

Particles and Universe

Lecture 7 Coupling constants

Maria Krawczyk, Aleksander F. Żarnecki

Faculty of Physics UW

I. Comparison of coupling constants

II. Feynman diagrams

III. Running coupling constants, asymptotic freedom

Interactions

In macro- and micro scales:

- **gravitation** – act between all massive particles, only attraction, responsible for Sun system, large astronomical objects, etc.
- **electromagnetism (e-m, el-mag)** – electric charge of both signs, attraction and repulsion, atoms ...

In microworld in addition interactions:

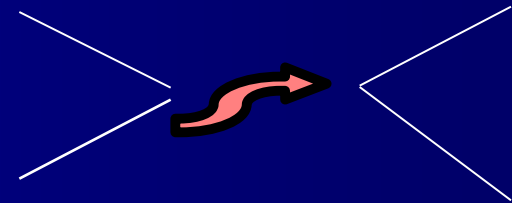
- **strong (nuclear)** - bounding nucleons in nuclei (**pions exchange**)
range 10^{-15} m
- strong fundamental (color)** - between quarks (**gluons exchange**),
range 10^{-15} m
- **weak** (nuclear), eg. neutron decay, range smaller than for strong
(pointlike interaction)
- weak fundamental** between quarks and leptons (exchange of gauge boson W/Z), range 10^{-18} m

Range of interactions

$$c = \hbar = 1$$

- Interaction in microworld = emission and absorption of bosons (photon, W/Z, gluons..) → exchange of particles
- Range** (Heisenberg, Yukawa) is related to the mass of exchanged particle (carrier of interactions)

$$x \sim 1/M$$



- gravitation and el-mag **infinite range** → graviton mass? photon mass = 0
- color (strong) int. : range ~ **proton radius 10^{-15} m** (although mass of gluons zero, **confinement!**)
- weak int. **range 10^{-18} m**, related to the mass of bosons

$$W/Z \sim 80-90 \text{ GeV} \quad 3$$

Strength of interactions

- Long range forces gravitation and el-mag very different - gravitation very weak
(gravitation between two protons 10^{36} times weaker than el-mag interaction)
- Strength's hierarchy at low* energies:
strong > electromagn. > weak > gravitation
- * low energies: 1 GeV up to 100 GeV
in the Standard Model – no gravitation!*
- Parameter of strength of elementary action
→ **coupling constant**

Coupling constants

Strength of elementary act of interaction = **coupling constant**

el-m: $e^- \rightarrow e^- \gamma$, $e^- \gamma \rightarrow e^-$ e (el. charge)

weak fund.: g ('weak' charge)

$e^- \rightarrow \nu_e W^-$, $\nu_e \rightarrow e^- W^+$

$d \rightarrow u W^-$, $t \rightarrow b W^+$

$d \rightarrow d Z$, $Z \rightarrow \nu \bar{\nu}$

strong fund., color. g_s ('strong' charge, color charge)

$u_R \rightarrow u_G + g_{R, \text{anty } G}$

Probability of elementary processes^{*},^{**}

el-m

$$\alpha = \alpha_{el} = e^2 / 4 \pi \simeq 1/137$$

weak fund.

$$\alpha_w = g^2 / 4 \pi \simeq 1/32$$

strong fund, color

$$\alpha_s = g_s^2 / 4 \pi \simeq 1$$

^{*} called coupling constant as well, ^{**} for energy ~ 1 GeV

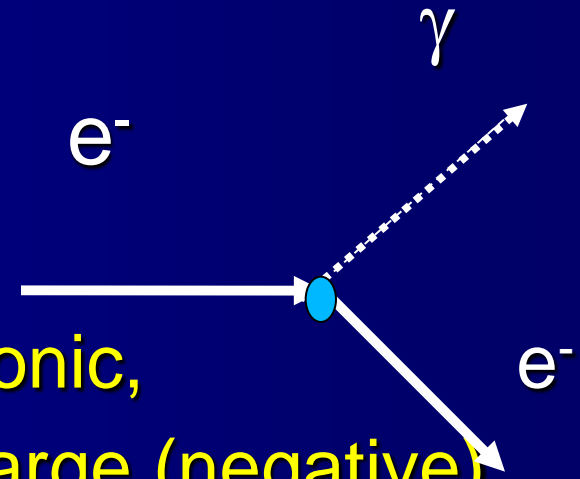
Feynman diagrams

Feynman diagrams –

particles are represented by different lines,

act of elementary interaction - by a vertex

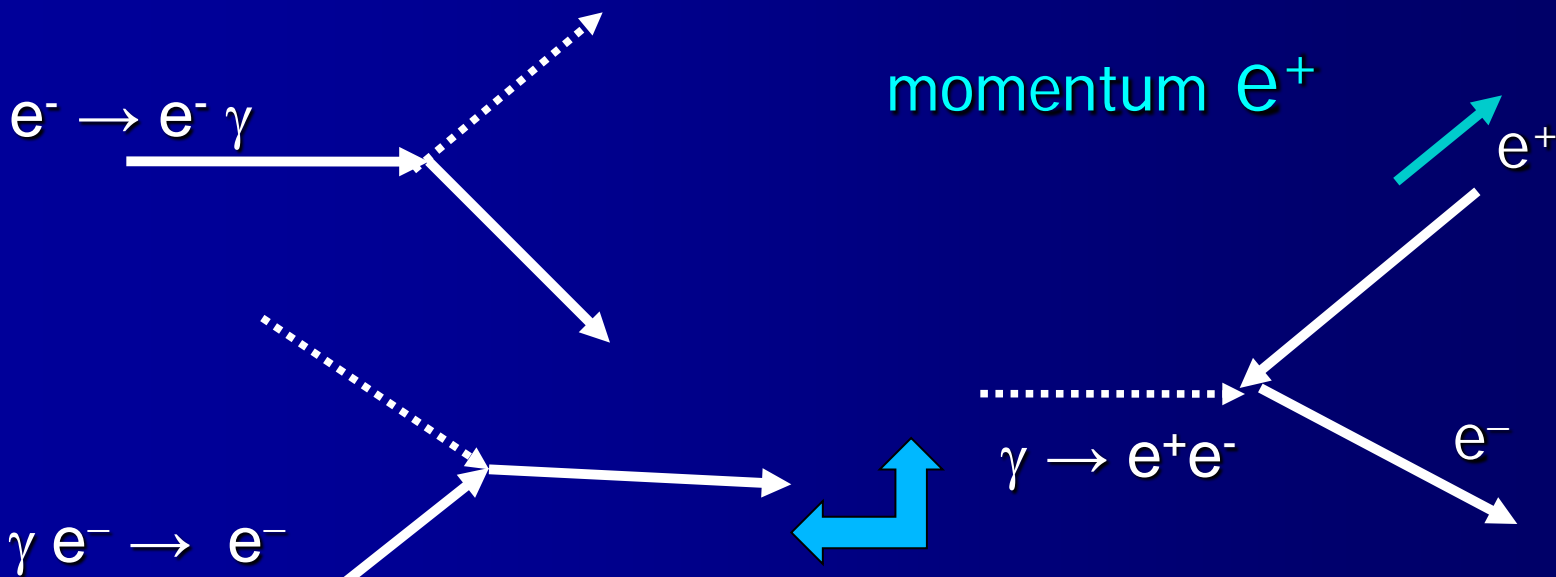
eg. emission of the photon by electron



Arrows on a continuous line (fermionic, here for e^-) \rightarrow flow of electric charge (negative) and momentum, while arrow on the photonic line (here dashed) \rightarrow only momentum

Feynman diagrams for crossing processes

Crossing processes involving $e e \gamma$
 (e represents e^+ and e^-)



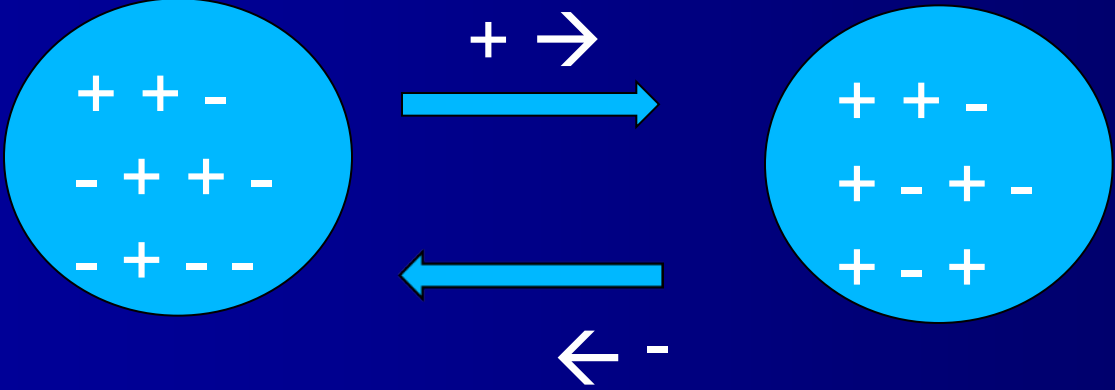
For e^+ - a flow of negative charge in opposite direction than momentum

