# Particles and Universe
## Lecture 9  Perturbative calculation

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I. Theory of elementary particles – description of events; Perturbative calculation; Renormalization

II. Probability of processes at high energy – a need for  Higgs particle

# Description of events

- Relativistic and quantum effects. To describe creation and annihilation of particles and mixing as well as decays → formalism of

  Quantum Field Theory,QFT

  *(applied first to the QED)*

- Technique of QFT

  - perturbative calculation:

    the lowest order of pert. series

    quantum corrections: trees and loops

  - Feynman diagrams

# Theory of elementary particles

- 1948 – new phase of quantum mechanics (QFT); precise measurements → a need of more precise calculations

- Feynman method: diagrams and Feynman's rules for calculation – today the universal tool of particle physics

*first application in QED*

- QED describes interaction of electron and photons. Difficulties since quantum corrections infinite!

  Method of removing them → renormalization procedure.

- Weak interaction – even more difficult situation. Proposal: new interaction and new particles → EW interaction with gauge bosons W/Z with Higgs boson - renormalizable !

  Nobel prize: Glashow, Salam, Weinberg 1979 (W/Z) t'Hooft, Veltman 1999 (renormalizability)

# Relativistic effects
## - free and virtual particles

# Relativistic effects (special theory of relativity )

- **Einstein postulates:**

  - Invariance of laws with respect to change of reference system (for the inertial systems)

  - Constant velocity of light c in the inertial systems

- Lorentz transformation relates measurements of two inertial reference frames

- The most „visible" relativistic effects
  – lifetime for particles moving with velocity close to c longer (*time dilatation*)

    *Muons born in the upper atmosphere arrive on the surface of Earth only due this effect (660 m)*

  -- similar effect for space (*length (Lorentz) contraction*)

# Lorentz and Poincare invariance

- Invariance with respect to rotations (in space-time) → Lorentz invariance

  If in addition invariance with respect to translation in space and time → Poincare invariance

Not everything is relative !!

If decay (process) is forbidden in some reference frame it is forbidden in any frame

- However – for a description of a particular process → a particular system maybe more useful.

# Forbidden processes - examples

- For a *free* electron process $e \rightarrow e\,\gamma$ is forbbiden *Why?*

  Electron at rest has the lowest possible energy $(E = mc^2)$, and cannot have lower energy after emission of photon. If it is not possible in one frame then it is not possible in any
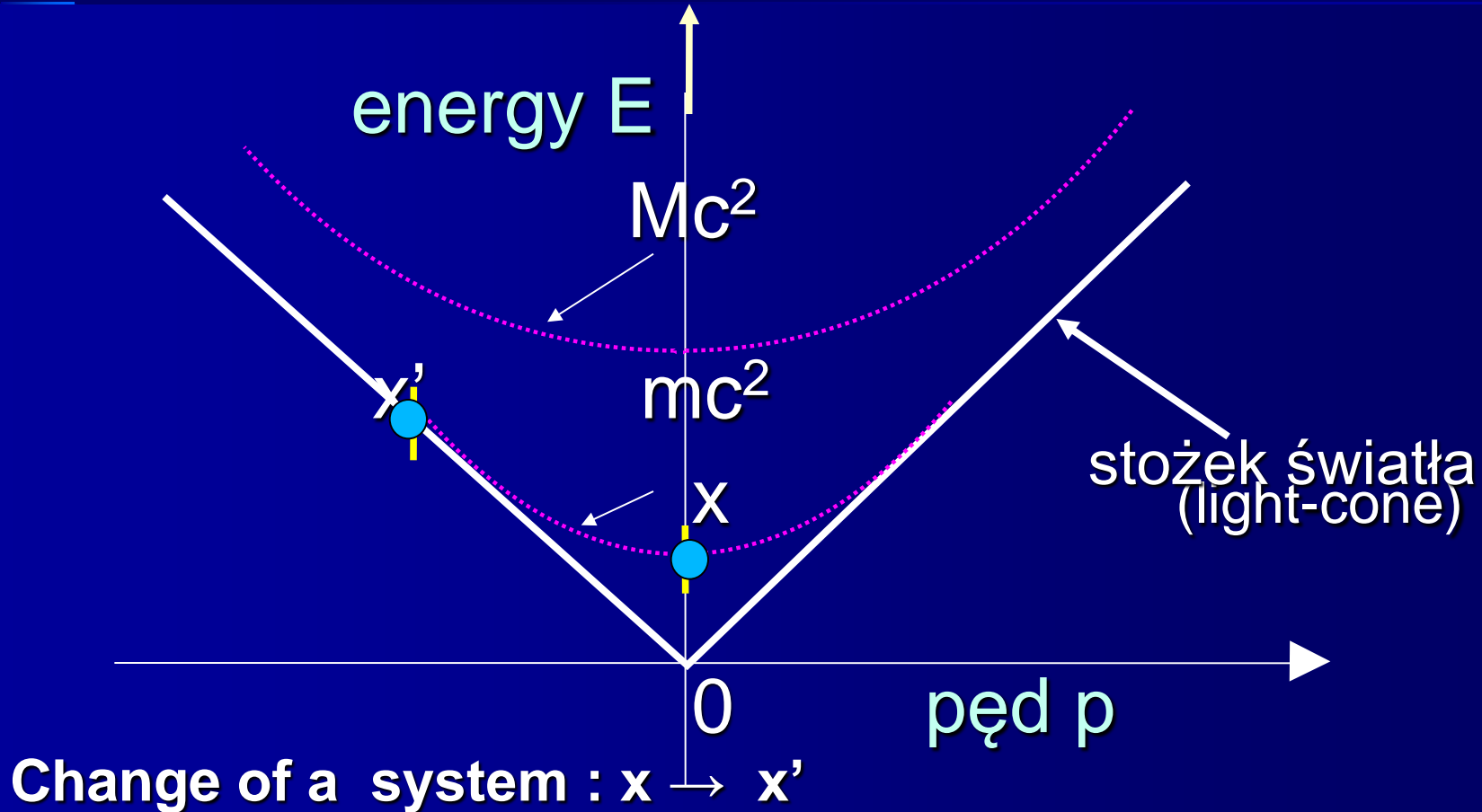
  *It is possible in matter*

- For a *free* photon the decay $\gamma \rightarrow e^+e^-$ is not possible - observer moving towards a photon see it with a smaller momentum/energy $(E = pc)$...

