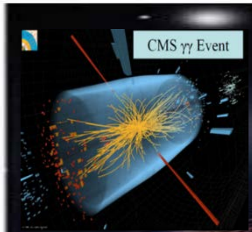

# Particles and Universe

# Lecture 11 Mixing

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Faculty of Physics UW

I. Mixing of quarks and not only…

II. Neutrinos

# Mixing

Mixing is natural in quantum mechanics –

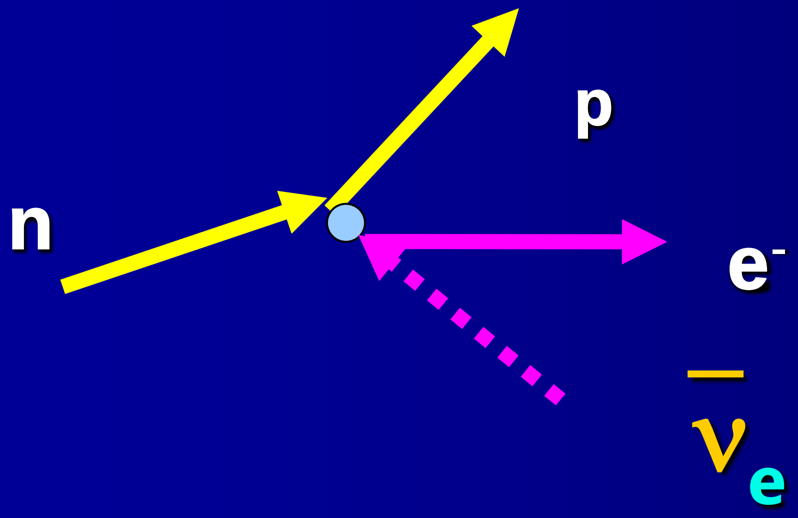
it is due to wave nature of particles

- We have discussed interference of various channels for processes
- Mixing - here one state mixes with another state

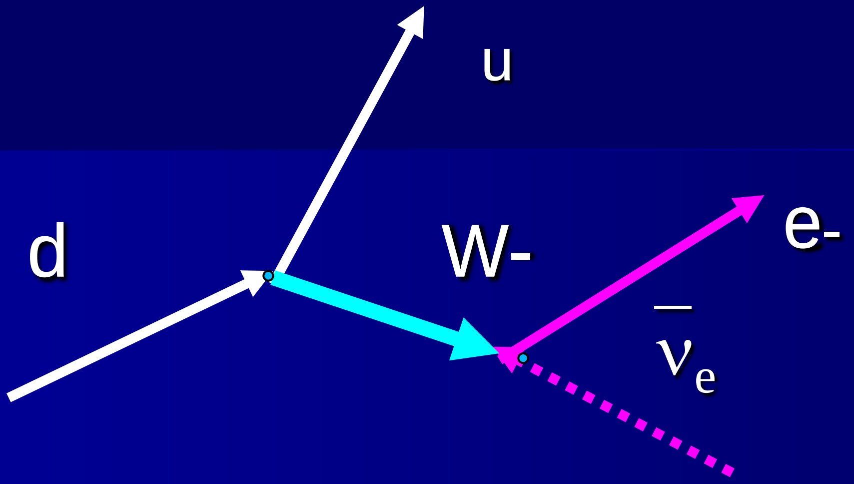
# Mixing for weak interaction

Pair of (p,n) and of (electron neutrino, electron) - *doublets* in the neutron decay

β decay



# Fundamental decay

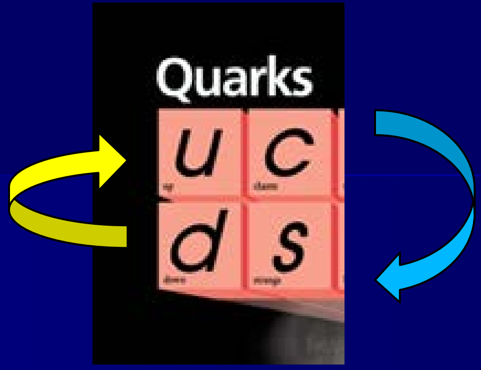


elementary acts of interaction :