Flow of charges

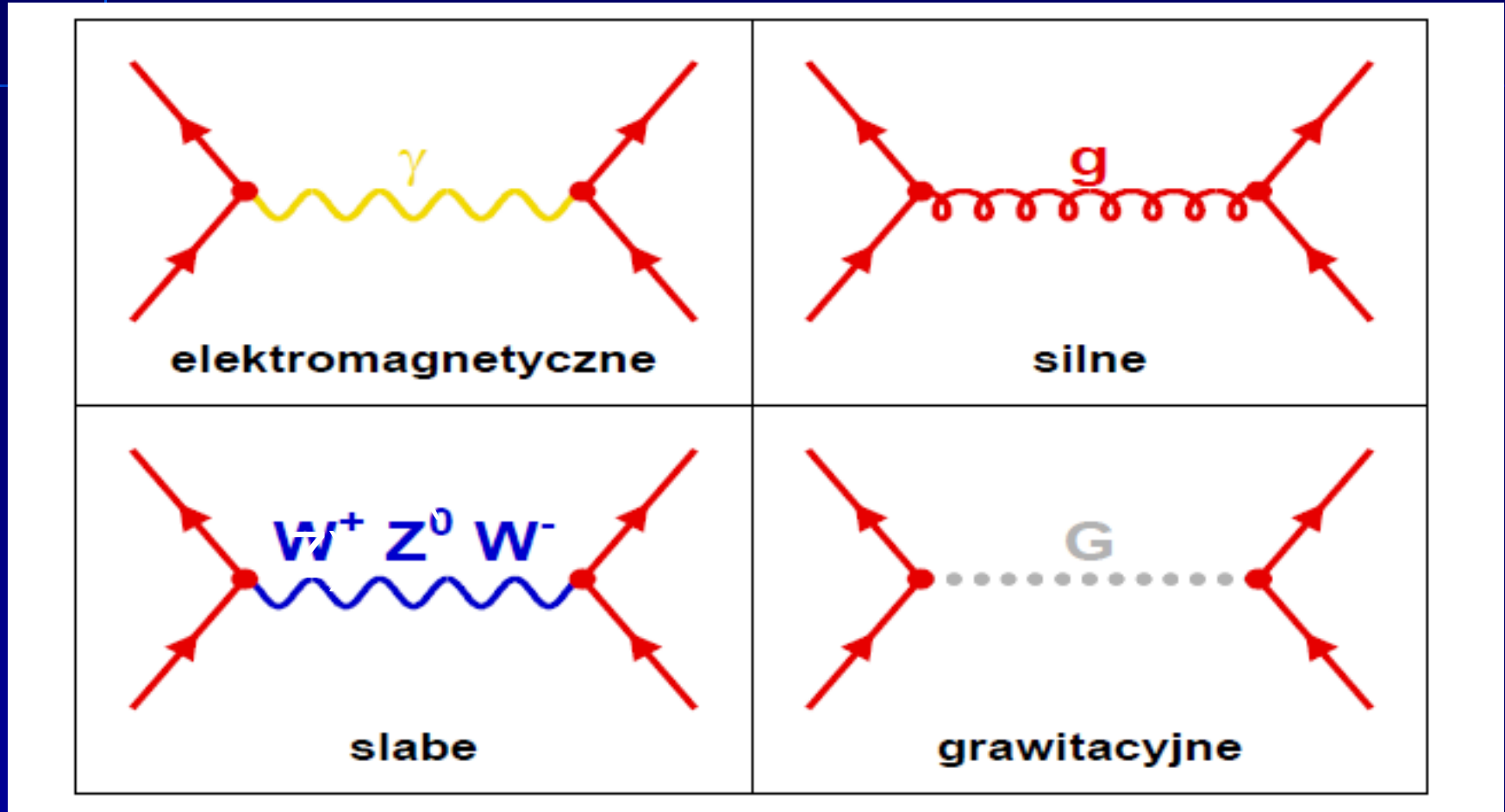
charge of
a system

$$N_+ = n_+ - n_-$$

$$N_- = n_- - n_+$$



Feynman diagrams



Electromagnetism and gravity

- Why gravity, so weak compared to electromagnetism was known first?
- Gravity only added while el-magn interaction canceled out for big
- The force for electron (with mass m) and proton (mass M) in the hydrogen atom H

$$F_{el} = e^2/r^2 \quad F_{gr} = GMm/r^2$$

- Ratio $GMm/e^2 = 10^{-40}$

Fundamental constants

Relation to physical phenomena

c – relativistic physics

velocity of light

\hbar – quantum physics

Planck constant $\hbar = h/2\pi$

G – gravitation

gravitational constant (Newton)

Subtle coupling constant

Electric charge e

$\alpha = e^2/4 \pi \hbar c \sim 1/137$ – a subtle coupling constant, introduced to describe interaction of electrons with photons by Sommerfeld in 1916 (*in subtle emission spectrum of hydrogen and silver*)

→ important in relativistic (c), quantum (\hbar) theory of electric charge (e)

quantum electrodynamics (funded in 20-30 XX), where α (or α_{em} , α_{el}) – measures „strength” of el-mag interaction of electrons and photons (→ coupling constant)

Note: formally we often take $\hbar c = 1$, eg.. on page 5 in definition of various coupling constants

Gravitation – PLANCK scales

- We neglect gravitation for individual particles at current energies
- When gravitation important in microworld?
From G , h and c we can construct quantity
 $(\hbar c/G)^{1/2}$ - Planck mass
- Planck's scales :
Planck's mass (energy) = 10^{19} GeV
Planck's length = 10^{-35} m
- For these scales \rightarrow relativistic quantum gravity. We are still looking for such theory...

Gravity contra electromagnetism

 ep in H atom

$$\frac{\text{Gravity P.E.}}{\text{Electromag}} \approx 10^{-40}$$

c.f. size of proton $\approx 10^{-15}$ m.

size of univ. $\leq 10^{10}$ yr. * 10^{16} m yr⁻¹
 $\leq 10^{26}$ m.

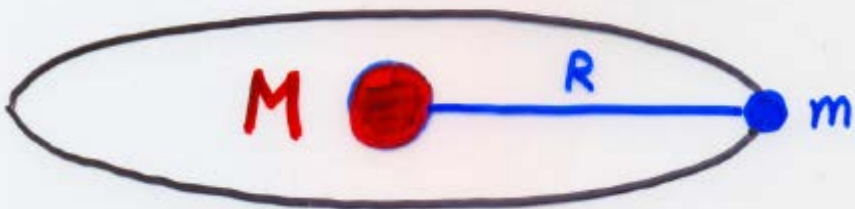
$$10^{-40} \approx \frac{\text{Radius of proton}}{\text{Radius of Universe}}$$

GRAVITY



Lecture by
F. Close

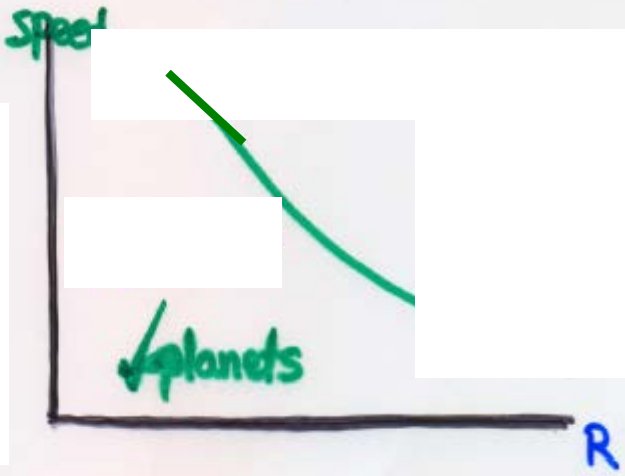
Velocity of object with mass m in the movement due to the gravitational attraction by mass M



