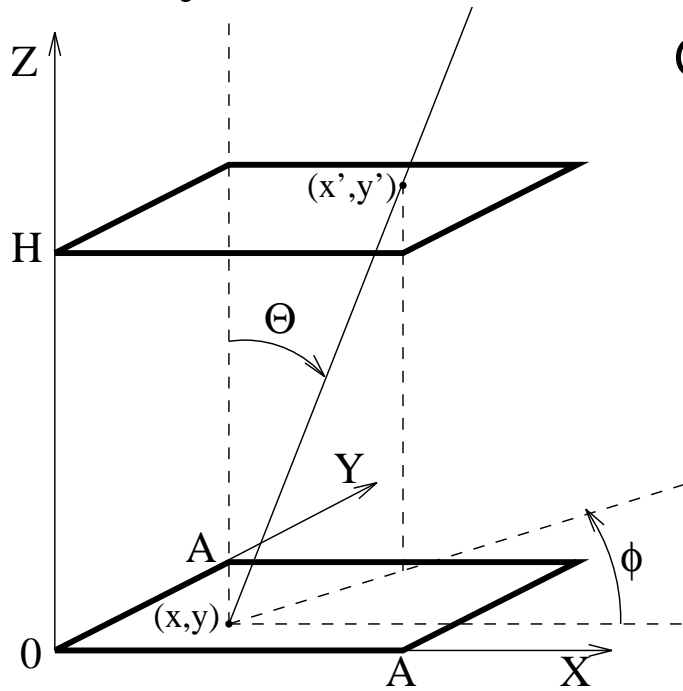$$y' \;=\; y + H \cdot tg(\theta) \cdot sin(\phi)$$

Kąt bryłowy dla całego układu możemy określić jako średnią po powierzchni dolnego licznika:

$$\Omega \;=\; \frac{1}{A^2} \int_0^A dx \int_0^A dy \int_0^1 d(cos\Theta) \int_0^{2\pi} d\phi \cdot F(x',y')$$

Wprowadzenie

Przykład

Metodą Monte Carlo możemy wyznaczyć Ω dużo prościej/szybciej.

Generujemy dużą liczbę N przypadków:

- losujemy punkt (x, y) przejścia cząstki przez dolny licznik (rozkład płaski w x i y)

- losujemy kąty określające kierunek lotu cząstki (θ, ϕ) (rozkład płaski w ϕ i $cos(\Theta)$)

- ekstrapolujemy tor cząstki do punktu przecięcia z płaszczyzną drugiego licznika (x', y')

- zliczamy przypadki dla których tor cząstki przeszedł przez oba liczniki (i.e. wartość $F(x', y')$).

Ω wyznaczone jest przez ułamek przypadków przechodzących przez oba liczniki ($\times 2\pi$, czyli obszar całkowania). Błąd wyniku symulacji maleje jak $1/\sqrt{N}$.

Monte Carlo

Losowanie parametrów

Rozważane parametry losowe rzadko maja płaskie rozkłady prawdopodobieństwa.

$\Rightarrow$ musimy umieć losować parametry z dowolnie skomplikowanych rozkładów.

Naogół stosujemy dedykowane algorytmy...

Metoda analityczna

Prawdopodobieństwo $\rho(x)$ dla zmiennej losowej x z przedziału $[x_1, x_2]$.

Dystrybuanta rozkładu:

$$D(x) \;=\; \int_{x_1}^{x} dx' \cdot \rho(x')$$

Z definicji $u = D(x)$ jest zmienną losową o rozkładzie płaskim w przedziale $[0, 1]$. Zmienną losową x możemy generować jako wartość funkcji odwrotnej do $D(x)$, liczonej dla zmiennej losowej u (losowanej z rozkładu płaskiego):

$$x \;=\; D^{-1}(u)$$

Monte Carlo

Losowanie parametrów

Metoda analityczna jest przydatna zwłaszcza wtedy, gdy zmienna jest określona na nieskończonym przedziale wartości.

Pozwala sprowadzić losowanie z wielu różnych rozkładów (opisanych analitycznie) do losowania z rozkładu płaskiego (standardowy generator liczb losowych).

Przykład: losowanie czasu t po jakim rozpadnie się cząstka o średnim czasie życia τ.

$$\rho(t) = \frac{1}{\tau} \cdot e^{-t/\tau}$$

$$D(t) = 1 - e^{-t/\tau}$$

$$\Rightarrow \quad t = D^{-1}(u)$$

$$t = -\tau \cdot ln(1 - u)$$

Monte Carlo

Losowanie parametrów

Przykład 2: rozkaład normalny (rozkład Gaussa o średniej 0 i dyspersji 1)

"Trick": rozważmy dwie nieskorelowane zmienne - dwuwymiarowy rozkład:

$$\rho(x,y) = \rho(x) \cdot \rho(y) = \frac{1}{2\pi} \cdot e^{-\frac{1}{2}(x^2+y^2)}$$

Przechodząc do współrzędnych biegunowych:

$$\rho(r,\phi) = \frac{1}{2\pi} \cdot r \cdot e^{-\frac{1}{2}r^2}$$

$$\Rightarrow \quad \rho(\phi) = \frac{1}{2\pi} \quad , \quad \rho(r^2) = \frac{1}{2} \cdot e^{-\frac{1}{2}r^2}$$

Losując ϕ z rozkładu płaskiego i r^2 z rozkładu eksponencjalnego ($u_\phi,\ u_r \in [0,1[$):

$$\phi = 2\pi \cdot u_\phi \quad , \quad r = \sqrt{-2 \cdot ln(1-u_r)}$$

$$\Rightarrow \quad x = \sqrt{-2 \cdot ln(1-u_r)} \cdot cos(2\pi \cdot u_\phi) \quad , \quad y = \sqrt{-2 \cdot ln(1-u_r)} \cdot sin(2\pi \cdot u_\phi)$$

Monte Carlo

Losowanie parametrów

Metoda ważenia

Najczęściej mamy do czynienia z rozkładami prawdopodobieństwa,
dla których nie możemy wyznaczyć dystrybuanty/funkcji odwrotnej do dystrybuanty.

$\rho(x)$ - rozkład zmiennej losowej x z przedziału $[x_1, x_2]$.

$\rho_0(x)$ - rozkład pomocniczy,dla którego potrafimy losować analitycznie

$$w(x) \; = \; \frac{1}{W} \cdot \frac{\rho(x)}{\rho_0(x)} \qquad w(x) \stackrel{df}{\in} [0,1]$$

gdzie W jest czynnikiem zapewniającym odpowiednią normalizację $W = \frac{\rho(x)}{\rho_0(x)}\big|_{max}$

- Losujemy zmienną x z rozkładu $\rho_0(x)$.

- Losujemy zmienną u z rozkładu płaskiego w przedziale $[0,1]$.

- Jeśli $u \leq w(x)$ przyjmujemy wartość x jako wylosowaną z rozkładu $\rho(x)$,
  w przeciwnym wypadku powtarzamy cały algorytm

Monte Carlo

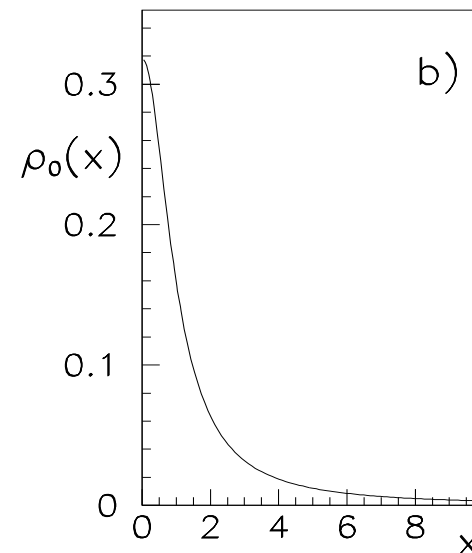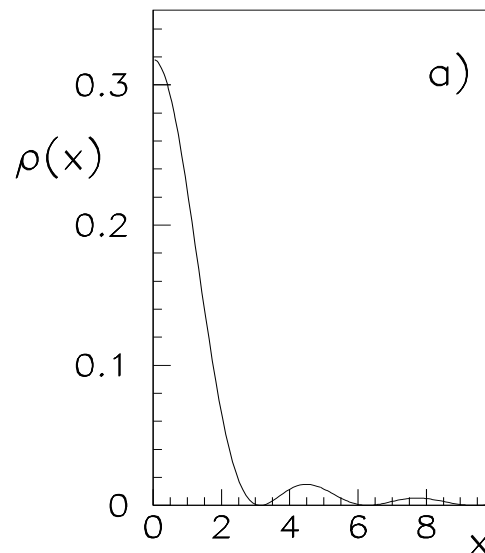## Losowanie parametrów

Przykład:

$$\rho(x) = \frac{1}{\pi} \cdot \left(\frac{sin(x)}{x}\right)^2$$

$$\rho_0(x) = \frac{1}{\pi} \cdot \frac{1}{x^2 + 1}$$

$$\Rightarrow \quad w(x) = \frac{2}{3} \cdot \frac{x^2 + 1}{x^2} \cdot sin^2(x)$$

$$\Rightarrow \quad x = tg(\pi \cdot (u_1 - \frac{1}{2}))$$

jeśli $w(x) \geq u_2$
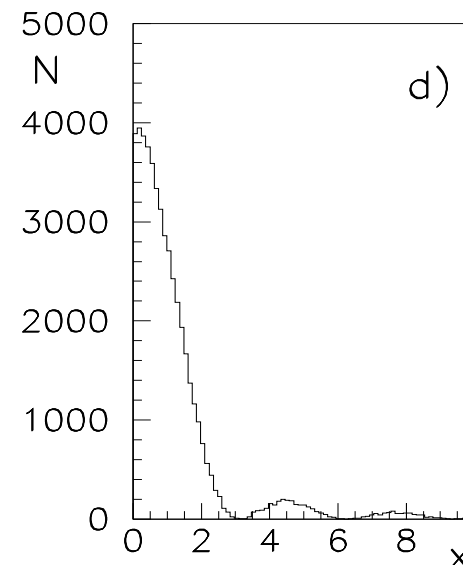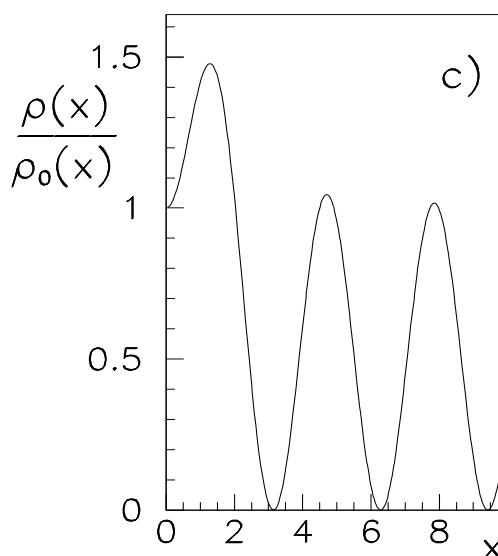
Monte Carlo

Losowanie parametrów

Tabularyzacja

Jeśli mamy zmienną losową, która przyjmuje wartości dyskretne i skończone.
Np: wybór kanału rozpadu dla cząstki.

Losowanie wprost:

- porównujemy zmienną losową u z sumami prawdopodobieństw $s_i = \sum_0^i p_j$

- bierzemy pierwsze i, dla którego $u \leq s_i$

Losowanie z wagami: (przydatne przy dużej liczbie możliwości)

- wybieramy przedział i z rozkładu płaskiego ($i = N * u_1$)

- bierzemy ten przedział jeśli $N * p_i > u_2$, w przeciwnym wypadku $i = C(i)$
  ($C(i)$ - tabela "sprzężonych przedziałów)

Można też stosować do generowania liczb z rozkładu ciągłego, jeśli nie jest potrzebna duża precyzja (tabularyzacja wartości prawdopodobieństwa w przedziałach wartości).

Monte Carlo integration

Convergence of numerical integration

- Consider $I = \int\limits_0^1 \mathrm{d}x^D f(\vec{x})$.

- Convergence behavior crucial for numerical evaluations.
  For integration (N = number of evaluations of f):
  - Trapezium rule $\simeq 1/N^{2/D}$
  - Simpson's rule $\simeq 1/N^{4/D}$
  - Central limit theorem $\simeq 1/\sqrt{N}$.

- Therefore: Use central limit theorem.

Monte Carlo integration

Monte Carlo integration

- Use random vectors $\vec{x}_i \longrightarrow$:
  Evaluate estimate of the integral $\langle I \rangle$ rather than I.
  $$\langle I(f) \rangle = \frac{1}{N} \sum_{i=1}^{N} f(\vec{x}_i).$$
  (This is the original meaning of Monte Carlo: Use random numbers for integration.)

- Quality of estimate given by error estimator (variance)
  $$\langle E(f) \rangle^2 = \frac{1}{N-1} \left[\langle I^2(f) \rangle - \langle I(f) \rangle^2 \right].$$

- Name of the game: Minimize $\langle E(f) \rangle$.

- Problem: Large fluctuations in integrand f

- Solution: Smart sampling methods

Monte Carlo integration

Importance sampling

Basic idea: Put more samples in regions, where f largest

$\implies$ improves convergence behavior

(corresponds to a Jacobian transformation).

- Assume a function $g(\vec{x})$ similar to $f(\vec{x})$;

- obviously then, $f(\vec{x})/g(\vec{x})$ is comparably smooth, hence $\langle E(f/g) \rangle$ is small.

