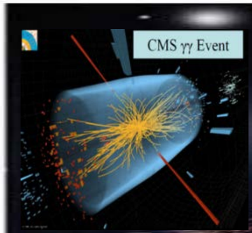

Particles and Universe

Lecture 9 Perturbative calculation

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I.Theory of elementary particles – description of events; Perturbative calculation; Renormalization

II. Probability of processes at high energy –
a need for a Higgs particle

Description of events

- Relativistic and quantum effects. To describe creation and annihilation of particles and mixing as well as decays → formalism of

  Quantum Field Theory,QFT

  (applied first in the QED)

- Technique of QFT

  - perturbative calculation:

    the lowest order of pert. series

    quantum corrections: trees and loops

  - Feynman diagrams

Theory of elementary particles

- 1948 – new phase of quantum mechanics (QFT); precise measurements → a need of more precise calculations
- Feynman method: diagrams and Feynman's rules for calculation – today the universal tool of particle physics

first application for QED

- QED describes interaction of electron and photons. Difficulties since quantum corrections - infinite.

Method of removing them → renormalization procedure.

- Weak interaction – even more difficult situation. Proposal: new interaction and new particles → EW interaction with gauge bosons W/Z with Higgs boson - renormalizable !

Nobel prize: Glashow,Salam,Weinberg 1979 (W/Z)
t'Hooft, Veltman 1999 (renomalizability)

Relativistic effects
- free and virtual particles

Relativistic effects (special theory of relativity)

- Einstein postulates:

  - Invariance of laws with respect to change of reference system (for inertial systems)

  - Constant velocity of light c in the intertial systems

- Lorentz transformation

- The most „visible" relativistic effect – lifetime for particles moving with velocity close to c longer (*time dilatation*)

  Muons born in the upper atmosphare arrive at the surpface of Earth only due this effect (660 m)

- Similar effect for space - contraction (Lorentz)

Lorentz and Poincare invariance

- Invariance with respect to rotations (in space-time) → Lorentz invariance

  If in addition invariance with respect to translation in space and time → Poincare invariance

Not everything is relative !!

If decay (process) is forbidden in some reference frame it is forbidden in any frame

- However – for a description of a process a particular system maybe more useful.

Forbidden processes - example

- For a *free* electron *process* $e \to e\,\gamma$ is forbbiden *Why?*

  Electron at rest has the lowest possible energy ($E=mc^2$), and cannot have lower energy after emission of photon. If it is not possible in one frame then it is not possible in any

  It is possible in matter

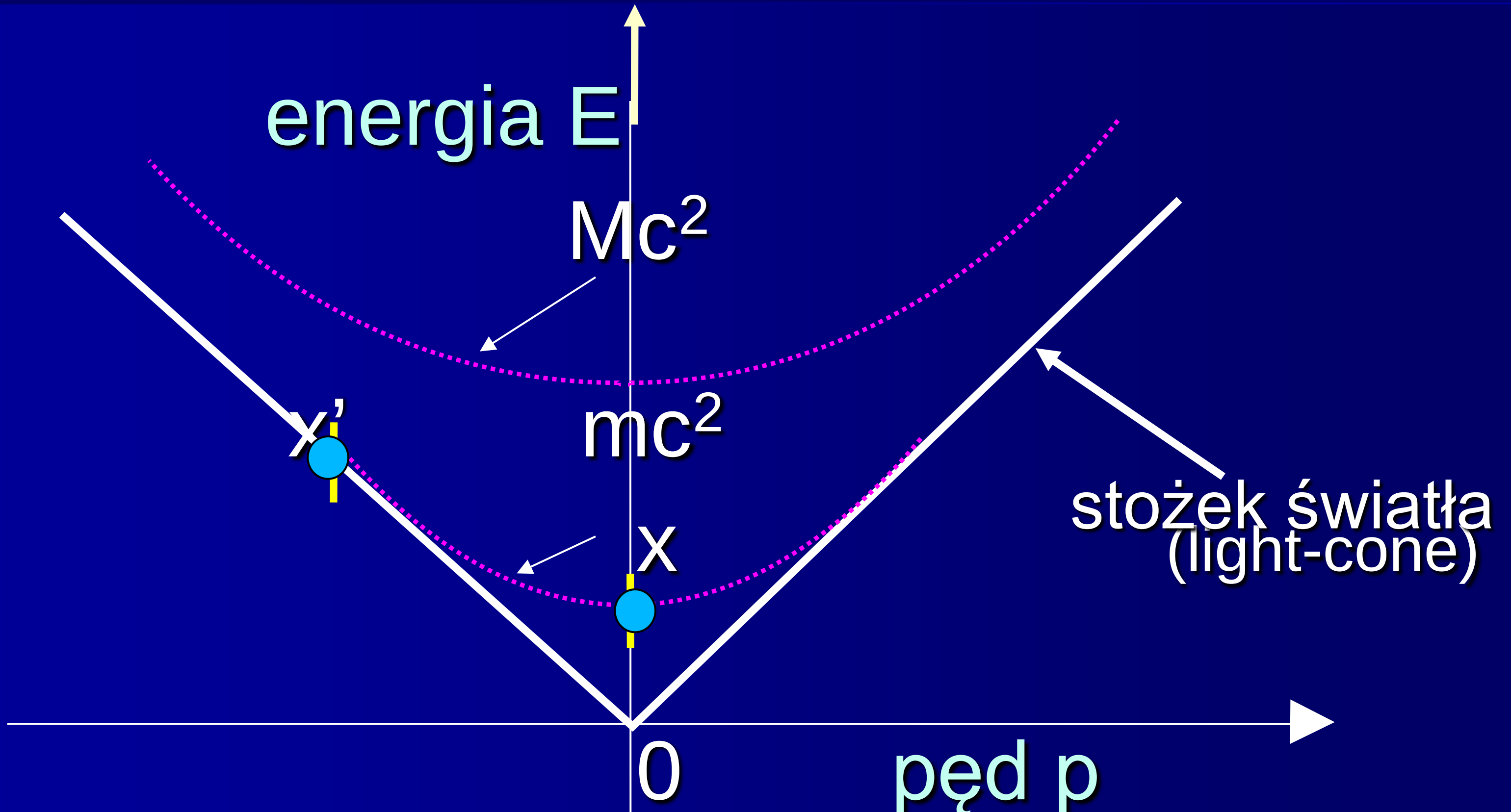
- For a *free* photon the decay $\gamma \to e^+e^-$ is not possible - observer moving towards a photon see it with a smaller momentum/energy ($E = pc$)...

