Particles and Universe
Lecture 11 Mixing

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I. Mixing of quarks and not only…

II. Neutrinos

Mixing

Mixing is natural in quantum mechanics –

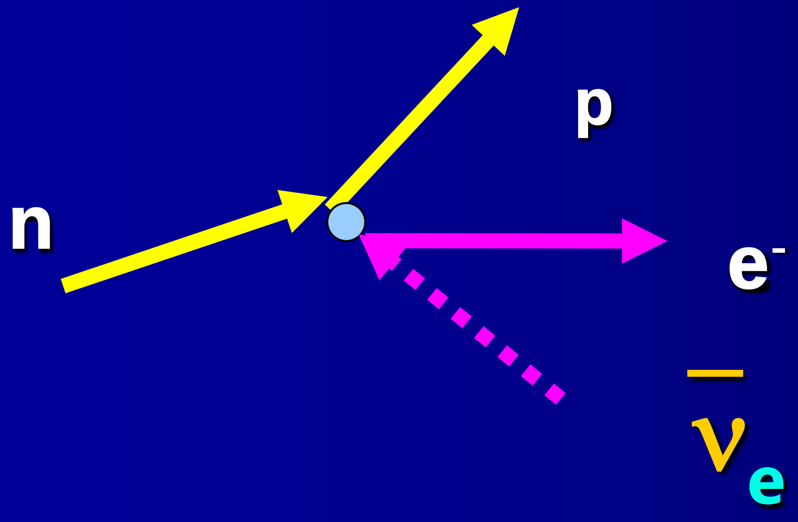
due to wave nature of particles

- We have discussed interference of various channels for processes
- Mixing - here one state is mixed with another state

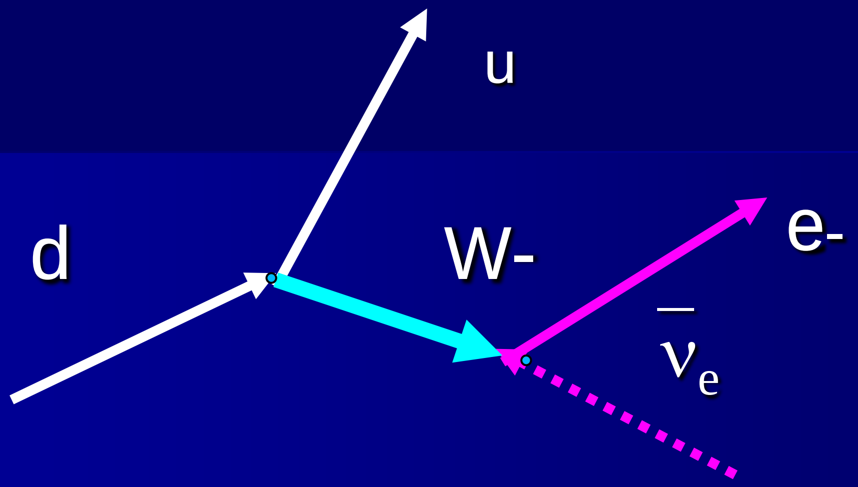
Mixing for weak interaction

Pair of (p,n) and of (electron neutrino, electron) - *doublets* in the neutron decay

β decay



Fundamental decay



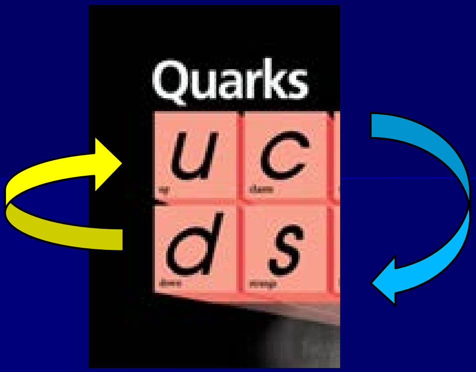
elementary acts of interaction :

$d \rightarrow u\, W^-$ and $W^- \rightarrow e^-\, \bar{\nu}_e$

g (g -„weak charge”) $\alpha_w = g^2/4\pi = 1/32$

Mixing in two light families

Quarks	el. charge	2/3	u	c
		-1/3	d	s
Leptons		0	ν_e	ν_μ
		-1	e	μ

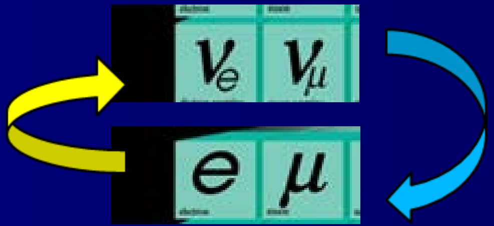


Formaly transition between quarks IN a given doublet

$u \leftrightarrow d$, $c \leftrightarrow s$

with gauge boson W-, eg.

$d \rightarrow u\, W^-$, $c \rightarrow s\, W^+$



Transition in the leptonic doublet

$e- \rightarrow \nu_e\, W^-$, $\nu_e \rightarrow e-\, W^+$

Probability for transition

EW theory:

Absolute value of weak coupling = g the same for all vertices (eg. u → d W+, W^- → e- ν_e), but the coupling itself can have positive and negative sign (as in the el-m interaction - two signs of el. charge)

Let L- an absolute value of the transition amplitude
L = | transition amplitude |
for quarks or leptons in the corresponding doublet

Since L is proportional to g,
a probability for transition process $L^2 \sim g^2$

Transition for quarks and for leptons (exchange of W+/-)

u	c	ν_e	
↕ L	↕ L	↕ L	$W^{\pm}$
d	s	e	

- Theory SU(2) → transition probability($\mathcal{P}$) = L^2
- Experiment (1963)→ difference between quarks and leptons..