Newton: $F = G \frac{M m}{R^2} = \frac{m v^2}{R}$

➔ $v^2 = \frac{GM}{R}$

Speed goes down as \sqrt{R}

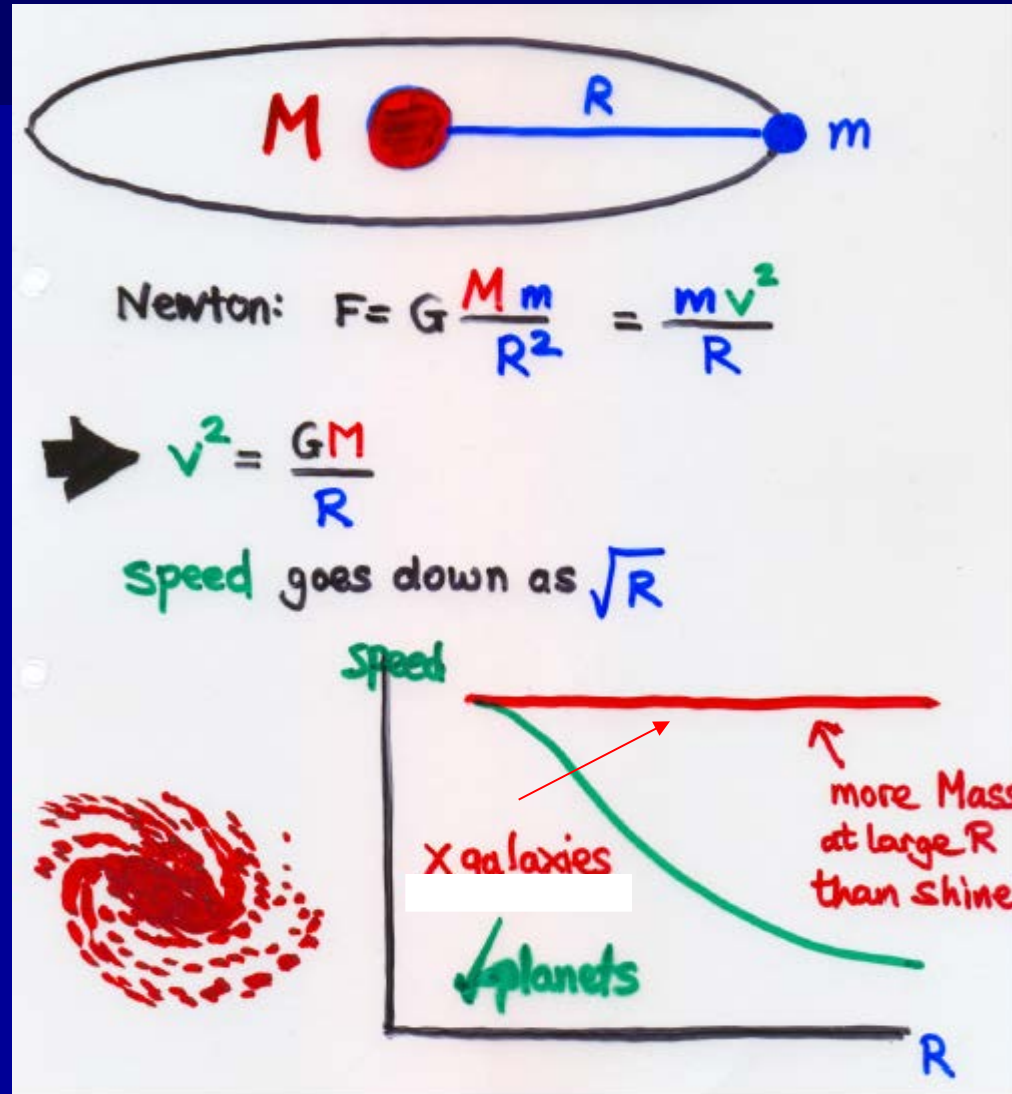


Velocity decreases for larger radius R

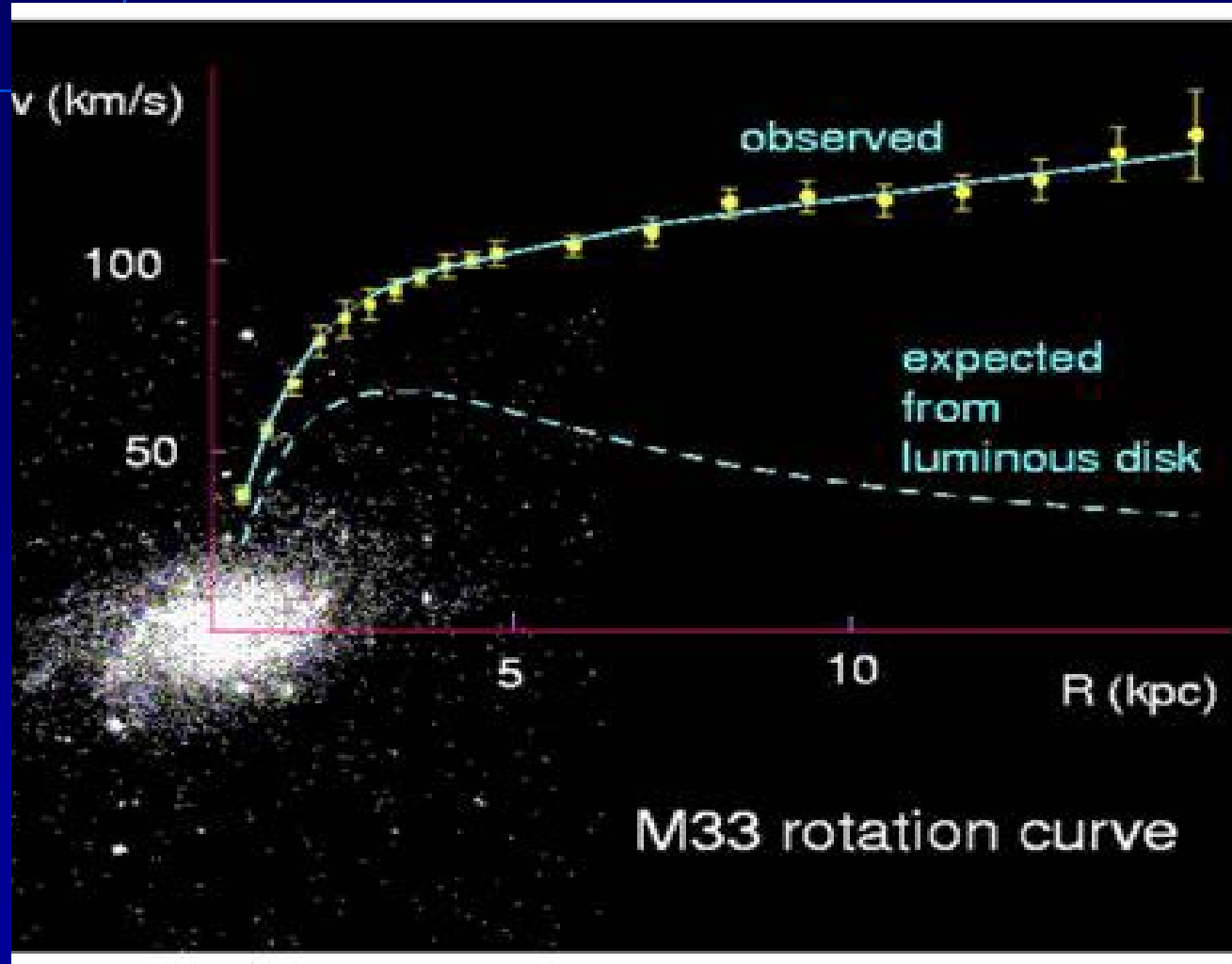
For planets !

Velocity of particles in galaxies?

Dark Matter



Rotation curve



Dark matter?

- ❖ We do not know what it is, but it must be neutral and:
- ❖ cold dark matter – heavy dark matter (small kinetic energy)

or

- ❖ hot dark matter – light dark matter (large kinetic energy)

more – next lectures

Electromagnetic interaction contra strong (color) interaction

Electrostatics

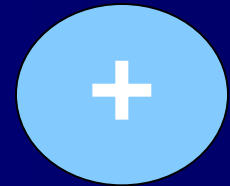
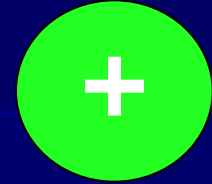
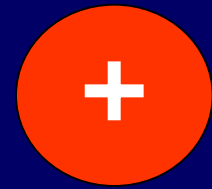
- Two types of electric charges
positive (+) and negative (–)

CHROMOSTATICS

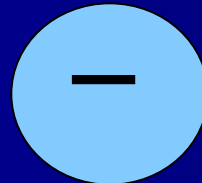
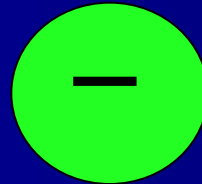
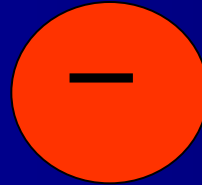
- Three types of color charges (colors),
each „positive” (+) and „negative” (–) →
means color and anticolor

3 colors

quarks



antiquarks

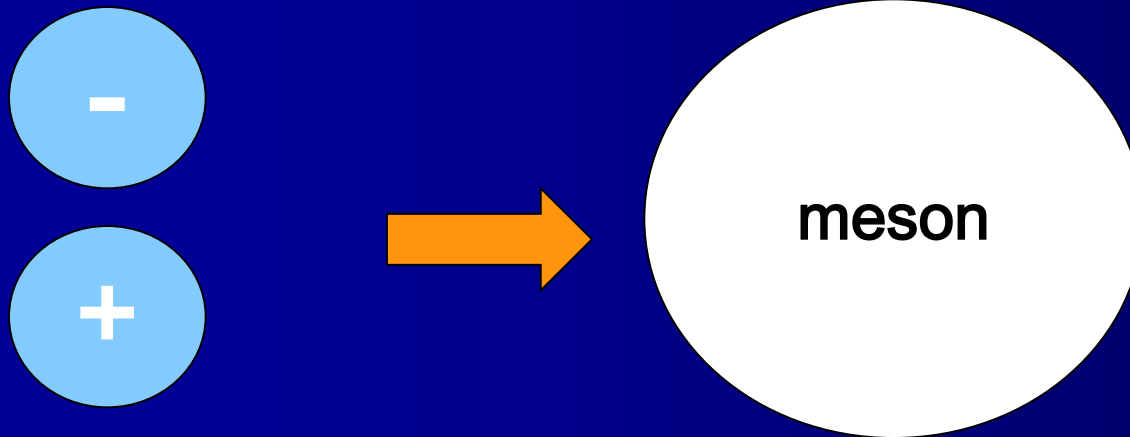


Known rule:

“The same colors
repulsive,
opposite colors- attractive”

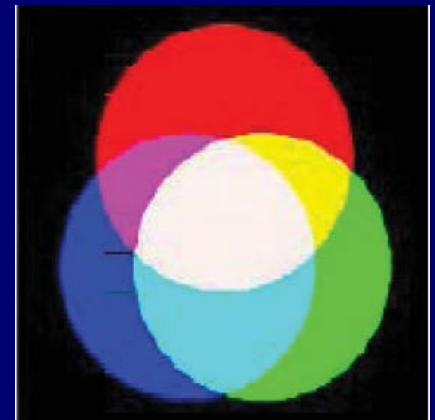
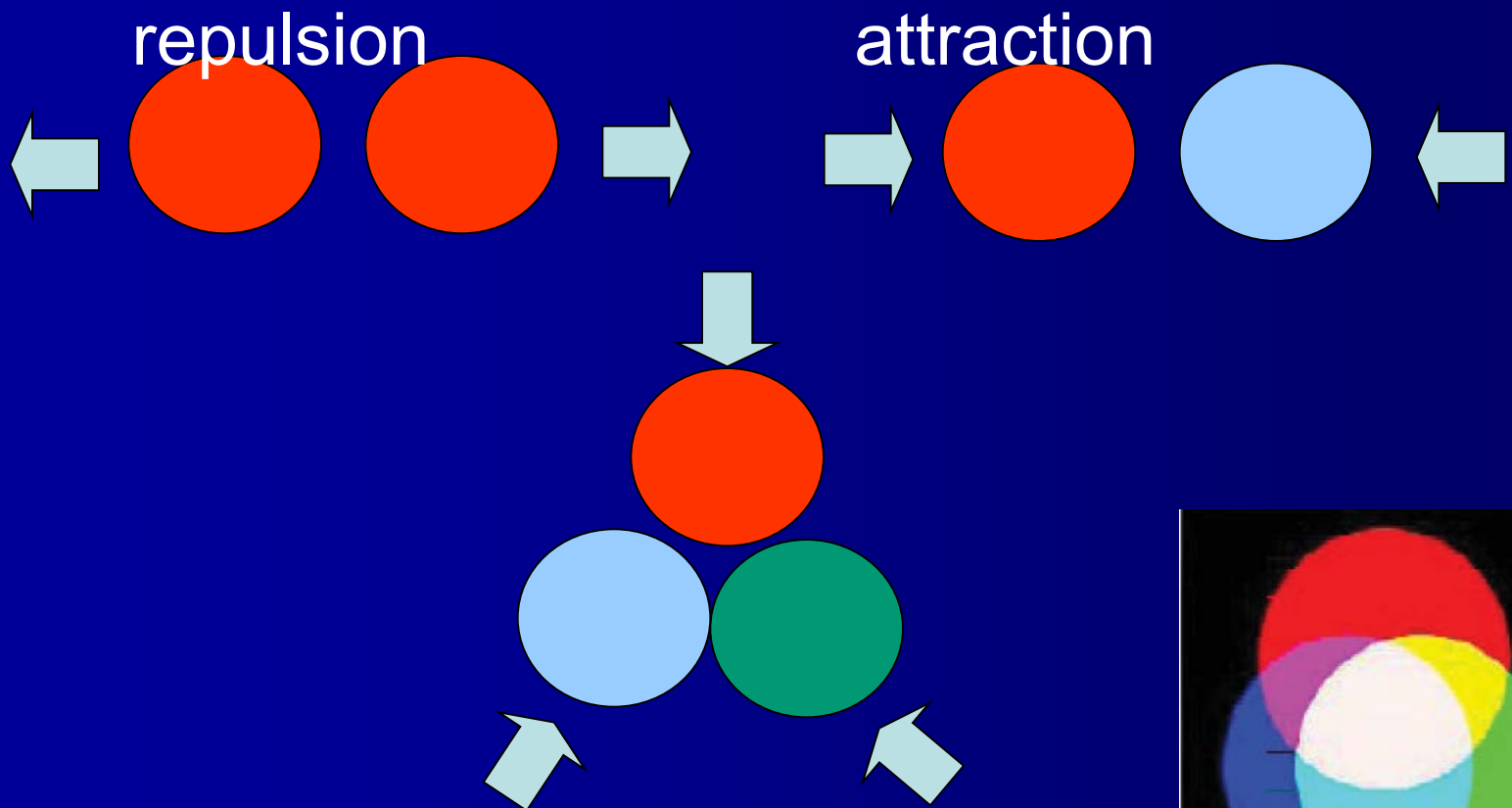
The simplest system: meson= quark+antiquark

Colors vanish, eg.