$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$
$$= 0.637 \pm 0.308/\sqrt{N}$$

$$I = \int_0^1 dx (1-x^2)\frac{\cos\frac{\pi}{2}x}{1-x^2}$$
$$= \int d\rho \frac{\cos\frac{\pi}{2}x}{1-x^2}[x(\rho)]$$
$$= 0.637 \pm 0.032/\sqrt{N}$$
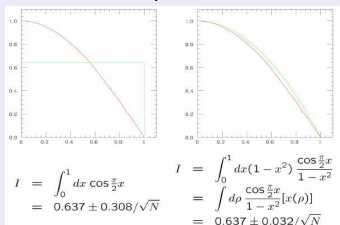
Monte Carlo integration

Stratified sampling

Basic idea: Decompose integral in M sub-integrals
$$\langle I(f)\rangle = \sum_{j=1}^{M}\langle I_j(f)\rangle, \quad \langle E(f)\rangle^2 = \sum_{j=1}^{M}\langle E_j(f)\rangle^2$$
Then: Overall variance smallest, if "equally distributed".
$\implies$ Sample, where the fluctuations are.

- Divide interval in bins;
- adjust bin-size or weight per bin such that variance identical in all bins.
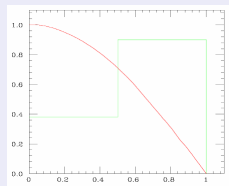
$\langle I\rangle = 0.637 \pm 0.147/\sqrt{N}$

Monte Carlo integration

Example for stratified sampling: VEGAS

- Assume m bins in each dimension of $\vec{x}$.

- For each bin k in each dimension $\eta \in [1, n]$ assume a **weight (probability)** $\alpha_k^{(\eta)}$ for x_k to be in that bin.

  Condition(s) on the weights:
  $\alpha_k^{(\eta)} \in [0, 1]$, $\sum_{k=1}^{m} \alpha_k^{(\eta)} = 1$.

- For each bin in each dimension calculate $\langle I_k^{(\eta)} \rangle$ and $\langle E_k^{(\eta)} \rangle$.

  Obviously, for all η, $\langle I \rangle = \sum_{k=1}^{m} \langle I_k^{(\eta)} \rangle$, but error estimates different.

- In each dimensions, iterate and update the $\alpha_k^{(\eta)}$; example for updating:

  $$\alpha_k^{(\eta)}(\text{rm new}) \propto \alpha_k^{(\eta)}(\text{rm old}) \left(\frac{E_k^{(\eta)}}{E_{\text{tot.}}^{(\eta)}} \right)^{\kappa}.$$

- Problem with this simple algorithm:
  Gets a hold only on fluctuations $\parallel$ to binning axes.
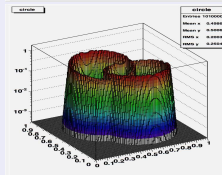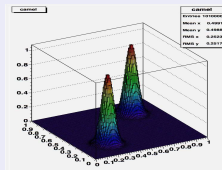
Monte Carlo integration

Multichannel sampling

Basic idea: Use a sum of functions $g_i(\vec{x})$ as Jacobian $g(\vec{x})$.

$\implies$ $g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x})$;

$\implies$ condition on weights like stratified sampling;

("Combination" of importance & stratified sampling).

Algorithm for one iteration:

- Select g_i with probability $\alpha_i \to \vec{x}_j$.

- Calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$

- Add $f(\vec{x}_j)/g(\vec{x}_j)$ to total result and $f(\vec{x}_j)/g_i(\vec{x}_j)$ to partial (channel-) results.

- After N sampling steps, update a-priori weights.
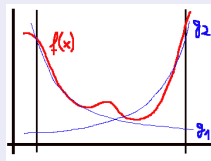
This is the method of choice for parton level event generation!

Monte Carlo integration

Selecting after sampling: Unweighting efficiency

Basic idea: Use hit-or-miss method;

Generate $\vec{x}$ with integration method,

compare actual $f(\vec{x})$ with maximal value during sampling;

$\implies$ "Unweighted events".

Comments:

- unweighting efficiency, $w_{\mathrm{eff}} = \langle f(\vec{x}_j)/f_{\mathrm{max}} \rangle =$ number of trials for each event.

- Good measure for integration performance.

- Expect $\log_{10} w_{\mathrm{eff}} \approx 3 - 5$ for good integration of multi-particle final states at tree-level.

- Maybe acceptable to use $f_{\mathrm{max,eff}} = K f_{\mathrm{max}}$ with $K > 1$.
  Problem: what to do with events where $f(\vec{x}_j)/f_{\mathrm{max,eff}} > 1$?
  Answer: Add $\mathrm{int}[f(\vec{x}_j)/f_{\mathrm{max,eff}}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\mathrm{max,eff}} - k$.

Programy symulacji

Choć stosują podobne metody programy symulacji dzielą się zasadniczo na:

- **Symulacja procesu fizycznego**, który chcemy badać
  Wynikiem symulacji jest wartość przekroju czynnego
  oraz zbiór wygenerowanych przypadków (przypadek: lista cząstek oraz ich pędów)

- **Symulacja działania detektora**
  Wynikiem jest zbiór pomiarów odpowiadający danym jakie zbierane są z detektora.
  $\Rightarrow$ można na nich "zapuścić" programy rekonstrukcji i analizy

Dodatkowo wydzielamy też czasami:

- **Preselekcja** - wstępna ocena przypadków fizycznych, przed symulacją detektora, pod kątem planowanej analizy fizycznej. Cel: zmniejszenie liczby przypadków do pełnej symulacji detektora (zwłaszcza tła).

- **Symulacja układu wyzwalania** - aby sprawdzić czy dany przypadek zostałby rzeczywiście przez eksperyment zarejestrowany.

Symulacja procesu fizycznego

Przykład   Proces Drella-Yana
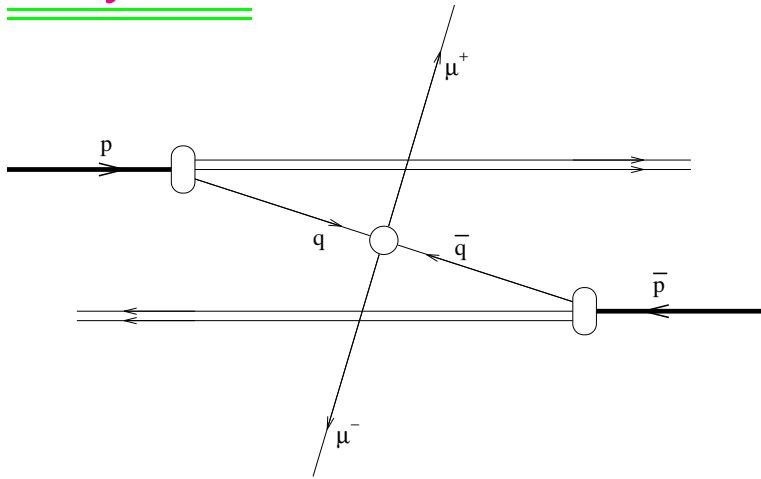
Przekrój czynny na poziomie partonowym (LO QED)

$$\frac{d\sigma^{q\bar{q}\to\mu^+\mu^-}}{d\cos\theta_{cm}} = \frac{\pi\alpha_{em}^2}{2s}\cdot e_q^2\cdot\left(1+\cos^2\theta_{cm}\right)$$

s jest kwadratem masy inwariantnej układu $q\bar{q}$

Różniczkowy przekrój czynny dla procesu $p+\bar{p}\to\mu^+\mu^-+X$

$$\frac{d\sigma(p\bar{p}\to\mu^+\mu^-X)}{dM_{\mu\mu}} =$$

$$\frac{1}{N_c}\sum_q\int dx\cdot f_{q/p}(x)\int dy\cdot f_{q/p}(y)\int d\cos\theta_{cm}\cdot\frac{d\sigma^{q\bar{q}\to\mu^+\mu^-}}{d\cos\theta_{cm}}\cdot\delta\left(M_{\mu\mu}-\sqrt{4xyE_p^2}\right)$$

Musimy losować q, x, y i $\cos\theta_{cm}$. Problem: bardzo różne wartości wagi $\left(\frac{d\sigma}{d\cos\theta}\right)$

Parton level simulations

Stating the problem(s)

- Multi-particle final states for signals & backgrounds.
- Need to evaluate $\mathrm{d}\sigma_N$:

$$\int_{\text{cuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^3 q_i}{(2\pi)^3 2E_i} \right] \delta^4 \left(p_1 + p_2 - \sum_i q_i \right) \left| \mathcal{M}_{p_1 p_2 \to N} \right|^2 .$$

- Problem 1: Factorial growth of number of amplitudes.
- Problem 2: Complicated phase-space structure.
- Solutions: Numerical methods.

Parton level simulations

Factorial growth: $e^+e^- \to q\bar{q} + ng$

n	$\#_{\mathrm{diags}}$
0	1
1	2
2	8
3	48
4	384
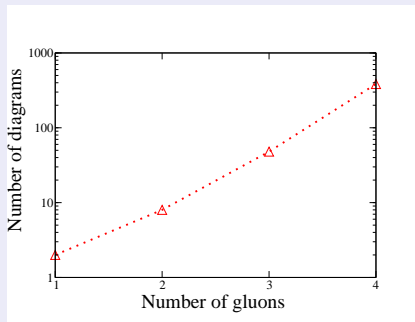
Survey of existing parton-level tools

Standard Model (and beyond) tools @ tree-level

- AlpGen M.L.Mangano *et al.*, JHEP **0307** (2003) 001;
- AMEGIC++ F.K., R.Kuhn, G.Soff, JHEP **0202** (2002) 044;
- CompHEP/CalcHEP E.Boos *et al.* Nucl. Instrum. Meth. A **534** (2004) 250;
- HELAC A.Kanaki and C.G.Papadopoulos, Comput. Phys. Commun. **132** (2000) 306;
- MadGraph/MadEvent F.Maltoni and T.Stelzer, JHEP **0302**, 027 (2003);
- O'Mega+WHIZARD M.Moretti, T.Ohl and J.Reuter, arXiv:hep-ph/0102195.

All tools here are completely self-contained and automated and provide amplitudes and integrators of their own.

Survey of existing parton-level tools

Comparison of tree-level tools

	Models	$2 \to n$	Ampl.	Integ.	public?	lang.
Alpgen	SM	$n = 8$	rec.	Multi	yes	Fortran
Amegic	SM,MSSM,ADD	$n = 6$	hel.	Multi	yes	C++
CompHep	SM,MSSM	$n = 4$	trace	1Channel	yes	C
HELAC	SM	$n = 8$	rec.	Multi	yes	Fortran
MadEvent	SM,MSSM	$n = 6$	hel.	Multi	yes	Fortran
O'Mega	SM,MSSM,LH	$n = 8$	rec.	Multi	yes	O'Caml

Limitations of parton level simulation

General

- Fixed order parton level (LO, NLO, . . .) implies fixed multiplicity

- No control over potentially large logs
  (appear when two partons come close to each other).

- Parton level is parton level
  experimental definition of observables relies on hadrons.

  Therefore: Need hadron level event generators!

Simulation's paradigm

Basic strategy

Divide event into stages,
separated by different scales.

- Signal/background:

  Exact matrix elements.

- QCD-Bremsstrahlung:

  Parton showers (also in initial state).

- Multiple interactions:

  Beyond factorization: Modeling.

- Hadronization:

  Non-perturbative QCD: Modeling.

Sketch of an event
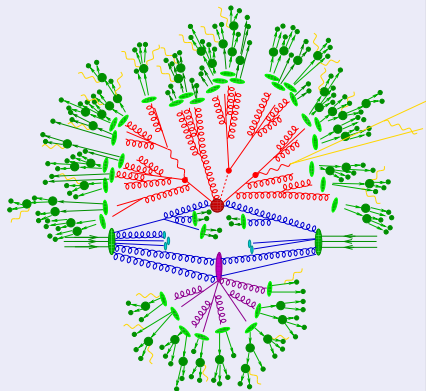
Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons
  Photons split into electron-positron pairs
- QCD: Quarks (colored) emit gluons
  Gluons split into quark pairs
- Difference: Gluons are colored (photons are not charged)
  Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower

Occurrence of large logarithms

$e^+e^- \to$ jets

- Differential cross section:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}x_1 \mathrm{d}x_2} = \sigma_{ee\to 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Singular for $x_{1,2} \to 1$.

- Rewrite with opening angle θ_{qg}
  and gluon energy fraction $x_3 = 2E_g/E_{\mathrm{c.m.}}$:

$$\frac{\mathrm{d}\sigma_{ee\to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee\to 2j} \frac{C_F \alpha_s}{\pi} \left[\frac{2}{\sin^2\theta_{qg}} \frac{1+(1-x_3)^2}{x_3} - x_3 \right]$$

Singular for $x_3 \to 0$ ("soft"), $\sin\theta_{qg} \to 0$ ("collinear").

Occurrence of large logarithms

Collinear singularities

- Use

$$\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta^2_{qg}}{\theta^2_{qg}} + \frac{\mathrm{d}\theta^2_{\bar{q}g}}{\theta^2_{\bar{q}g}}$$

- Independent evolution of two jets (q and $\bar{q}$):

$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2_{jg}}{\theta^2_{jg}} P(z) \; ,$$

where $P(z) = \frac{1+(1-z)^2}{z}$ (DGLAP splitting function)

Occurrence of large logarithms

Many emissions

- Iterate emissions (jets)

  Maximal result for $t_1 > t_2 > \ldots t_n$:
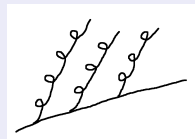
$$\mathrm{d}\sigma \propto \sigma_0 \int\limits_{Q_0^2}^{Q^2} \frac{\mathrm{d}t_1}{t_1} \int\limits_{Q_0^2}^{t_1} \frac{\mathrm{d}t_2}{t_2} \ldots \int\limits_{Q_0^2}^{t_{n-1}} \frac{\mathrm{d}t_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

- How about Q^2? Process-dependent!

