Metody eksperymentalne w fizyce wysokich energii

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Wykład XIII

- Metody Monte Carlo
- ⇒ Symulacja procesów fizycznych
- ⇒ Symulacja detektorów

MC methods - introduction

- Invented by Stanislaw Ulam, while working on Manhattan project
- When invented they were of purely academic interest, there were no computers in those days
- With the advent of computers became very useful
- Applied in many areas: science, financial world
- The Mathematics of Financial Derivatives : A Student Introduction <u>Paul Wilmott</u>, <u>Sam Howison</u>, <u>J.</u> <u>Dewynne</u>

Motywacja

Bardzo rzadko, we współczesnych eksperymentach, interesująca nas wielkość fizyczna może być zmierzona bezpośrednio w detektorze.

Wyniki pomiarów obarczone są:

- fluktuacjami w przebiegu obserwowanego zjawiska (np. rozwoju kaskady, wielokrotnego rozpraszania cząstek, ISR)
- wpływem tła (np. pile-up lub nakładające się przypadki)
- błędami pomiarowymi (rozdzielczość aparatury)
- wpływem nieefektywności (akceptacja geometryczna, nieefektywności liczników, progi układu wyzwalania)
- błędami rekonstrukcji i identyfikacji cząstek
- niepewnościami systematycznymi (kalibracji, pozycjonowania, itp.)

Motywacja

Często też badany proces jest tak skomplikowany lub tak różnorodny, że interesująca nas wielkość (np. stała sprzężenia) nie może być wprost wyrażona przez wyniki pomiarów. Dlatego też przeprowadza się symulację komputerową wykonanego doświadczenia, opisującą zarówno przebieg samego procesu fizycznego jak i działanie detektora.

Symulację **Monte Carlo** możemy stosować jedynie gdy znamy (ew. założymy) wszystkie szczegóły procesu fizycznego oraz działania aparatury pomiarowej.

Nie jest to sposób na uproszczenie zagadnienia, czy "zakrycie" naszej niewiedzy.

Teoretycznie, wykorzystując te same informacje moglibyśmy przedstawić oczekiwane wyniki pomiaru w postaci zbioru formuł analitycznych.

Metoda Monte Carlo jest sposobem na policzenie (przesumowanie lub przecałkowanie) takich formuły z dowolną dokładnością, praktycznie ograniczoną jedynie przez czas i szybkość działania komputera.

Simplest example: calculate area of a figure

- Cover the figure by a grid, calculate the number of grid cells which are inside and this gives you the area
- Shoot at random at the figure. Count the bullets that hit it. The area of then figure is
- S=(Nhit/Ntotal)*S(rectangle)





Przykład

Wyznaczenie akceptacji układu dwóch liczników



Kąt bryłowy "widziany" przez element dolnego licznika:

$$d\Omega(x,y) = \int_0^1 d(\cos\Theta) \int_0^{2\pi} d\phi \cdot F(x',y')$$

Funkcja F(x', y') wyraża warunek przechodzenia toru przez górny licznik (0 < x' < A & 0 < y' < A), a

 $x' = x + H \cdot tg(\theta) \cdot cos(\phi)$ $y' = y + H \cdot tg(\theta) \cdot sin(\phi)$

Kąt bryłowy dla całego układu możemy określić jako średnią po powierzchni dolnego licznika:

$$\Omega = \frac{1}{A^2} \int_0^A dx \int_0^A dy \int_0^1 d(\cos\Theta) \int_0^{2\pi} d\phi \cdot F(x',y')$$

Przykład

Metodą Monte Carlo możemy wyznaczyć Ω dużo prościej/szybciej.



Generujemy dużą liczbę *N* przypadków:

- losujemy punkt (x, y) przejścia cząstki przez dolny licznik (rozkład płaski w x i y)
- losujemy kąty określające kierunek lotu cząstki (θ, φ) (rozkład płaski w φ i cos(Θ))
- ekstrapolujemy tor cząstki do punktu przecięcia z płaszczyzną drugiego licznika (x', y')
- zliczamy przypadki dla których tor cząstki przeszedł przez oba liczniki (i.e. wartość F(x', y')).

 Ω wyznaczone jest przez ułamek przypadków przechodzących przez oba liczniki (×2 π , czyli obszar całkowania). Błąd wyniku symulacji maleje jak $1/\sqrt{N}$.

Przykład

Dla szybkozmiennej funkcji, nawet w jednym wymiarze, całka liczona metodą Monte Carlo może być wyznaczana z podobną efektywnością co przy całkowaniu numerycznym.

Wyniki całkowania w przedziale [0,1] "zwariowanej" funkcji:

$$f(x) = 1 - \sin\left(0.1 \cdot \exp\left(\frac{1}{x}\right)\right)$$





Area lights

Directional form of reflection equation (many shadow rays don't intersect light source) Surface form of reflection equation (all shadow rays intersect light source)





Same number of samples (shadow rays)

Convergence of numerical integration

• Consider
$$I = \int_{0}^{1} \mathrm{d}x^{D} f(\vec{x}).$$

- Convergence behavior crucial for numerical evaluations. For integration (N =number of evaluations of f):
 - Trapezium rule $\simeq 1/N^{2/D}$
 - Simpson's rule $\simeq 1/N^{4/D}$
 - Central limit theorem $\simeq 1/\sqrt{N}$.
- Therefore: Use central limit theorem.

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Monte Carlo integration

• Use random vectors $\vec{x_i} \longrightarrow$: Evaluate estimate of the integral $\langle I \rangle$ rather than *I*. $\langle I(f) \rangle = \frac{1}{N} \sum_{i=1}^{N} f(\vec{x_i}).$

(This is the original meaning of Monte Carlo: Use random numbers for integration.)