  *Possible in the matter*

**Relation defines the mass shell for a particle with mass m**

$$E = c\sqrt{p^2 + m^2 c^2}$$

energy E

$Mc^2$

$mc^2$

x'

x

stożek światła
(light-cone)

0    pęd p

**Change of a system : x → x'**

**Particle on a mass shell = free particle**
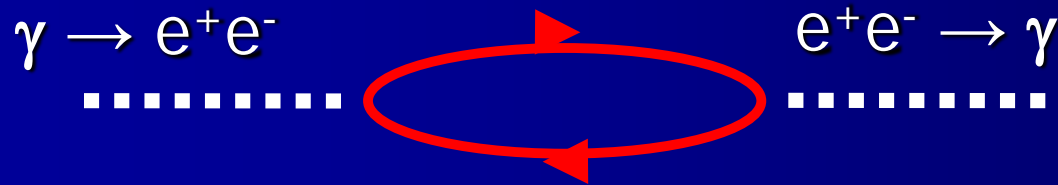
# Inner particles in diagrams = virtual particles

Quantum mechanics allows for

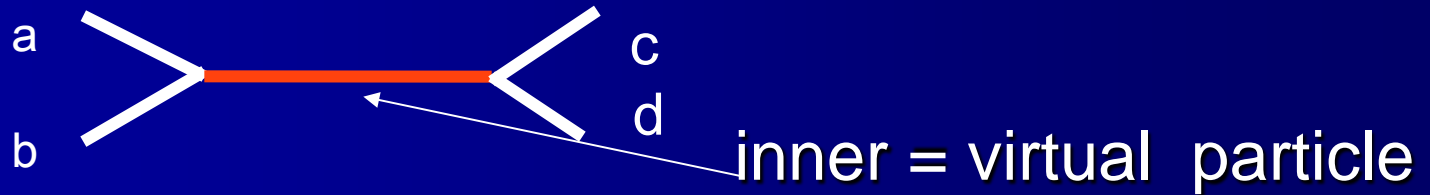virtual particles  for which
$$E^2 \neq p^2 + m^2 \ (c=1)$$

(particles *off*  mass shell)

They exist for a while and are not observed directly
eg. they appear in the loops

$$\gamma \rightarrow e^+ e^- \qquad\qquad e^+ e^- \rightarrow \gamma$$

# Examples- virtual particles

- In a collision of two particles   a,b

two particles are produced c, d:  a+b → c+d

a

b

c

d

inner = virtual  particle

There are various channels
(= various virtual exchange)   *like two split experiments*

- Some processes are possible due to them–
like light-on-light scattering

γ  γ → γ γ

*el. charged virtual particle*
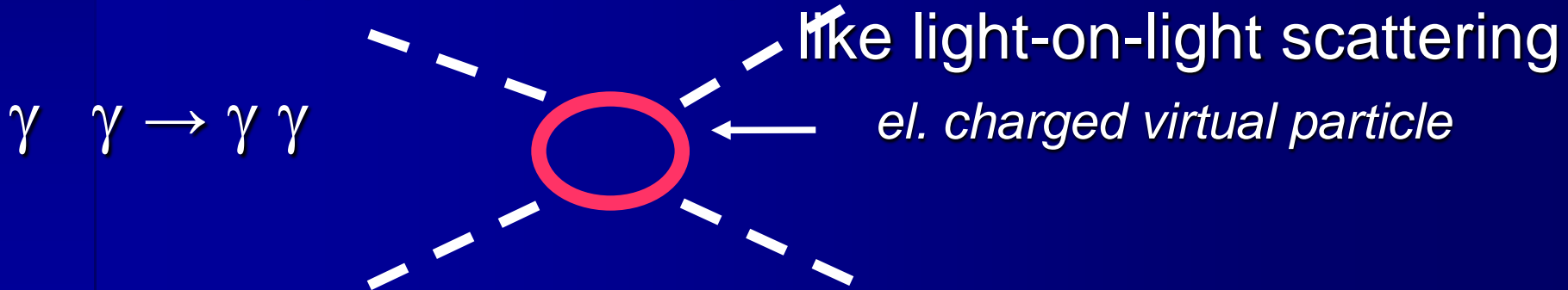
*photons are neutral  and can not interact with themseves*

# Probability of processes

# Probability amplitude and probability of processes

- Knowing properties of particles we can calculate probability of processes they are involved in

- Initial particles we produce or prepare, but the final particles we can only observe – but there are many ways from the initial to the final particles

  (*channels of processes* )

- We calculate *probability amplitudes for each channel* → interference of the amplitudes

- Feynman diagrams for all channels of process → we sum probability amplitudes <u>not</u> probabilities!

$$|A|^2 = |\Sigma \, A_i|^2$$

*Question which channel is like a question - which slit a photon used*

# Infinities

- Description of the process - all channels should be incuded, with possible virtual particles

- Virtual particles may have various energies, in loops even arbitrary energies and all these cases should be included.

- If contributions are not damped – a problem
  -> infinite probabilities for a given process!

- ... → problems with spin 1 particles, damping smaller for larger spin

*(problem with graviton even worst)*

# QED: infinities and renormalization

- Problems with photon (spin 1 )? – In QED infinite contributions from various diagrams cancel - as was shown in 1948 by Feynman, Tomonaga, Schwinger (Nobel 1964) → renormalization procedure

  *(introduced by Kramers in 1938)*

- QED gives finite predictions - very precise, eg. for anomalous magnetic moment

# Perturbative calculation

- Prediction for a process – we need to sum various contributions  - with many vertices
- In principle ->  series of  infinite number of terms…

  - it is OK if  next terms small

    (small perturbation, perturbation series)

- Eg. in  QED  small coupling constant

  $$\alpha = e^2/(4\,\pi\,) \sim 1/137$$

  so next term in perturbative expansion   $\sim \alpha^2$  -  1%

- More terms in the pertubation series –

  including higher orders in the  expansion in the coupling

  constant (higher powers of the coupling constant)

    –> higher precision of theoretical prediction

# Anomalous magnetic moment for muon

# Anomalous magnetic moment for muon (or g-2|$_\mu$)

The magnetic moment $\mu$ proportional to the spin *s*

*spin and magn. moment*

*- vector quantities*

$$\vec{\mu} = g_\mu \, \frac{e\hbar}{2m_\mu c} \, \vec{s} \; ; \quad g_\mu = 2\,(1 + a_\mu)$$