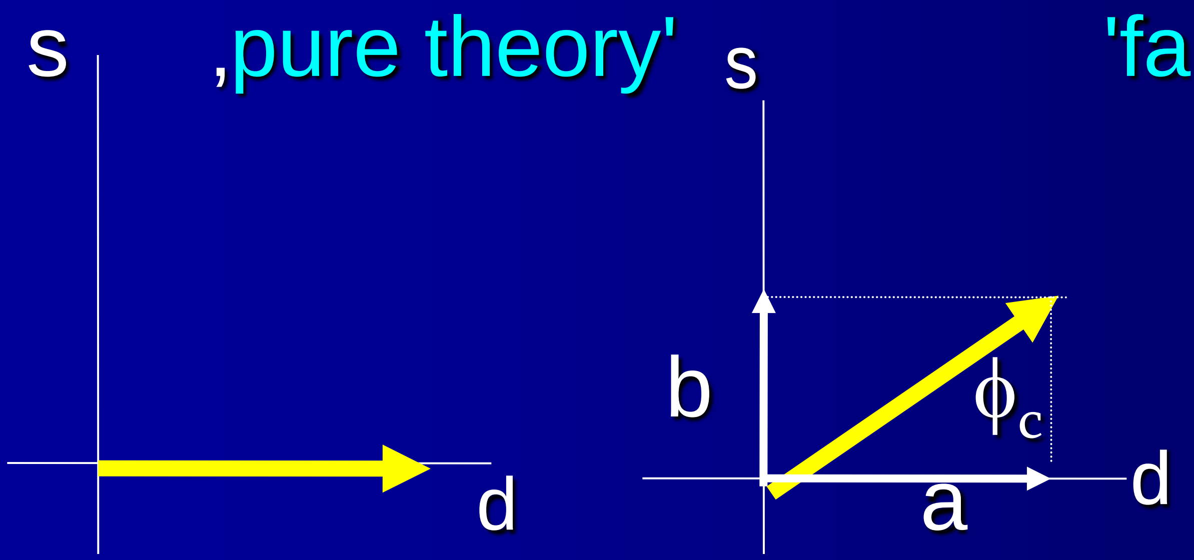
$$\mathcal{P}[u \to d\, W^+] + \mathcal{P}[u \to s\, W^+] = \mathcal{P}[\nu_e \to e\, W^+]$$

Mixing of s quark to the emission of W^+ by u quark !

Mixing of s with d -> Cabibbo angle ϕ_c

So u quark couples both to d and s

Definition: $L^2 = \mathcal{P}[\nu_e \rightarrow e^- W^+]$, vector L



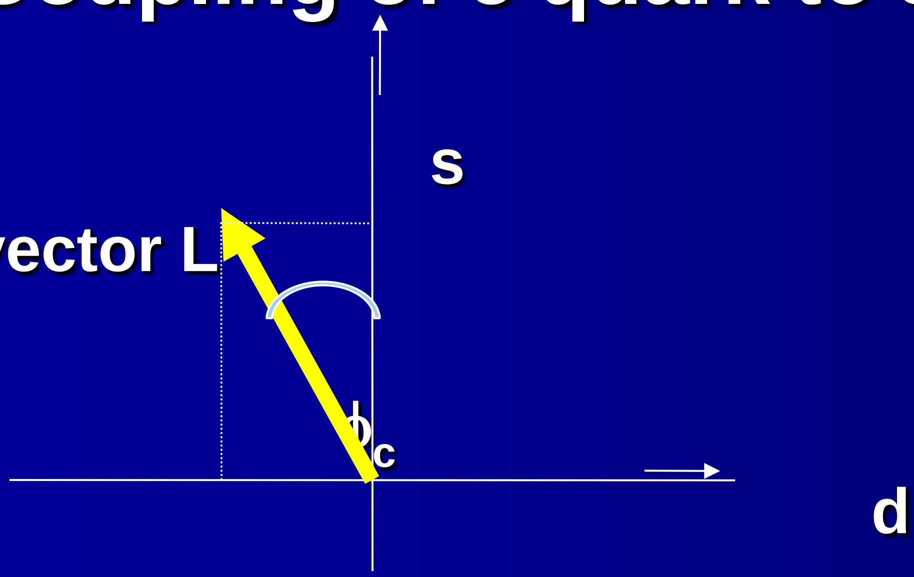
$a^2+b^2=L$

-> couplings usW ~b=g sin (ϕ_c) and udW ~a=g cos (ϕ_c)

Exp: ϕ_c=12.7° *(Nicola Cabibbo 1963)*

Decay of c quark to s and d

Coupling of c quark to s and d (facts)



So, couplings cdW ~ - g sin (ϕ_c) and csW ~ g cos (ϕ_c)

minus!

Cabibbo angle – mixing between the 1 & 2 family of quarks

- Cabibbo angle needed to describe data –
  Theory ? – no prediction

- In fact a mixing among 3 families.. 3 x 3 matrix (unitary)
  → Cabibbo–Kobayashi–Maskawa matrix (CKM)
  4 parameters:
  3 angles (including Cabibbo angle) and phase

Kobayashi and Maskawa postulate such matrix in 1973 r before discovery of the 3d family.

Data needed a phase.. (CP violation)

Matrices for mixing of quarks

- 2x2 matrix (rotation of vektor L)

$$\begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} \cos\phi_c & \sin\phi_c \\ -\sin\phi_c & \cos\phi_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 x 3 matrix

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} \rightarrow \begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Similar mixing for up quarks: u, c, t

Kobayashi & Maskawa

Nobel' 2008

.