Occurrence of large logarithms

Ordering the emissions : Radiation pattern
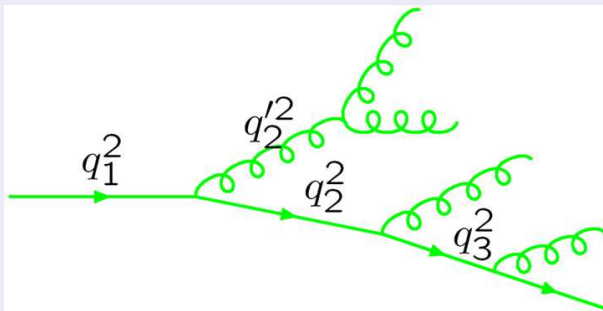
$$q_1^2 > q_2^2 > q_3^2, \ q_1^2 > q_2'^2$$

Survey of existing showering tools

Tools	evolution	AO/Coherence
Ariadne	k_perp-ordered	by construction
Herwig	angular ordering	by construction
Herwig++	improved angular ordering	by construction
Pythia	old: virtuality ordered	by hand
	new: $k_\perp$-ordered	by construction
Sherpa	virtuality ordered	by hand
	(like old Pythia)	

PDFs and factorization

Space-time picture of hard interactions

Partons "collinear" with hadron: $k_\perp \ll 1/R_{\mathrm{had}}$.

Lifetime of partons $\tau \sim 1/x$, $r \sim 1/Q$.

Hard interaction at scales $Q_{\mathrm{hard}} \gg 1/R_{\mathrm{had}}$.

Hard interaction at scale Q>>1/Rhad,
breaks recombination of partons
==> hadron break up (loss of coherence)
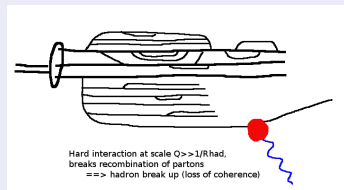
- Too "fast" for color field - only one parton takes part.
- Other partons feel absence only when trying to recombine.
- Universality (process-independence) of PDFs.
- Collinear factorization.

Hadronization

Confinement

- Consider dipoles in QED and QCD

QED:

QCD:
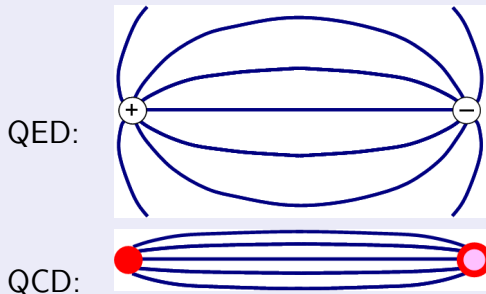
Hadronization

Linear QCD potential in quarkonia
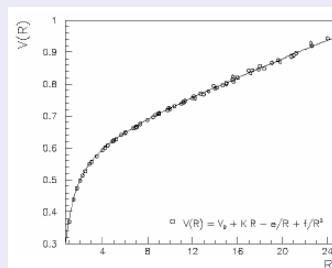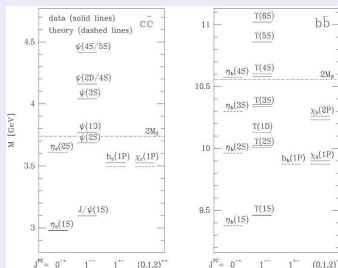
Hadronization

Dynamical strings in $e^+e^- \to q\bar{q}$

B.Andersson, G.Gustafson, G.Ingelman and T.Sjostrand, Phys. Rept. **97** (1983) 31.

- Ignoring gluon radiation: Point-like source of string.

- Intense chromomagnetic field within string:
  More $q\bar{q}$ pairs created by tunnelling.

- Analogy with QED (Schwinger mechanism):
  $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t \exp\left(-\pi m_q^2/\kappa\right)$, $\kappa =$ "string tension".
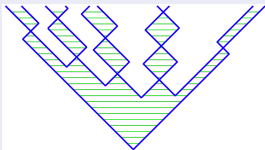
Hadronization

Some experimental facts → naive parameterizations

- In $e^+e^- \to$ hadrons: Limits $p_\perp$, flat plateau in y.
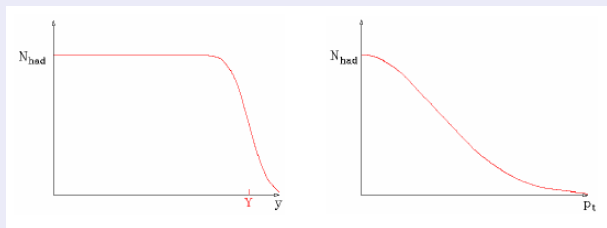
- Try "smearing": $\rho(p_\perp^2) \sim \exp(-p_\perp^2/\sigma^2)$

Hadronization

Implementation of naive parameterizations

- Feynman-Field independent fragmentation.

  R.D.Field and R.P.Feynman, Nucl. Phys. B **136** (1978) 1

- Recursively fragment $q \to q' +$ had, where

  - Transverse momentum from (fitted) Gaussian;
  - longitudinal momentum arbitrary (hence from measurements);
  - flavor from symmetry arguments + measurements.

- Problems: frame dependent, "last quark", infrared safety, no direct link to perturbation theory, ...

Hadronization

Preconfinement

- Underlying: Large N_c-limit (planar graphs).
- Follows evolution of color in parton showers:
  at the end of shower color singlets close in phase space.
- Mass of singlets: peaked at low scales $\approx Q_0^2$.
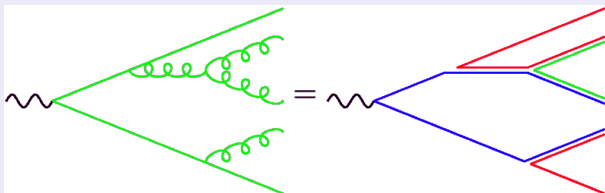
Hadronization

Primordial cluster mass distribution

- Starting point: Preconfinement;
- split gluons into $q\bar{q}$-pairs;
- adjacent pairs color connected, form colorless (white) clusters.
- Clusters ("$\approx$ excited hadrons) decay into hadrons
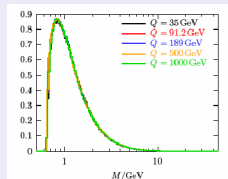
Underlying Event

Multiple parton scattering?
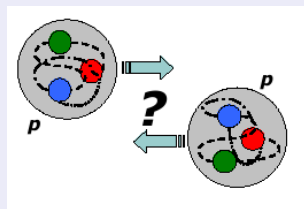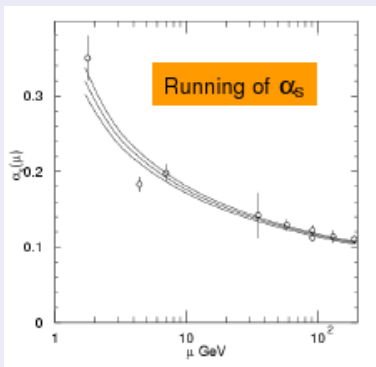
- Hadrons = extended objects!
- No guarantee for one scattering only.
- Running of α_S
  $\implies$ preference for soft scattering.

Underlying Event

Definition(s)
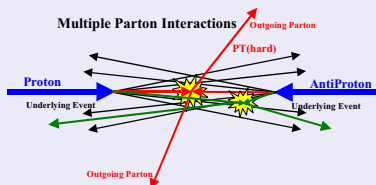
1. Everything apart from the hard interaction including IS showers, FS showers, remnant hadronization.

2. Remnant-remnant interactions, soft and/or hard.

3. Lesson: hard to define

Underlying event

Old Pythia model: Algorithm, simplified

T.Sjostrand and M.van Zijl, Phys. Rev. D **36** (1987) 2019.

- Start with hard interaction, at scale Q_{hard}^2.
- Select a new scale $p_\perp^2$
  (according to $f = \frac{\mathrm{d}\sigma_{2\to 2}(p_\perp^2)}{\mathrm{d}p_\perp^2}$ with $p_\perp^2 \in [p_{\perp,\mathrm{min}}^2, \, Q^2]$)
- Rescale proton momentum   ("proton-parton = proton with reduced energy").
- Repeat until below $p_{\perp,\mathrm{min}}^2$.
- May add impact-parameter dependence, showers, etc..
- Treat intrinsic $k_\perp$ of partons ($\to$ parameter)
- Model proton remnants ($\to$ parameter)

Orientation
○○○○○●
Introduction

ME corrections

MC@NLO

CKKW

Orders in ME and PS

ME vs. PS

- Matrix elements good for: hard, large-angle emissions; take care of interferences.

- Parton shower good for: soft, collinear emissions; resums large logarithms.

- Want to combine both! Avoid double-counting.

α_s vs. Log
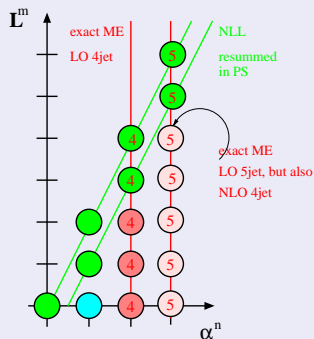
Correcting the parton shower

Example: $e^+e^- \rightarrow q\bar{q}g$

ME : $\left| \begin{array}{c}\end{array} + \begin{array}{c}\end{array} \right|^2$

PS : $\left| \begin{array}{c}\end{array} \right|^2 + \left| \begin{array}{c}\end{array} \right|^2$
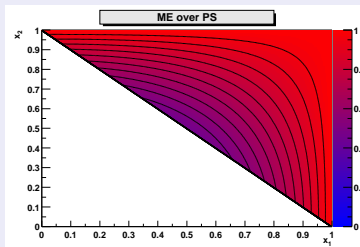
Combining MEs & PS

Basic principles

- Want:
    - All jet emissions correct at tree level $+$ LL,
    - Soft emissions correctly resummed in PS
- Method:
    - Separate Jet-production/evolution by Q_{jet} ($k_\perp$ algorithm).
    - Produce jets according to LO matrix elements
    - re-weight with Sudakov form factor $+$ running α_s weights,
    - veto jet production in parton shower.
- Process-independent implementation.

Combining MEs & PS

Independence on Q_{jet}

Example: $p_\perp$ of W in $p\bar{p} \to W + X$ @ Tevatron

in F.K., A.Schälicke, S.Schumann and G.Soff, Phys. Rev. D **70** (2004) 114009
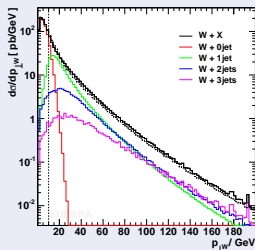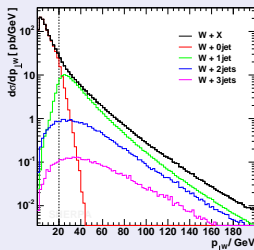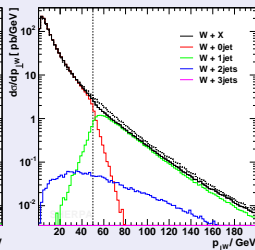
Combining MEs & PS

Comparison with data from Tevatron

$p_\perp$ of Z-bosons in $p\bar{p} \to Z + X$
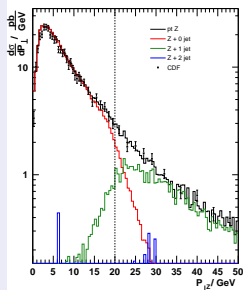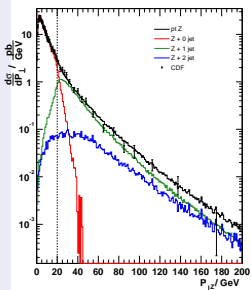
Detector Simulation - General

- General characteristics of a detector simulation system
  - You specify the geometry of a particle detector
  - Then the software system automatically transports the particle you shoot into the detector by simulating the particle interactions in matter based on the Monte Carlo method

- The heart of the simulation: the Monte Carlo method
  - A method to search for solutions to a mathematical problem using a statistical sampling with random numbers

The role of simulation

- Simulation plays a fundamental role in various domains and phases of an experimental physics project
  - design of the experimental set-up
  - evaluation and definition of the potential physics output of the project
  - evaluation of potential risks to the project
  - assessment of the performance of the experiment
  - development, test and optimisation of reconstruction and physics analysis software
  - contribution to the calculation and validation of physics results

- The scope of these lectures (and of Geant4) encompasses the simulation of the passage of particles through matter
  - there are other kinds of simulation components, such as *physics event generators*, *electronics response* generation, etc.
  - often the simulation of a complex experiment consists of several of these components interfaced to one another

Basic requirements for a simulation system

- Modeling the experimental set-up
- Tracking particles through matter
- Interaction of particles with matter
- Modeling the detector response
- Run and event control
- Accessory utilities *(random number generators, PDG particle information etc.)*
- Interface to event generators
- Visualisation of the set-up, tracks and hits
- User interface
- Persistency

Geant 4

The zoo

EGS4, EGS5, EGSnrc
Geant3, Geant4
MARS
MCNP, MCNPX, A3MCNP, MCNP-DSP, MCNP4B
MVP, MVP-BURN
Penelope
Peregrine
Tripoli-3, Tripoli-3 A, Tripoli-4

...and I probably forgot some more

DPM
EA-MC
FLUKA
GEM
HERMES
LAHET
MCBEND
MCU
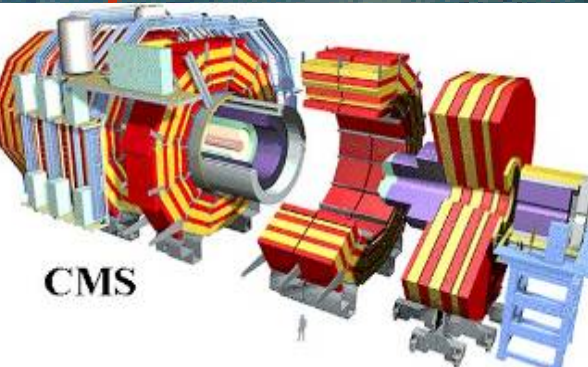MF3D
NMTC
MONK
MORSE
RTS&T-2000
SCALE
TRAX
VMC++