For the fundamental particle with spin ½ the simplest act of the el-(magnetic) interaction → g = 2 (Dirac 1925) so deviation (or g-2) is called the anomalous magnetic moment
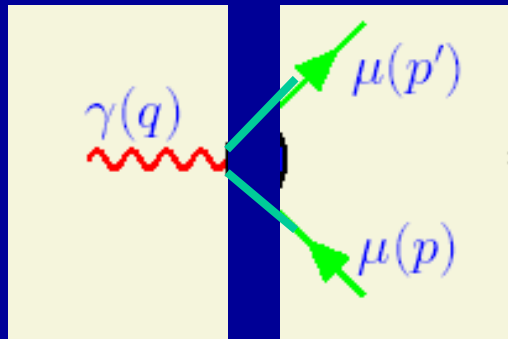
First measurement for electron in 1922! Next in 1948r

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2\,(1.00119 \pm 0.00005)$

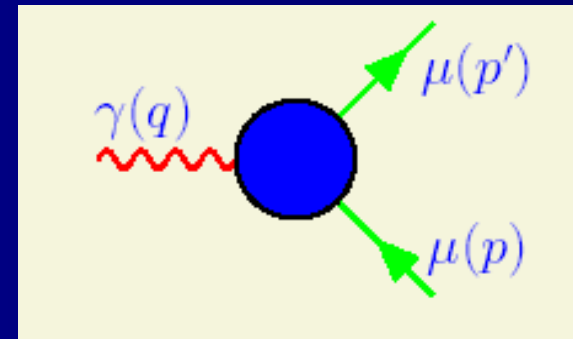# Origin of a$_\mu$
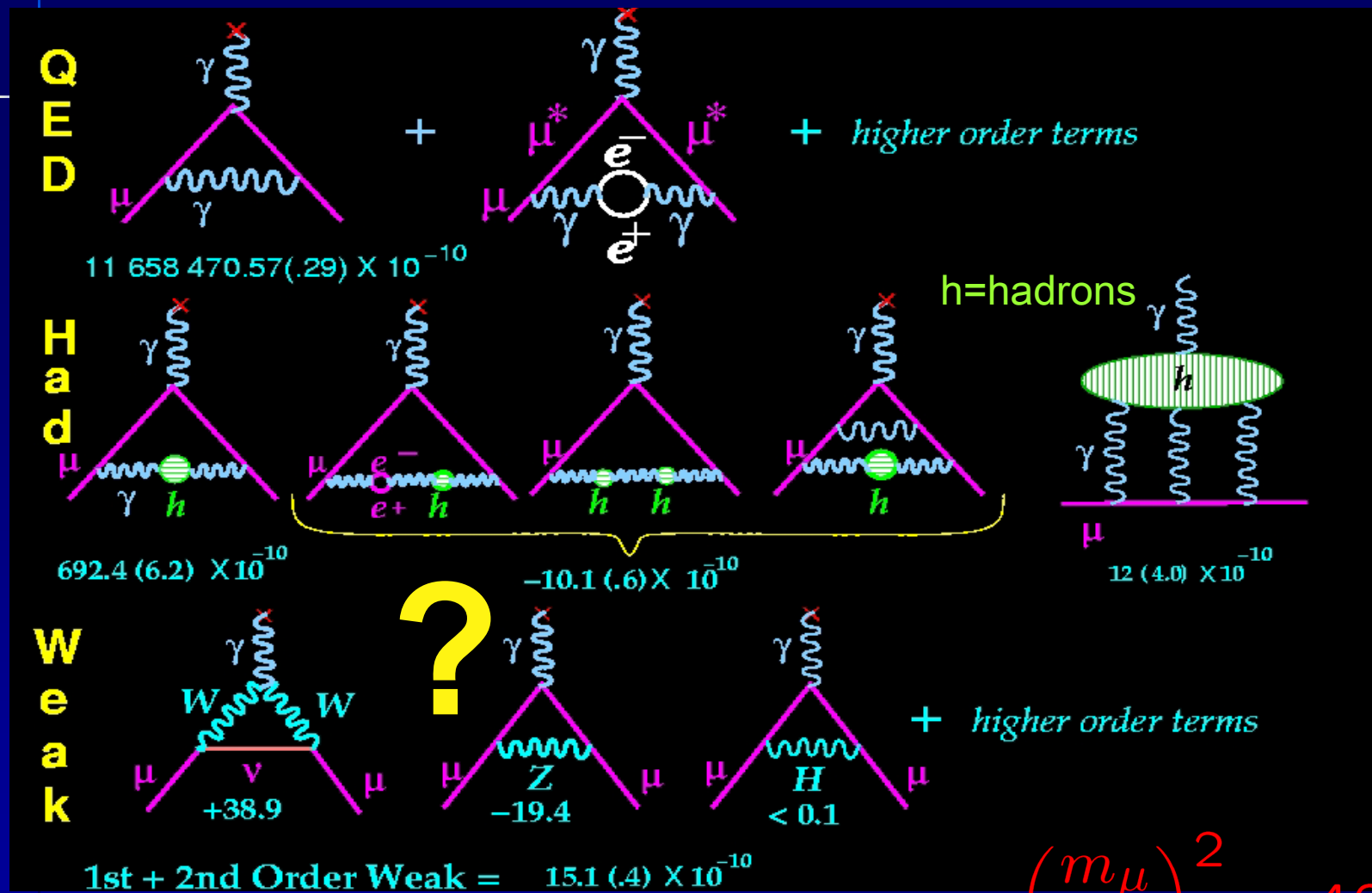
## (for muon)