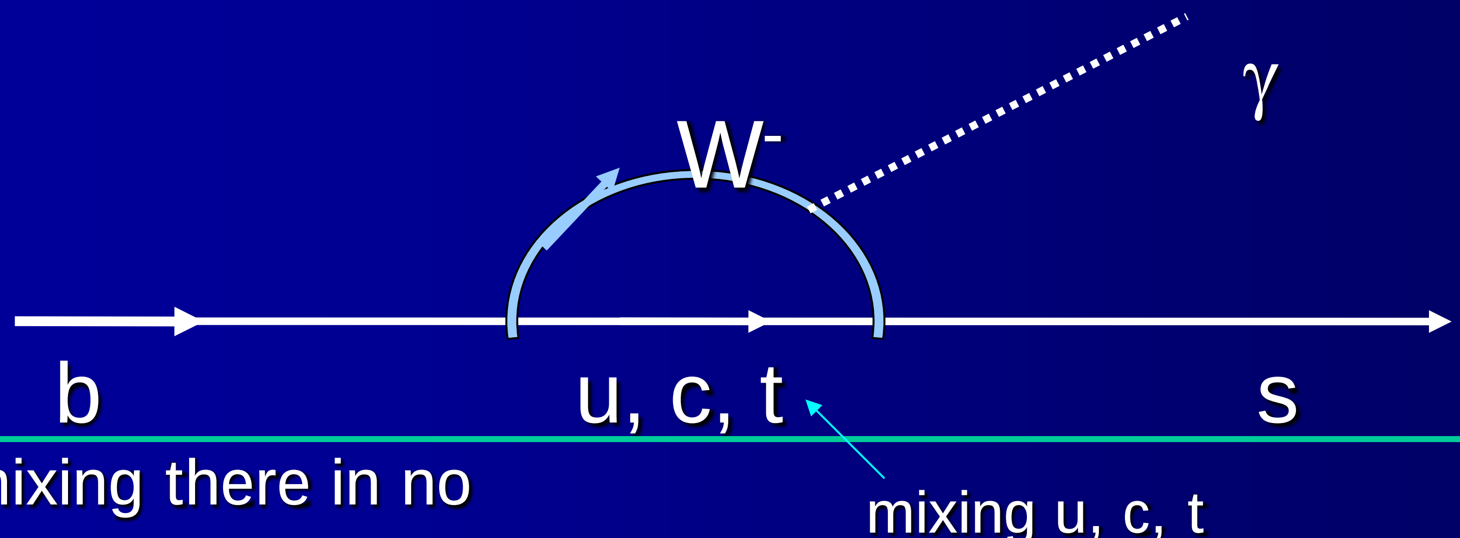
Why Cabibbo did not get Nobel prize ?!

The prize was not for a mixing but for observation that starting with 3 family of quarks there appear a phase needed for violation of CP in kaon decays

Example: decay $b \to s \gamma$

Very precise measurement and SM prediction

($Br \sim 10^{-4}$)



Without mixing there in no such process!

Besides $b\,W^-\,t$ we have $b\,W^-u$ and bW^-c (and sW^-c, sW^-u, sW^-t)

V_{CKM}

K.A. Olive *et al.* (PDG), Chin. Phys. **C38**, 090001 (2014

- Matrix (with one phase)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\bar{\eta} = 0.354 \pm 0.015$$

- For the magnitudes

$$V_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

No mixing for transition via Z bozon

Transition without changing of el. charge

$d \to d Z$

Mixing ? $d \to s Z$? NO!

(Flavour changing neutral current FCNC)

Why? No answer

Mixing of leptons? NO if neutrinos massless

A comment – mixing of gauge bosons?

Yes,

Z boson and photon are combinations of the initial gauge bosons of grups SU(2) i U(1) →

Weinberg angle to describe this mixing

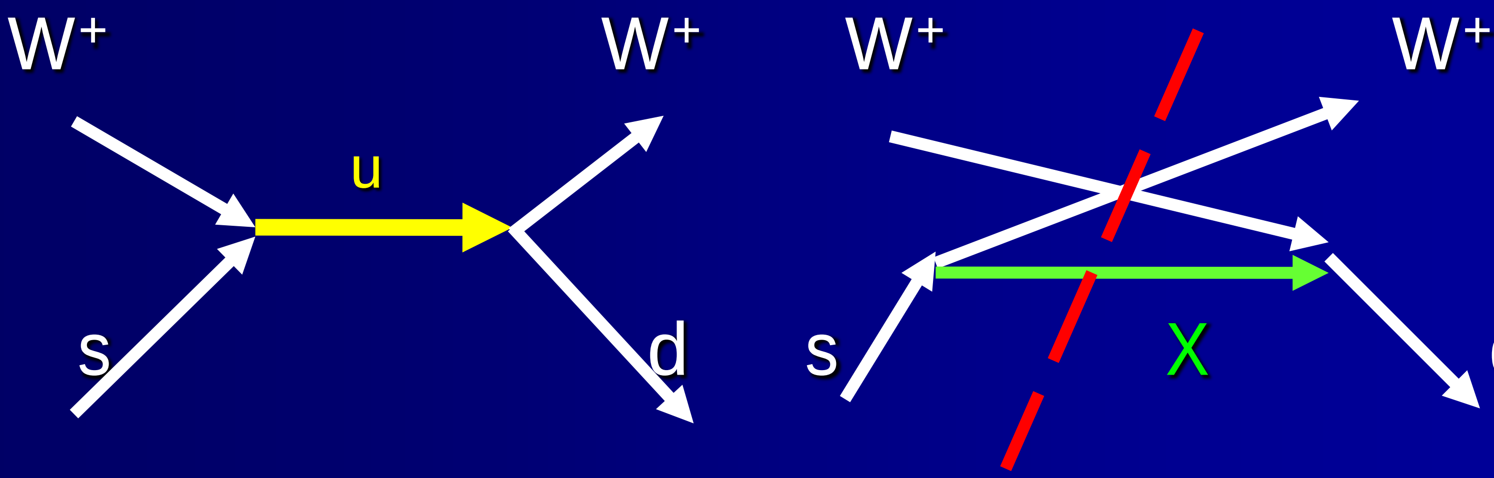
„Mixture Z" couples to neutrins,
while „mixture photon" does not
(destruction of the corresponding prob. amplitude !)