Monte Carlo codes presented at the MC200 Conference, Lisbon, October 2000
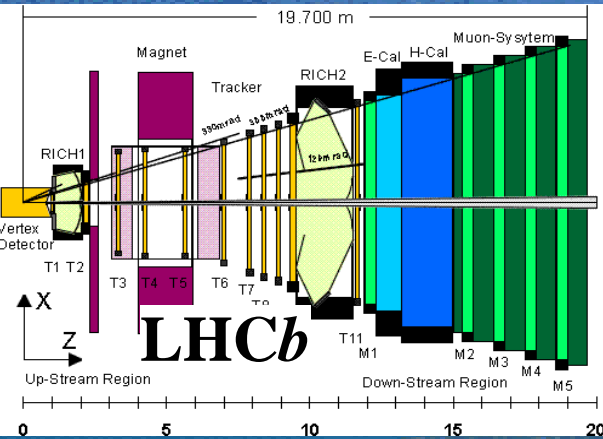
Geant 4

**Complex physics
Complex detectors
20 years**
software life-span

LHC

LHCb

ATLAS

CMS

Maria Grazia Pia, *INFN Genova*
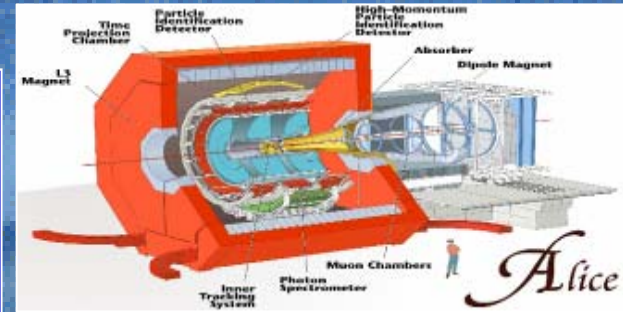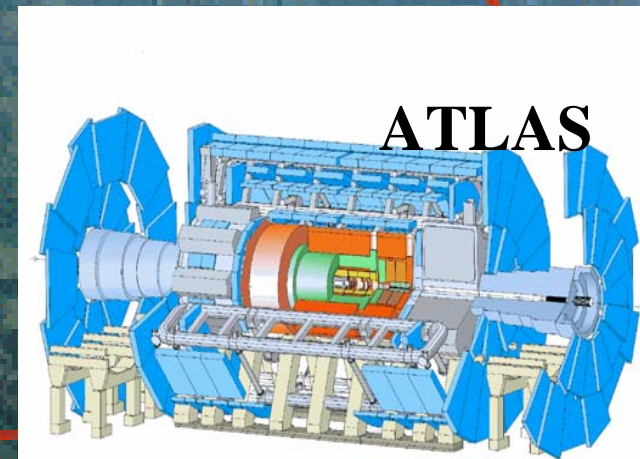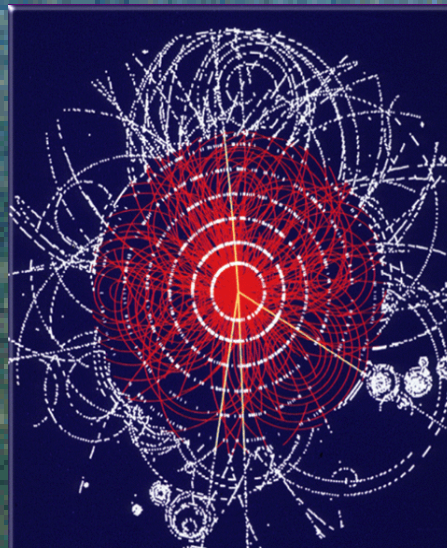
Geant4 Collaboration

MoU based

Distribution, Development and User Support of Geant4

CERN, ESA, KEK, SLAC, TRIUMF, TJNL
INFN, IN2P3, PPARC

Barcelona Univ., Budker Inst., Frankfurt Univ., Karolinska Inst., Helsinki Univ., Lebedev Inst., LIP, Northeastern Univ. *etc.*

What Can Geant4 Do for You?

■ Transports a particle step-by-step by taking into account the interactions with materials and external electromagnetic fields until the particle

- **loses its kinetic energy to zero,**
- **disappears by an interaction,**
- **comes to the end of the simulation volume**

■ Provides a way for the user to access the transportation process and grab the simulation results

- **at the beginning and end of transportation,**
- **at the end of each stepping in transportation,**
- **at the time when the particle is going into the sensitive volume of the detector**
- **etc.**
- ➡ *These are called "User Actions"*

Geant 4

What You Have to Do for Geant4?

■ Three essential information you have to provide:

- **Geometrical information of the detector**
- **Choice of physics processes**
- **Kinematical information of particles going into the detector**

■ Auxiliary you have to prepare:

- **Magnetic and electric field**
- **Actions you want to take when you access the particle transportation**
- **Actions you want to take when a particle goes into a sensitive volume of the detector**
- **etc.**

Geant 4

Geometry

ATLAS
5.2 M volume objects
110 K volume types
Courtesy of ATLAS Collaboration

- ## Role
  - – detailed detector description
  - – efficient navigation

- ## Three conceptual layers
  - – Solid: shape, size
  - – LogicalVolume: material, sensitivity, daughter volumes, etc.
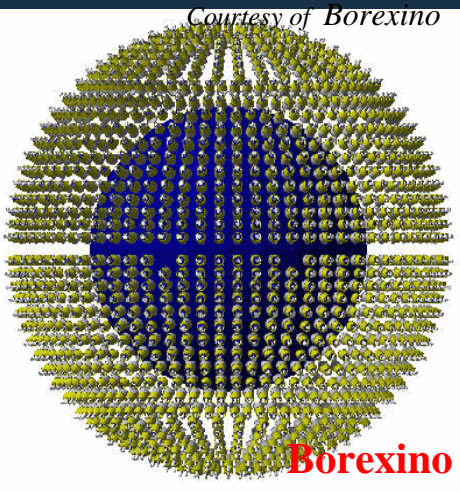 - – PhysicalVolume: position, rotation

- ## One can do fancy things with geometry…
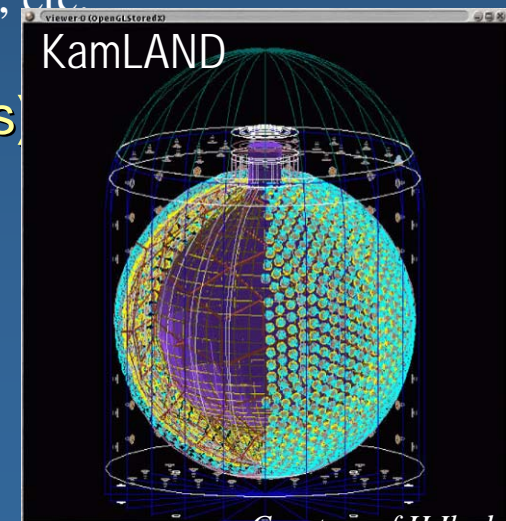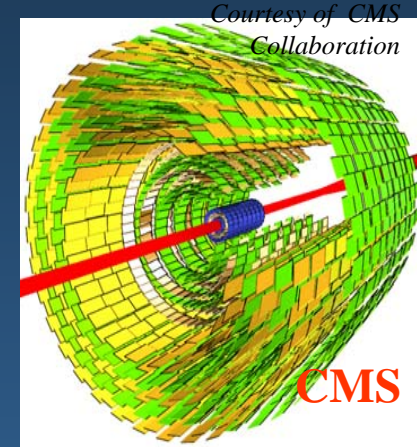
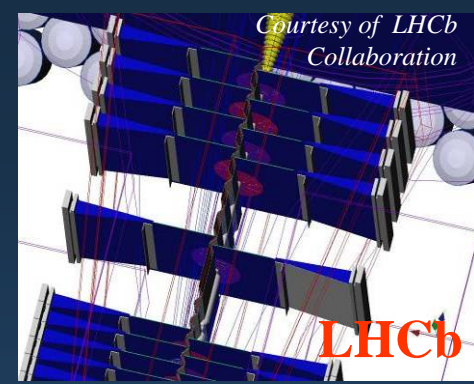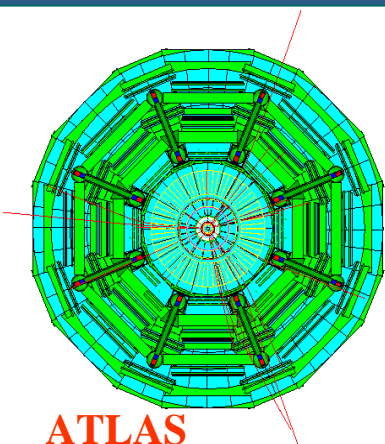Boolean operations
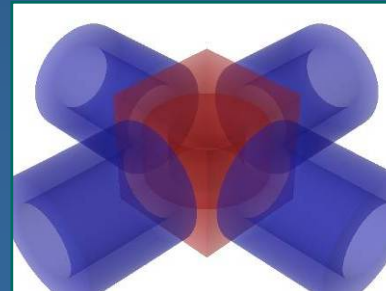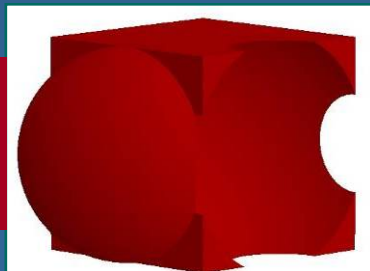
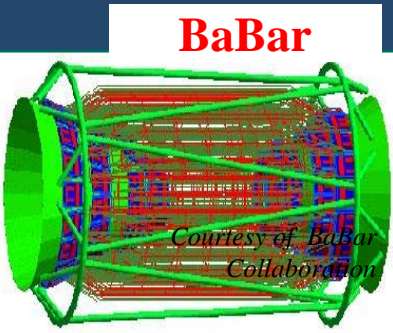Transparent solids
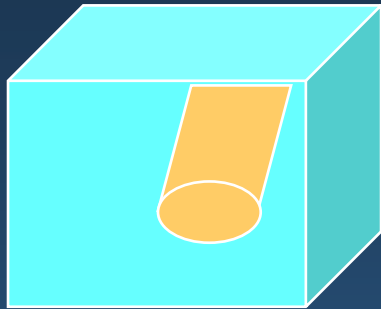
16
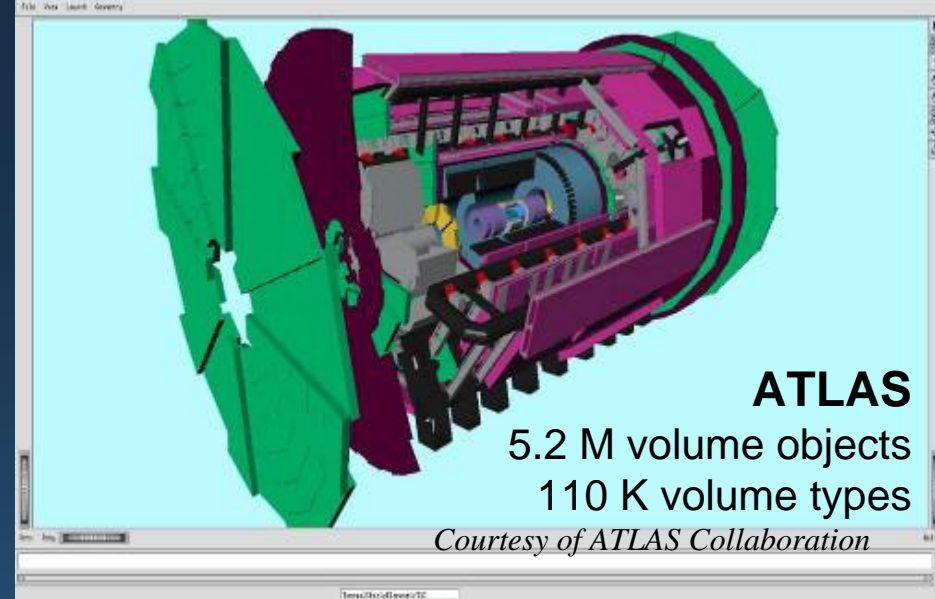
Solids

Multiple representations
Same abstract interface

- CSG (**C**onstructed **S**olid **G**eometries)
  – simple solids

- STEP extensions
  – polyhedra, spheres, cylinders, cones, toroids, etc.