- Quality of estimate given by error estimator (variance) $\langle E(f) \rangle^2 = \frac{1}{N-1} [\langle I^2(f) \rangle - \langle I(f) \rangle^2].$
- Name of the game: Minimize $\langle E(f) \rangle$.
- Problem: Large fluctuations in integrand f
- Solution: Smart sampling methods

Importance sampling

Basic idea: Put more samples in regions, where f largest

- \implies improves convergence behavior (corresponds to a Jacobian transformation).
- Assume a function g(x) similar to f(x);
- obviously then, f(x)/g(x) is comparably smooth, hence ⟨E(f/g)⟩ is small.



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Losowanie parametrów

Rozważane parametry losowe rzadko maja płaskie rozkłady prawdopodobieństwa.

 \Rightarrow musimy umieć losować parametry z dowolnie skomplikowanych rozkładów.

Naogół stosujemy dedykowane algorytmy...

Metoda analityczna

Prawdopodobieństwo $\rho(x)$ dla zmiennej losowej x z przedziału $[x_1, x_2]$. Dystrybuanta rozkładu:

$$D(x) = \int_{x_1}^x dx' \cdot \rho(x')$$

Z definicji u = D(x) jest zmienną losową o rozkładzie płaskim w przedziale [0, 1]. Zmienną losową x możemy generować jako wartość funkcji odwrotnej do D(x), liczonej dla zmiennej losowej u (losowanej z rozkładu płaskiego):

$$x = D^{-1}(u)$$



Losowanie parametrów

Metoda analityczna jest przydatna zwłaszcza wtedy, gdy zmienna jest określona na nieskończonym przedziale wartości.

Pozwala sprowadzić losowanie z wielu różnych rozkładów (opisanych analitycznie) do losowania z rozkładu płaskiego (standardowy generator liczb losowych).

Przykład: losowanie czasu t po jakim rozpadnie się cząstka o średnim czasie życia τ .

$$\rho(t) = \frac{1}{\tau} \cdot e^{-t/\tau}$$

$$D(t) = 1 - e^{-t/\tau}$$

$$\Rightarrow t = D^{-1}(u)$$

$$t = -\tau \cdot \ln(1-u)$$

Losowanie parametrów

Przykład 2: rozkaład normalny (rozkład Gaussa o średniej 0 i dyspersji 1) "Trick": rozważmy dwie nieskorelowane zmienne - dwuwymiarowy rozkład:

$$\rho(x,y) = \rho(x) \cdot \rho(y) = \frac{1}{2\pi} \cdot e^{-\frac{1}{2}(x^2 + y^2)}$$

Przechodząc do współrzędnych biegunowych:

$$\rho(r,\phi) = \frac{1}{2\pi} \cdot r \cdot e^{-\frac{1}{2}r^2}$$

$$\Rightarrow \quad \rho(\phi) = \frac{1}{2\pi} \quad , \quad \rho(r^2) = \frac{1}{2} \cdot e^{-\frac{1}{2}r^2}$$

Losując ϕ z rozkładu płaskiego i r^2 z rozkładu eksponencjalnego ($u_{\phi}, u_r \in [0, 1[$):

$$\phi = 2\pi \cdot u_{\phi}$$
 , $r = \sqrt{-2 \cdot ln(1-u_r)}$

 $\Rightarrow \quad x = \sqrt{-2 \cdot \ln(1 - u_r)} \cdot \cos(2\pi \cdot u_\phi) \quad , \quad y = \sqrt{-2 \cdot \ln(1 - u_r)} \cdot \sin(2\pi \cdot u_\phi)$

Losowanie parametrów

Metoda ważenia

Najczęściej mamy do czynienia z rozkładami prawdopodobieństwa,

dla których nie możemy wyznaczyć dystrybuanty/funkcji odwrotnej do dystrybuanty.

 $\rho(x)$ - rozkład zmiennej losowej x z przedziału [x_1, x_2].

 $\rho_0(x)$ - rozkład pomocniczy, dla którego potrafimy losować analitycznie

$$w(x) = \frac{1}{W} \cdot \frac{\rho(x)}{\rho_0(x)} \qquad w(x) \stackrel{df}{\in} [0, 1]$$

gdzie W jest czynnikiem zapewniającym odpowiednią normalizację $W = \frac{\rho(x)}{\rho_0(x)}\Big|_{max}$

- Losujemy zmienną x z rozkładu $\rho_0(x)$.
- Losujemy zmienną u z rozkładu płaskiego w przedziale [0, 1].
- Jeśli $u \le w(x)$ przyjmujemy wartość x jako wylosowaną z rozkładu $\rho(x)$, w przeciwnym wypadku powtarzamy cały algorytm

Losowanie parametrów

Przykład:

$$\rho(x) = \frac{1}{\pi} \cdot \left(\frac{\sin(x)}{x}\right)^2$$
$$\rho_0(x) = \frac{1}{\pi} \cdot \frac{1}{x^2 + 1}$$

$$\Rightarrow \quad w(x) = \frac{2}{3} \cdot \frac{x^2 + 1}{x^2} \cdot \sin^2(x)$$

$$\Rightarrow \quad x \; = \; tg(\pi \cdot (u_1 - \frac{1}{2}))$$

jeśli $w(x) \ge u_2$





Losowanie parametrów

Tabularyzacja

Jeśli mamy zmienną losową, która przyjmuje wartości dyskretne i skończone. Np: wybór kanału rozpadu dla cząstki.

Losowanie wprost:

- porównujemy zmienną losową u z sumami prawdopodobieństw $s_i = \sum_{0}^{i} p_j$
- bierzemy pierwsze *i*, dla którego $u \leq s_i$

Losowanie z wagami: (przydatne przy dużej liczbie możliwości)

- wybieramy przedział i z rozkładu płaskiego ($i = N * u_1$)
- bierzemy ten przedział jeśli $N * p_i > u_2$, w przeciwnym wypadku i = C(i)(C(i) - tabela "sprzężonych przedziałów)

Można też stosować do generowania liczb z rozkładu ciągłego, jeśli nie jest potrzebna duża precyzja (tabularyzacja wartości prawdopodobieństwa w przedziałach wartości).