Cabibbo angle: u → d W+ versus ν_e → e W+
Weinberg angle: u → d W+ versus u → u Z

Quark mixing and high energy behaviour

Scattering of boson W on quark- behaviour for large energies

Replacing e^- by s quark

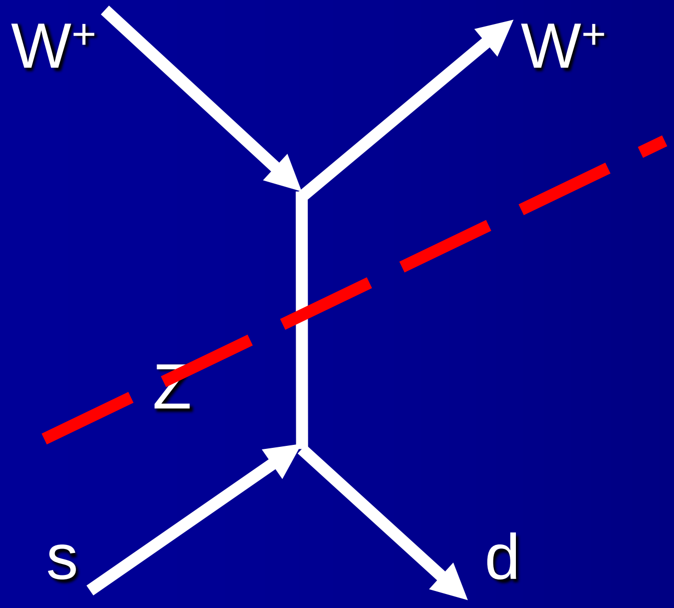


El. charge conservation: particle X with el. charge - 4/3 does not exist !

So, bad behaviour (amplitude): $E^2 (\sqrt{E})^2 \, 1/E = E^2$

Diagram with Z boson – no help here!

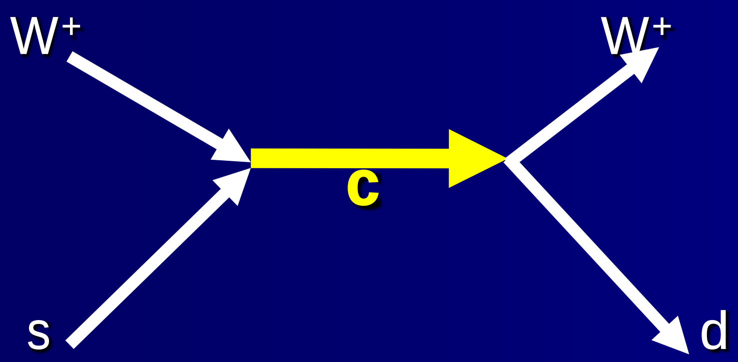
Diagram with Z boson Z



This process does not exist –

„absence of FCNC"!

New diagram with c quark
(possible only if mixing of s and d quarks)



Couplings:

cs W	$\cos(\phi_c)$
cd W	$-\sin(\phi_c)$

due to minus sign a cancelation of bad high energy behaviour for a process

$W+\ s \rightarrow u \rightarrow W+\ d$

Comment on discovery of c quark

- So to the process $W+ s \to u \to W+ d$

we add process with c - quark

$W^+ s \to c \to W^+ d$

(various channels of the process W+s→ W+d)

- In fact in 1964 this was only hypothesis

about existence of c quark with fixed properties, (including Cabibbo angle) – so that terms $\sim E^2$ cancel.

- c quark discovered in 1974 has these properties Sucess of theory!!!

Quark mixing

- Exists
- Important
- Description – but not understanding

→

- Flavour problem

Neutrinos

Introduced by Pauli to preserve energy conservation in β decays

eg.: $^{60}_{28}Co \rightarrow\ ^{60}_{29}Ni^{\star} + e^{-} + \bar{\nu}_e$

on particle level:

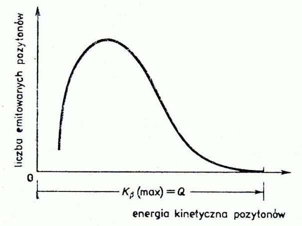
$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Required properties:

- very weak interactions
- negligible mass

We assumed that neutrinos are massless...

Continuous β energy spectra:



Direct mass constraints (95% CL):

$$m_{\nu_e} < 2.2\ eV \approx 4.3 \cdot 10^{-6}\ m_e$$
$$m_{\nu_\mu} < 170\ keV \approx 0.0018\ m_\mu$$
$$m_{\nu_\tau} < 15.5\ MeV \approx 0.01\ m_\tau$$

Much stronger mass constraints from astrophysics and cosmology...