- BREPS (**B**oundary **REP**resented **S**olids)
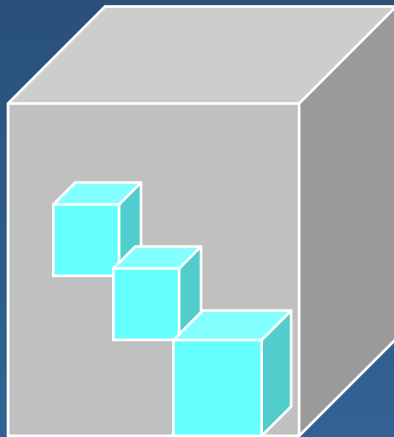 – volumes defined by boundary surfaces

CAD exchange

Courtesy of Borexino
Borexino

BaBar
Courtesy of BaBar Collaboration

ATLAS

Courtesy of LHCb Collaboration
LHCb

Courtesy of CMS Collaboration
CMS

KamLAND
Courtesy of H.Ikeda

Maria Grazia Pia, *INFN Genova*
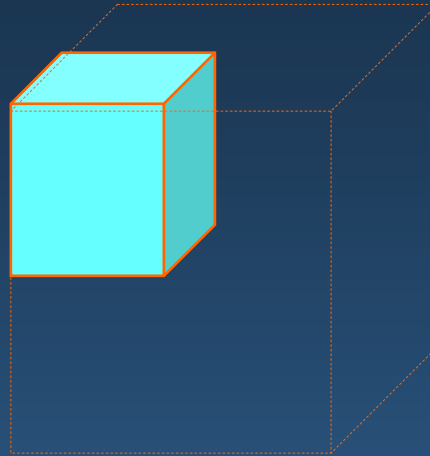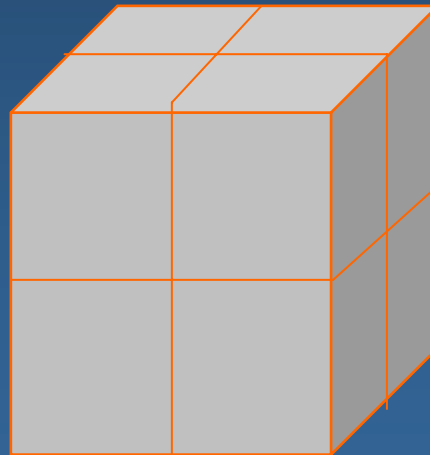
Physical Volumes

placement

parameterised

replica
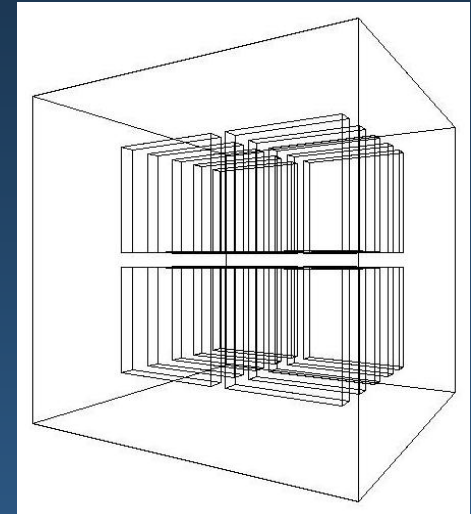
assembled

Versatility to describe
complex geometries

Maria Grazia Pia, *INFN Genova*

Electric and magnetic fields of variable non-uniformity and differentiability

CMS
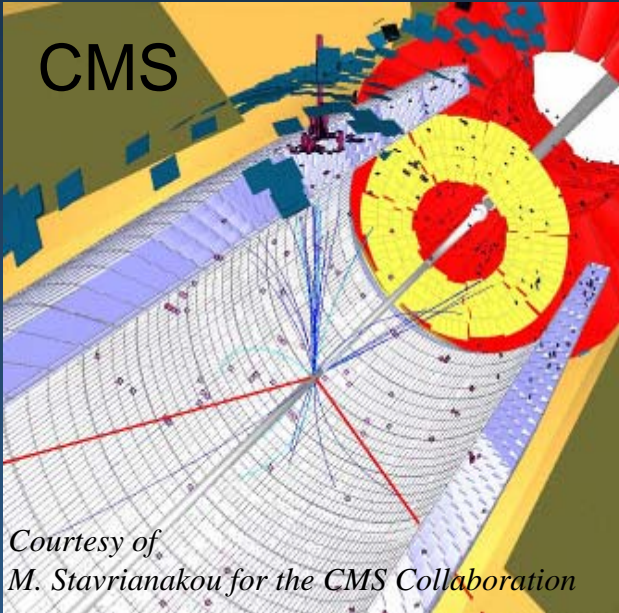
Geant4 field ~ 2 times faster
than FORTRAN/GEANT3

Courtesy of
M. Stavrianakou for the CMS Collaboration

MOKKA

Linear
Collider
Detector

1 GeV proton in the Earth's geomagnetic field

Courtesy Laurent Desorgher, University of Bern
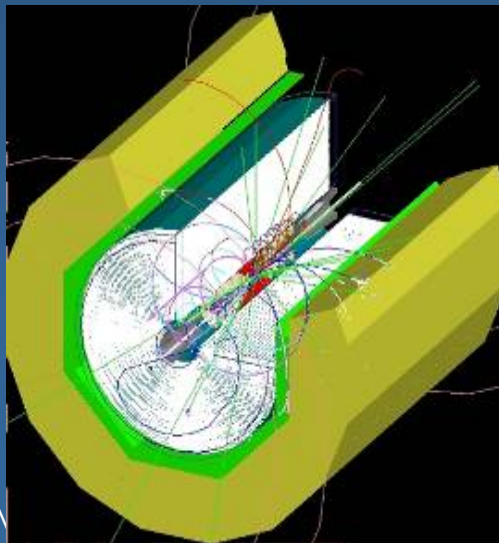
Physics

- **Abstract interface** to physics processes
  - **Tracking independent from physics**
  - Uniform treatment of electromagnetic and hadronic processes

- Distinction between **processes** and **models**
  - multiple models for the same physics process *(complementary/alternative)*

- **Transparency** (supported by *encapsulation* and *polymorphism*)
  - Calculation of cross-sections independent from the way they are accessed (data files, analytical formulae etc.)
  - Calculation of the final state independent from tracking

- Explicit use of units throughout the code

- Open system
  - Users can easily create and use their own models

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Electromagnetic physics

- electrons and positrons
- γ, X-ray and optical photons
- muons
- charged hadrons
- ions

Comparable to Geant3 already in the α release (1997)

Further extensions *(facilitated by the OO technology)*

High energy extensions
– needed for LHC experiments, cosmic ray experiments…

Low energy extensions
– fundamental for space and medical applications, dark matter and ν experiments, antimatter spectroscopy etc.

Alternative models for the same process

energy loss
- **Multiple scattering**
- **Bremsstrahlung**
- **Ionisation**
- **Annihilation**
- **Photoelectric effect**
- **Compton scattering**
- **Rayleigh effect**
- **γ conversion**
- **e⁺e⁻ pair production**
- **Synchrotron radiation**
- **Transition radiation**
- **Cherenkov**
- **Refraction**
- **Reflection**
- **Absorption**
- **Scintillation**
- **Fluorescence**
- **Auger**
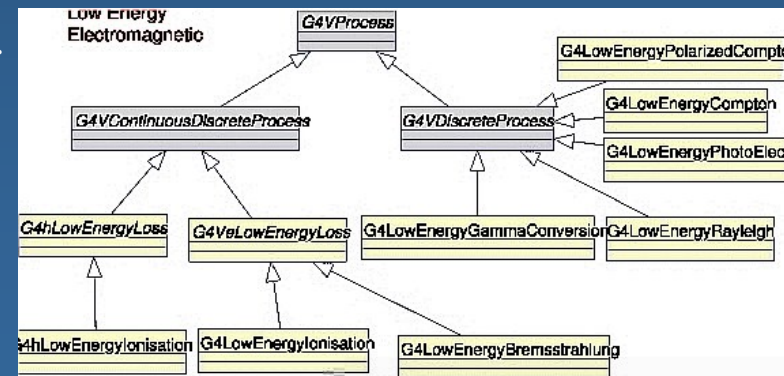
All obeying to the same abstract Process interface <u>transparent to tracking</u> 24

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Calorimetry

Single crystal containment: E_{1x1}/E_{3x3} versus position

Courtesy of
M. Stavrianakou for the CMS Collaboration
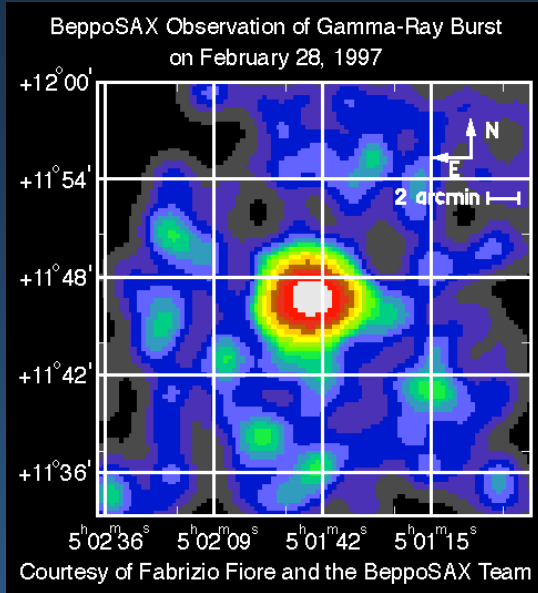
Tracking

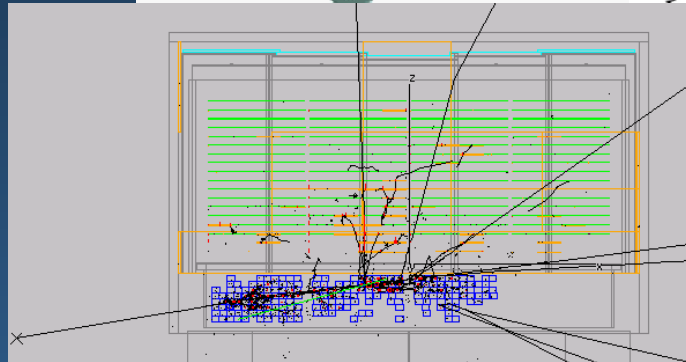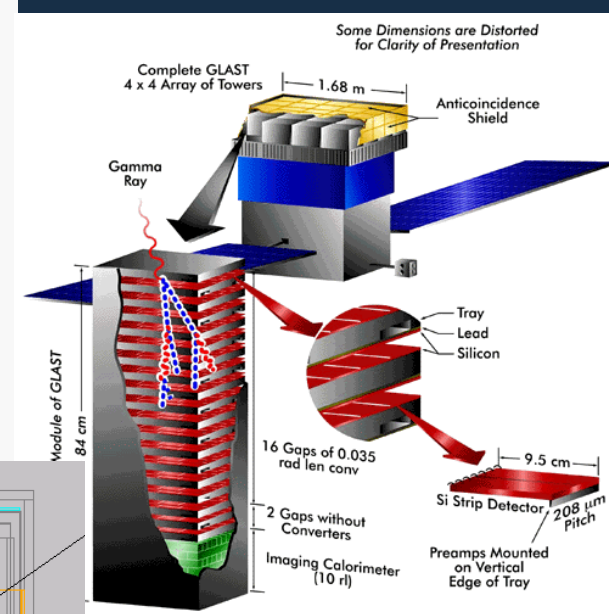
Courtesy of
BaBar
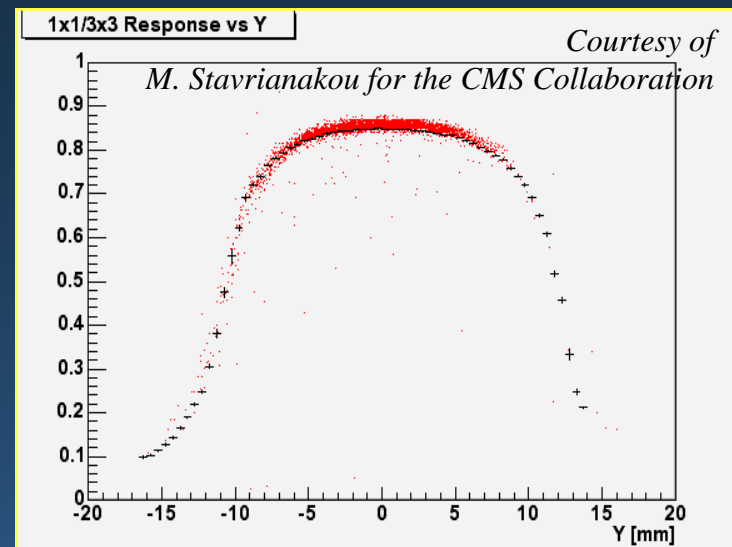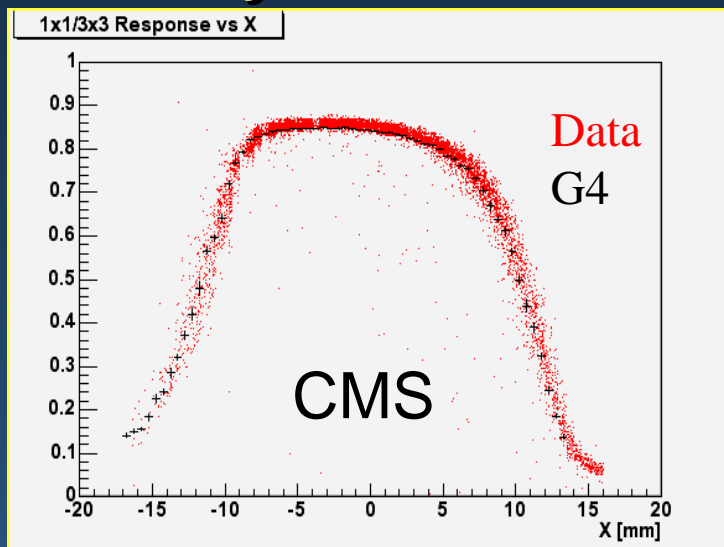
Geant4
Standard
Electromagnetic
Physics

Maria Grazia Pia, *INFN Genova*

25

γ astrophysics

γ-ray bursts

BeppoSAX Observation of Gamma-Ray Burst on February 28, 1997

Courtesy of Fabrizio Fiore and the BeppoSAX Team

AGILE

Courtesy of AGILE

Complete GLAST
4 x 4 Array of Towers

Some Dimensions are Distorted for Clarity of Presentation

Anticoincidence Shield

Gamma Ray

Tray
Lead
Silicon

16 Gaps of 0.035 rad len conv

9.5 cm

Si Strip Detector 208 μm Pitch

2 Gaps without Converters

Preamps Mounted on Vertical Edge of Tray

Imaging Calorimeter (10 rl)

Module of GLAST 84 cm

GLAST

Typical telescope:
Tracker
Calorimeter
Anticoincidence

- γ conversion
- electron interactions
- multiple scattering
- δ-ray production
- charged particle tracking

GLAST Hits Display

File Name HITS_0.dat
Event ID 15
Hits N. 72

Previous
View XZ
View YZ
Zoom
Unzoom
New Center
Reset 3D
Marker +
Marker −
Save as Gif
View X3D