3 colors

Needed to get white baryons (3 quarks)
(eg. proton)



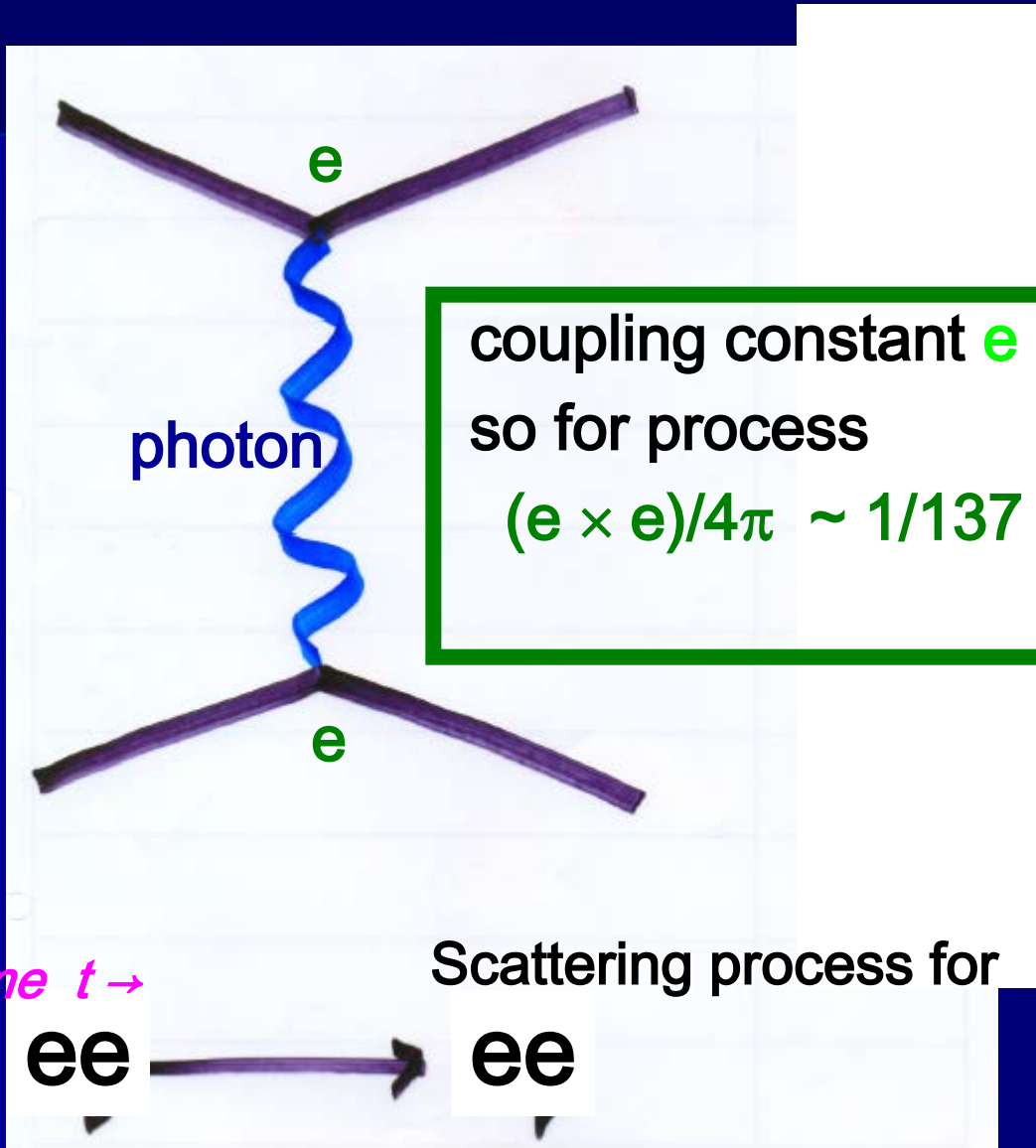
Quantum Electrodynamics: QED



Quantum Chromodynamics: QCD



Feynman's diagram for electromagnetic interaction



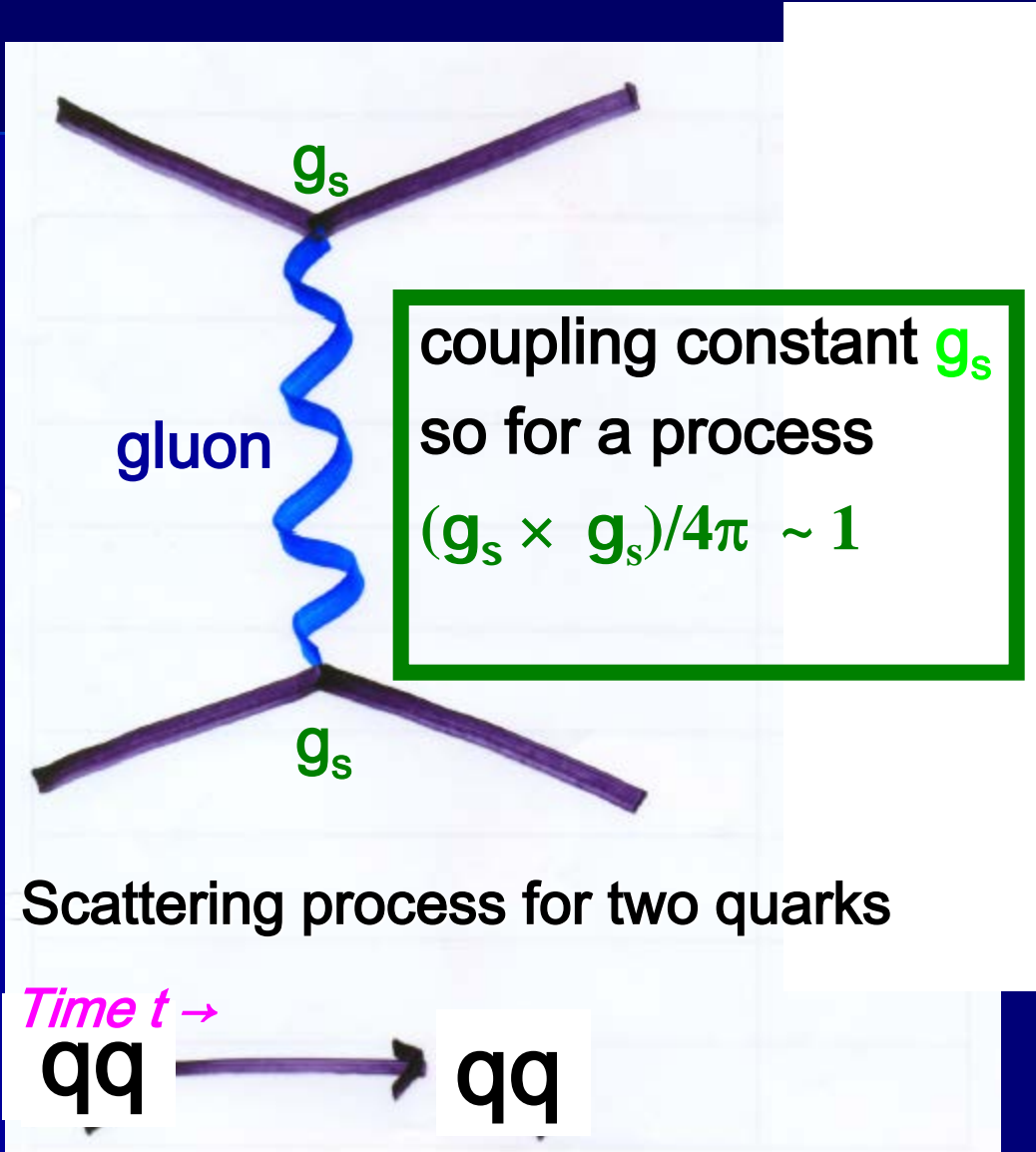
α

Time $t \rightarrow$

Scattering process for two electrons

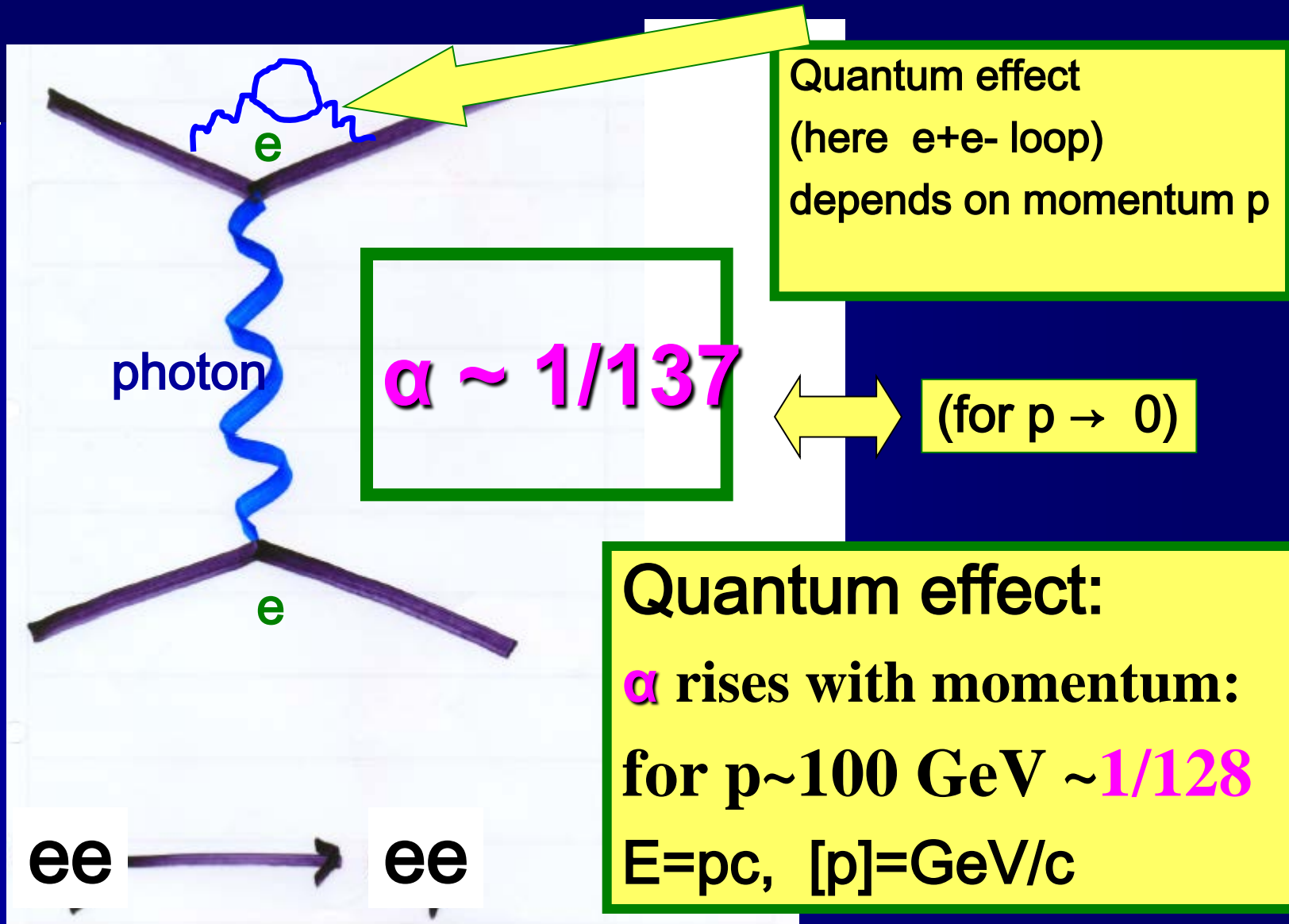


Feynman's diagram for color interaction



α_s

Electromagnetic interaction



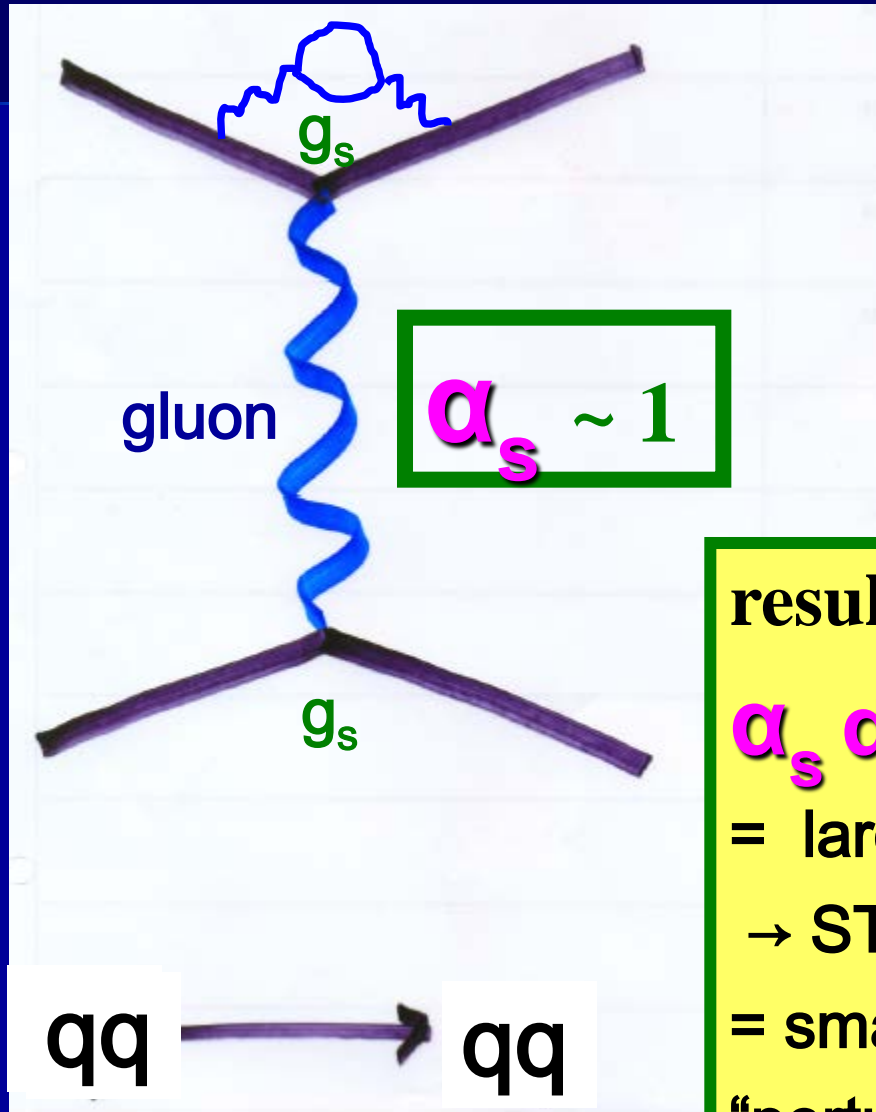