Standard Model

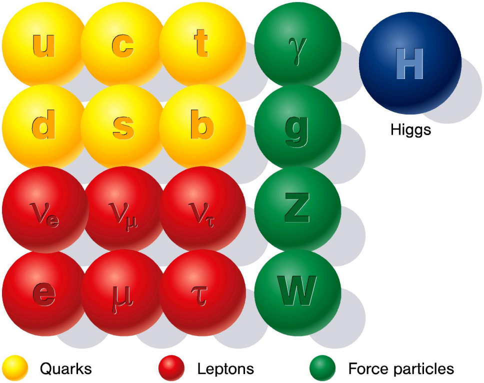
Laboratory measurements have shown that each charged lepton couples to distinct neutrino

⇒ three neutrinos flavours

Until recently we believed that

- all neutrinos are massless
- lepton flavour is conserved

But conservation of the lepton number is not imposed by theory - it was assumed based on experimental evidence...



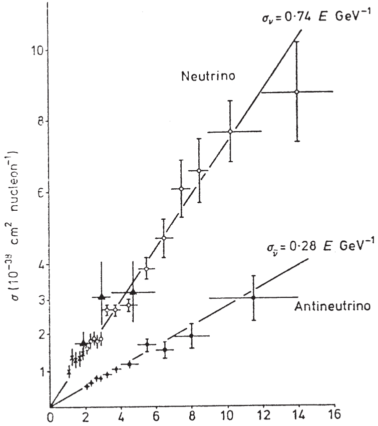
Interactions

Cross section for neutrino interaction in matter is incredibly small.
For energy of the order of 1 MeV

$$\sigma_{\nu N} \sim 10^{-43}\ cm^2$$

$\Rightarrow$ average free path in matter (!) of the order of light years !!!
For comparison $\sigma_{NN} \sim 10^{-24}\ cm^2$

Cross section grows with energy, but only linearly...



We need very strong sources and very large detectors to study neutrinos...

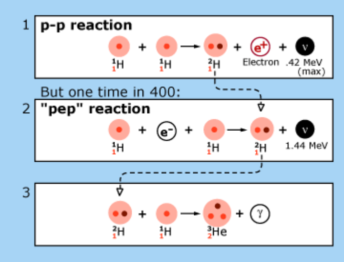
Sun

Sun is an extremely intense neutrino source.
Most neutrinos come from **p–p** reaction:

$$p + p \quad \rightarrow \quad D + e^+ + \nu_e \; (E_\nu \leq 0.42\ MeV)$$

Slightly higher energies in **"pep"** process:

$$p + e^- + p \quad \rightarrow \quad D + \nu_e \; (E_\nu \approx 1.44\ MeV)$$



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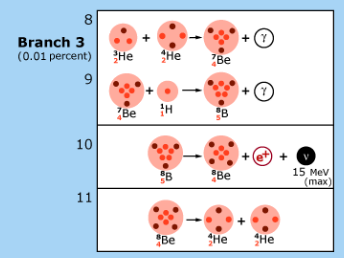
Highest energies result from ^8B decay

$$^8_5B \rightarrow {}^8_4Be + e^{+} + \nu_e$$

where neutrino energy can reach 15 MeV

Only these neutrinos can be observed in particle detectors...



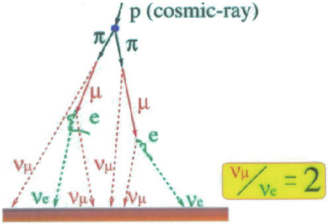
Cosmic rays

Primary cosmic rays consist mainly of high energy protons and light nuclei, with energies reaching 10^{12} GeV (10^{21} eV).

They interact with O and N nuclei in the atmosphere creating a cascade of secondary particles, most of them are pions $\pi^{\pm}$.

Neutrinos result from decay chain:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$



Neutrino sources

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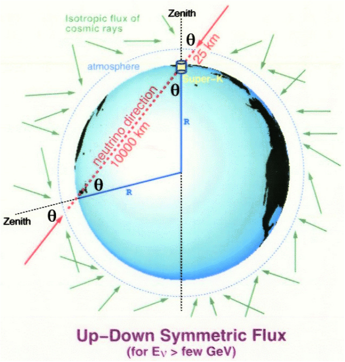

As the primary cosmic radiation is isotropic, we expect that atmospheric neutrino flux will also be isotropic !

Nuclear reactors

Fission of single ^{238}U nucleus produces energy of about 200 MeV
On average, six electron anti-neutrinos are produced (per fission), mainly in neutron decay: $n \rightarrow p + e^{-} + \bar{\nu}_e$

Reactor with 1GW power $\Rightarrow \sim 10^{20}$ fissions per second $\Rightarrow \sim 6 \cdot 10^{20}\ \nu/s$

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Accelerators

We can produce high energy, high current proton beams.
When interacting with dense target, protons produce secondary pions and kaons (as in the cosmic ray interactions). Muon neutrinos are produced in:

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ K^+ &\rightarrow \mu^+ + \nu_\mu \end{aligned}$$

By focusing produced pions and kaons prior to their decay, we can obtain high energy neutrino (or anti-neutrino) beams...

Comparison

source	flavour	energies	flux
Sun	ν_e	≤ 15 MeV	$6 \cdot 10^{10} \frac{1}{s\ cm^2}$ (on Earth)
Cosmic rays	ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$	100 MeV - TeV	$3 \cdot 10^{7} \frac{1}{s\ cm^2}$ (above 10 MeV)
Reactors	$\bar{\nu}_e$	MeV range	$5 \cdot 10^{9} \frac{1}{s\ cm^2}$ (1 GW from 1 km)
Beams	ν_μ, $\bar{\nu}_\mu$	GeV range	10^{20} (total sample)

All these sources are used nowadays to study neutrino properties

Detector

Neutrino detector built in an old mine, under Mt Kamioka in Japan.