$d \to u\, W^-$ and $W^- \to e^-\, \bar{\nu}_e$

g (g -„weak charge”) $\alpha_W = g^2/4\pi = 1/32$

# Mixing in two light families



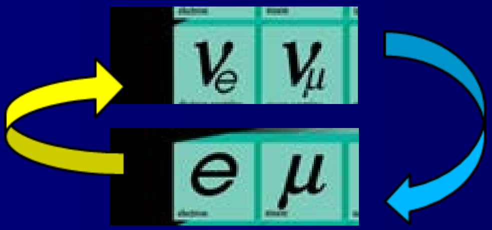
| | | | | |
|---|---|---|---|---|
| Quarks | el. charge | 2/3 | u | c |
| | | -1/3 | d | s |
| Leptons | | 0 | $\nu_e$ | $\nu_\mu$ |
| | | -1 | e | $\mu$ |

Formaly transition between quarks IN a given doublet

$$u \leftrightarrow d,\ c \leftrightarrow s$$

due to exchange of gauge boson W-, eg.

$$d \rightarrow u W^-,\ c \rightarrow s W^+$$



Transition in the leptonic doublet

$$e- \rightarrow \nu_e W^-,\ \nu_e \rightarrow e- W^+$$

# Probability for transition

EW theory:

Absolute value of weak coupling = g the same for all vertices (eg. u → d W+, $W^-$ → e- $\nu_e$), but the coupling itself can have positive and negative sign (as in the el-m interaction - two signs of el. charge )

**Let L- an absolute value of the transition amplitude**

**L = | transition amplitude |**

**for quarks or leptons in the corresponding doublet**

**Since L is proportional to g,**

**a probability for transition process $L^2 \sim g^2$**

# Transition for quarks and for leptons (exchange of W+/-)

- Theory SU(2) → transition probability($\mathcal{P}$) = $L^2$
- Experiment (1963)→ difference between quarks and leptons..

$$\mathcal{P}[u \to d\, W^+] + \mathcal{P}[u \to s\, W^+] = \mathcal{P}[\nu_e \to e\, W^+]$$

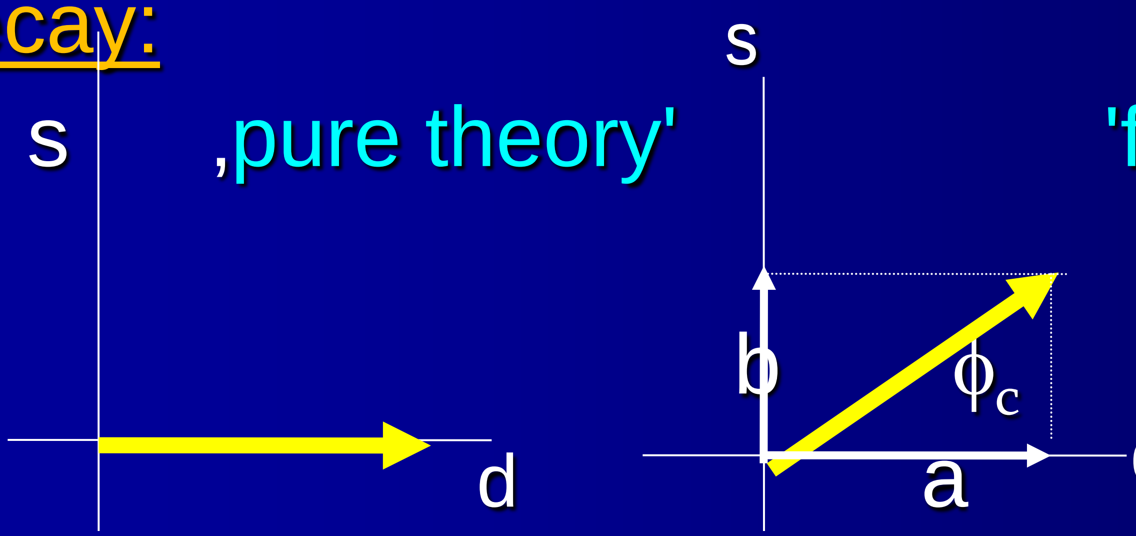
Mixing of s quark to the emission of $W^+$ by u quark !