Courtesy of GLAST

Maria Grazia Pia, *INFN Genova*
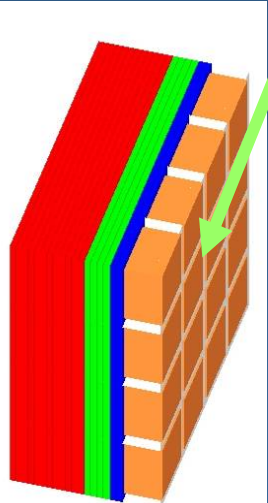
Geant 4

Optical photons

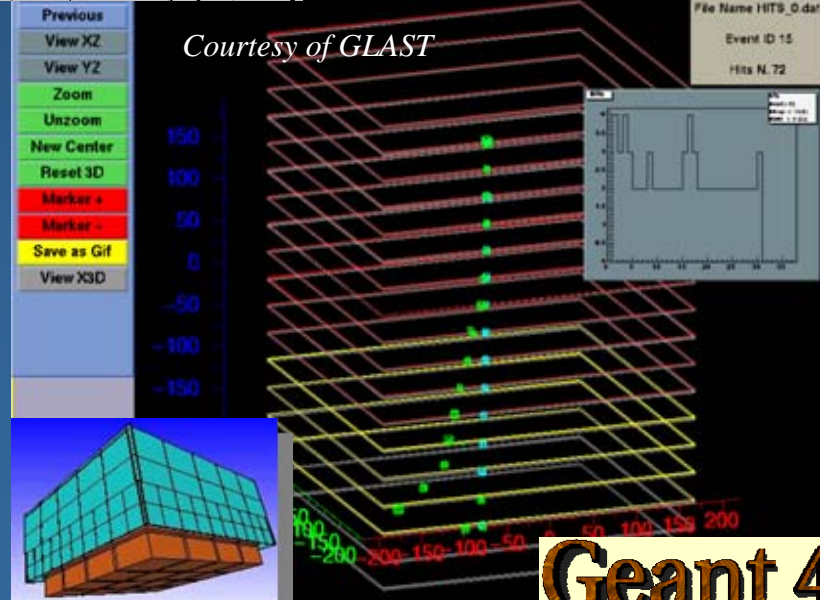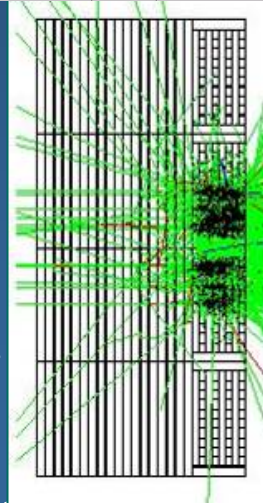*Production of optical photons in HEP detectors is mainly due to Cherenkov effect and scintillation*

Processes in Geant4:

- in-flight absorption
- Rayleigh scattering
- medium-boundary interactions (reflection, refraction)

Geant4 Optical Processes :
Scintillating Cells and WLS Fibers

Maria Grazia Pia, *INFN Genova*

Courtesy of J. Mc Cormick (SLAC)

Hadronic physics

- **Completely different approach w.r.t. the past (Geant3)**
  - native
  - transparent
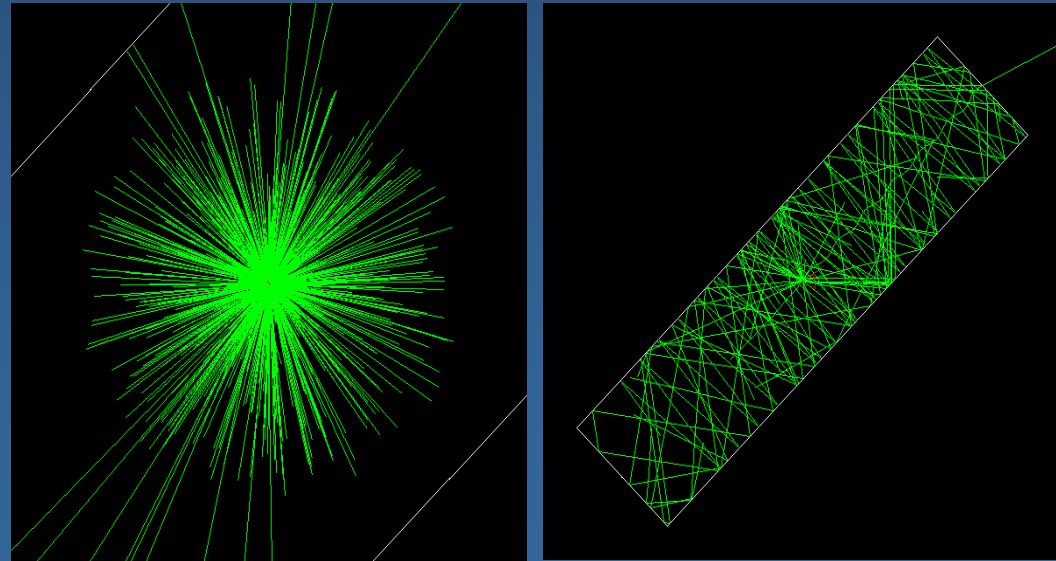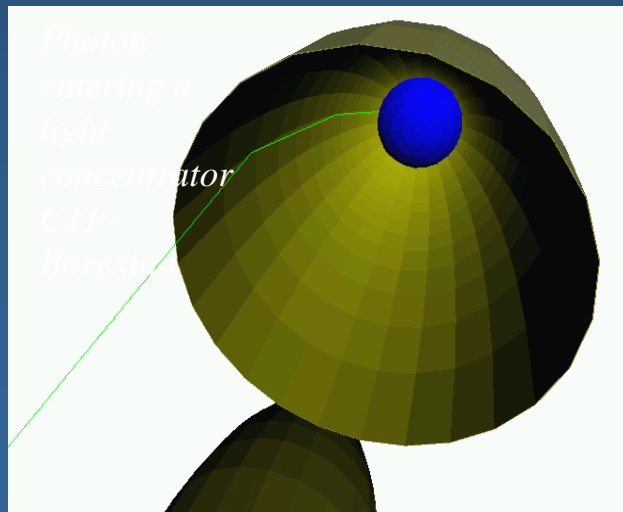 - no longer interface to external packages
  - clear separation between data and their use in algorithms

- **Cross section data sets**
  - transparent and interchangeable

- **Final state calculation**
  - models by particle, energy, material

- **Ample variety of models**
  - the most complete hadronic simulation kit on the market
  - alternative and complementary models
  - data-driven, parameterised and theoretical models

Maria Grazia Pia, *INFN Genova*

Hadronic model inventory
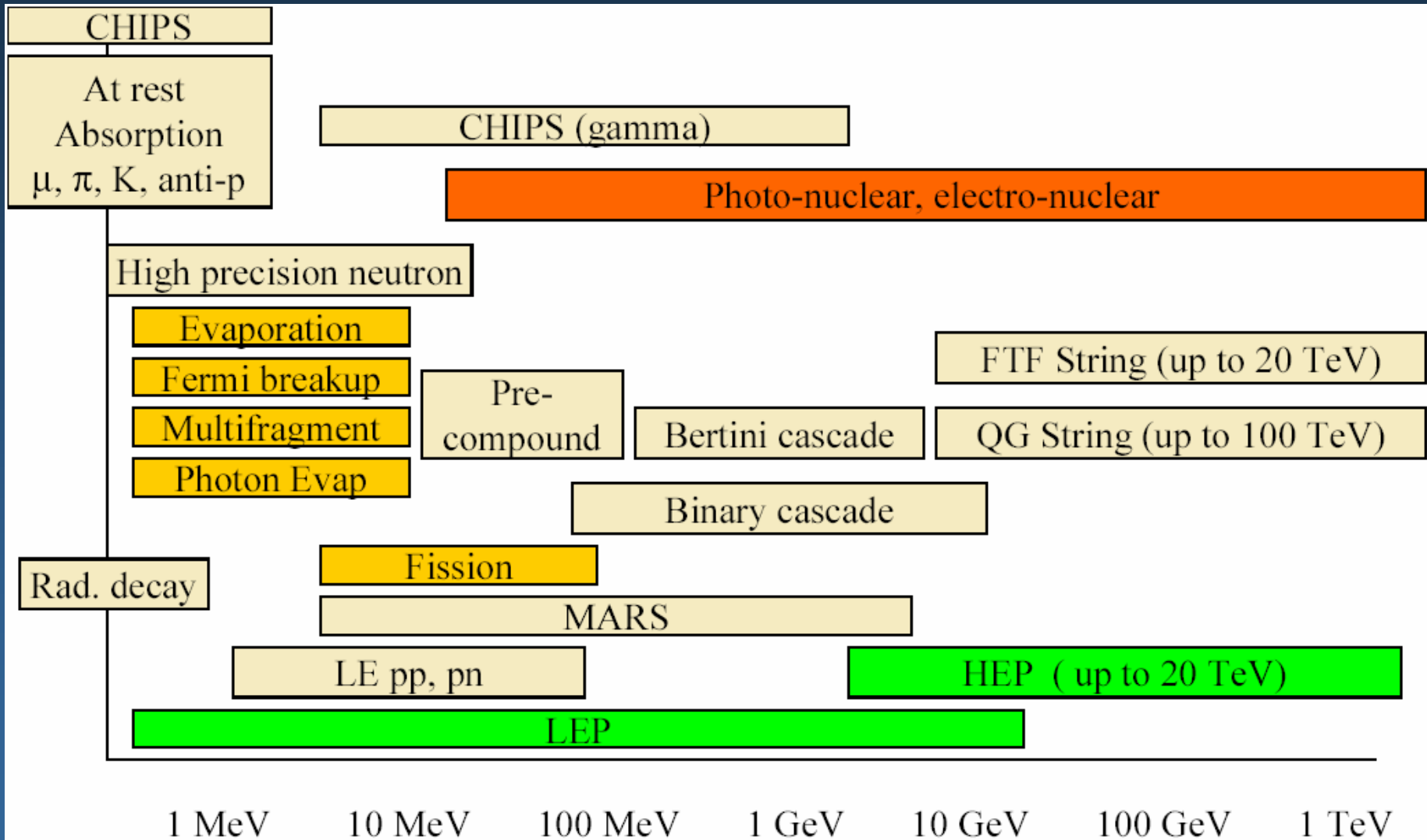
Parameterised and data-driven hadronic models (1)

Based on experimental data

- Some models originally from GHEISHA
  - completely reengineered into OO design
  - refined physics parameterisations
- New parameterisations
  - pp, elastic differential cross section
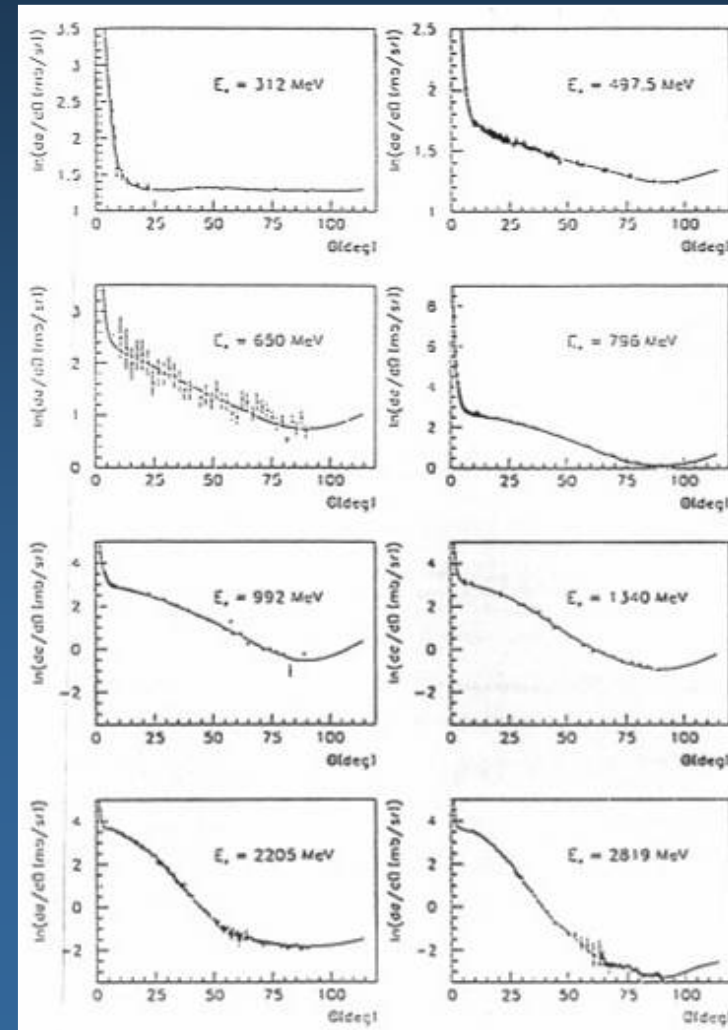 - nN, total cross section
  - pN, total cross section
  - np, elastic differential cross section
  - πN, total cross section
  - πN, coherent elastic scattering

p elastic scattering on Hydrogen
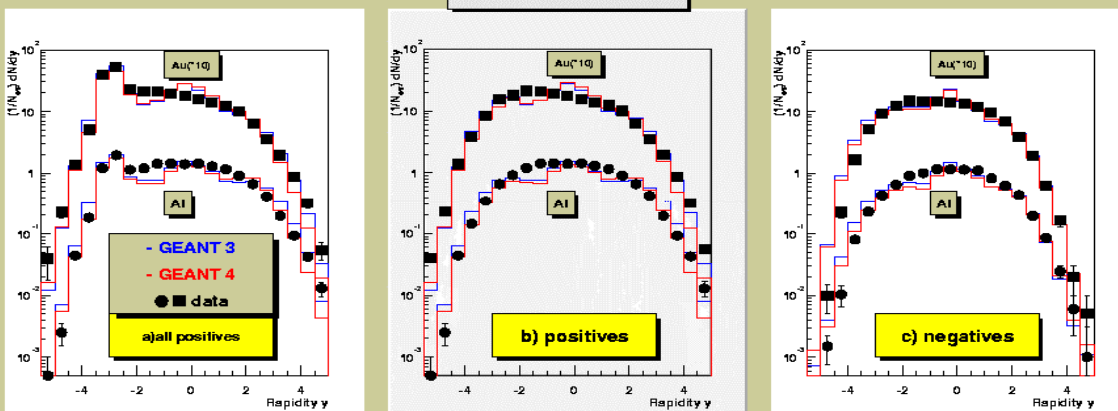
Parameterised and data-driven hadronic models (2)