Losowanie parametrów

Optymalizacja

Wynik całkowania nie zmienia się jeśli:

- całkowaną funkcję przedstawić w postaci sumy funkcji bardziej elementarnych np. kinematykę rozpadu generujemy niezależnie dla każdego kanału
- obszar całkowania podzielić na części, zależnie od charakteru zmienności funkcji

W obu przypadkach kluczowy jest wybór podziału oraz określenie wag, z jakimi poszczególne funkcje składowe lub przedziały będą wybierane.

Istnieją dedykowane algorytmy i programy, które optymalizują podział.

Dobór wag: na ogół iteracyjnie, wagi są korygowane w trakcie działania algorytmu.

W ogólnym przypadku zmienność całkowanej funkcji nie jest dokładnie znana przed przystąpieniem do całkowania.

Stratified sampling

Basic idea: Decompose integral in M sub-integrals $\langle I(f) \rangle = \sum_{j=1}^{M} \langle I_j(f) \rangle$, $\langle E(f) \rangle^2 = \sum_{j=1}^{M} \langle E_j(f) \rangle^2$ Then: Overall variance smallest, if "equally distributed". \implies Sample, where the fluctuations are.

- Divide interval in bins;
- adjust bin-size or weight per bin such that variance identical in all bins.



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Example for stratified sampling: VEGAS

- Assume *m* bins in each dimension of *x*.
- For each bin k in each dimension η ∈ [1, n] assume a weight (probability) α^(η)_k for x_k to be in that bin.

Condition(s) on the weights: $\alpha_{k}^{(\eta)} \in [0, 1], \sum_{k=1}^{m} \alpha_{k}^{(\eta)} = 1.$

• For each bin in each dimension calculate $\langle I_k^{(\eta)} \rangle$ and $\langle E_k^{(\eta)} \rangle$.

Obviously, for all $\eta,~\langle I\rangle=\sum_{k=1}^m\langle I_k^{(\eta)}\rangle,$ but error estimates different.

In each dimensions, iterate and update the $\alpha_k^{(\eta)}$; example for updating:

$$\alpha_k^{(\eta)}(\mathsf{rm new}) \propto \alpha_k^{(\eta)}(\mathsf{rm old}) \left(rac{E_k^{(\eta)}}{E_{\mathrm{tot.}}(\eta)}
ight)^{\kappa}.$$

Problem with this simple algorithm: Gets a hold only on fluctuations || to binning axes.



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Multichannel sampling

Basic idea: Use a sum of functions $g_i(\vec{x})$ as Jacobian $g(\vec{x})$. $\implies g(\vec{x}) = \sum_{i=1}^{N} \alpha_i g_i(\vec{x});$ \implies condition on weights like stratified sampling; ("Combination" of importance & stratified sampling).

Algorithm for one iteration:

- Select g_i with probability $\alpha_i \rightarrow \vec{x_j}$.
- Calculate total weight $g(\vec{x}_j)$ and partial weights $g_i(\vec{x}_j)$
- Add f(x_j)/g(x_j) to total result and f(x_j)/g_i(x_j) to partial (channel-) results.
- After N sampling steps, update a-priori weights.



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This is the method of choice for parton level event generation!

Selecting after sampling: Unweighting efficiency

Basic idea: Use hit-or-miss method;

Generate \vec{x} with integration method, compare actual $f(\vec{x})$ with maximal value during sampling; \implies "Unweighted events".

Comments:

- unweighting efficiency, $w_{eff} = \langle f(\vec{x}_i) / f_{max} \rangle$ = number of trials for each event.
- Good measure for integration performance.
- Expect log₁₀ w_{eff} ≈ 3 5 for good integration of multi-particle final states at tree-level.

• Maybe acceptable to use $f_{\max, eff} = K f_{\max}$ with K > 1. Problem: what to do with events where $f(\vec{x}_j)/f_{\max, eff} > 1$? Answer: Add $\inf[f(\vec{x}_j)/f_{\max, eff}] = k$ events and perform hit-or-miss on $f(\vec{x}_j)/f_{\max, eff} - k$.

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Programy symulacji

Choć stosują podobne metody programy symulacji dzielą się zasadniczo na:

- Symulacja procesu fizycznego, który chcemy badać
 Wynikiem symulacji jest wartość przekroju czynnego
 oraz zbiór wygenerowanych przypadków (przypadek: lista cząstek oraz ich pędów)
- Symulacja działania detektora
 - Wynikiem jest zbiór pomiarów odpowiadający danym jakie zbierane są z detektora. → można na nich "zapuścić" programy rekonstrukcji i analizy

Dodatkowo wydzielamy też czasami:

- Preselekcja wstępna ocena przypadków fizycznych, przed symulacją detektora, pod kątem planowanej analizy fizycznej. Cel: zmniejszenie liczby przypadków do pełnej symulacji detektora (zwłaszcza tła).
- Symulacja układu wyzwalania aby sprawdzić czy dany przypadek zostałby rzeczywiście przez eksperyment zarejestrowany.

Hadron-hadron processes

In hard hadron-hadron scattering, constituent partons from each incoming hadron interact at short distance (large momentum transfer Q^2).



For hadron momenta P_1, P_2 ($S = 2P_1 \cdot P_2$), form of cross section is

$$\sigma(S) = \sum_{i,j} \int dx_1 dx_2 D_i(x_1, \mu^2) D_j(x_2, \mu^2) \hat{\sigma}_{ij}(\hat{s} = x_1 x_2 S, \alpha_S(\mu^2), Q^2/\mu^2)$$

where μ^2 is factorization scale and $\hat{\sigma}_{ij}$ is subprocess cross section for parton types i, j.

- ★ Notice that factorization scale is in principle arbitrary: affects only what we call part of subprocess or part of initial-state evolution (parton shower).
- ★ Unlike e^+e^- or ep, we may have interaction between spectator partons, leading to soft underlying event and/or multiple hard scattering.