# Mixing of s with d → Cabibbo angle $\phi_c$

So, u quark couples both to d and s

Definition: $L^2 = \mathcal{P}[\nu_e \rightarrow e^- W^+]$, vector L

u decay:



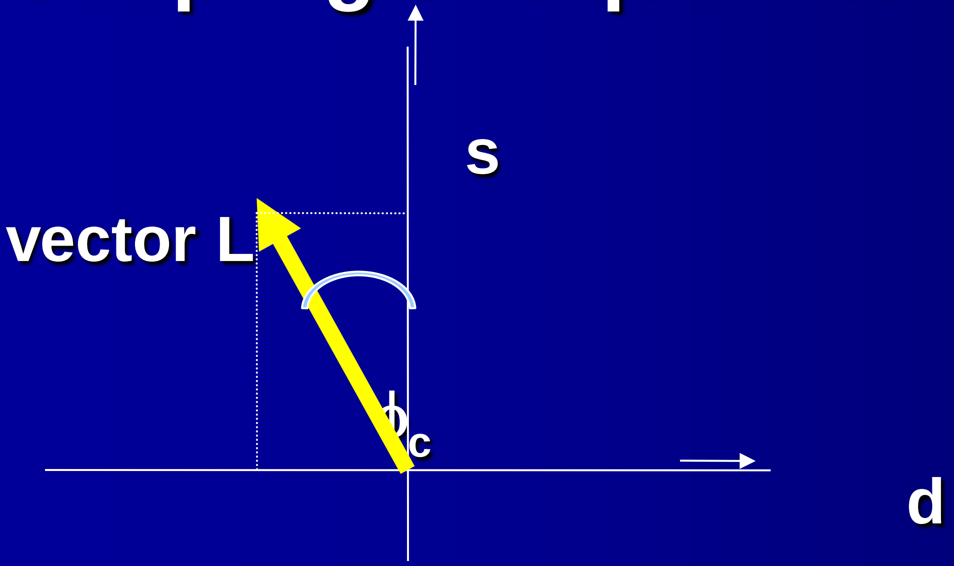
$a^2+b^2=L$

-> couplings usW ~b=g sin ($\phi_c$) and udW ~a=g cos ($\phi_c$)

Exp: $\phi_c=12.7^o$ *(Nicola Cabibbo 1963)*

# Decay of c quark to s <u>and</u> d

## Coupling of c quark to s and d (facts)



So, couplings cdW ~ - g sin ($\phi_c$) and csW ~ g cos ($\phi_c$)

minus!

# Cabibbo angle – mixing between the 1 & 2 family of quarks

- Cabibbo angle needed to describe data – Theory ? – no prediction

- In fact a mixing among 3 families.. 3 x 3 matrix (unitary)

  → Cabibbo–Kobayashi–Maskawa matrix (CKM)

  4 parameters:

  3 angles (including Cabibbo angle) and phase

Kobayashi and Maskawa postulate such matrix in 1973 r before discovery of the 3d family.

Data needed a phase.. (→ CP violation)

# Matrices for mixing of quarks

- 2x2 matrix (rotation of vektor L)

$$\begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} \cos\phi_c & \sin\phi_c \\ -\sin\phi_c & \cos\phi_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 x 3 matrix

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} \rightarrow \begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Similar mixing for up quarks: u, c, t