Quantum effect
(here e^+e^- loop)
depends on momentum p

$$\alpha \sim 1/137$$

(for $p \rightarrow 0$)

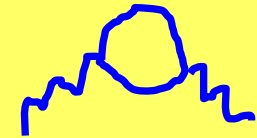
Quantum effect:
 α rises with momentum:
for $p \sim 100$ GeV $\sim 1/128$
 $E=pc$, $[p]=\text{GeV}/c$

Color interaction



Quantum effect in QCD
as in

QED:



quark loop

In QCD in addition
gluon loop



!

result:

α_s decreases with p !

= large for small p

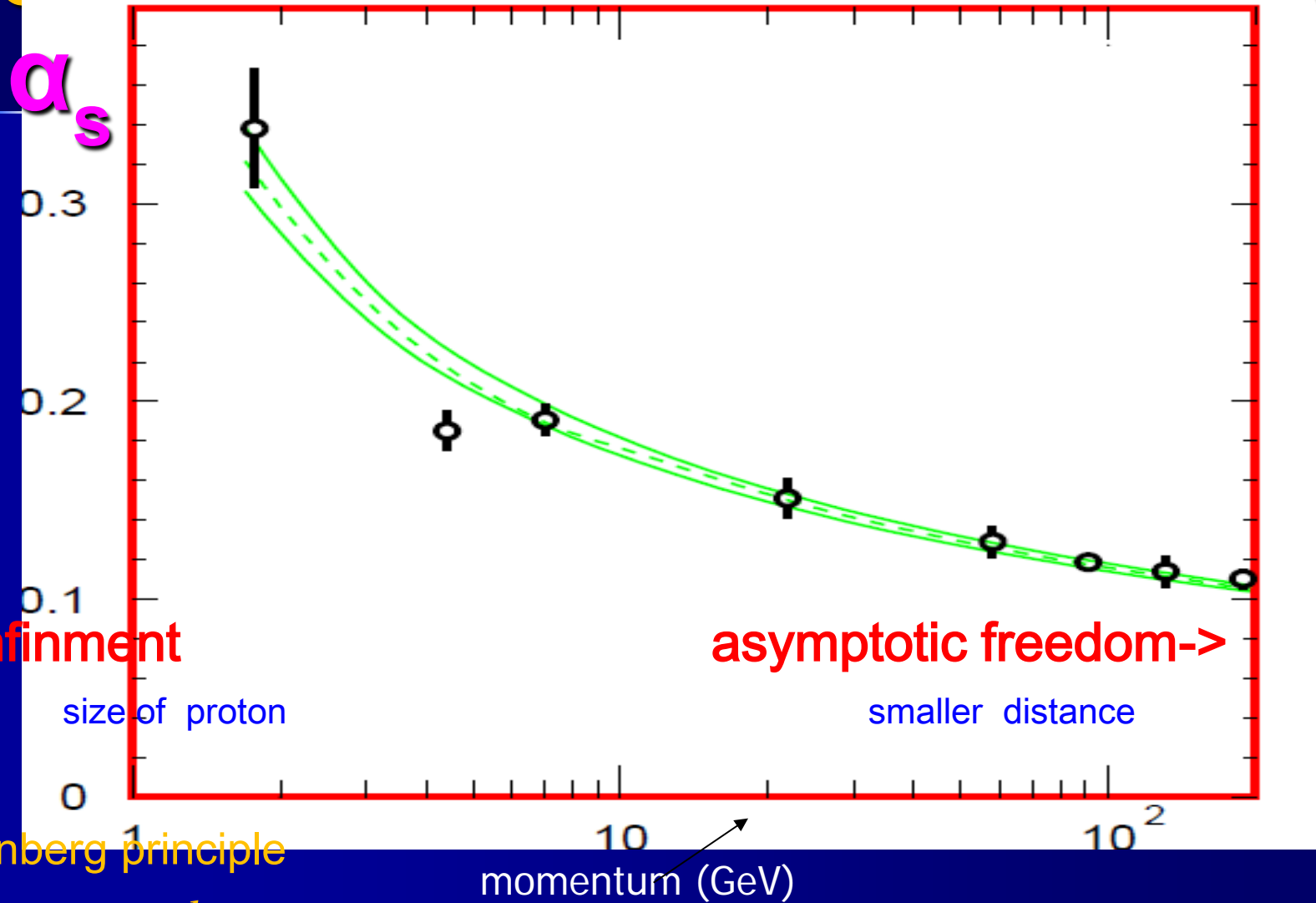
→ STRONG INTERACTION!