From extra interactions  …



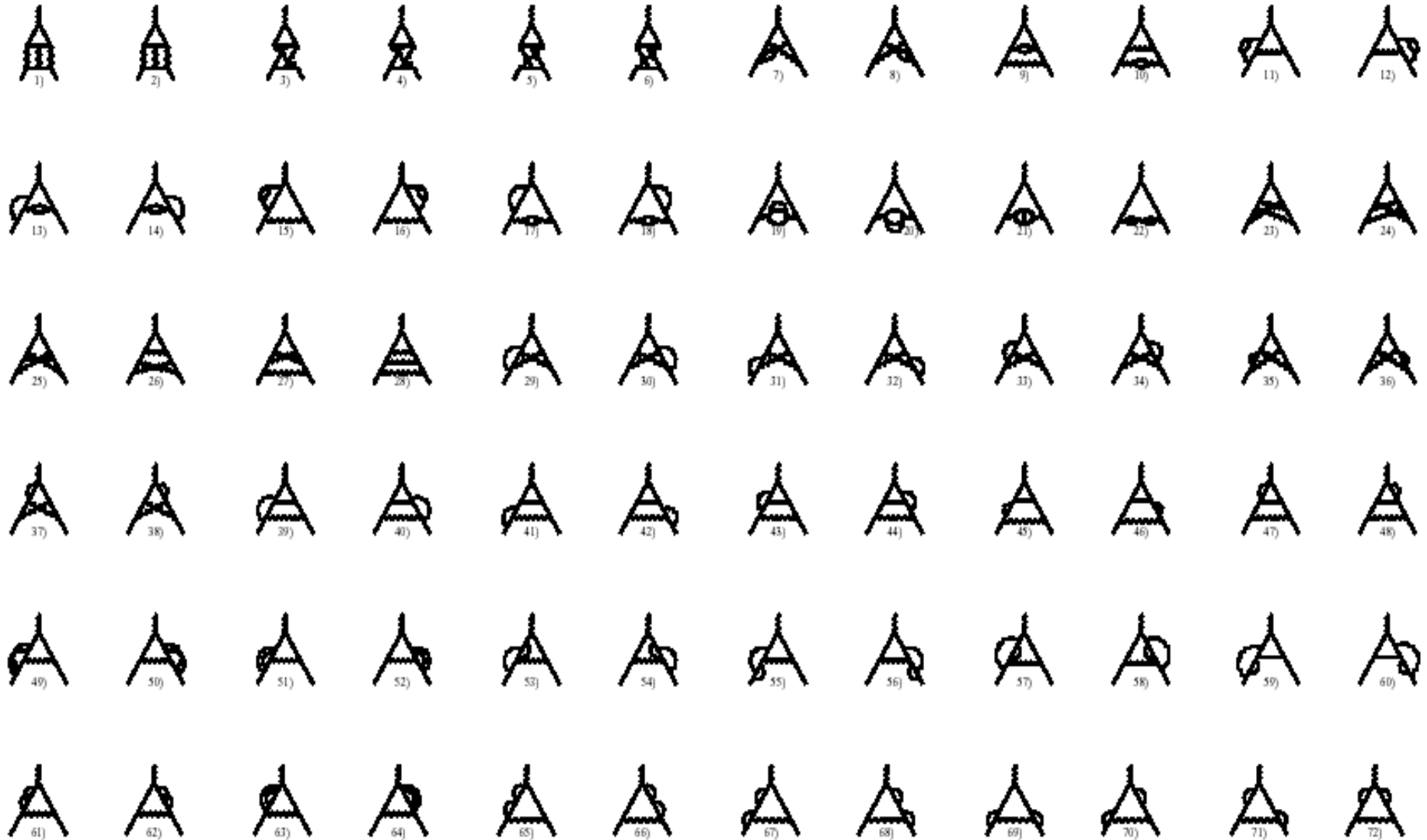g = 2  if only this act                    a = g-2 ≠ 0  if more actions

foton z pędem q, mion z pędem p, mion z pędem p',  p'=p+q
(dokładniej to są 4-pędy: (energia, 3-pęd))

# Standard Model : QED, hadronic (h) and EW (W/Z i H) contributions

$$a_\mu^{\mathrm{QED}} \cdot 10^{10} = \Sigma C_i \left(\frac{\alpha}{\pi}\right)^i = \quad 11614097.3 \ (\text{1-loop})$$

$$+ \qquad 41321.8 \ (\text{2-loop})$$

$$+ \qquad 3014.2 \ (\text{3-loop})$$

$$+ \qquad 38.1 \ (\text{4-loop})$$

$$+ \qquad 0.4 \ (\text{5-loop})$$

Terms up to $\alpha^3$ are known analytically, a recent more accurate numerical calculation of the $\alpha^4$ terms and the leading log $\alpha^5$ terms gave (T. Kinoshita and M. Nio, 2005; A.L. Kataev, 2006):

$$a_\mu^{\mathrm{QED}} = (116584719.4 \pm 1.4) \cdot 10^{-11}.$$

From the latest value of $a_e$ (G. Gabrielse et al., 2006; M. Passera, 2006):

$$\alpha^{-1} = 137.035999710(96), \ a_\mu^{\mathrm{QED}} = (116584718.09 \pm 0.14 \pm 0.08) \cdot 10^{-11}.$$

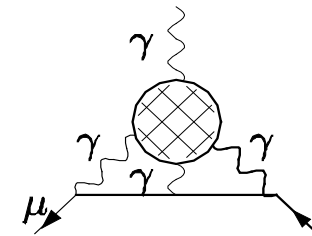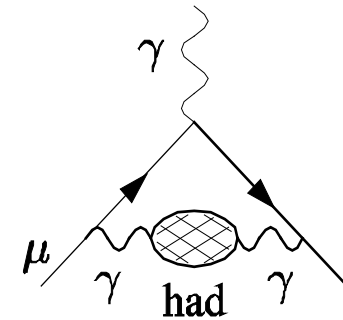The errors are due to: a/ $\mathcal{O}(\alpha^5)$, b/ $\alpha$