# Kobayashi & Maskawa

**Nobel' 2008**

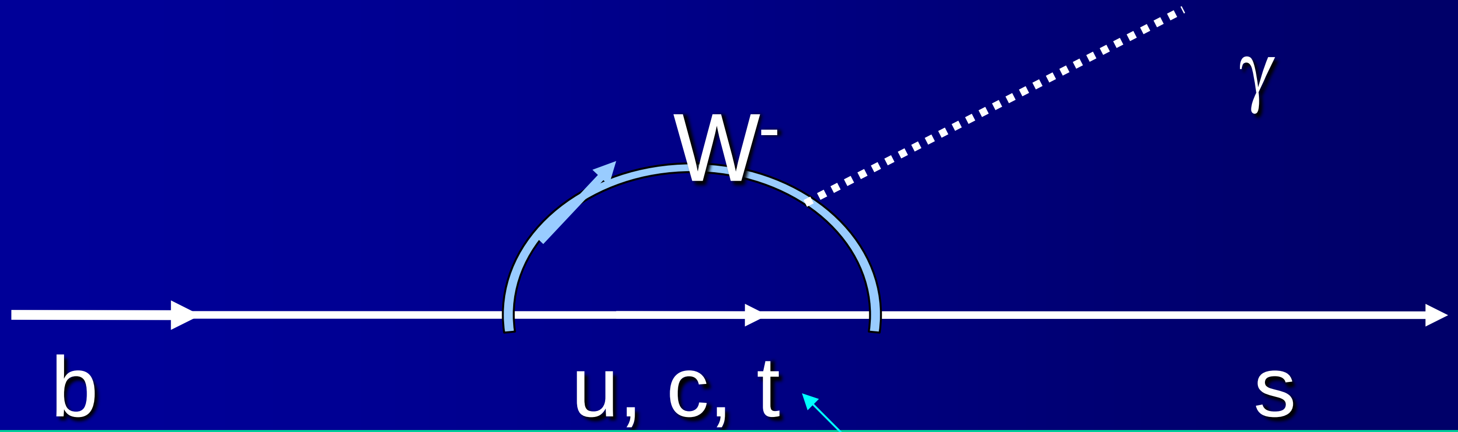
.

Why Cabibbo did not get Nobel prize ?!

The prize was not for a mixing but for observation that starting with 3 family of quarks there appear a phase in the mixing matrix needed for violation of CP in kaon decays

# Example: decay b → s γ

Very precise measurement and SM prediction
(Br ~ $10^{-4}$)



mixing u, c, t

Without mixing there in no such process!

Besides b $W^-$ t we have b $W^-$u and bW$^-$c (and sW$^-$c,sW$^-$u, sW$^-$t)

# $V_{CKM}$

K.A. Olive *et al.* (PDG), Chin. Phys. **C38**, 090001 (2014

- Matrix (with one phase)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\bar{\eta} = 0.354 \pm 0.015$$

- For the magnitudes

$$V_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

# No mixing for transition via Z bozon

Transition without changing of el. charge

$d \rightarrow d Z$

Mixing ? $d \rightarrow s Z$? NO!

*(Flavour changing neutral current FCNC)*

Why? No answer

Mixing of leptons? NO if neutrinos massless

# A comment – mixing of gauge bosons?

Yes,

Z boson and photon are combinations of the initial gauge bosons of two groups: SU(2) and U(1) →

Weinberg angle to describe this mixing

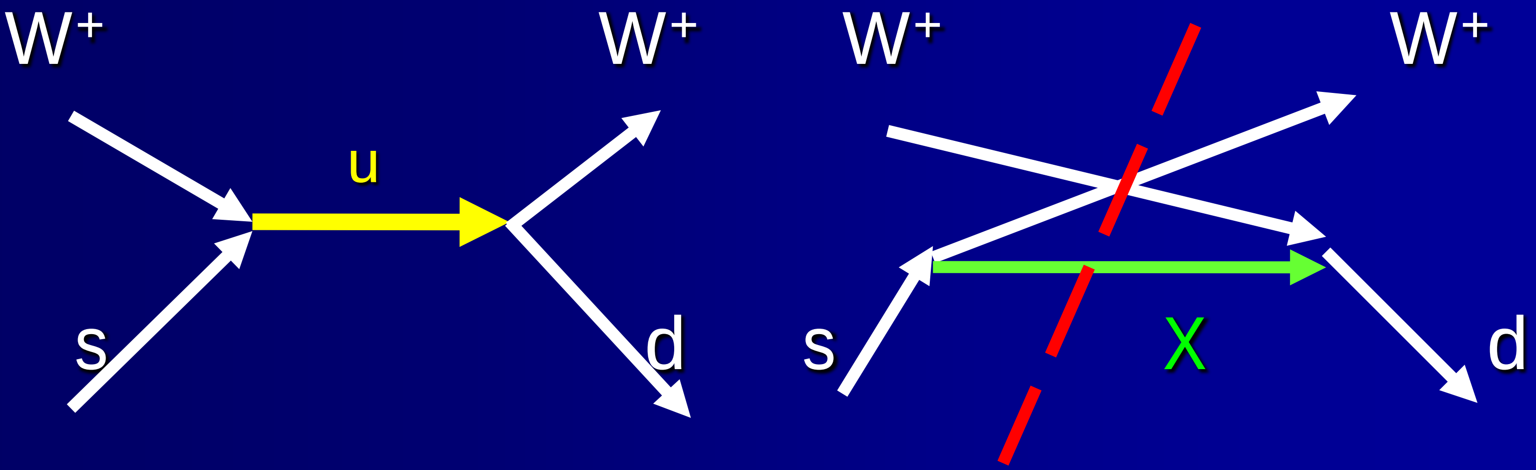
„Mixture called Z" couples to neutrins,
while „mixture called photon" does not
(destruction of the corresponding prob. amplitude !)