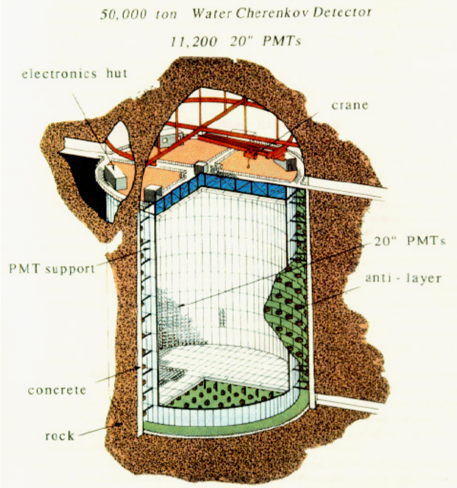
Steel tank with 50,000 tons of ultra-pure water.

Look for neutrino interactions:

$$\nu_l \; n \quad \rightarrow \quad l^- p$$

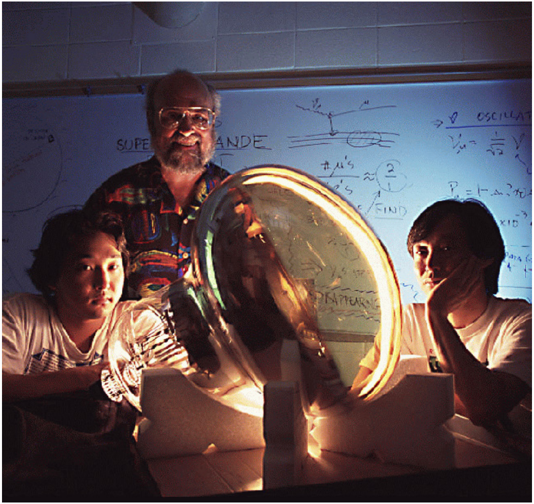
$\sim$11'000 photomultipliers installed to measure Cherenkov radiation

Radiation of charged particles traveling with velocity greater than speed of light (in water).

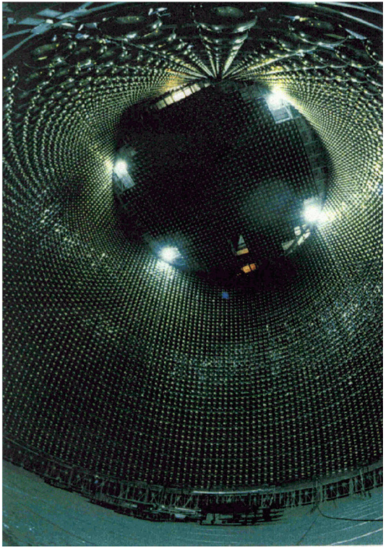


Super Kamiokande

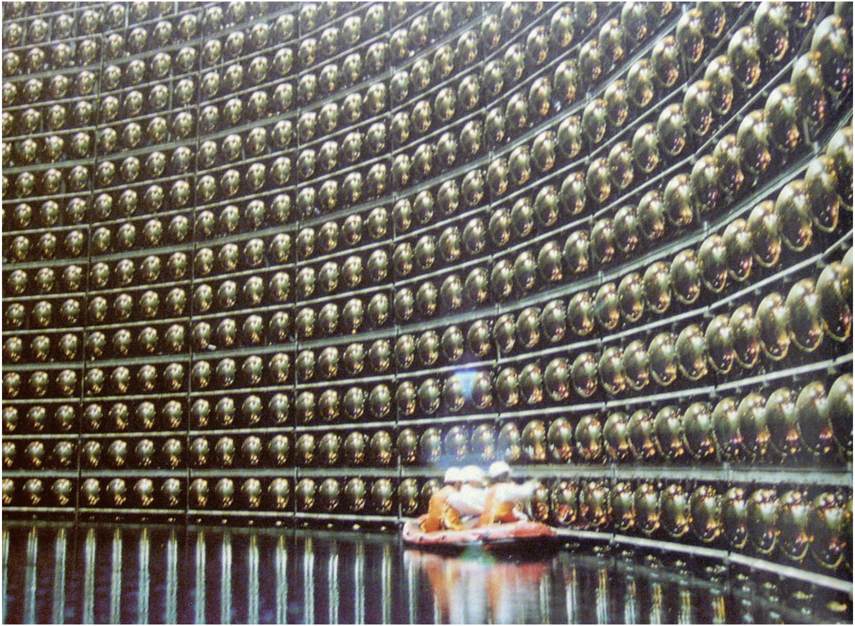
Photomultiplier



Detector



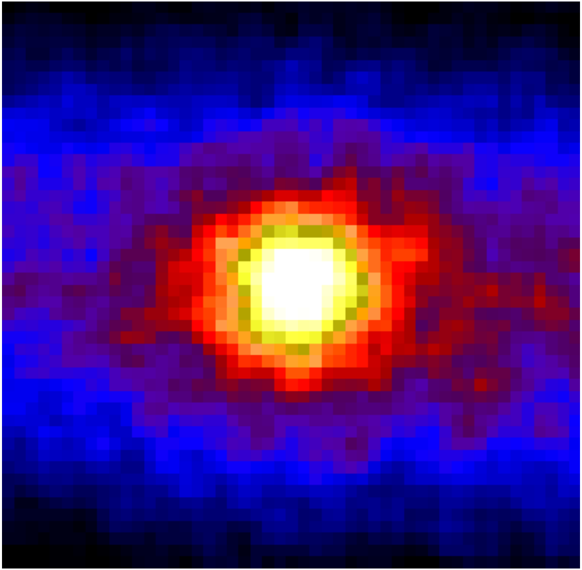
Super Kamiokande



Super Kamiokande

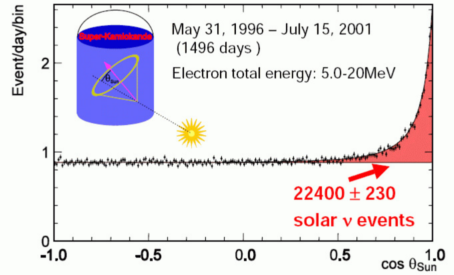
Image of Sun
reconstructed by Super Kamiokande from the measured neutrinos