Symulacja procesu fizycznego



Przekrój czynny na poziomie partonowym (LO QED) $\frac{d\sigma^{q\bar{q}\to\mu^+\mu^-}}{d\cos\theta_{cm}} = \frac{\pi\alpha_{em}^2}{2s} \cdot e_q^2 \cdot \left(1 + \cos^2\theta_{cm}\right)$ s jest kwadratem masy inwariantnej układu $q\bar{q}$

Różniczkowy przekrój czynny dla procesu $p + \bar{p} \rightarrow \mu^+ \mu^- + X$

$$\frac{d\sigma(p\bar{p}\to\mu^+\mu^-X)}{dM_{\mu\mu}} = \frac{1}{N_c}\sum_q \int dx \cdot f_{q/p}(x) \int dy \cdot f_{q/p}(y) \int d\cos\theta_{cm} \cdot \frac{d\sigma^{q\bar{q}\to\mu^+\mu^-}}{d\cos\theta_{cm}} \cdot \delta\left(M_{\mu\mu} - \sqrt{4xyE_p^2}\right)$$

Musimy losować q, x, y i cos θ_{cm} . Problem: bardzo różne wartości wagi $\left(\frac{d\sigma}{d\cos\theta}\right)$

Next-to-leading order



The contribution of the real diagrams (in four dimensions) is

$$|M|^{2} \sim g^{2} C_{F} \left[\frac{u}{t} + \frac{t}{u} + \frac{2Q^{2}s}{ut} \right] = g^{2} C_{F} \left[\left(\frac{1+z^{2}}{1-z} \right) \left(\frac{-s}{t} + \frac{-s}{u} \right) - 2 \right]$$

where $z = Q^2/s, s + t + u = Q^2$.

Note that the real diagrams contain collinear singularities, $u \to 0, t \to 0$ and soft singularities, $z \to 1$.

The coefficient of the divergence is the unregulated branching probability $\hat{P}_{qq}(z)$.

Ignore for simplicity the diagrams with incoming gluons.

Parton level simulations

Stating the problem(s)

- Multi-particle final states for signals & backgrounds.
- Need to evaluate $d\sigma_N$:

$$\int_{\text{tuts}} \left[\prod_{i=1}^{N} \frac{\mathrm{d}^{3} q_{i}}{(2\pi)^{3} 2 E_{i}} \right] \delta^{4} \left(p_{1} + p_{2} - \sum_{i} q_{i} \right) \left| \mathcal{M}_{p_{1} p_{2} \to N} \right|^{2}$$

- Problem 1: Factorial growth of number of amplitudes.
- Problem 2: Complicated phase-space structure.
- Solutions: Numerical methods.

Parton level simulations

Factorial growth: $e^+e^- \rightarrow q\bar{q} + ng$



Survey of existing parton-level tools

Standard Model (and beyond) tools @ tree-level

- AlpGen M.L.Mangano *et al.*, JHEP **0307** (2003) 001;
- AMEGIC++ F.K., R.Kuhn, G.Soff, JHEP 0202 (2002) 044;
- CompHEP/CalcHEP E.Boos et al. Nucl. Instrum. Meth. A 534 (2004) 250;
- HELAC A.Kanaki and C.G.Papadopoulos, Comput. Phys. Commun. 132 (2000) 306;
- MadGraph/MadEvent F.Maltoni and T.Stelzer, JHEP 0302, 027 (2003);
- O'Mega+WHIZARD M.Moretti, T.Ohl and J.Reuter, arXiv:hep-ph/0102195.

All tools here are completely self-contained and automated and provide amplitudes and integrators of their own.

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Survey of existing parton-level tools

Comparison of tree-level tools

	Models	$2 \rightarrow n$	Ampl.	Integ.	public?	lang.
Alpgen	SM	<i>n</i> = 8	rec.	Multi	yes	Fortran
Amegic	SM, MSSM, ADD	<i>n</i> = 6	hel.	Multi	yes	C++
CompHep	SM,MSSM	<i>n</i> = 4	trace	1Channel	yes	С
HELAC	SM	<i>n</i> = 8	rec.	Multi	yes	Fortran
MadEvent	SM,MSSM	<i>n</i> = 6	hel.	Multi	yes	Fortran
O'Mega	SM,MSSM,LH	<i>n</i> = 8	rec.	Multi	yes	O'Caml

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Limitations of parton level simulation

General

- Fixed order parton level (LO, NLO, ...) implies fixed multiplicity
- No control over potentially large logs (appear when two partons come close to each other).
- Parton level is parton level experimental definition of observables relies on hadrons.

Therefore: Need hadron level event generators!

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Simulation's paradigm

Basic strategy

Divide event into stages, separated by different scales.

• Signal/background:

Exact matrix elements.

• QCD-Bremsstrahlung:

Parton showers (also in initial state).

Multiple interactions:

Beyond factorization: Modeling.

• Hadronization:

Non-perturbative QCD: Modeling.



Shower Monte Carlos

Attempt to give a complete description of a hadron scattering event. Analysis of events at the Tevatron and the LHC will be performed using these programs.

- Large number of standard model hard scattering processes
- Inclusion of some BSM processes.
- Inclusion of real (and virtual) radiation using QCD-based parton shower approximation
- Fragmentation of partons into the observed hadrons.
- Model for resonance decay included



Motivation: Why parton showers?

Common wisdom

- Well-known: Accelerated charges radiate
- QED: Electrons (charged) emit photons Photons split into electron-positron pairs
- QCD: Quarks (colored) emit gluons Gluons split into quark pairs
- Difference: Gluons are colored (photons are not charged) Hence: Gluons emit gluons!
- Cascade of emissions: Parton shower

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Occurrence of large logarithms

- $e^+e^-
 ightarrow$ jets
 - Differential cross section:

$$\frac{\mathrm{d}\sigma_{ee \to 3j}}{\mathrm{d}x_1 \mathrm{d}x_2} = \sigma_{ee \to 2j} \frac{C_F \alpha_s}{\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

Singular for $x_{1,2} \rightarrow 1$.