= small for large momentum

“perturbative QCD”

Running coupling constant (2008)

α_s



α_s

← confinement

asymptotic freedom →

size of proton

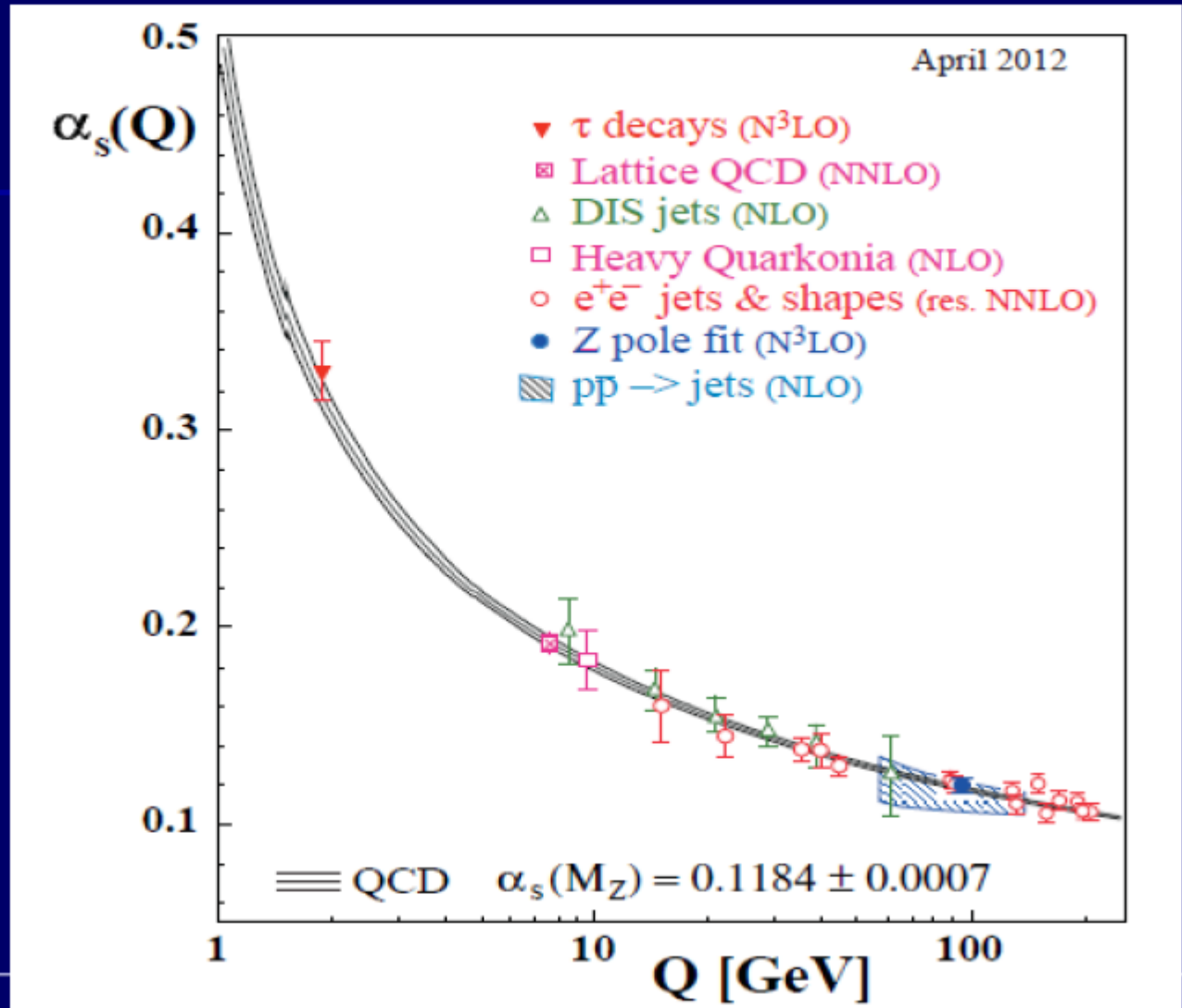
smaller distance

Heisenberg principle

$$\Delta x \sim 1/\Delta p \quad (\hbar c=1)$$

Running coupling constant

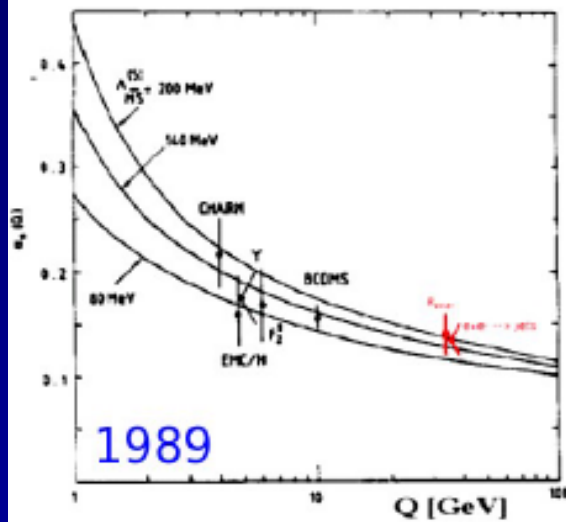
April 2012



Determination of the QCD coupling α_s

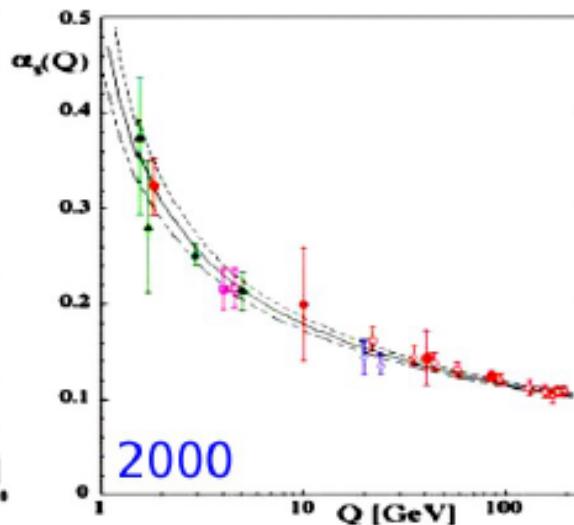
α_s = Single free parameter in QCD
(in the $m_q \rightarrow 0$ limit). Determined
at a given ref. scale (e.g. m_Z).
Decreases as $\sim \ln(Q^2/\Lambda^2)$,
with $\Lambda \sim 0.25$ GeV

- Least precisely known of all couplings:
 $\delta\alpha \sim 3 \cdot 10^{-10}$, $\delta G_F \sim 5 \cdot 10^{-8}$, $\delta G \sim 10^{-5}$, $\delta\alpha_s \sim 5 \cdot 10^{-3}$
- Impacts all LHC cross-sections.
- Key for SM precision fits
(e.g. uncertainties b,c Yukawa).
- BSM physics (e.g. couplings at GUT).



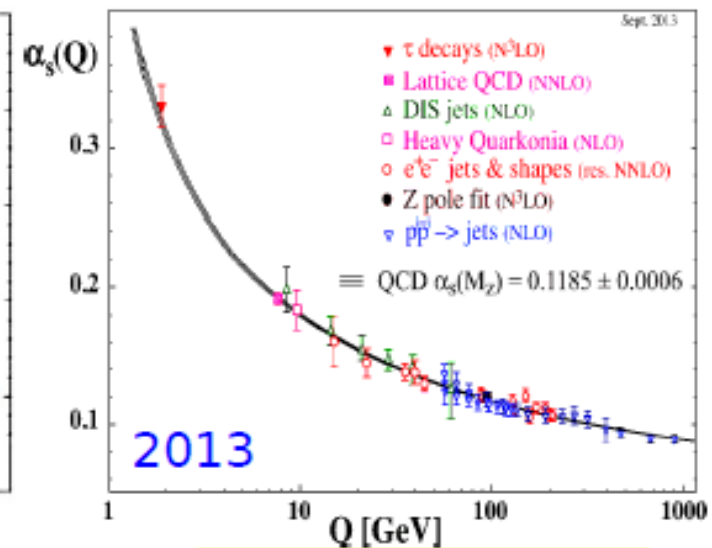
$$\alpha_s(M_Z) = 0.110^{+0.006}_{-0.008} \text{ (NLO)}$$

G. Altarelli, Ann. Rev. Nucl. Part. Sci. 39, 1989



$$\alpha_s(M_Z) = 0.1184 \pm 0.0031 \text{ (NNLO)}$$

S. B., J. Phys. G 26, 2000

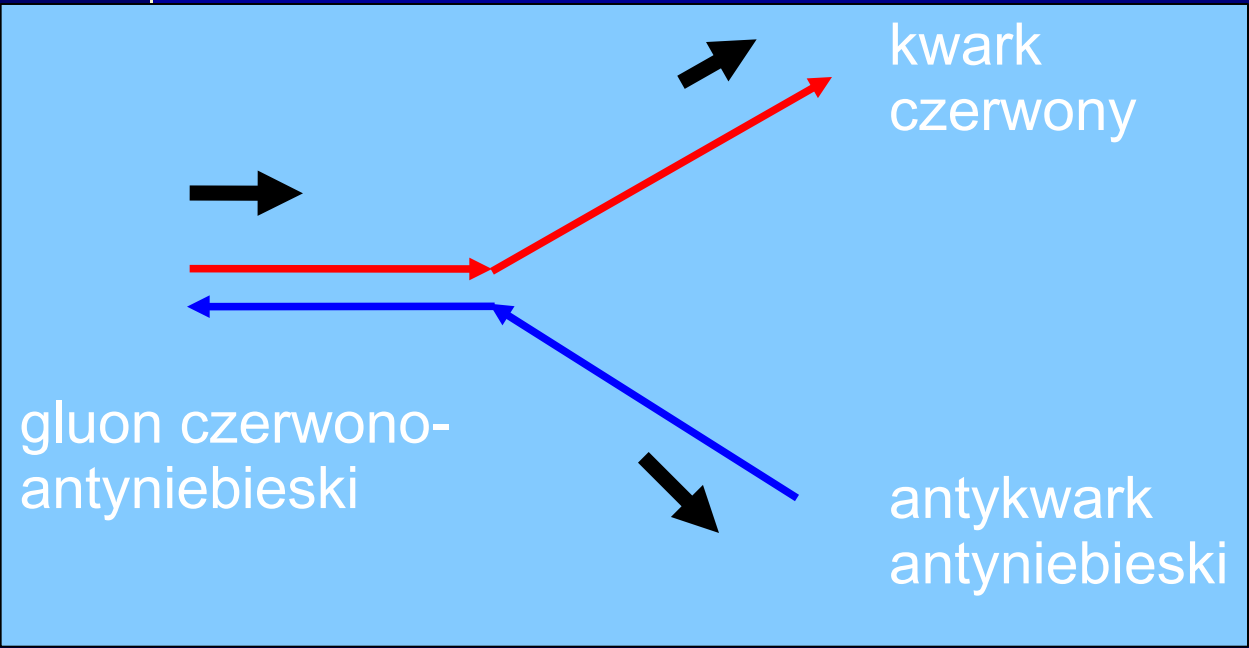



$$\alpha_s(M_Z) = 0.1185 \pm 0.0006 \text{ (NNLO)}$$

Current uncertainty: $\pm 0.5\%$
(lattQCD disputed by some: $\pm 1\%$)