  Possible in the matter

Energy and momentum for a free particle

Relation defines the mass shell for a particle with mass m

$$E = c\sqrt{p^2 + m^2c^2}$$



Change of a system : x → x'

Particle on a mass shell = free particle

Inner particles in diagrams = virtual particles

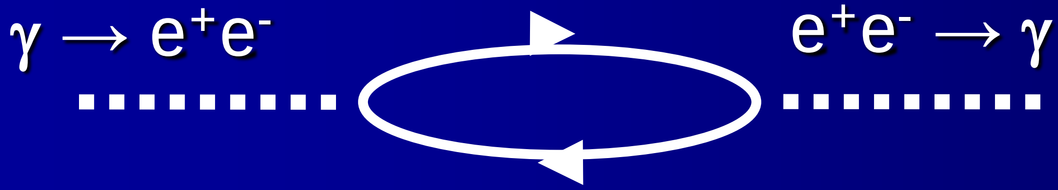
Quantum mechanics allows for

virtual particles for which

$E^2 \neq p^2 + m^2$ (c=1)

(particles off mass shell)

They exist for a while and are not observed directly eg, in loops



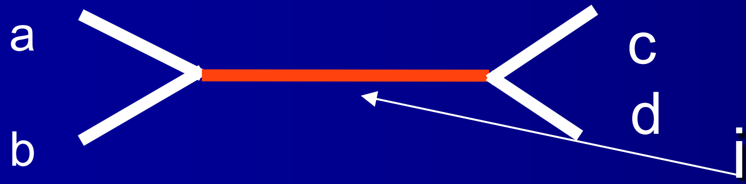
$\gamma \rightarrow e^+e^-$

$e^+e^- \rightarrow \gamma$

Examples- virtual particles

- In a collision of two particles a,b

two particles are produced c, d: a+b → c+d

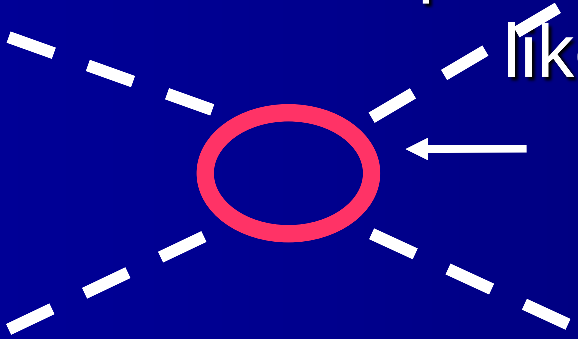


inner = virtual particle

There are various channels (= various virtual exchange) *like two split experiments*..

- Some processes are possible due to them– like light-on-light scattering

γ γ → γ γ



el. charged virtual particle

photons are neutral and can not interact with themseves

Probability of processes

Probability amplitude and probability of processes

- Knowing properties of particles we can calculate probability of processes they are involved in

- Initial particles we produce or prepared, but the final particles we can only observed
- Interference of the amplitudes, since there are many ways from the initial to final particles

  (*channels of processes*)
- Feynman diagrams for all channels of process → we sum probability amplitudes not probabilities!

$$|A|^2 = |\Sigma\ A_i|^2$$

Question which channel is like a question - which slit for a photon

Infinities

- Description of the process; all channels should be incuded, with possible virtual particles
- Virtual particles may have various energies, in loops even arbitrary energies and all these cases should be included.
- If contributions are not damped – a problem
  -> infinite probabilities for a given process!
- ... → problems with spin 1 particles, damping smaller for larger spin.
  (problem with graviton even worst)

QED:
infinities and renormalization

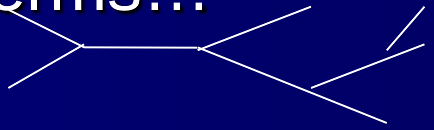
- Problems with photon (spin 1)? – In QED infinite contributions from various diagrams cancel - as shown in 1948 Feynman, Tomonaga, Schwinger (Nobel 1964) and renormalization procedure

  (Kramers 1938)

- QED gives finite predictions- very precise, eg. for anomalous magnetic moment

Perturbative calculation

- Prediction for a process – we need to sum various contributions - with many vertices
- In principle -> series of infinite number of terms…
  - it is OK if next terms small



(small perturbation)

- Eg. in QED small coupling constant

$$\alpha = e^2/(4\pi) \sim 1/137$$

so next term in perturbative expansion $\sim \alpha^2$ - 1%

- More terms in the pertubation series – including higher order in the expansion in the coupling constant (higher power of the coupling constant)

–> higher precision of theoretical prediction

Anomalous magnetic moment for muon

Anomalous magnetic moment for muon (or g-2$|_\mu$)

The magnetic moment μ proportional to the spin *s*

spin and magn. moment

- vectors

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s} \;; \quad g_\mu = 2\,(1 + a_\mu)$$

For the fundamental particle with spin ½ the simplest act of the el-(magnetic) interaction → g=2
so deviation (or g-2) is called the anomalous magnetic moment

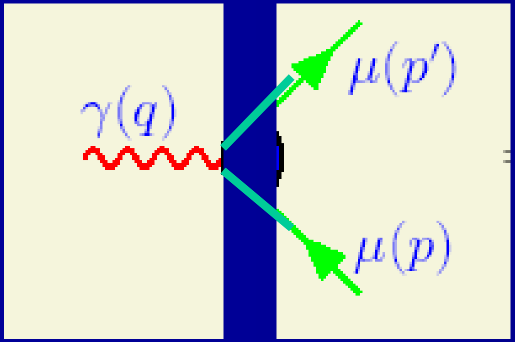
First measurement for electron in 1922! Then in 1948r