**Jegerlehner'07**
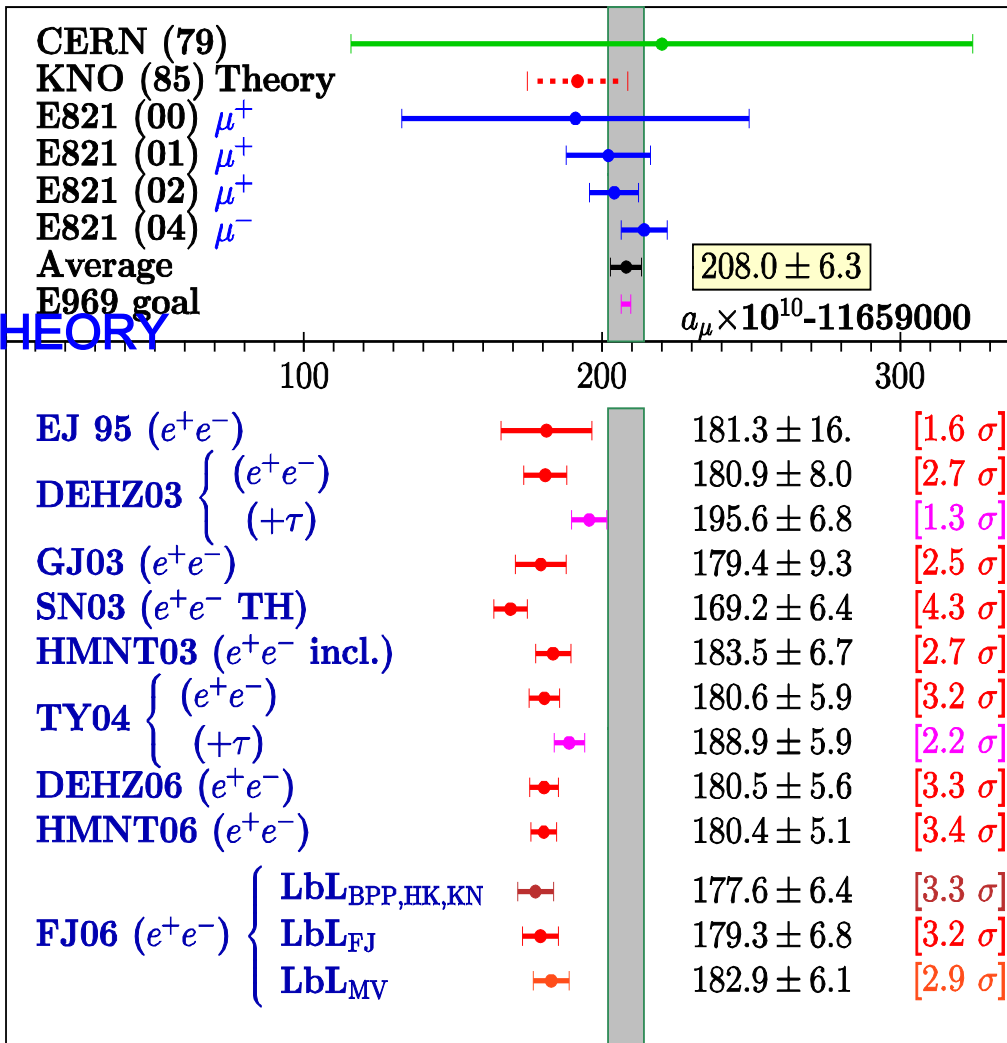
# g-2|$_\mu$ : exp-theory (MS)

**EXP**

**New Physics?**



$\delta\, a_\mu = (287 \pm 91)\, 10^{-11}$

**(3.2 σ)**

**THEORY**

| | | | $a_\mu \times 10^{10} - 11659000$ |
|---|---|---|---|
| CERN (79) | | | |
| KNO (85) Theory | | | |
| E821 (00) $\mu^+$ | | | |
| E821 (01) $\mu^+$ | | | |
| E821 (02) $\mu^+$ | | | |
| E821 (04) $\mu^-$ | | | |
| Average | | | $208.0 \pm 6.3$ |
| E969 goal | | | |

| | | | |
|---|---|---|---|
| EJ 95 ($e^+e^-$) | | $181.3 \pm 16.$ | [1.6 σ] |
| DEHZ03 ($e^+e^-$) | | $180.9 \pm 8.0$ | [2.7 σ] |
| DEHZ03 ($+\tau$) | | $195.6 \pm 6.8$ | [1.3 σ] |
| GJ03 ($e^+e^-$) | | $179.4 \pm 9.3$ | [2.5 σ] |
| SN03 ($e^+e^-$ TH) | | $169.2 \pm 6.4$ | [4.3 σ] |
| HMNT03 ($e^+e^-$ incl.) | | $183.5 \pm 6.7$ | [2.7 σ] |
| TY04 ($e^+e^-$) | | $180.6 \pm 5.9$ | [3.2 σ] |
| TY04 ($+\tau$) | | $188.9 \pm 5.9$ | [2.2 σ] |
| DEHZ06 ($e^+e^-$) | | $180.5 \pm 5.6$ | [3.3 σ] |
| HMNT06 ($e^+e^-$) | | $180.4 \pm 5.1$ | [3.4 σ] |
| FJ06 ($e^+e^-$) LbL$_{BPP,HK,KN}$ | | $177.6 \pm 6.4$ | [3.3 σ] |
| FJ06 ($e^+e^-$) LbL$_{FJ}$ | | $179.3 \pm 6.8$ | [3.2 σ] |
| FJ06 ($e^+e^-$) LbL$_{MV}$ | | $182.9 \pm 6.1$ | [2.9 σ] |

M.Krawczyk, AFZ  Particles and Universe 9

**Jegerlehner'07**

# New calculation in SM (2013)

## Dermisek, Rava – May 2013

The discrepancy between the measured value of the muon anomalous moment [17] and the SM prediction,

**3.4 σ**

$$\Delta a_\mu^{exp} = a_\mu^{exp} - a_\mu^{SM} = 2.7 \pm 0.80 \times 10^{-9}, \qquad (28)$$

Experiment  Brookhaven  2006

[17] G. W. Bennett *et al.* [Muon G-2 Collaboration], "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," Phys. Rev. D **73**, 072003 (2006) [hep-ex/0602035].

The most precise quantity…

Brookhaven,
 USA

In 2014

tranport to
Fermilab
(Chicago)
http://muon-g-2.fnal.gov/bigmove/

The BNL muon storage ring

Pomiar g-2
dla mionu

www.g-2.bnl.gov

Small, very precise experiment at low energies  - *www.g-2.bnl.gov*

# **Renormalization**

# Idea

- QED – infinities only in expressions containing mass and electric charge. Using quantities measured in experiment we can remove infinities as follows:

A – expression for physical quantity; let 1st order correction $\Delta$ contains an infinite contribution $\Delta$,

$$A = e_0(1+ \Delta) + ... = e_0(1+\Delta'+..)(1+\Delta''+..) = e_{fiz}(1+\Delta'' +...),$$

so the prediction for quantity $A$ finite ($\Delta''$ - finite)

$$(+...\text{higher terms})$$

For magnetic moments infinities are at the intermediate stages of calculations – but they are the same for electron and muon.
We can express them by each other and for electric charge use the exp. value.