actual Sun size $\sim \frac{1}{2}$ pixel



Solar neutrinos observation

Solar neutrinos clearly identified by their direction (w.r.t. Sun position)



Flux measured by SK:

$$\Phi_S^{(B)} = 2.4 \pm 0.1 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

Expected from Solar Model:

$$\Phi_S^{(B)} = 5.3 \pm 0.6 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

Two processes used in neutrino detection: at energies $\sim$ 10 MeV

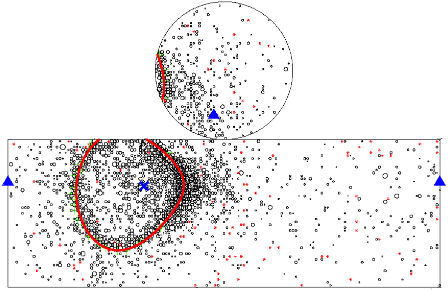
- Charged Current (CC) scattering: $\nu_e + N/e^- \rightarrow e^- + X \Rightarrow$ only ν_e
- Neutral Current (NC) scattering: $\nu_l + N/e^- \rightarrow \nu_l + X \Rightarrow$ all ν's

But NC cross section much smaller $\Rightarrow$ Flux measurement restricted to ν_e

Thanks to very sensitive readout and low background, discrimination between different neutrino flavours possible for high energy neutrinos

Electron neutrino

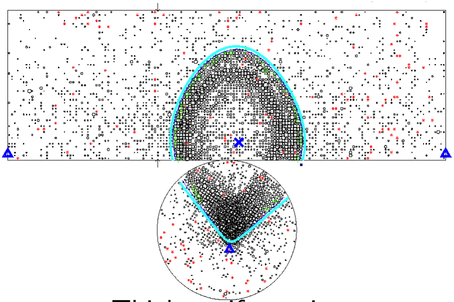
$\nu_e\ n \rightarrow e^- p$ event



Thin, irregular ring

Muon neutrino

$\nu_\mu\ n \rightarrow \mu^- p$ event



Thick, uniform ring

Electron is much lighter than muon and scatters/interacts much more

Atmospheric neutrino observations

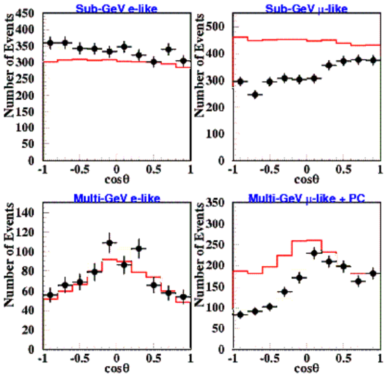
Measured neutrino direction for electron neutrinos (left) and muon neutrinos (right)

Similar number of electron neutrinos going down ($\cos\theta > 0$) and going up ($\cos\theta < 0$)
$\Rightarrow$ in agreement with predictions (red)

Clear deficit of muon neutrinos!

Direction measured best for high energy
$\Rightarrow$ deficit of up-going ν_μ

Can muon neutrino “disappear” in Earth ?!



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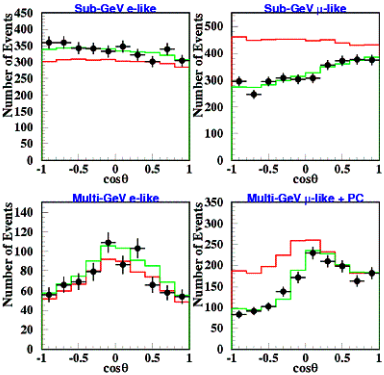

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Can muon neutrino "disappear" in Earth ?!

No! We can only explain it assuming neutrinos oscillate!
Predictions of oscillation model indicated by green histogram

Neutrino mixing for two neutrino flavours

We have to distinguish between two different neutrino state definitions

- flavour eigenstates - neutrinos with defined lepton flavour, as produced in weak interactions: ν_e, ν_μ
- mass eigenstates - free neutrinos with well defined mass, as propagating in vacuum: ν_1, ν_2

Flavour eingenstates can be described as a mixture of mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Assuming ν_e is produced at $t = 0$, the neutrino state can be written as:

$$|\nu\rangle_0 = \cos\theta_{12} |\nu_1\rangle_0 + \sin\theta_{12} |\nu_2\rangle_0 = |\nu_e\rangle$$

If neutrino were massless, $|\nu_1\rangle$ and $|\nu_2\rangle$ would travel with the same velocity, and neutrino state would not change: $|\nu\rangle_t \equiv |\nu_e\rangle$

Neutrino oscillations

Neutrino mixing for two neutrino flavours

Free states $|\nu_i\rangle$ evolve with time:

$$|\nu_i\rangle(t,\vec{x}) \;=\; |\nu_i\rangle_0 \cdot \exp\left(-\frac{i}{\hbar}(Et - \vec{p}\,\vec{x})\right)$$