Color interaction

Example gluon-> quark + antiquark



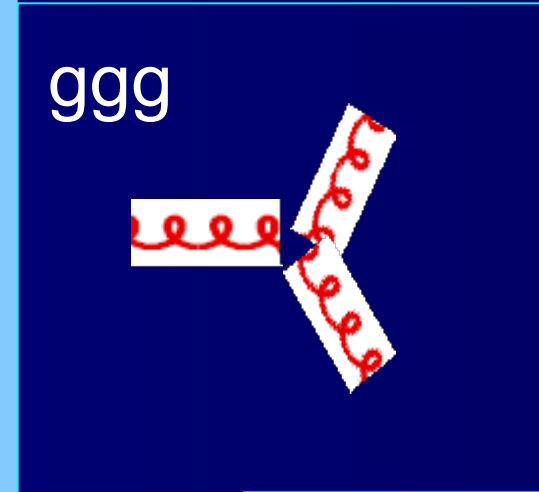
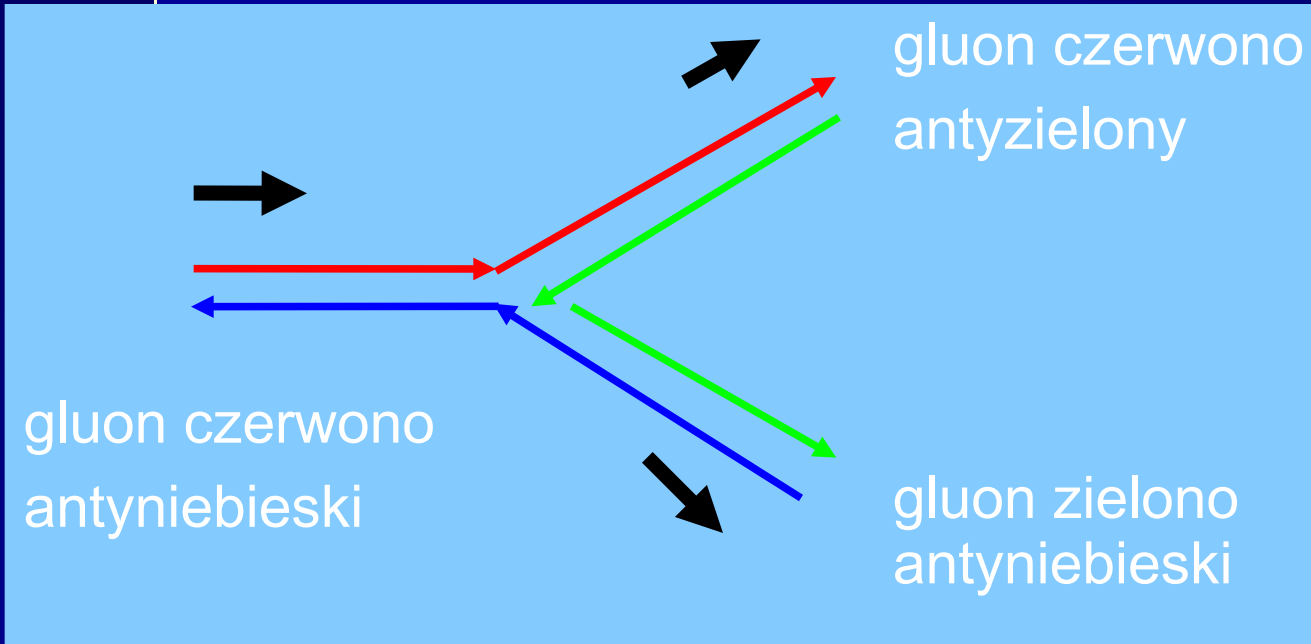
gluon line 

← here we follow a color

- color lines - flow of color (color conservation)
- black arrows – momenta of particles (momenta conservation)

Color interaction

Example gluon- \rightarrow gluon+ gluon

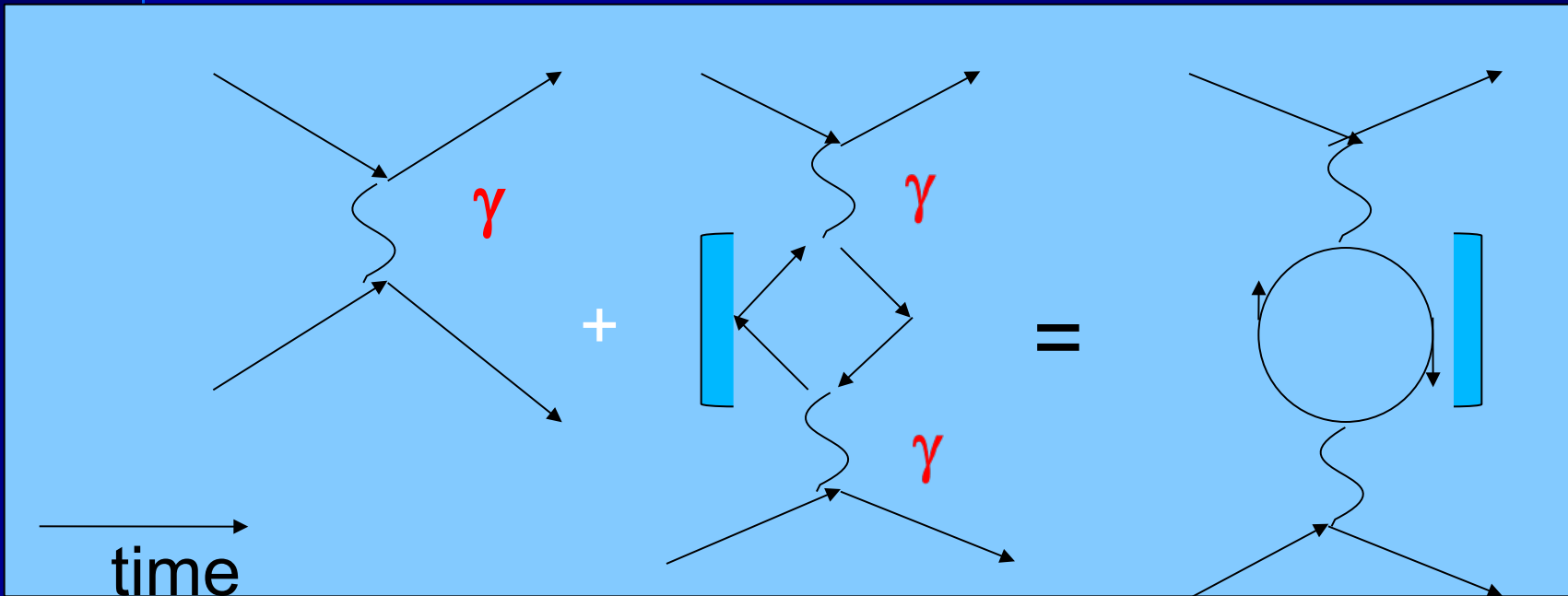


*Also
gggg*

- color lines - flow of color (color conservation)
- black arrows – momenta of particles (momenta conservation)

Extraction of α

Measurement of α in $e^-e^- \rightarrow e^-e^-$

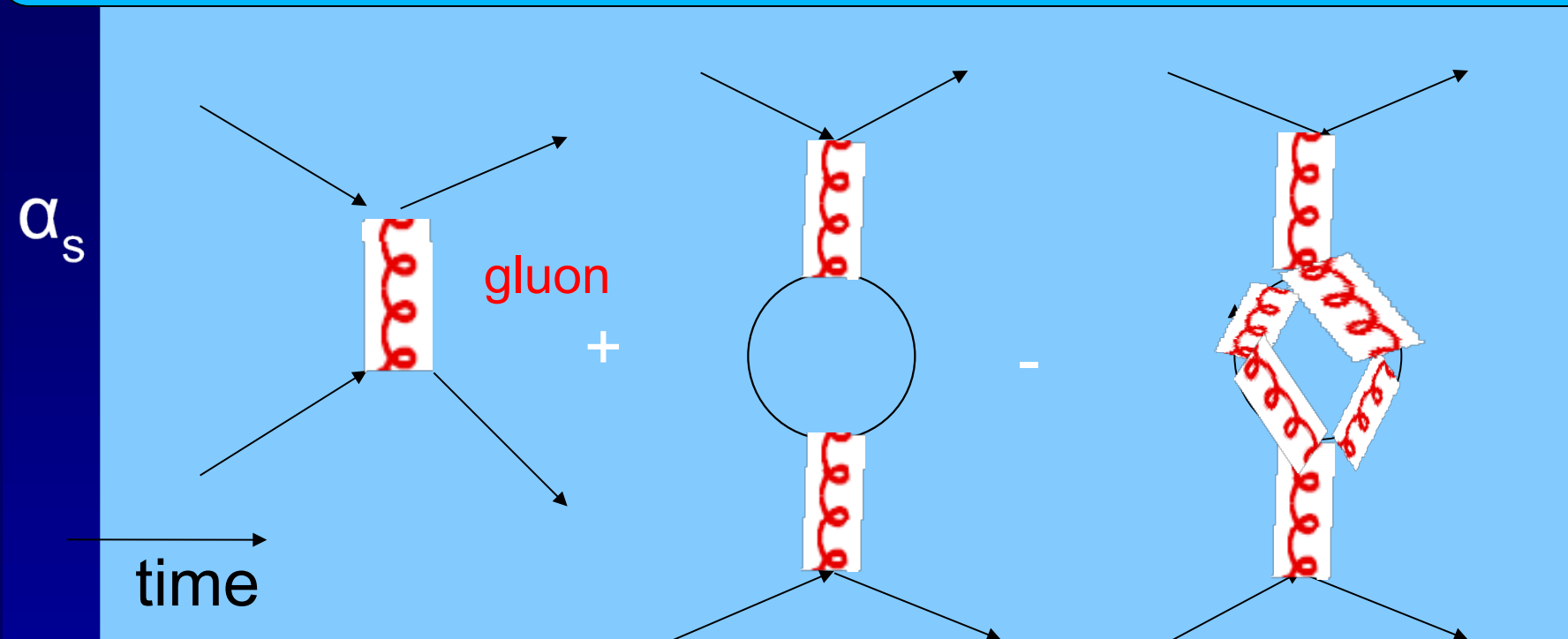


$$\alpha \times [1 + \text{corrections}(p)] = \alpha(p)$$

Electron loop $\rightarrow \alpha$ depends on momentum p (,runs'); is rising with energy (momentum)

Extraction of α_s

Scattering $qq \rightarrow qq$



Pętla kwarkowa - efekt podobny jak dla oddziaływań el-mag (powoduje wzrost stałej α_s). Tu dodatkowo pętla gluonowa, która ma **przeciwny** znak \rightarrow i w efekcie α_s maleje ze wzrostem pędu !!

Running coupling constants - unification?

- Couplings are running with energy (momentum) – this is an effect of quantum corrections
- Structure of interaction decides about rising or decreasing of coupling constants
 - key point - are carries of interactions „charged” or not (means do they interact with themselves), eg. photon neutral, while gluons „charged”*
- if some couplings are rising and other decreasing – at some energy they can have similar values

→ unified description?