Other models are completely new, such as:

stopping particles: π^-, K^-
(relevant for μ/π PID detectors)

Stopping π

absorption

nuclear deexcitation

— Geant4
● Exp. data

Energy

MeV

γs from 14 MeV neutron capture on Ur
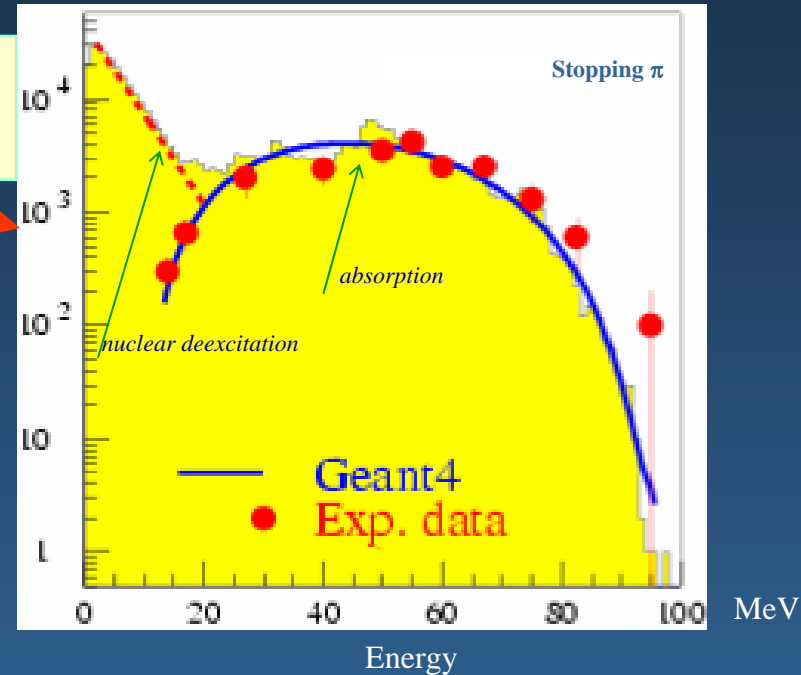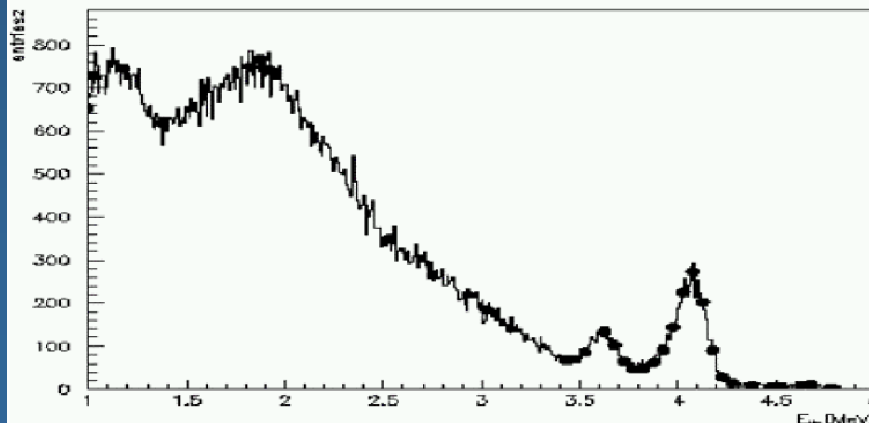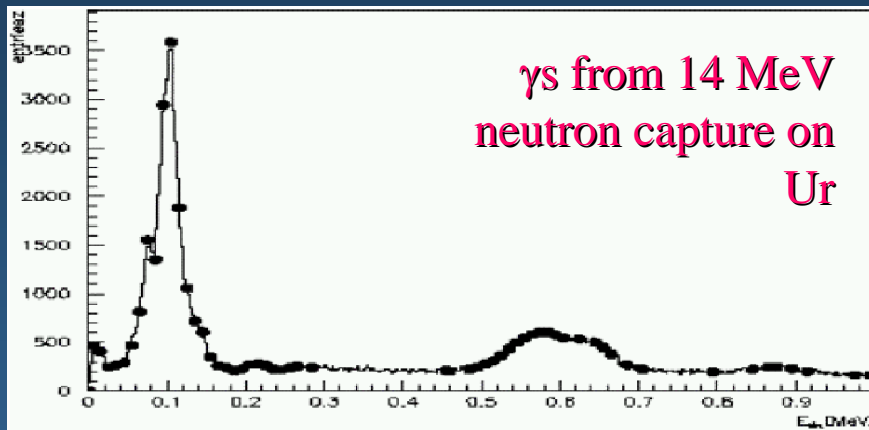
neutrons

All worldwide existing databases used in **neutron** transport

Brond, CENDL, EFF, ENDFB, JEF, JENDL, MENDL etc.

Maria Grazia Pia, *INFN Genova*

Theory-driven models

- Complementary and alternative models
- *Evaporation phase*
- Low energy range O(100 MeV): *pre-equilibrium*
- Intermediate energy range, O(100 MeV) to O(5 GeV): *intra-nuclear transport*
- High energy range: *hadronic generator* régime

Bertini cascade model: pion production from 730 MeV proton on Carbon

G4QGSModel: differential pion yields in pion-Mg

CCLRC
Rutherford Appleton Laboratory

Nature, July 1984

NEWS AND VIEWS
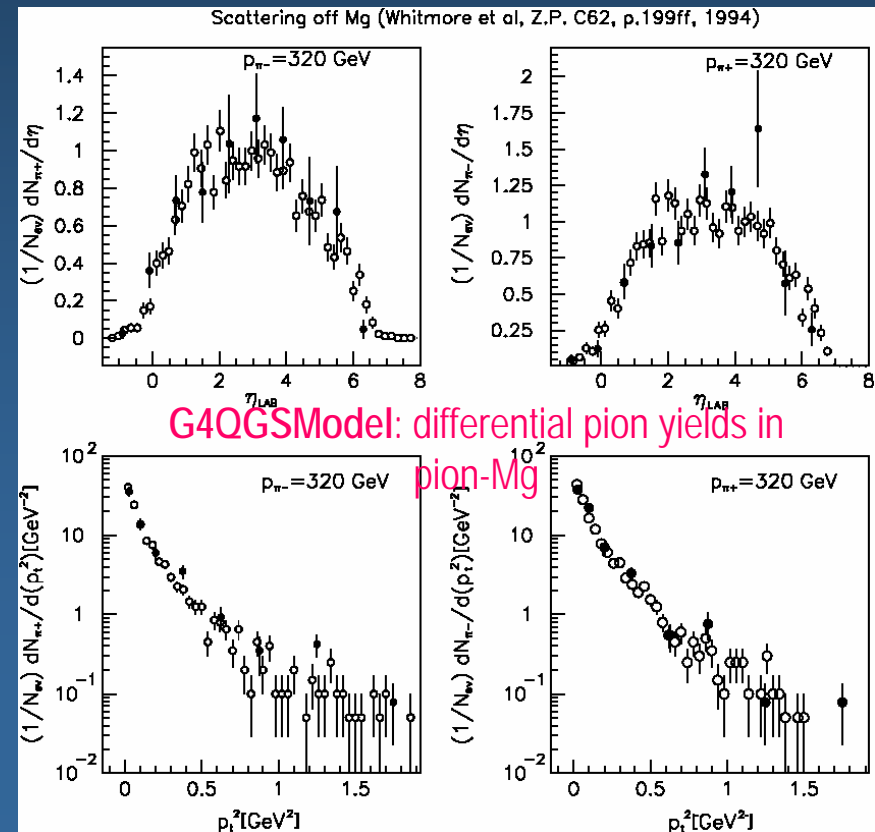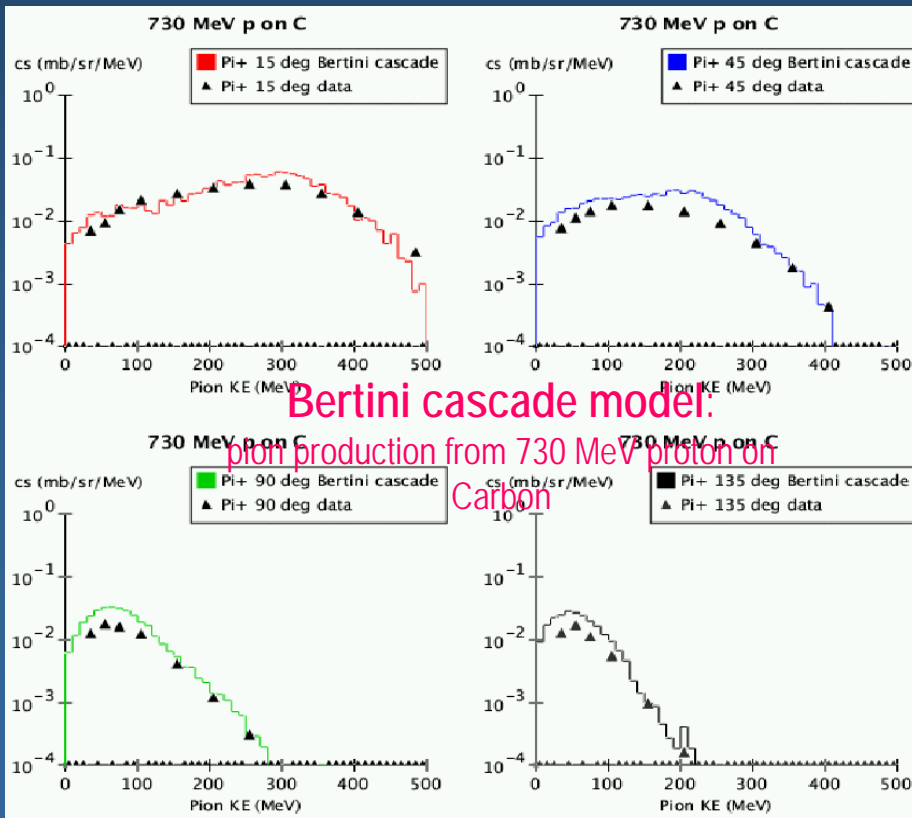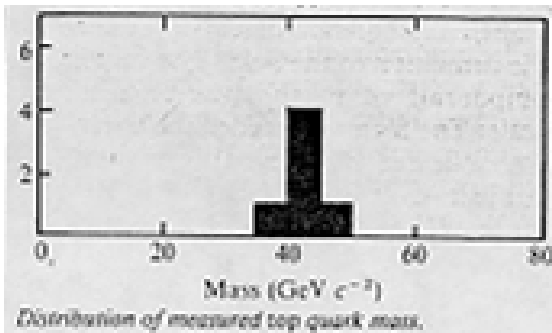
CERN comes out again on top

With the discovery of the electroweak bosons (W± and Z⁰) in the bag, CERN now announces the discovery of the quark called top. What will come next?

THE Matthew principle — "to him that hath shall be given" — is working in favour of CERN, the European high-energy physics laboratory at Geneva, and of the UA1 collaboration which, at the end of last year, announced the discovery of the W± and Z⁰ particles which mediate the electroweak interaction. Last week, the same 80-strong collaboration, under the leadership of Carlo Rubbia, announced the discovery of the missing sixth quark, called *top*, long-predicted but hitherto elusive. By doing so, they have put yet another cap on the electroweak theory while restoring a seemly symmetry to the evolving picture of quarks as the elementary constituents of the material Universe.

The new development at CERN follows almost exactly along the lines expected (and described, for example, by Dr F. Close in his comment on the electroweak bosons, see *Nature* 303, 656; 1983). The source of the sixth quark is a charged boson, W⁺ or W⁻, first recognized at CERN by their decay into an electron (with electrical charge of the same sign), with excess momentum carried away by a neutrino. Events of this kind accumulated at CERN in the past two years have amply confirmed that the mass of the W± particles is that predicted by the electroweak theory, the equivalent of 82 ± 2 GeV. The neutral member of the trio of heavy bosons, the Z⁰, is less frequently produced (by a factor of about 10) in the proton–antiproton collisions at CERN, has a greater mass (to the tune of an extra 12 GeV) and is chiefly recognizable by its decay into a pair of electrons, positive and negative.

Although the chief decay path for the W± bosons is that by which their existence was first recognized, it has from the outset been accepted that decay schemes leading to the production of quarks should be recognizable alternatives. Briefly, a W⁺ particle should be capable of yielding a *top* and the antimatter version of a *bottom* quark. (W⁻ would then yield anti-*top* and anti-*bottom*.) For the past two years, there has been general agreement on the way in which these particles could be recognized. The *bottom* quark (or anti-quark) would itself decay into a narrow jet of nuclear matter — pi-mesons for example. And the *top* quark, with a greater mass, would first decay into *bottom* and then yield another jet of particles, this time less tightly collimated. Since the first evidence for W± particles began to accumulate at CERN, people have been wondering whether some

of the events recorded by UA1 were signs of decay of this kind. Six events have now been unambiguously identified as the decay of W± into *top* and *bottom*; the mass of *top*, estimated at 40 GeV, remains substantially uncertain.