• Rewrite with opening angle θ_{qg} and gluon energy fraction $x_3 = 2E_g/E_{c.m.}$:

$$\frac{\mathrm{d}\sigma_{ee \to 3j}}{\mathrm{d}\cos\theta_{qg}\mathrm{d}x_3} = \sigma_{ee \to 2j}\frac{C_F\alpha_s}{\pi}\left[\frac{2}{\sin^2\theta_{qg}}\frac{1+(1-x_3)^2}{x_3}-x_3\right]$$

Singular for $x_3 \rightarrow 0$ ("soft"), sin $\theta_{qg} \rightarrow 0$ ("collinear").

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Occurrence of large logarithms

Collinear singularities

• Use

$$\frac{2\mathrm{d}\cos\theta_{qg}}{\sin^2\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{qg}}{1+\cos\theta_{qg}} = \frac{\mathrm{d}\cos\theta_{qg}}{1-\cos\theta_{qg}} + \frac{\mathrm{d}\cos\theta_{\bar{q}g}}{1-\cos\theta_{\bar{q}g}} \approx \frac{\mathrm{d}\theta_{qg}^2}{\theta_{qg}^2} + \frac{\mathrm{d}\theta_{\bar{q}g}^2}{\theta_{\bar{q}g}^2}$$

• Independent evolution of two jets $(q \text{ and } \bar{q})$:

$$\mathrm{d}\sigma_{ee\to 3j} \approx \sigma_{ee\to 2j} \sum_{j\in\{q,\bar{q}\}} \frac{C_F \alpha_s}{2\pi} \frac{\mathrm{d}\theta_{jg}^2}{\theta_{jg}^2} P(z) \; ,$$

where
$$P(z) = \frac{1+(1-z)^2}{z}$$
 (DGLAP splitting function)

Occurrence of large logarithms

Many emissions

Iterate emissions (jets)

Maximal result for $t_1 > t_2 > \ldots t_n$:



$$\mathrm{d}\sigma \propto \sigma_0 \int_{Q_0^2}^{Q^2} \frac{\mathrm{d}t_1}{t_1} \int_{Q_0^2}^{t_1} \frac{\mathrm{d}t_2}{t_2} \dots \int_{Q_0^2}^{t_{n-1}} \frac{\mathrm{d}t_n}{t_n} \propto \log^n \frac{Q^2}{Q_0^2}$$

• How about Q²? Process-dependent!

Occurrence of large logarithms

Ordering the emissions : Radiation pattern



Infrared cutoff

- In DGLAP equation, infrared singularities of splitting functions at z = 1 are regularized by plus-prescription. However, in above form we must introduce an explicit infrared cutoff, $z < 1 - \epsilon(t)$. Branchings with z above this range are unresolvable: emitted parton is too soft to detect. Sudakov form factor with this cutoff is probability of evolving from t_0 to t without any resolvable branching.
- Sudakov form factor sums enhanced virtual (parton loop) as well as real (parton emission) contributions. No-branching probability is the sum of virtual and unresolvable real contributions: both are divergent but their sum is finite.
- Infrared cutoff $\epsilon(t)$ depends on what we classify as resolvable emission. For timelike branching, natural resolution limit is given by cutoff on parton virtual mass-squared, $t > t_0$. When parton energies are much larger than virtual masses, we may write, ($n^2 = p^2 = p \cdot p_T = p \cdot p_T = 0, n \cdot p = 1$)

$$p_{a} = p^{\mu} + \frac{p_{a}^{2}}{2}n^{\mu}$$

$$p_{b} = zp^{\mu} + \frac{p_{T}^{2} + p_{b}^{2}}{2z}n^{\nu} + p_{T}^{\mu}$$

$$p_{c} = (1-z)p^{\mu} + \frac{p_{T}^{2} + p_{c}^{2}}{2(1-z)}n^{\mu} - p_{T}^{\mu}$$

Leading general purpose programs

Pythia, T. Sjöstrand, L. Lönnblad, S. Mrenna and P.Z. Skands, http://www.thep.lu.se/~torbjorn/Pythia.html

Pythia 8, T. Sjöstrand C++ http://www.thep.lu.se/~torbjorn/Pythia.html

HERWIG, G. Corcella, I.G. Knowles, G. Marchesini, S. Moretti, K. Odagiri, P. Richardson, M.H. Seymour, B.R. Webber, http://hepwww.rl.ac.uk/theory/seymour/herwig/

Herwig++

S. Gieseke, A. Ribon, P. Richardson, M.H. Seymour, P. Stephens, B.R. Webber http://projects.hepforge.org/herwig/

SHERPA,

Tanju Gleisberg, Frank Krauss, Andreas Schälicke, Steffen Schumann, Jan Winter http://www.sherpa-mc.de/

ISAJET,

F. Paige, S. Protopopescu, H. Baer and X. Tata http://www.phy.bnl.gov/~isajet

Orders in ME and PS

ME vs. PS

- Matrix elements good for: hard, large-angle emissions; take care of interferences.
- Parton shower good for: soft, collinear emissions; resums large logarithms.
- Want to combine both! Avoid double-counting.



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MC@NLC

Correcting the parton shower



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Combining MEs & PS

S.Catani, F.K., R.Kuhn and B.R.Webber, JHEP **0111** (2001) 063 F.K., JHEP **0208** (2002) 015

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Basic principles

- Want:
 - All jet emissions correct at tree level + LL,
 - Soft emissions correctly resummed in PS
- Method:
 - Separate Jet-production/evolution by $Q_{\rm jet}$ (k_{\perp} algorithm).
 - Produce jets according to LO matrix elements
 - $\bullet\,$ re-weight with Sudakov form factor + running $\alpha_{\rm s}$ weights,
 - veto jet production in parton shower.
- Process-independent implementation.

Combining MEs & PS



Combining MEs & PS

n 20 400 400

140 160 180 200

P_{1Z}/GeV

Comparison with data from Tevatron p_{\perp} of Z-bosons in $p\bar{p} \rightarrow Z + X$ t. 84 (2000) 845 _____Z+0 iet 습, <u>pb</u> 라, <u>GeV</u> - Z + 1 int CDF 10 10

5

0

10 15 20 25 30

35 45 50

40

P_{IZ}/GeV





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Orientation	PDFs	Hadronization	Underlying Event	

Dynamical strings in $e^+e^- \rightarrow q\bar{q}$

B.Andersson, G.Gustafson, G.Ingelman and T.Sjostrand, Phys. Rept. 97 (1983) 31.