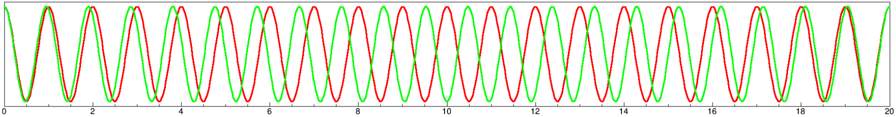
$\Rightarrow$ **If** $m_1 \neq m_2$ then the relative phase of $|\nu_1\rangle$ and $|\nu_2\rangle$ states changes and

$$|\nu\rangle(t,\vec{x}) \;=\; a(t)\,|\nu_1\rangle + b(t)\,|\nu_2\rangle \;\neq\; |\nu_e\rangle(t,\vec{x})$$

But we can decompose it into flavour eigenstates

$$|\nu\rangle(t,\vec{x}) \;=\; A(t)\,|\nu_e\rangle + B(t)\,|\nu_\mu\rangle$$

Flavour contributions change in time $\Rightarrow$ oscillations



Survival probability for two neutrino flavours

Probability that ν_e produced at $t_0 = 0$ interacts as ν_e after time t

$$P_{\nu_e \to \nu_e}(t) \quad = \quad |A(t)|^2$$

Assuming mass differences are small, $|m_1 - m_2| \ll m_1 \sim m_2 \ll E_\nu$

$$P_{\nu_e \to \nu_e}(t) \quad = \quad 1 - \sin^2(2\theta_{12}) \, \sin^2\left(\frac{(m_2^2 - m_1^2)\, t}{4E} \cdot \frac{c^4}{\hbar}\right)$$

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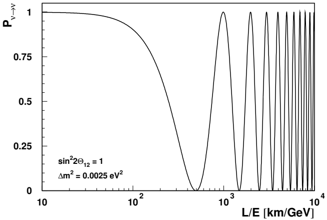
$$P_{\nu_e \to \nu_e}(t) = |A(t)|^2$$

Assuming mass differences are small, $|m_1 - m_2| \ll m_1 \sim m_2 \ll E_\nu$

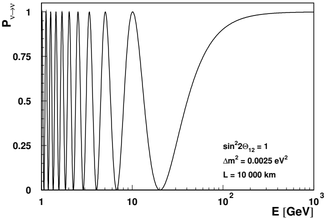
$$P_{\nu_e \to \nu_e}(t) = 1 - \sin^2(2\theta_{12}) \sin^2\left(1.27 \cdot \Delta m^2 \frac{L}{E}\right)$$

$L = ct$ [km], Δm^2[eV^2] i E [GeV].

As a function of distance:



As a function of energy: $(L \sim 2R_Z)$



For very large L/E $(E/L \ll \Delta m^2)$ we get: $P_{\nu_e \to \nu_e} \approx 0.5$

Neutrino mixing

For three flavours:

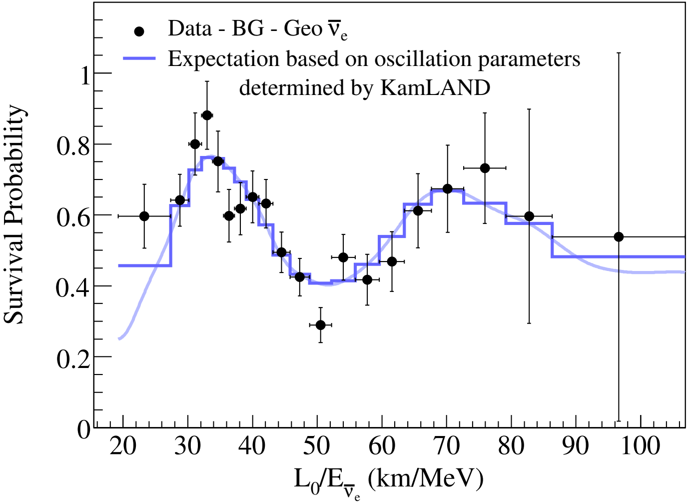
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS - Pontecorvo-Maki-Nakagawa-Sakata matrix
corresponding to CKM matrix for quarks

$$U = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\substack{\nu_e \leftrightarrow \nu_\mu \\ \text{solar neutrinos} \\ \theta_{12} \approx 35^\circ}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\substack{\nu_\mu \leftrightarrow \nu_\tau \\ \text{atmospheric} \\ \theta_{23} \approx 45^\circ}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}_{\substack{\nu_e \leftrightarrow \nu_\tau \\ \text{reactor, } \theta_{13} \sim 8^\circ \\ \text{established only in 2012!}}}$$

KamLAND

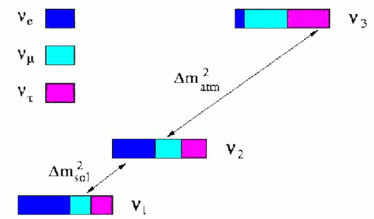
First direct observation of neutrino “regeneration” for reactor neutrinos



Summary

Neutrinos are produced as ν_e, ν_μ or ν_τ

But they propagate as superposition of mass eigenstates ν_1, ν_2 and ν_3



⇒ lepton flavour is not conserved, neutrinos oscillate

Oscillations are described by 3 mixing angles, 2 mass squared differences and a phase, which could be a source of CP violation in lepton sector.

Oscillations needed to describe a wide range of experimental results:

- deficit of solar electron neutrinos
- deficit of up-going muon atmospheric neutrinos
- oscillations of electron anti-neutrino flux in reactor experiments
- muon neutrino disappearance in accelerator experiments
- electron and taon neutrino appearance in accelerator experiments