Stern, Gerlach 22: $g_e = 2$; **Kusch, Foley 48:** $g_e = 2\,(1.00119 \pm 0.00005)$

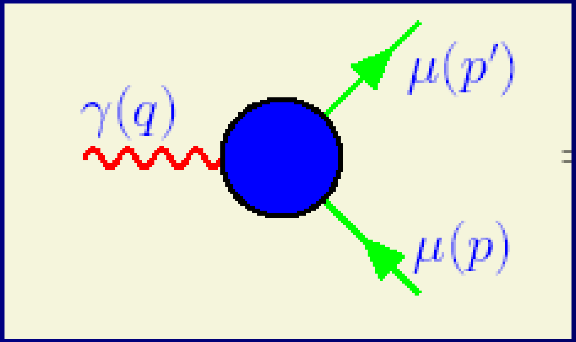
Origin of a_μ

(for muon)

From extra interactions …



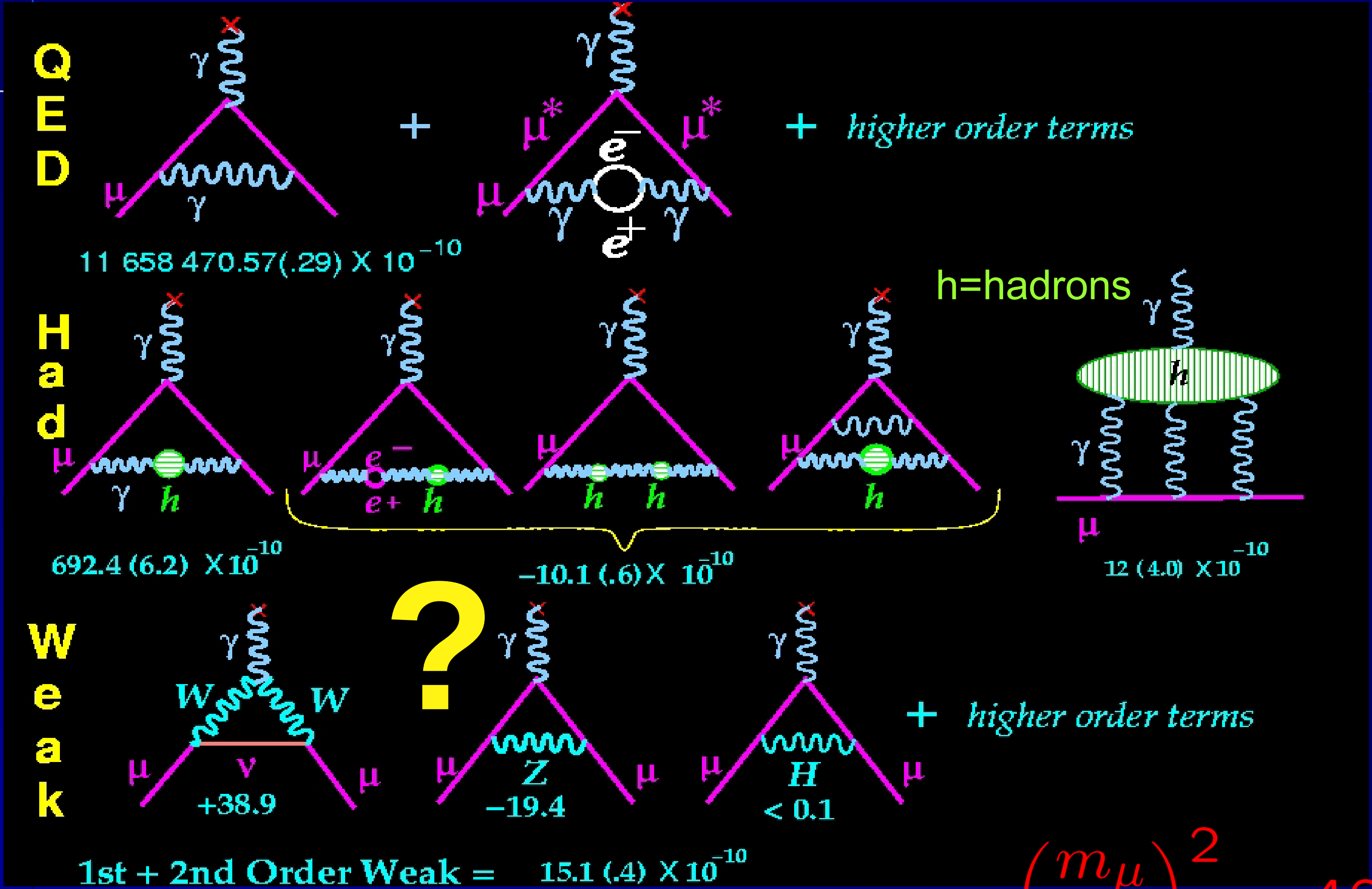
g=2 if only this act



a=g-2 ≠ 0 if more actions

foton z pędem q, mion z pędem p, mion z pędem p’, p’=p+q
(dokładniej to są 4-pędy: (energia, 3-pęd))