- Ignoring gluon radiation: Point-like source of string.
- Intense chromomagnetic field within string: More $q\bar{q}$ pairs created by tunnelling.
- Analogy with QED (Schwinger mechanism): $\mathrm{d}\mathcal{P} \sim \mathrm{d}x\mathrm{d}t\exp\left(-\pi m_a^2/\kappa\right)$, $\kappa =$ "string tension".



F. Krauss

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Implementation of naive parameterizations

• Feynman-Field independent fragmentation.

R.D.Field and R.P.Feynman, Nucl. Phys. B 136 (1978) 1

- Recursively fragment q
 ightarrow q'+ had, where
 - Transverse momentum from (fitted) Gaussian;
 - longitudinal momentum arbitrary (hence from measurements);
 - flavor from symmetry arguments + measurements.
- Problems: frame dependent, "last quark", infrared safety, no direct link to perturbation theory,

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Orientation	PDFs	Hadronization	Underlying Event	

Preconfinement

- Underlying: Large N_c-limit (planar graphs).
- Follows evolution of color in parton showers: at the end of shower color singlets close in phase space.
- Mass of singlets: peaked at low scales $\approx Q_0^2$.



A (10) × (10) × (10)

Primordial cluster mass distribution

- Starting point: Preconfinement;
- split gluons into qq
 -pairs;
- adjacent pairs color connected, form colorless (white) clusters.
- Clusters ("≈ excited hadrons) decay into hadrons



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Underlying Event

Multiple parton scattering?





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Orientation	PDFs	Hadronization	Underlying Event	

Underlying Event





- Everything apart from the hard interaction including IS showers, FS showers, remnant hadronization.
- **2** Remnant-remnant interactions, soft and/or hard.
- Sesson: hard to define

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Orient		PDFs	Hadronization	Underlying Event	
Ur	nder	lying event			
	Old	Pythia model: A	lgorithm	, simplified	
		T.Sjostrand and M.van Zijl, Phys. Re	v. D 36 (1987) 20	119.	
	٩	Start with hard inter	action, at	scale $Q^2_{ m hard}$.	
	۹	Select a new scale p	2 ⊥		
		(according to $f = \frac{\mathrm{d}\sigma_{2 \to 2}(p_{\perp}^{2})}{\mathrm{d}p_{\perp}^{2}}$ with	$p_{\perp}^2 \in [p_{\perp,\min}^2,$	Q ²])	
	٩	Rescale proton mom	entum	("proton-parton = proton with reduced e	energy").
	٩	Repeat until below μ	$p_{\perp,\min}^2$.		
	٩	May add impact-par	ameter de	pendence, showers, etc	
	٩	Treat intrinsic k_{\perp} of	partons (ightarrow parameter)	
	٩	Model proton remna	nts (\rightarrow p	arameter)	

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Detector Simulation - General

- General characteristics of a detector simulation system
 - You specify the geometry of a particle detector
 - Then the software system automatically transports the particle you shoot into the detector by simulating the particle interactions in matter based on the Monte Carlo method
- The heart of the simulation: the Monte Carlo method
 - A method to search for solutions to a mathematical problem using a statistical sampling with random numbers



The role of simulation

- Simulation plays a fundamental role in various domains and phases of an experimental physics project
 - design of the experimental set-up
 - evaluation and definition of the potential physics output of the project
 - evaluation of potential risks to the project
 - assessment of the performance of the experiment
 - development, test and optimisation of reconstruction and physics analysis software
 - contribution to the calculation and validation of physics results
- The scope of these lectures (and of Geant4) encompasses the simulation of the passage of particles through matter
 - there are other kinds of simulation components, such as *physics event generators*, *electronics response* generation, etc.
 - often the simulation of a complex experiment consists of several of these components interfaced to one another



Basic requirements for a simulation system

- Modeling the experimental set-up
- Tracking particles through matter
- Interaction of particles with matter
- Modeling the detector response
- Run and event control
- Accessory utilities (random number generators, PDG particle information etc.)
- Interface to event generators
- Visualisation of the set-up, tracks and hits
- User interface
- Persistency





EGS4, EGS5, EGSnrc Geant3, Geant4 MARS MCNP, MCNPX, A3MCNP, MCNP-DSP, MCNP4B MVP, MVP-BURN Penelope Peregrine Tripoli-3, Tripoli-3 A, Tripoli-4

...and I probably forgot some more

DPM FA-MC FLUKA GEM HERMES LAHET **MCBEND** MCU MF3D NMTC MONK MORSE RTS&T-2000 SCALE TRAX VMC++

Monte Carlo codes presented at the MC200 Conference, Lisbon, October 2000



Maria Grazia Pia, INFN Genova



PP∙\RC













Stanford Linear Accelerator



Geant4 Collaboration

MoU based

Distribution, Development and User Support of Geant4





CERN, ESA, KEK, SLAC, TRIUMF, TJNL INFN, IN2P3, PPARC

Barcelona Univ., Budker Inst., Frankfurt Univ., Karolinska Inst., Helsinki Univ., Lebedev Inst., LIP, Northeastern Univ. etc.



enter













What Can Geant4 Do for You?

- Transports a particle step-by-step by taking into account the interactions with materials and external electromagnetic fields until the particle
 - Ioses its kinetic energy to zero,
 - disappears by an interaction,
 - comes to the end of the simulation volume
- Provides a way for the user to access the transportation process and grab the simulation results
 - at the beginning and end of transportation,
 - at the end of each stepping in transportation,
 - at the time when the particle is going into the sensitive volume of the detector
 - etc.
 - These are called "User Actions"

Geant 4

What You Have to Do for Geant4?