# Trick very useful

- Electric charge is a *free (initial)* parameter of the QED; the same for the electron mass $m_e$

- Theory does not predict values of these parameters – we can hide infinities in them

- If infinities only in expressions for free parameters of the theory $\rightarrow$ theory is renormalizable and provide definite predictions for physical processes

- It is not satisfactory, but works OK

# Non-renormalizability

- There are theories with infinities not only in free parameters

- For long time it was considered that theories with spin-1 particles, but different than a photon, are non-renormalizable (eg. Fermi theory for weak int.).

- Today we known that theories with spin-1 particles are renormalizable if they are gauge theories, (moreover – non-abelian Yang-Mills theories)

*Gravitation: gauge theory, but not renormalizable*

# Gauge transformation- a reminder

Schrodinger equation, based on $E = ( \bar{p}^2/2m + V )$

- $i \, \partial \, \psi( \bar{x},t) / \partial \, t = - 1/2m \; \bar{\nabla}^2 \, \psi( \bar{x},t)$

  (for potential $V=0$ and using $\hbar = 1$)

- $E \rightarrow i \, \partial_0 \qquad\qquad \partial^\mu = (\partial_0, - \bar{\nabla})$

  $\bar{p} \rightarrow - i \; \bar{\nabla}$

- local transformation, phase $\alpha( \bar{x},t)$

  $\psi'( \bar{x},t) = e^{\, i \, \alpha( \bar{x},t)} \psi( \bar{x},t)$

$|\psi( \bar{x},t)|^2$ probability does not change, but

  invariance of equation only if we add interaction !

# Predictions for larger energies

# Probability of processes for large energies

- Bad, if probability of processes rises with energy (bigger than 100 % !?)

- QED: Compton scattering – at the lowest order of pert. calculation two diagrams, each rising with energy but the sum of diagrams – OK (cancelation)

- Cancelation results from the structure of the theory (gauge symmetry)

# Calculation of probability using Feynman rules

■ To each line and vertices in Feynman diagram a factor is assigned. Here we track only the energy E.

- Incoming or outgoing photon
(and each spin 1 particle) – a factor E
Virtual photon (spin1 particle) – a factor $1/E^2$

- Incoming or outgoing spin ½ particle - a factor $\sqrt{E}$,
virtual spin ½ particle – a factor $1/E$

- Incoming or outgoing spin 0 particle – a factor 1,
virtual spin 0 particle – a factor $1/E^2$

- Additional factors from couplings ….,
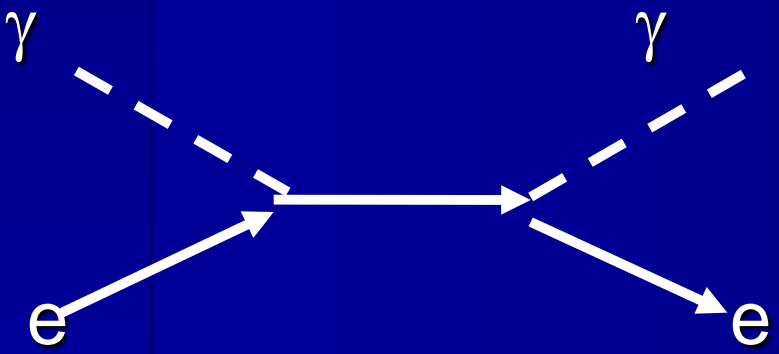
**Product of factors→probability amplitude A**
$$\text{(probability} = |A|^2)$$

# Electroweak interaction

## Decay $d \rightarrow u\ e\ \bar{\nu}_e$

$d \xrightarrow{\quad} W^- \quad e^-$
$u \qquad \bar{\nu}_e$
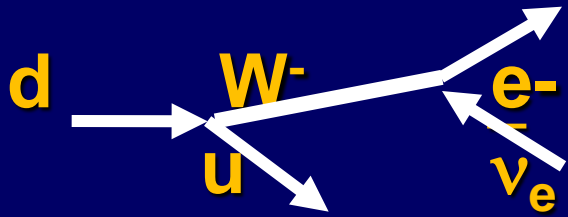
- W boson   is  very massive (80.4 GeV), so in decay of quark d (mass ~MeV) it is very virtual (far off mass-shell)

- Boson W  has spin 1→ problem with  renormalizability

- To understand this problem, we analyse first Comptona scattering $\gamma$ e →  $\gamma$ e (QED), two diagrams:

$\gamma \qquad\qquad\qquad \gamma \qquad\qquad \gamma \qquad\qquad\qquad \gamma$

$e \qquad\qquad\qquad\qquad e \qquad\qquad e \qquad\qquad\qquad\qquad e$
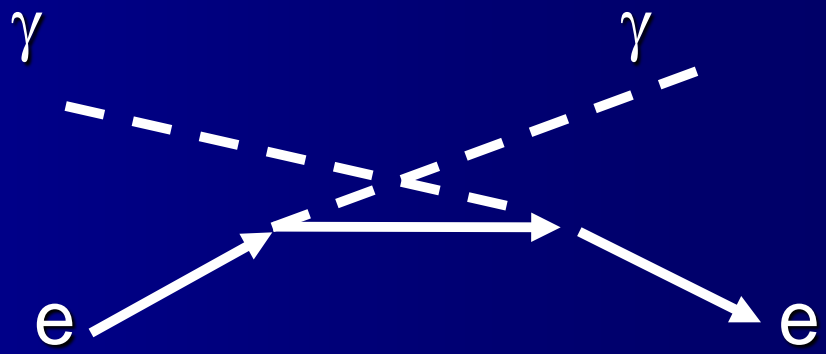
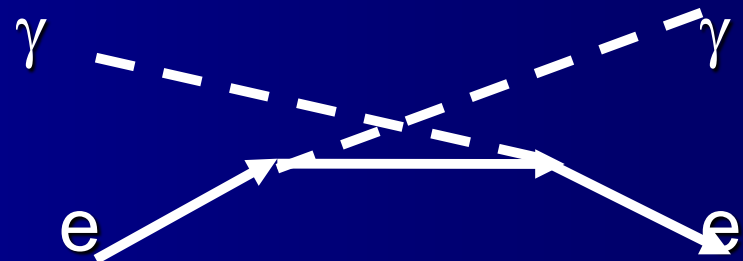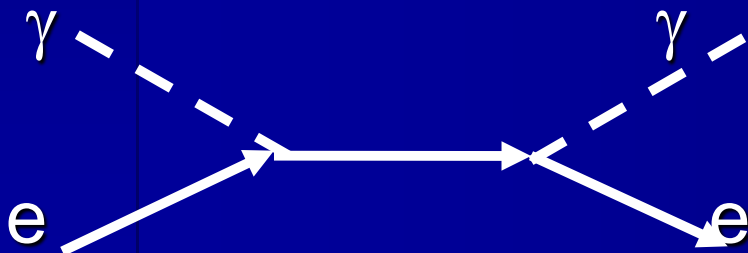# Calculation of the probability amplitude for the Compton process