For the time being, however, the proof that *top* exists is enough to be going on with. In the simplest terms, the asymmetry that has now been removed is that between the set of known electron-like particles and the set of quarks. For reasons which are frankly not understood, the natural world contains not just one material lepton, the electron (and its anti-particle, the positron), but two others, the muon and the tauon (each with its oppositely charged anti-particle). With each of these three leptons is associated a distinctive neutrino, recognizably different in the mechanisms by which they interact with matter but, on present form, not otherwise distinguishable — they have no electrical charge and no mass. But neutrinos are, like electrons, true leptons — they are involved symmetrically with electrons, muons and tauons in the working of the weak nuclear interaction (as in beta decay).
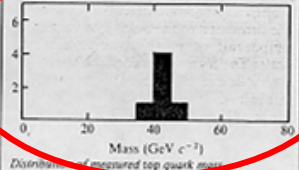
The idea that quarks should also come in pairs, and that there should be as many pairs of quarks as there are pairs of leptons, is more an act of faith than a consequence of theoretical expectation. To be sure, if the world is symmetrical in this way, it is possible to build neater theories, more symmetrical than would otherwise be the case. But that is merely a sign that, in its foundations, theoretical physics remains Pythagorean.

Phenomenologically, the need for symmetry has nevertheless been urgent since the late 1940s. The recognition of the difference between the pi-meson and the muon first raised the puzzle of the apparently superfluous lepton. The discovery (in cosmic rays) at the same time of a new kind of hadronic (nuclear) matter, called *strange* because that is what it was, set the scene for Gillman's radical proposal that mesons such as the pi-meson, but also

the *strange* particles themselves, are pairs of quarks — the pi-meson is a pair called *up* and *down* for example. But nucleons, such as protons and neutrons, and other baryons, are combinations of three quarks — the proton, for example, is two *up* quarks and one *down*. The partner of *strange*, discovered only in 1975, is *charm*. Evidence for *bottom*, also known as *beauty*, was found in 1977 in the proton–proton collisions arranged at Fermilab, where a meson whose mass exceeds the equivalent of 9.4 GeV was surmised to be a bound state of *bottom* and anti-*bottom*.

The quark called *top* (and also, sometimes, *truth*) is thus the missing member of the series. Its appearance has been expected for some time, but is no less welcome to the closet-Pythagoreans on that account. What will, in the short term, matter more is that the steady refinement of the mass now on the cards should make possible a degree of certainty about the nature of some still disputed hadronic particles and resonances. While the electroweak theory itself has been further confirmed, CERN and its UA1 collaboration have provided a more stringent test both of theories of quantum chromodynamics (theories of the strong nuclear interaction) and of Grand Unified Theories (which would roll that together with the electroweak theory but not — yet — with gravitation). Only time will tell whether the outcome is any confirmation of some version or another surprise — yet another pair of leptons or quarks, for example.

Inevitably, the question will arise in Britain whether the discovery of the *top* quark at the collaborative high-energy physics laboratory will bear on the decision, now delegated to a committee under Sir John Kendrew, on whether Britain should continue to collaborate. The arguments run both ways. The discovery of *top* means that CERN's list of unattained achievements has been reduced by one, but at the same time the laboratory's reputation for success has been enhanced. It is, however, unlikely that the committee's recommendations will be determined by scalp-counting of this kind, while high-energy physicists will properly draw attention to the need, now, for the careful understanding of the relationships between the six quarks that will come only from more careful measurements of the decay schemes now recognized, and of the alternatives still to be found.

John Maddox

Distribution of measured top quark mass.

John Womersley

Top at UA1

- **Associated Production of an isolated, large transverse momentum lepton (electron or muon) and two jets at the CERN $\bar{p}p$ Collider**
  G. Arnison et al., Phys. Lett. 147B, 493 (1984)

- **Looking for**

  $$\bar{p}p \rightarrow W$$
  $$\llcorner \rightarrow b\,t$$
  $$\llcorner \rightarrow bl\nu$$
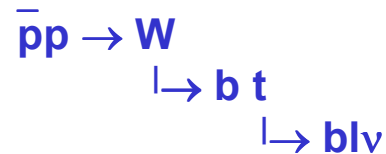
- **Signature is isolated lepton plus ME$_T$ and two jets**
  - **Mass (jlν) should peak at m$_t$**
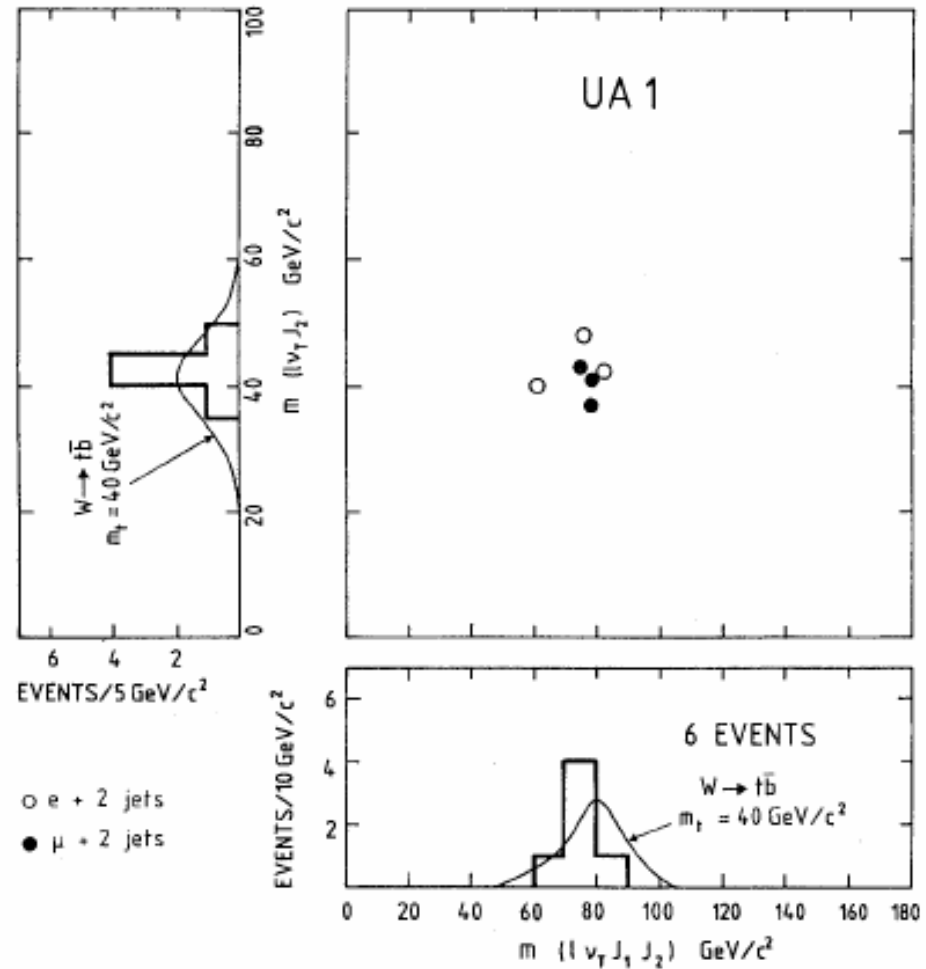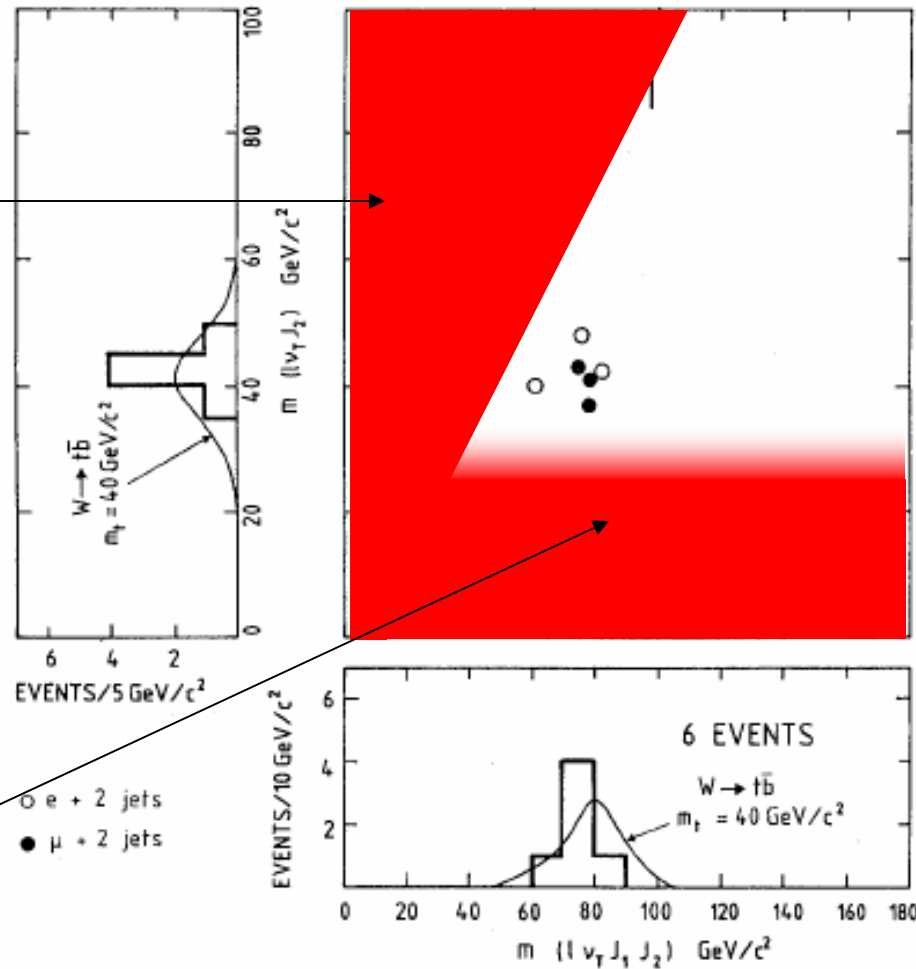 - **Mass (jjlν) should peak at m$_W$**

$\llcorner \rightarrow$

CCLRC
Rutherford Appleton Laboratory
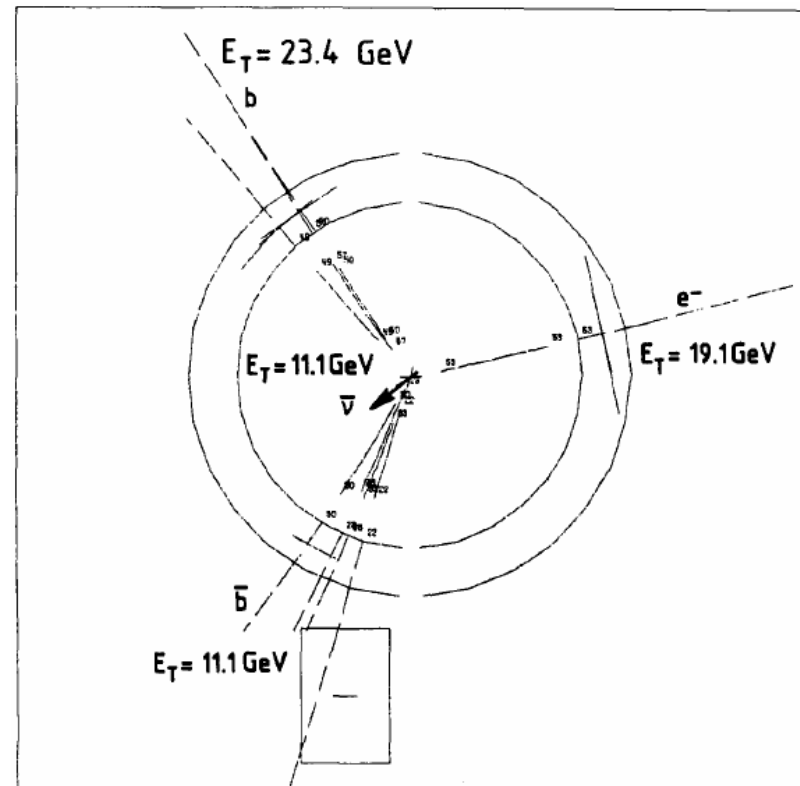
6 events observed

0.5 expected

JW (a young, naïve student):

"This looks pretty convincing!"

My advisor (older and wiser):

"Not necessarily…"

John Womersley

Hard to get $m(l\nu j_1 j_2)$ below $m(l\nu j_2)$ + 8 GeV
(since p_T^{j1} > 8 GeV)

Hard to get $m(l\nu j_2)$ below 24 GeV
(since p_T^l > 12 GeV)

In fact 30–50 GeV is typical for events just passing the p_T cuts

John Womersley

The moral

- **If the kinematic cuts tend to make events lie in the region where you expect the signal, you are really doing a "counting experiment" which depends on absolute knowledge of backgrounds**

efficiency   x   background   =   peak
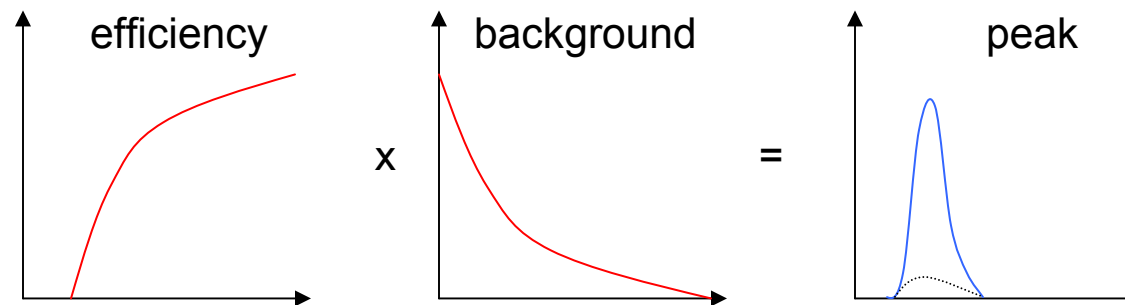
- **UA1 claim was later retracted after analysis of more data and better understanding of the backgrounds (J/ψ, Y, bb and cc)**
  - **In fairness, the knowledge of heavy flavor cross sections and the calculational toolkit available at that time were much less complete**
  - **Final limits from UA2 (UA1): m_t > 69 (60) GeV**