Three essential information you have to provide:

- Geometrical information of the detector
- Choice of physics processes
- Kinematical information of particles going into the detector

Auxiliary you have to prepare:

- Magnetic and electric field
- Actions you want to take when you access the particle transportation
- Actions you want to take when a particle goes into a sensitive volume of the detector
- etc.



Geometry

Role

- detailed detector description
- efficient navigation
- Three conceptual layers
 - Solid: shape, size



- LogicalVolume: material, sensitivity, daughter volumes, etc.
- PhysicalVolume: position, rotation

One can do fancy things with geometry...















Multiple representations Same abstract interface CSG (Constructed Solid Geometries) - simple solids

- **STEP** extensions
 - polyhedra, spheres, cylinders, cones, toroids, etc.
- BREPS (Boundary REPresented Solids)
 - volumes defined by boundary surfaces

CAD exchange







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Physical Volumes



Electric and magnetic fields of variable non-uniformity and differentiability



Geant4 field ~ 2 times faster than FORTRAN/GEANT3

M. Stavrianakou for the CMS Collaboration

MOKKA Linear Collider Detector



1 GeV proton in the Earth's geomagnetic field

Courtesy Laurent Desorgher, University of Bern

Maria Grazia Pia, IN



P. Mora de Freitas / H. Videau, L.L.R. – Ecole polytechnique



P. Mora de Freitas / H. Videau, L.L.R. – Ecole polytechnique

Mokka: a Common Geometry Access API (F77, C++,C)



Implements some reconstruction utilities. Java API coming soon. G. Musat

P. Mora de Freitas / H. Videau, L.L.R. – Ecole polytechnique



geometry for reconstruction

GEometry **A**PI for **R**econstruction

- high level abstract interface:
- per subdetector type (Hcal,TPC,...) parameters/quantities for reco
 - geometry + some navigation
 - implementation uses xml files writter from Mokka (simulation)
- abstract interface for detailed geometry &materials:
 - point properties
 - path properties
 - implementation based on geant4



MokkaGear

 enforce only one source of geometry: the simulation program creates the geometry xml files used in reconstruction

(recently improved by K.Harder et al)
software framework architecture



Physics

Abstract interface to physics processes

- Tracking independent from physics
- Uniform treatment of electromagnetic and hadronic processes
- Distinction between processes and models
 - multiple models for the same physics process (complementary/alternative)

Transparency (supported by *encapsulation* and *polymorphism*)

- Calculation of cross-sections independent from the way they are accessed (data files, analytical formulae etc.)
- Calculation of the final state independent from tracking
- Explicit use of units throughout the code
- Open system
 - Users can easily create and use their own models

Electromagnetic physics

- electrons and positrons
- y, X-ray and optical photons
- muons
- charged hadrons
- ions

Comparable to Geant3 already in the α release (1997) Further extensions (*facilitated by the OO technology*)

- High energy extensions
 - needed for LHC experiments, cosmic ray experiments...
- Low energy extensions
 - fundamental for space and medical applications, dark matter and v experiments, antimatter spectroscopy etc.
- Alternative models for the same process

All obeying to the same abstract Process interface <u>transparent to tracking</u> Maria Grazia Pia, INFN Genova

- Multiple scattering
- Bremsstrahlung
- Ionisation

energy

loss

- Annihilation
- Photoelectric effect
- Compton scattering
- Rayleigh effect
- γ conversion
- e⁺e⁻ pair production
- Synchrotron radiation
- Transition radiation
- Cherenkov
- Refraction
- Reflection
- Absorption
- Scintillation
- Fluorescence
- Auger



Calorimetry

Single crystal containment: E_{1x1}/E_{3x3} versus position



Mass Jpsi mm

Physics Maria Grazia Pia, INFN Genova

K_s mass



Optical photons

Production of optical photons in HEP detectors is mainly due to Cherenkov effect and scintillation

Processes in Geant4:

- in-flight absorption
- Rayleigh scattering
- medium-boundary interactions (reflection, refraction)

Geant4 Optical Processes : Scintillating Cells and WLS Fibers





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Courtesy of J. Mc Cormick (SLAC)

Hadronic physics

Completely different approach w.r.t. the past (Geant3)

- native
- transparent
- no longer interface to external packages
- clear separation between data and their use in algorithms

Cross section data sets

- transparent and interchangeable
- Final state calculation
 - models by particle, energy, material

Ample variety of models

- the most complete hadronic simulation kit on the market
- alternative and complementary models
- data-driven, parameterised and theoretical models

Hadronic model inventory



Parameterised and data-driven hadronic models (1)

Based on experimental data

- Some models originally from GHEISHA
 - completely reengineered into OO design
 - refined physics parameterisations

New parameterisations

- pp, elastic differential cross section
- nN, total cross section
- pN, total cross section
- np, elastic differential cross section
- $-\pi N$, total cross section
- $-\pi N$, coherent elastic scattering







p elastic scattering on Hydrogen



Parameterised and data-driven hadronic models (2) Other models are completely new, such as:



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Theory-driven models

- Complementary and alternative models
- Evaporation phase
- Low energy range O(100 MeV): pre-equilibrium
- Intermediate energy range, O(100 MeV) to O(5 GeV): *intra-nuclear transport*
- High energy range: hadronic generator régime





Nature, July 1984



CERN comes out again on top

-NEWS AND VIEWS-

With the discovery of the electroweak bosons (W[±] and Z⁰) in the bag, CERN now announces the discovery of the quark called top. What will come next?

THE Matthew principle -- "to him that | of the events recorded by UAI were signs of | the strange particles themselves, are pairs hath shall be given" - is working in favour of CERN, the European high-energy physics laboratory at Geneva, and of the UA1 collaboration which, at the end of last year, announced the discovery of the Wa and Z⁰ particles which mediate the electroweak interaction. Last week, the same 80-strong collaboration, under the leadership of Carlo Rubbia, announced the discovery of the missing sixth quark, called top, long-predicted but hitherto elusive. By doing so, they have put yet another cap on the electroweak theory while restoring a seemly symmetry to the evolving picture of quarks as the elementary constituents of the material Universe.