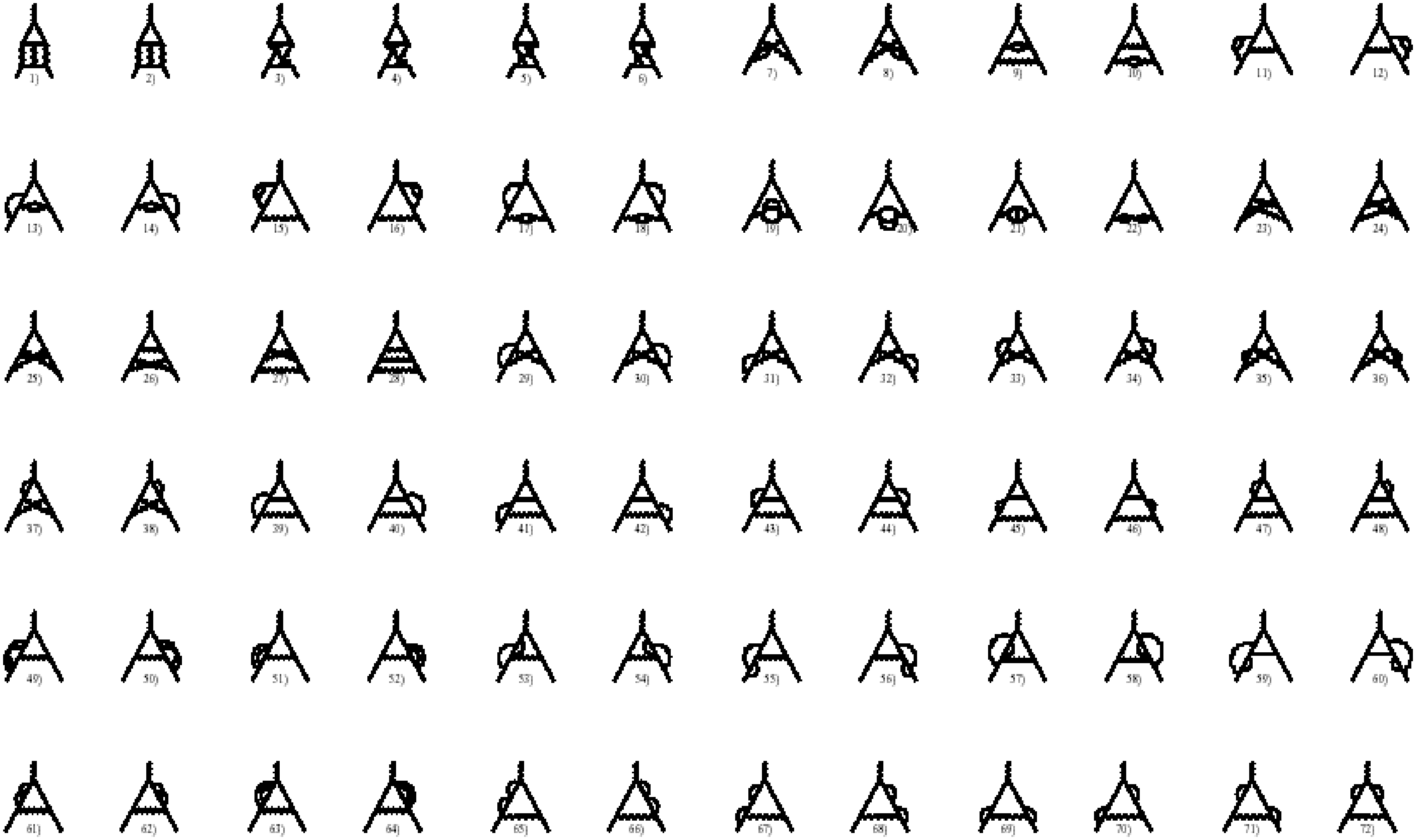
Standard Model : QED, hadronic (h) and EW (W/Z i H) contributions



e vs. μ : relative contribution of heavier things

$$\left(\frac{m_\mu}{m_e}\right)^2 \simeq 40,000$$

3- order of pert. calculation (QED)



QED Contribution a_μ^{QED}

$$\begin{aligned} a_\mu^{\mathrm{QED}} \cdot 10^{10} = \Sigma C_i \left(\tfrac{\alpha}{\pi}\right)^i = & \quad 11614097.3 \text{ (1-loop)} \\ + & \quad 41321.8 \text{ (2-loop)} \\ + & \quad 3014.2 \text{ (3-loop)} \\ + & \quad 38.1 \text{ (4-loop)} \\ + & \quad 0.4 \text{ (5-loop)} \end{aligned}$$

Terms up to α^3 are known analytically, a recent more accurate numerical calculation of the α^4 terms and the leading log α^5 terms gave (T. Kinoshita and M. Nio, 2005; A.L. Kataev, 2006):

$$a_\mu^{\mathrm{QED}} = (116584719.4 \pm 1.4) \cdot 10^{-11}.$$

From the latest value of a_e (G. Gabrielse et al., 2006; M. Passera, 2006):

$\alpha^{-1} = 137.035999710(96)$, $a_\mu^{\mathrm{QED}} = (116584718.09 \pm 0.14 \pm 0.08) \cdot 10^{-11}$.