For this process the amplitude rises with the energy as
$$E^2 (\sqrt{E})^2 \, 1/E = E^2$$

(for probability $E^4$)

Bad behaviour for the individual diagrams, but in sum cancelation and the final amplitude does not grow with energy
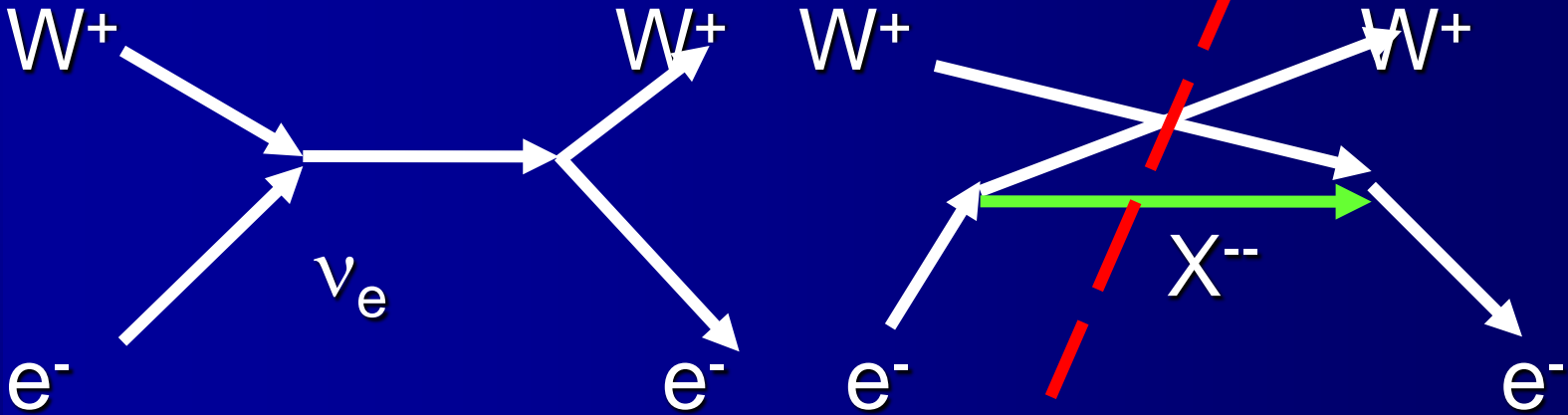
$$\gamma \, e \rightarrow \gamma \, e$$

# Scattering W e → eW⁻⁺

Tu strzałki na liniach W
oznaczają pędy

■ Exchange γ  by W:

W⁺            W⁺    W⁺            W⁺

$\nu_e$

X⁻⁻

e⁻             e⁻    e⁻             e⁻
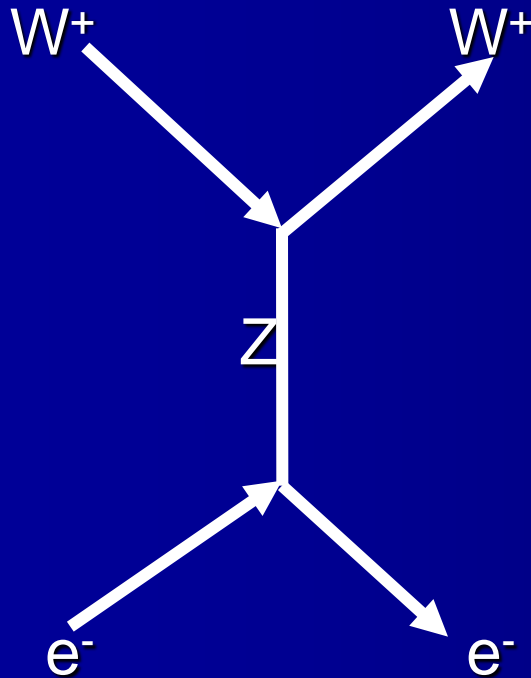
Charge conservation → X⁻⁻ double charged

particle – not observed !