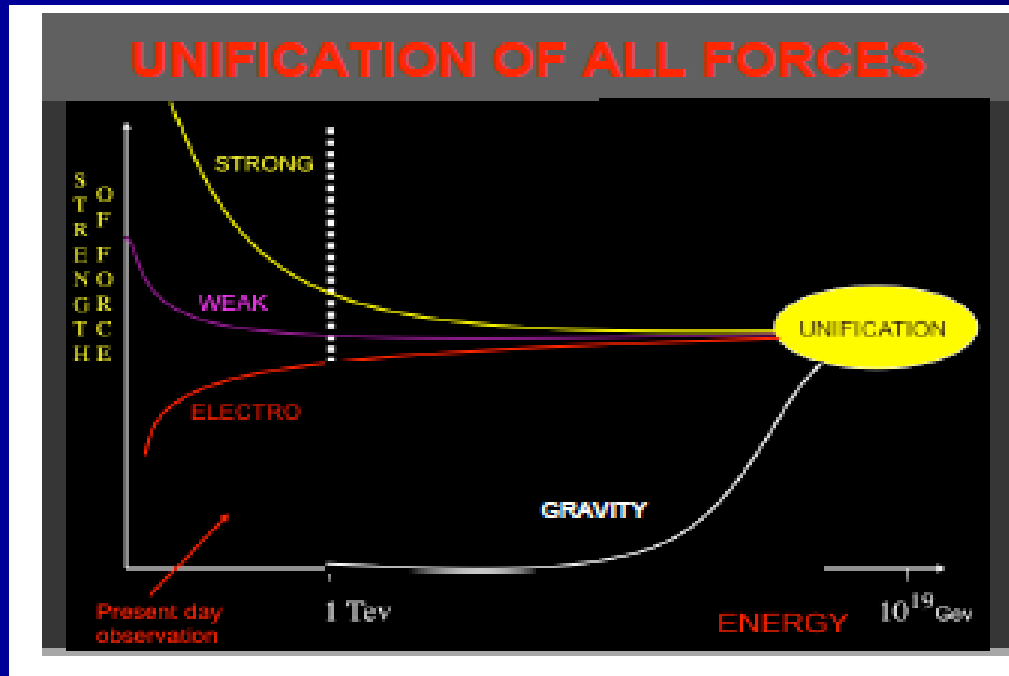
(,Running couplings constants)!

For larger energy: strong interaction weaker
weak interaction - weaker
el-mag interaction stronger

α_s

α_w

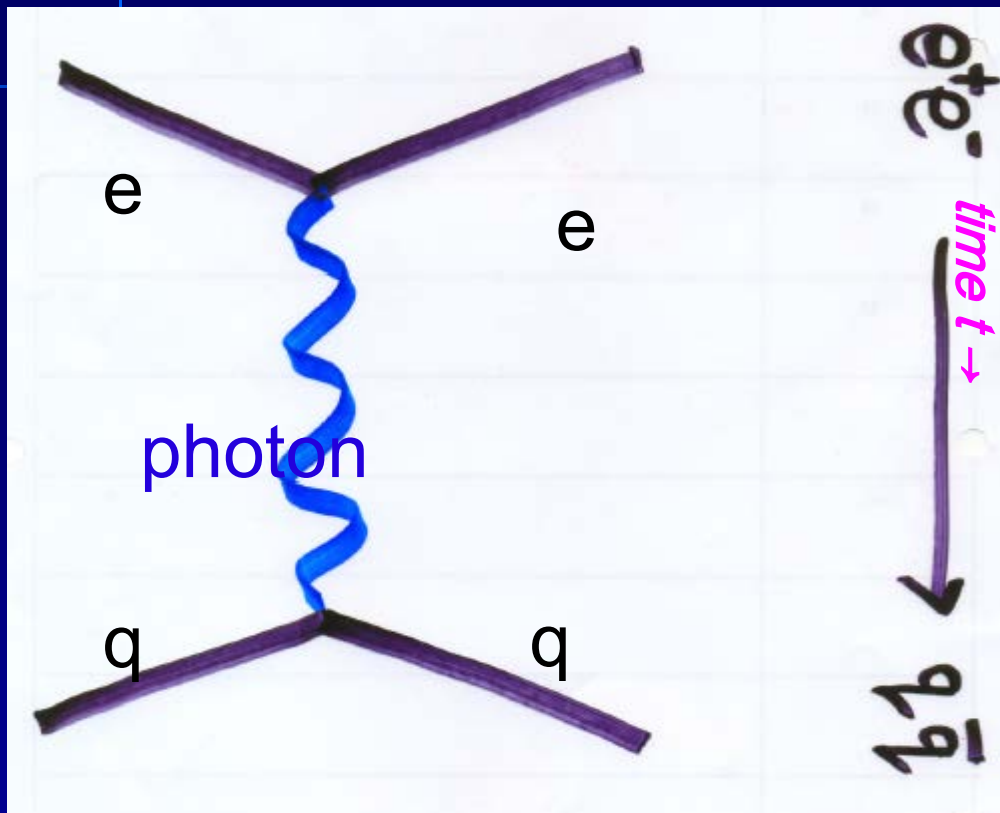
α_{el}



D. Gross,
Photon 2005

Gravity ???

Crossing for fixed external particles



Here 2 e
 (e- e- lub e+ e+, lub e-e+)
 And 2 quarks q
 (q q , q anty-q,
 anty-q anty-q)

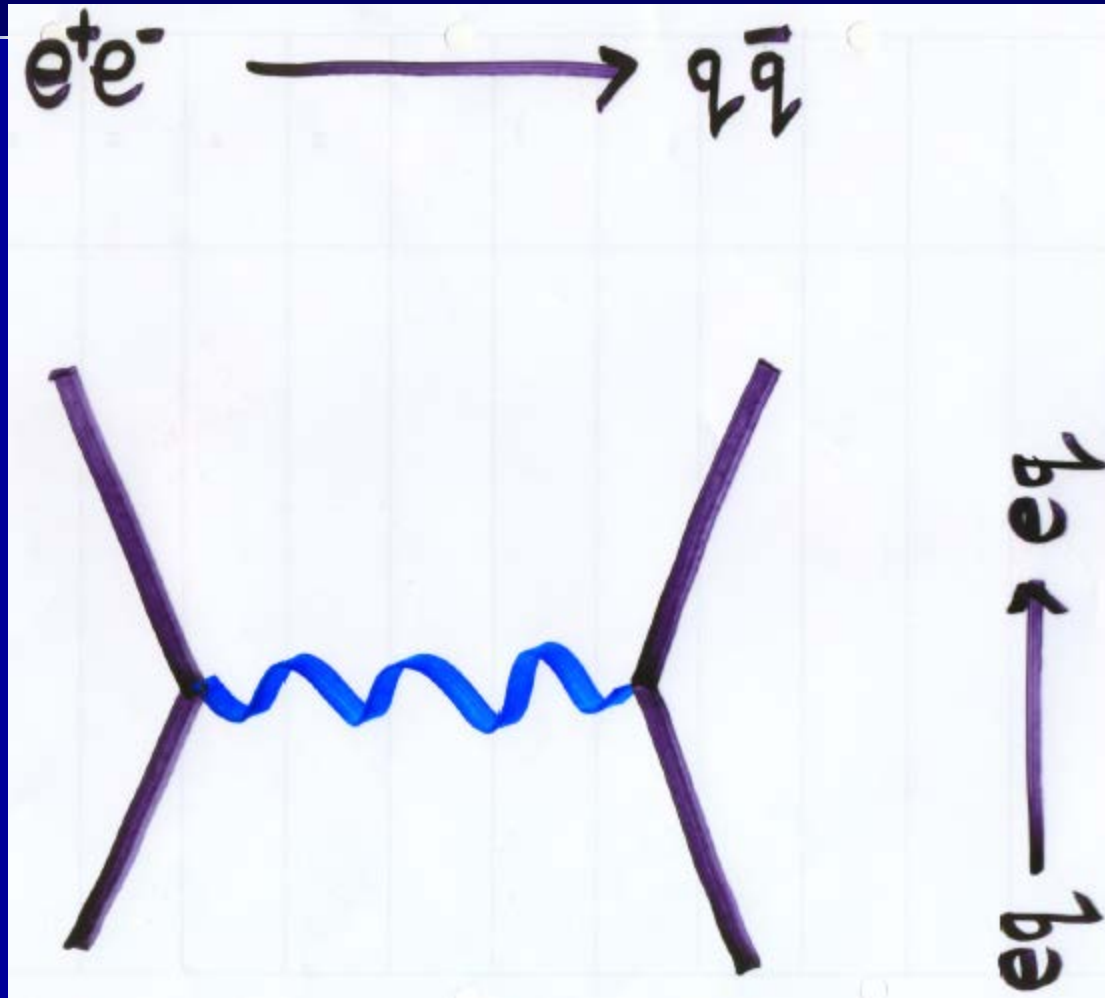
Time t →

$$e^- q \rightarrow e^- q$$

Crossing processes

positron in <-> electron out
 anti-q out <-> q in

Feynman diagram



Crossing processes – example:

$e^-e^- \rightarrow e^-e^-$

(time – from left to right)

- Other processes (crossing processes) we got exchange replacing initial particles with the final particles simultaneous replacing particles with antiparticles
- Yellow particle \rightarrow to be transferred to the future (as antiparticle) and pink particle \leftarrow to be transferred to the past (as an antiparticle):
- $e^-e^- \rightarrow e^-e^- \rightleftharpoons e^-e^+ \rightarrow e^-e^+$, and next
 - $e^-e^+ \rightarrow e^-e^+ \rightleftharpoons e^+e^+ \rightarrow e^+e^+$

Question to lecture 7

- Is the range of weak forces larger or smaller than the range of strong interactions?
- Which particles interact using nuclear forces, which particles using the color forces .
- Is gravitation important in the microworld for low energies?
- What is the value of the Planck length? What is value of the Planck mass?
- Write 3 elementary acts of interactions between particles from the first family
- What is the value of subtle coupling constant for momentum $p \rightarrow 0$?, for momentum $p = 100 \text{ GeV}$?
- Do two electrons interact stronger or weaker for larger energies (momenta)?
- What is value of strong coupling constant for momentum (energy) about 1 GeV ? For which momentum α_s is equal to $1/10$?
- When quarks are more free: for small or large energies?
- Write 2 processes obtained by crossing of the scattering process $u d \rightarrow u d$
- To what quarks does the green-antired gluon decay ?
- What is the reason of a running of coupling constants? Why the subtle coupling constant is growing while the strong coupling constant decreases with a grow of energy?