ATURE VOL 310 12 JULY 1984

The new development at CERN follows almost exactly along the lines expected (and described, for example, by Dr F. Close in his comment on the electroweak bosons, see Nature 303, 656; 1983). The source of the sixth quark is a charged boson, W+ or W-, first recognized at CERN by their decay into an electron (with electrical charge of the same sign), with excess momentum carried away by a neutrino. Events of this kind accumulated at CERN in the past two years have amply confirmed that the mass of the W* particles is that predicted by the electroweak theory, the equivalent of 82 ± 2 GeV. The net tral memberoful member of the trie of neavy bosons, th is less frequently produced (by a facto Z⁰. about 10) in the proton-antiproto

collisions at CERN, has a greater mass (to the tune of an extra 12 GeV) and is chiefly recognizable by its decay into a pair of electrons, positive and negative. Although the chief decay path for the

W= bosons is that by which their existence was first recognized, it has from the outset been accepted that decay schemes leading to the production of quarks should be recognizable alternatives. Briefly, a W* particle should be capable of yielding a top and the antimatter version of a bottom quark. (W- would then yield anti-top and anti-bottom.) For the past two years, there has been general agreement on the way in which these particles could be recognized. The bottom quark (or anti-quark) would itself decay into a narrow jet of nuclear matter - pi-mesons for example. And the top quark, with a greater mass, would first decay into bottom and then yield another jet of particles, this time less tightly collimated. Since the first evidence for W* particles began to accumulate at CERN, people have been wondering whether some decay of this kind. Six events have now been unambiguously identified as the decay of W* into top and bottom; the mass of top, estimated at 40 GeV, remains substantially uncertain.

For the time being, however, the proof that top exists is enough to be going on with. In the simplest terms, the asymmetry that has now been removed is that between the set of known electron-like particles and the set of quarks. For reasons which are frankly not understood, the natural world contains not just one material lepton, the electron (and its anti-particle, the positron), but two others, the muon and the tauon (each with its oppositely charged anti-particle). With each of these three leptons is associated a distinctive neutrino, recognizably different in the mechanisms by which they interact with matter but, on present form, not otherwise distinguishable - they have no electrical charge and no mass. But neutrinos are, like electrons, true leptons - they are involved symmetrically with and tauons in the working of the weak nuclear 1 ction (as in beta decay).



The idea that quarks should also come in pairs, and that there should be as many pairs of quarks as there are pairs of leptons, is more an act of faith than a consequence of theoretical expectation. To be sure, if the world is symmetrical in this way, it is possible to build neater theories, more symmetrical than would otherwise be the case. But that is merely a sign that, in its foundations, theoretical physics remains Pythagorean.

Phenomenologically, the need for symmetry has nevertheless been urgent since the late 1940s. The recognition of the difference between the pi-meson and the muon first raised the puzzle of the apparently superfluous lepton. The discovery (in cosmic rays) at the same time of a new kind of hadronic (nuclear) matter, called strange because that is what it was, set the scene for Gillman's radical proposal that mesons such as the pi-meson, but also

of quarks - the pi-meson is a pair called up and down for example. But nucleons, such as protons and neutrons, and other baryons, are combinations of three quarks - the proton, for example, is two up quarks and one down. The partner of strange, discovered only in 1975, is charm. Evidence for bottom, also known as beauty, was found in 1977 in the protonproton collisions arranged at Fermilab, where a meson whose mass exceeds the equivalent of 9.4 GeV was surmised to be a bound state of bottom and anti-bottom.

The quark called top (and also, sometimes, truth) is thus the missing member of the series. Its appearance has been expected for some time, but is no less welcome to the closet-Pythagoreans on that account. What will, in the short term, matter more is that the steady refinement of the mass now on the cards should make possible a degree of certainty about the nature of some still disputed hadronic particles and resonances. While the electroweak theory itself has been further confirmed, CERN and its UA1 collaboration have provided a more stringent test both of theories of antum chromodynamics (theories of the strong nuclear interaction) and of Grand United Theories (which would roll that together with the electroweak theory but not - yet - with gravitation). Only time tell whether the outcome is any will nfirmation of some version or another surprise - yet another pair of leptons or quarks, for example,

Inevitably, the question will arise in Britain whether the discovery of the top quark at the collaborative high-energy physics laboratory will bear on the decision, now delegated to a committee under Sir John Kendrew, on whether Britain should continue to collaborate. The arguments run both ways. The discovery of top means that CERN's list of unattained achievements has been reduced by one, but at the same time the laboratory's reputation for success has been enhanced. It is, however, unlikely that the committee's recommendations will be determined by scalp-counting of this kind, while highenergy physicists will properly draw attention to the need, now, for the careful understanding of the relationships between the six quarks that will come only from more careful measurements of the decay schemes now recognized, and of the alternatives still to be found. John Maddoy

John Womersley



Top at UA1

- Associated Production of an isolated, large transverse momentum lepton (electron or muon) and two jets at the CERN pp Collider
 G. Arnison et al., Phys. Lett. 147B, 493 (1984)
- Looking for

 \rightarrow

 $\overline{p}p \rightarrow W$ $\rightarrow b t$ $\rightarrow b lv$

- Signature is isolated lepton plus ME_T and two jets
 - Mass (jlv) should peak at m_t
 - Mass (jjlv) should peak at m_W





What they found

6 events observed 0.5 expected

JW (a young, naïve student): "This looks pretty convincing!"

My advisor (older and wiser): "Not necessarily..."



John Womersley





John Womersley



The moral

 If the kinematic cuts tend to make events lie in the region where you expect the signal, you are really doing a "counting experiment" which depends on absolute knowledge of backgrounds



- UA1 claim was later retracted after analysis of more data and better understanding of the backgrounds (J/ ψ , Y, bb and cc)
 - In fairness, the knowledge of heavy flavor cross sections and the calculational toolkit available at that time were much less complete
 - Final limits from UA2 (UA1): $m_t > 69$ (60) GeV