Bad high energy behaviour    $E^2 (\sqrt{E})^2 \, 1/E = E^2$

# Z boson needed!

- Using diagram with Z  boson



Behaviour  $E^2 (\sqrt{E})^2$ (vertex WWZ~E)/$E^2 = E^2$, and a proper sign of coupling  WWZ – cancelation like for  the Compton process!
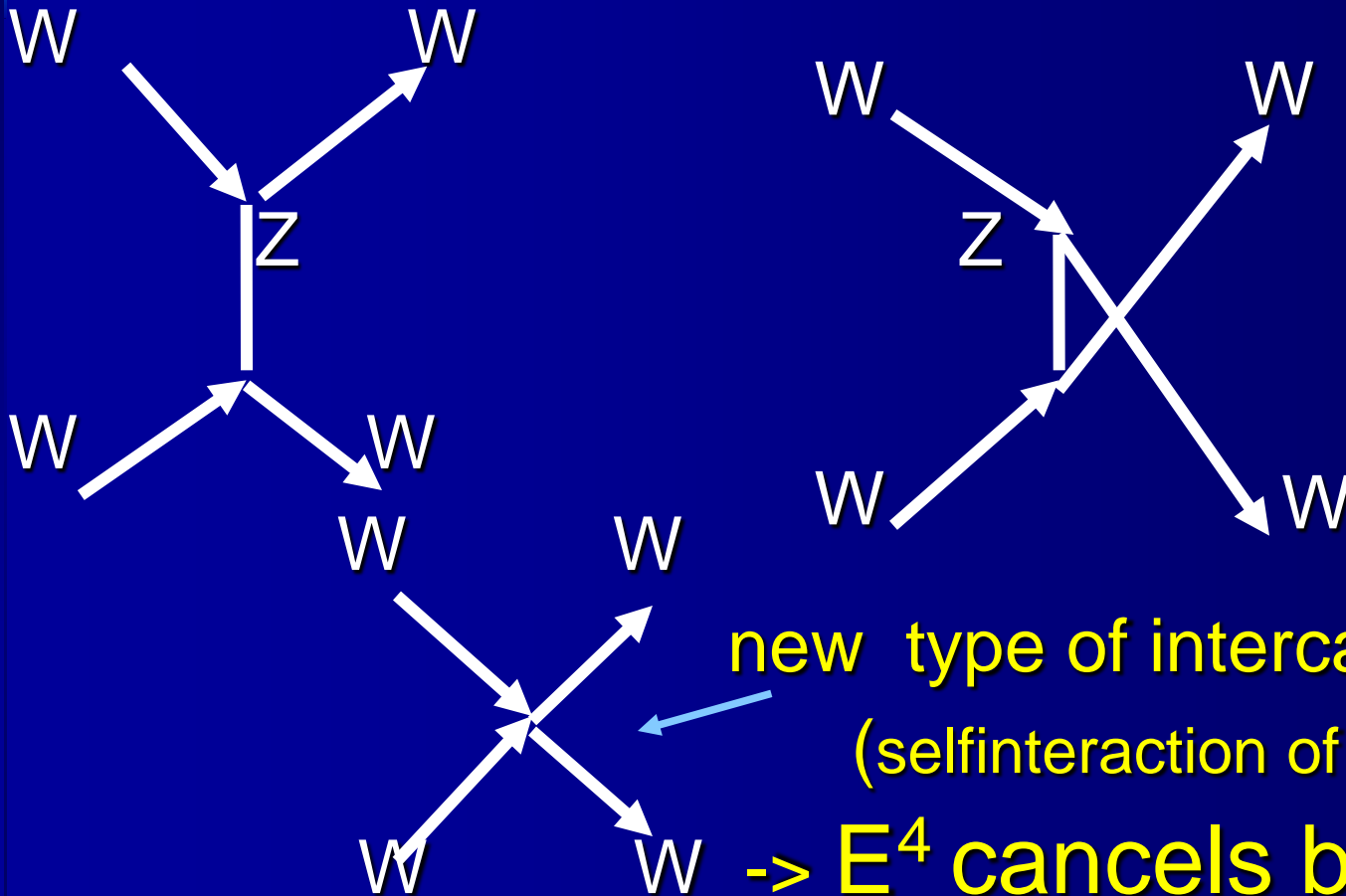
# Price for a good behaviour for E:
## it must exist neutral particle Z with
### a define interaction (coupling) !

It was found  – this is a Z boson !

$\rightarrow$  A success of theory

# Scattering WW -> WW
## ~ E⁴ (since E⁴ E²/E²) - even worst...

$$\sim E^4 \text{ (since } E^4 \, E^2/E^2\text{)}$$

W      W

Z

W      W

W      W

Z

W      W

W      W

new type of intercation

(selfinteraction of W)

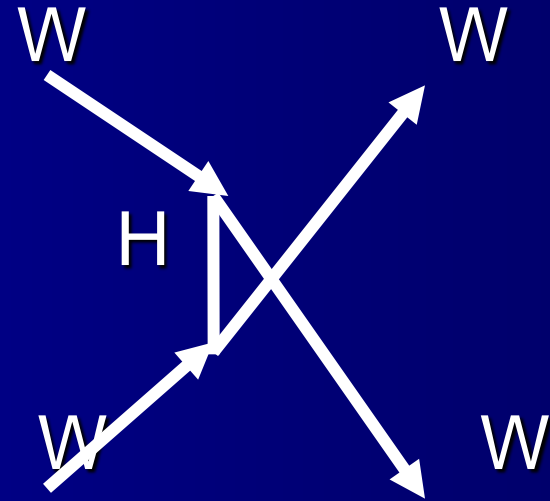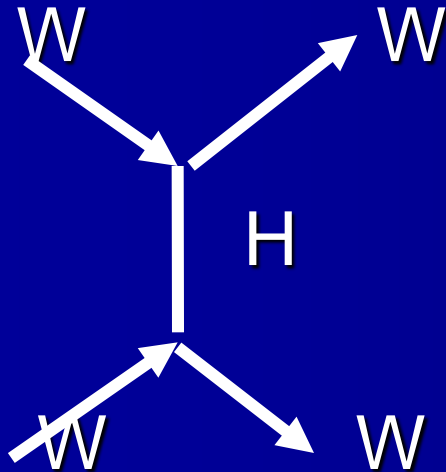W      W    -> $E^4$ cancels but not $E^2$

# E² term, in fact  E² M²

- In the amplitude - wrong term $E^2 M^2$ ,

    where M- mass of W or Z

- New contribution needed to cancel this bad term → the simplest  diagram with exchange of spinie-0  particle, which couples to W/Z proportionally  to mass

## Higgs boson needed !

# Scattering of W on W: H contribution

W       W           W       W

H               H

W       W           W       W

Couplings proportional to mass of particles
to which H couples, and as a result $\rightarrow$
a good  high energy behaviour  WW $\rightarrow$ WW !

# Mass generation and consistent description of processes at high energies

- Good high energy behaviour -  if  H exists and couples to  W/Z  proportionally  to their masses
- Higgs particle is related to the mass generation in SSB

→properties of H particle related to the mass generation and  those needed  for a good  high energy behaviour  WW → WW

- THE SAME !!!

- Expected relation to gravity (mass..)

# The theory ends here  - Veltman

- *„The theory ends here. We need help.*
  *Experiments must clear up this mess."*
  $\rightarrow$ LHC pp, ILC e$^+$e$^-$ (PLC $\gamma$ $\gamma$, e$\gamma$)

**July 2012 :**

**LHC -  Higgs particle with mass 125 GeV**

- Does fact of decaying of a particle depend on the reference frame?

- Do *a free particle* and *a particle on the mass shell* mean the same?

- Why a free positron can not emit a photon and remain on the mass shell?

- What does it mean – *a virtual particle*?

- Does QED have infinite expressions at intermediate stages of calculation?

- Compare the one- and two –loop QED corrections for the anomalous magnetic moments for muon?

- What is a precision we know (in 2013) the anomalous magnetic moments for muon (experiment minus theory (SM))?

- When the theory is renormalizable?

- Proper high energy behaviour of the probability of EW processes demads existence of spin 0 particle. How such a particle couples to gauge bozon Z?

- Is the Higgs boson discovered?