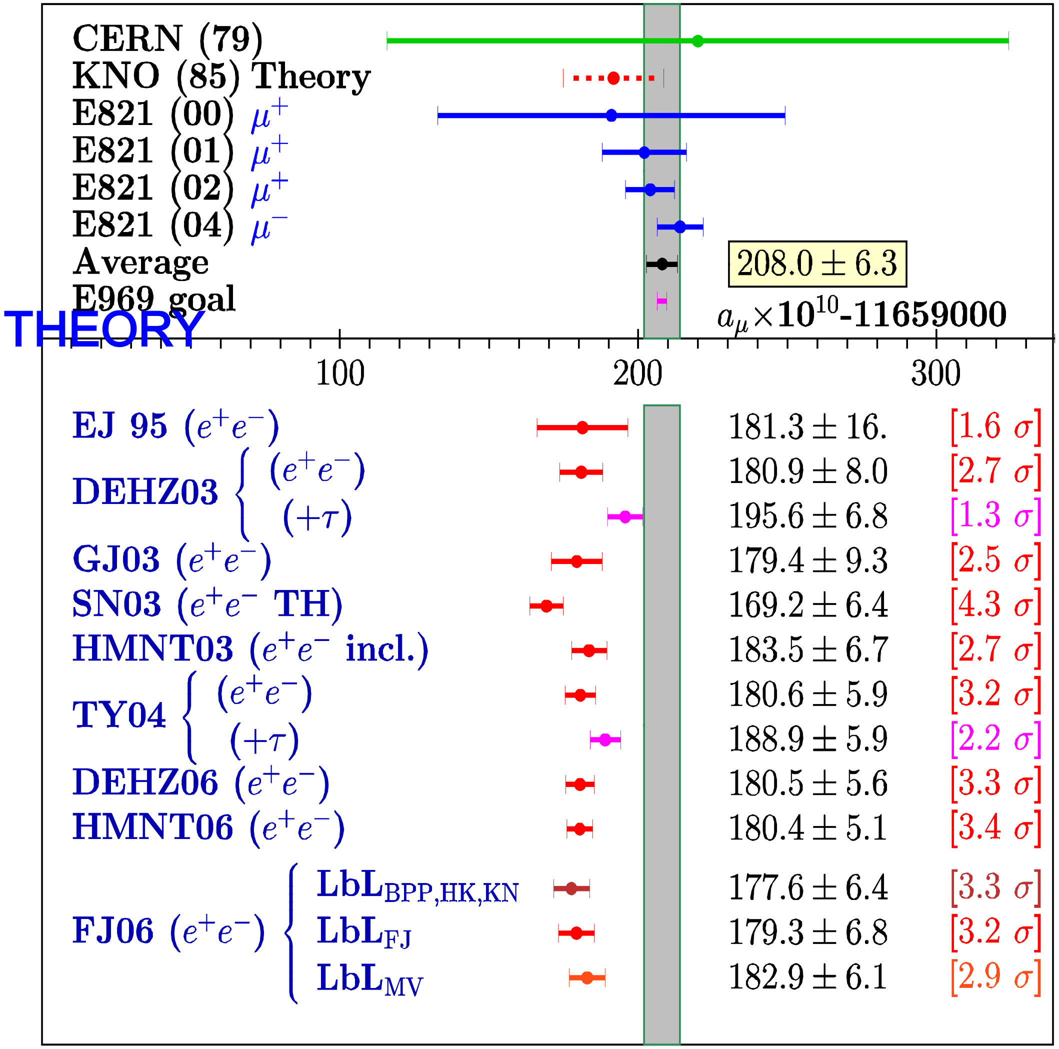
The errors are due to: a/ $\mathcal{O}(\alpha^5)$, b/ α

Jegerlehner'07

g-2|$_\mu$: exp-theory (MS)

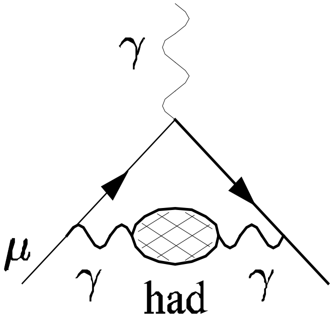
EXP

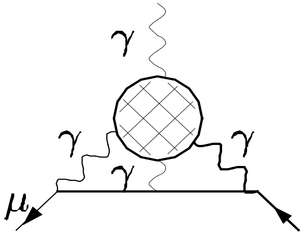
New Physics?



$$\delta\, a_\mu = (287 \pm 91)\ 10^{-11}$$

($3.2\ \sigma$)

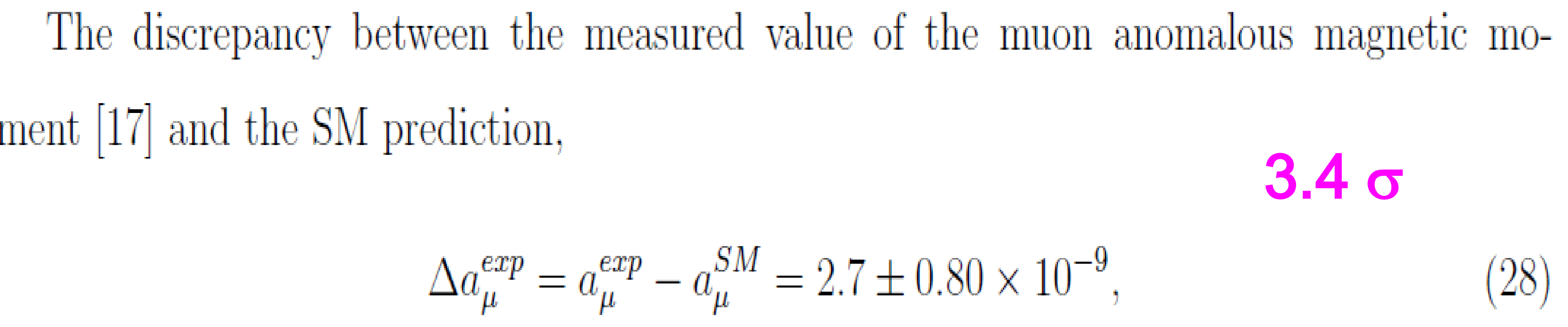




Jegerlehner’07

New calculation in SM (2013)

Dermisek, Rava – May 2013

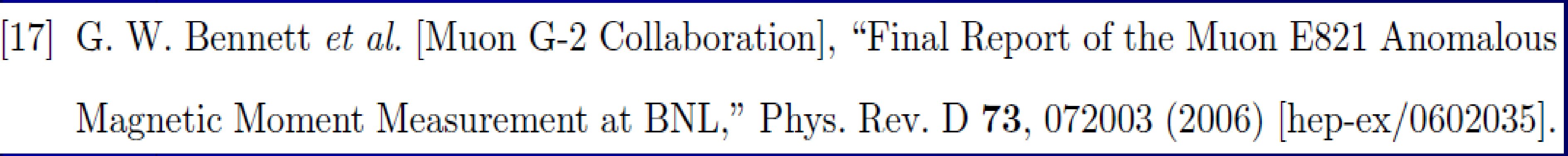


The discrepancy between the measured value of the muon anomalous magnetic moment [17] and the SM prediction,

$$\Delta a_\mu^{exp} = a_\mu^{exp} - a_\mu^{SM} = 2.7 \pm 0.80 \times 10^{-9}, \qquad (28)$$

3.4 σ

Experiment Brookhaven 2006



[17] G. W. Bennett *et al.* [Muon G-2 Collaboration], "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," Phys. Rev. D **73**, 072003 (2006) [hep-ex/0602035].

The most precise quantity…

Brookhaven, USA

In 2014

tranport to Fermilab (Chicago)
http://muon-g-2.fnal.gov/bigmove/


The BNL muon storage ring

Pomiar g-2
dla mionu

Small, very precise experiment at low energies - *www.g-2.bnl.gov*

Renormalization

Idea

- QED – infinities only in a few expressions, containing mass and electric charge. Using quantities measured in experiment we can remove infinities as follows

A – expression for physical quantity; let 1st order correction Δ contains an infinite contribution Δ',

$$A=e_0(1+\Delta)+\ldots=e_0(1+\Delta'+..)(1+\Delta''+..)=e_{fiz}(1+\Delta''+\ldots),$$

so the prediction for quantity A finite (Δ'' - finite)

(+...higher terms)

For magnetic moments infinities are at the intermediate stages of calculations – but they are the same for electron and muon. We can express them by each other and for electric charge use the exp. value.

Trick very useful

- Electric charge is a *free (initial)* parameter of the QED; the same for the electron mass m_e
- Theory does not predict values of these parameters – we can hide infinities in them
- If infinities only in expressions for a free parameters of the theory → theory is renormalizable and provide definite predictions for physical processes
- It is not satisfactory, but works OK

Non-renormalizability

- There are theories with infinities not only in free parameters
- For long time it was considered that theories with spin-1 particles, but different than a photon, are non-renormalizable (eg. Fermi theory for weak int.).
- Today we known that theories with spin-1 particles are renormalizable if they are gauge theories, (moreover – non-abelian Yang-Mills theories)

GraVitation: gauge theory, but not renormalizable

Gauge transformation- a reminder

Schrodinger equation, based on $E = (\bar{p}^2/2m + V)$

- $i \, \partial \psi(\bar{x},t) / \partial t = - 1/2m \; \bar{\nabla}^2 \psi(\bar{x},t)$

  (for potential V=0 and using $\hbar = 1$)

- $E \to i \, \partial_0$ $\qquad \partial^\mu = (\partial_0, - \bar{\nabla})$

  $\bar{p} \to - i \, \bar{\nabla}$

- local transformation, phase $\alpha(\bar{x},t)$

  $\psi'(\bar{x},t) = e^{\, i \, \alpha(\bar{x},t)} \psi(\bar{x},t)$

$|\psi(\bar{x},t)|^2$ probability does not change, but

invariance of equation only if we add interaction

Predictions for larger energies

Probability of processes for large energies

- Bad, if probability of processes rises with energy (bigger than 100 % !?)
- QED:Compton scattering – at the lowest order of pert. calculation two diagrams, each rising with energy but the sum of diagrams – OK (cancelation)
- Cancelation results from the structure of the theory (gauge symmetry)

Calculation of probability - using Feynman rules

- To each line and vertices in Feynman diagram a factor is assigned. Here we track only the energy E.

- Incoming or outgoing photon
(and each spin 1 particle) – a factor E
Virtual photon (spin1 particle) – a factor $1/E^2$

- Incoming or outgoing spin ½ particle - a factor $\sqrt{E}$,
virtual spin ½ particle – a factor 1/E

- Incoming or outgoing spin 0 particle – a factor 1,
virtual spin 0 particle – a factor $1/E^2$

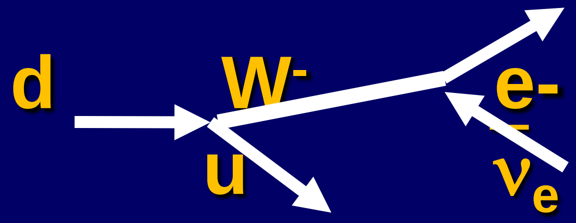
- Additional factors from couplings ….,

Product of factors→probability amplitude A

(probability = $|A|^2$)

Electroweak interaction

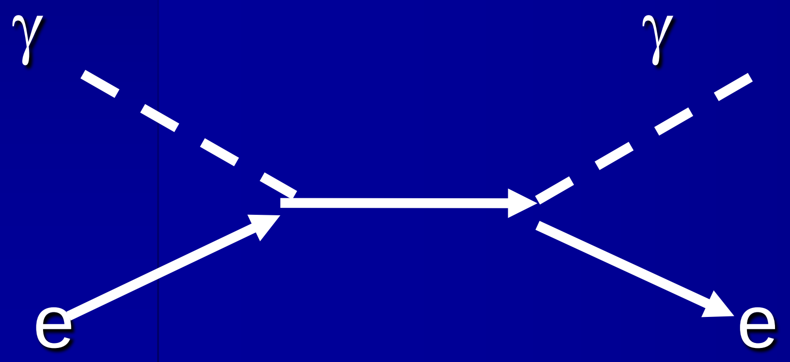
Decay d → u e $\bar{\nu}_e$

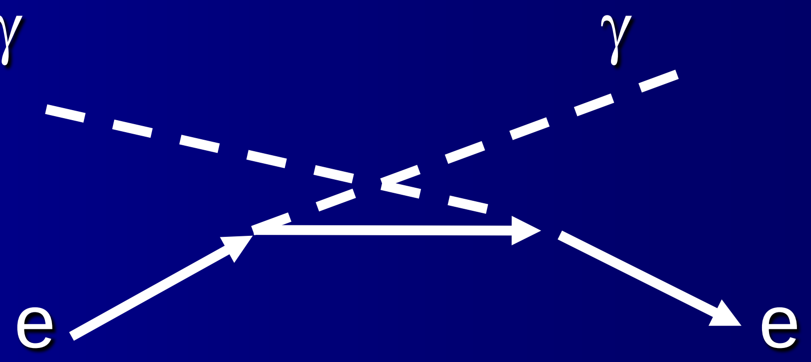


- W boson is very massive (80.4 GeV), so in decay of quark d (mass ~MeV) it is very virtual (far off mass-shell)

- Boson W has spin 1→ problem with renormalizability

- To understand this problem, we analyse first Comptona scattering γ e → γ e (QED), two diagrams:





Calculation of the probability amplitude for the Compton process

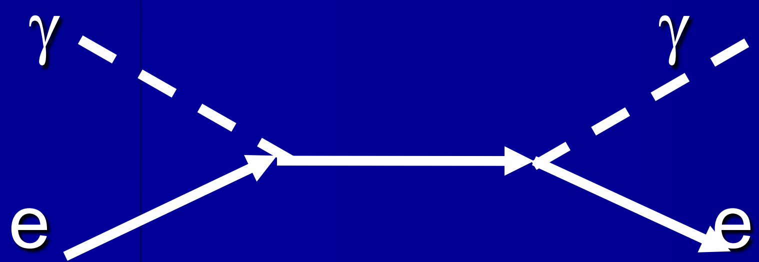
For this process the amplitude rises with the energy as

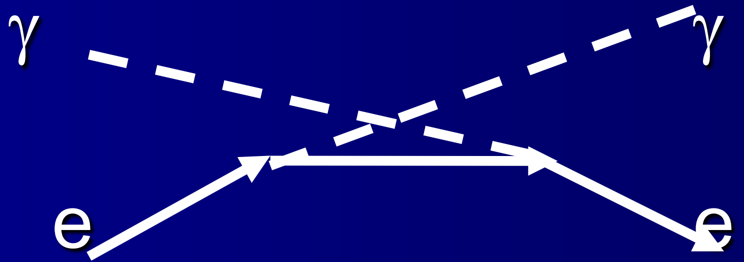
$E^2\ (\sqrt{E}\ )^2\ 1/E = E^2$

(for probability E^4)

Bad behaviour for the individual diagrams, but in sum cancelation and the final amplitude does not grow with energy

$\gamma\ e \rightarrow\ \gamma\ e$

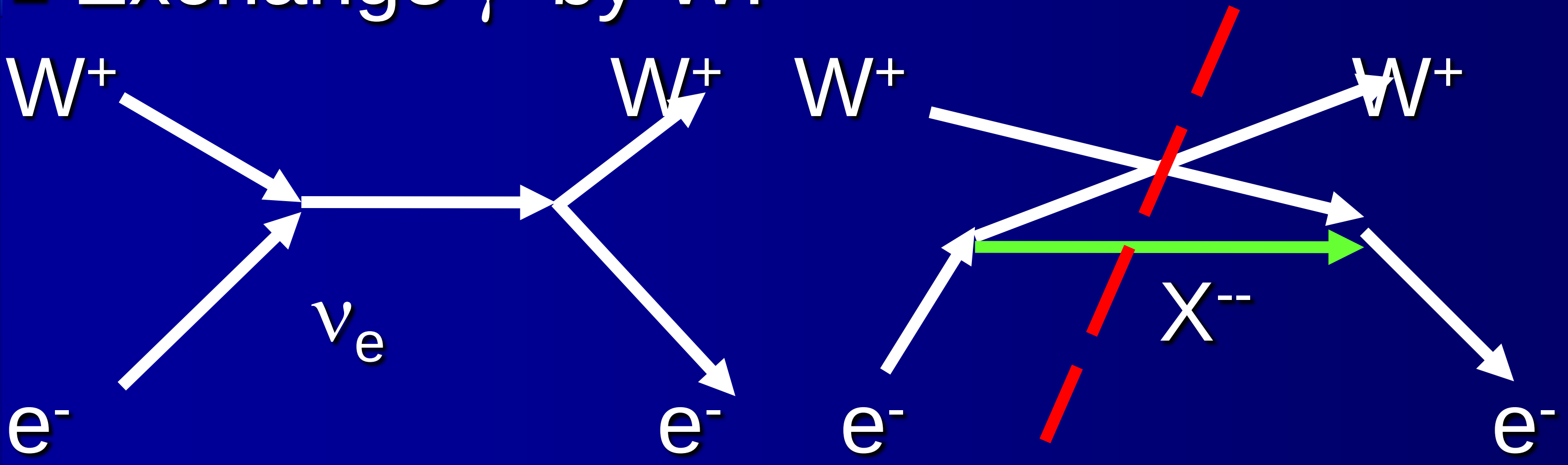




Scattering W e -> eW$_+$

Tu strzałki na liniach W
oznaczają pędy

- Exchange γ by W:

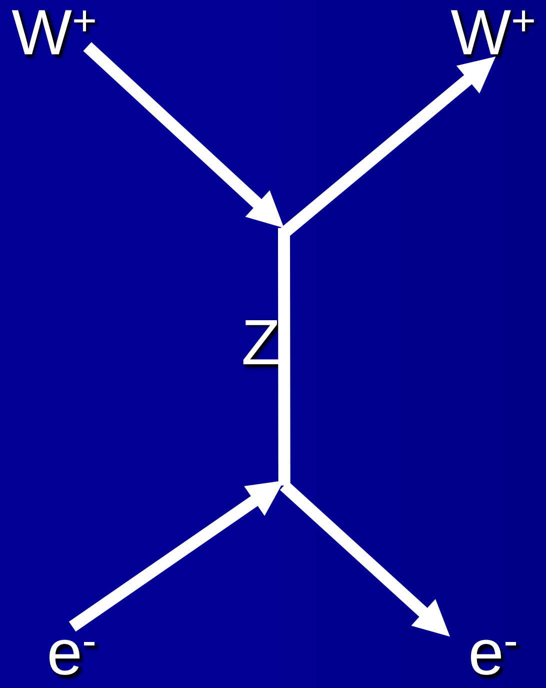


Charge conservation → X^{--} double charged particle – not observed !

Bad high energy behaviour $E^2 (\sqrt{E})^2 \, 1/E = E^2$

Z boson needed!

- Using diagram with Z boson



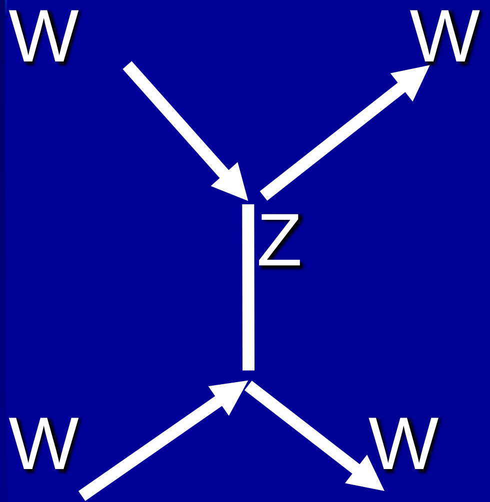
Behaviour $E^2\ (\sqrt{E})^2\ (\underline{\text{vertex WWZ} \sim E})/E^2 = E^2$, and a proper sign of coupling WWZ – cancelation like for the Compton process!

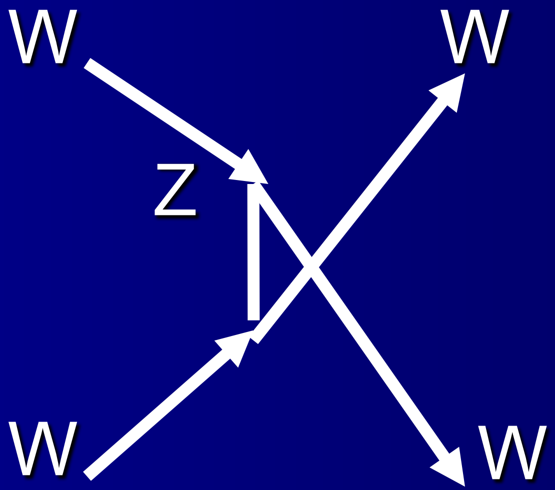
Price for a good behaviour for E: it must exist neutral particle Z with a define interaction (coupling) !

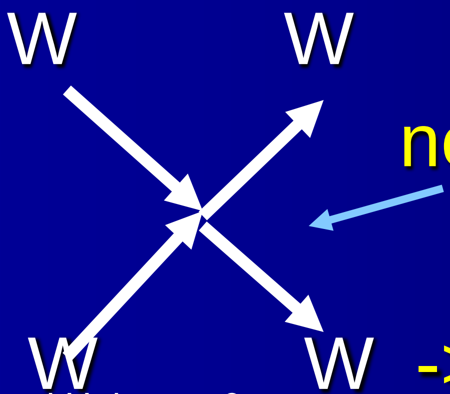
It was found – this is a Z boson !

→ A success of theory

Scattering WW -> WW
~ E^4 (since E^4 E^2/E^2) - even worst…







new type of intercation

(selfinteraction of W)

-> E^4 cancels but not E^2

E^2 term, in fact $E^2 M^2$

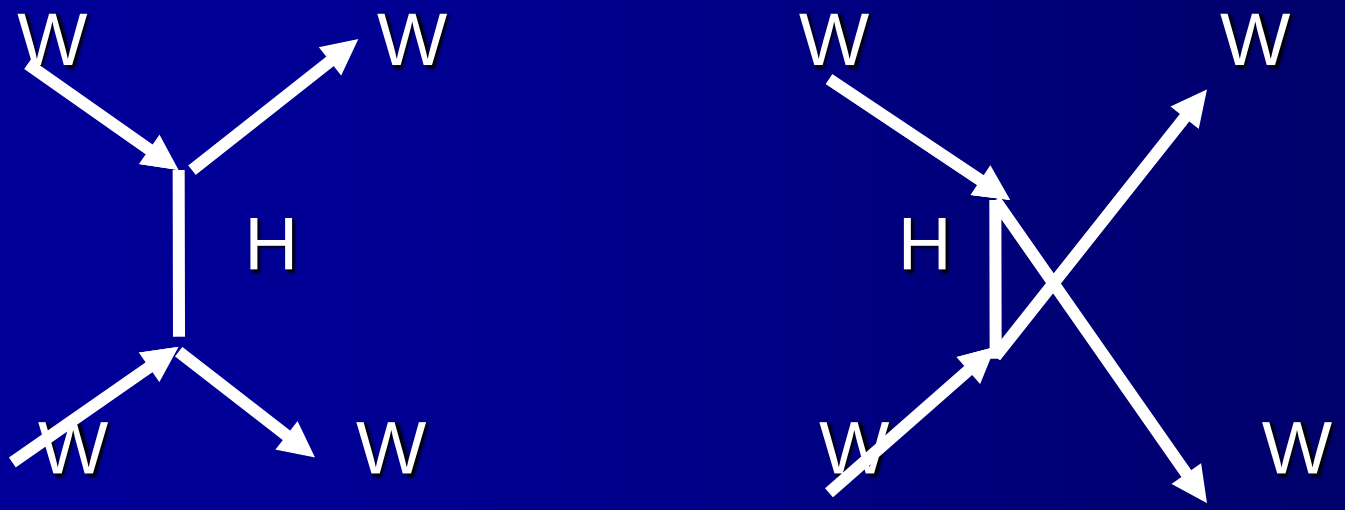
- In the amplitude - wrong term $E^2 M^2$,

  where M- mass of W or Z

- New contribution needed to cancel this bad term → the simplest diagram with exchange of spinie-0 particle, which couples to W/Z proportionally to mass

Higgs boson needed !

Scattering of W on W: H contribution



Couplings proportional to mass of particles to which H couples, and as a result →

a good high energy behaviour WW → WW !

Mass generation and consistent description of processes at high energies

- Good high energy behaviour - if H exists and couples to W/Z proportionally to their masses
- Higgs particle is related to the mass generation in SSB

→properties of H particle related to the mass generation and those needed for a good high energy behaviour WW → WW

- **THE SAME !!!**

- Expected relation to gravity (mass..)

The theory ends here - Veltman

Facts and Mysteries in Elementary Particle Physics, 2003

- *„The theory ends here. We need help. Experiments must clear up this mess.”*

  →LHC pp, ILC e^+e^- (PLC $\gamma\ \gamma$, $e\gamma$)

July 2012 :

LHC - Higgs particle with mass 125 GeV