Cabibbo angle: u → d W+ versus $\nu_e$ → e W+
Weinberg angle: u → d W+ versus u → u Z

# Quark mixing and high energy behaviour

# Scattering of boson W on quark- behaviour at large energies

Replacing $e^-$ by s quark

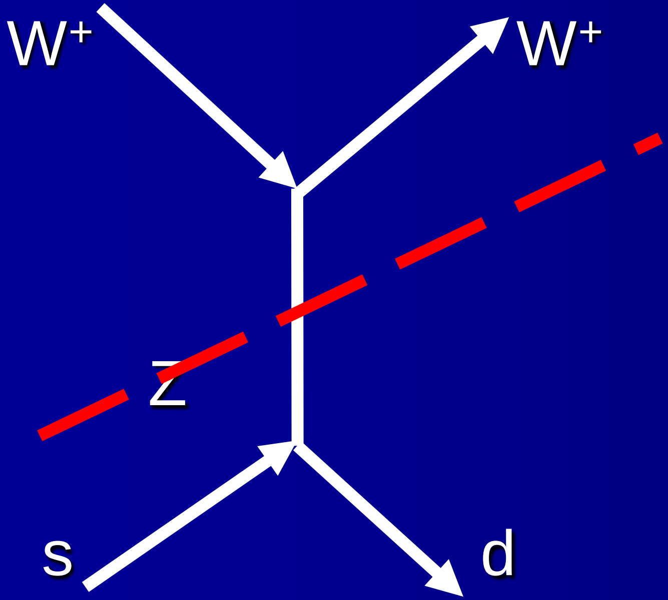


El. charge conservation: particle X with el. charge - 4/3 does not exist !

So, bad behaviour (amplitude): $E^2 (\sqrt{E})^2 1/E = E^2$

# Diagram with Z boson – no help here!

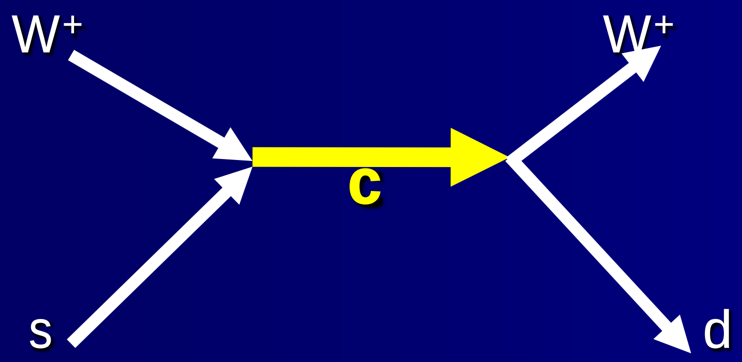
**Diagram with Z boson Z?**



This process does not exist –

„absence of FCNC”!

# New diagram with c quark (possible only if mixing of s and d quarks)



Couplings:

| | |
|---|---|
| cs W | $\cos(\phi_c)$ |
| cd W | $-\sin(\phi_c)$ |

due to minus sign a cancelation of bad high energy behaviour for a process

W+ s →u → W+ d

# Comment on discovery of c quark

- So to the process $W^+ s \rightarrow u \rightarrow W^+ d$

we add process with c - quark

$W^+ s \rightarrow c \rightarrow W^+ d$

(these are various channels of the process $W^+s \rightarrow W^+d$)

- In fact in 1964 this was only a hypothesis about existence of c quark with fixed properties, (including Cabibbo angle) – so that terms $\sim E^2$ cancel.
- c quark discovered in 1974 has these properties

Sucess of theory!!!

# Quark mixing

- Exists
- Important
- Described – but not understand

→

- Flavour problem !

# Questions to lecture 11

- Does Cabibbo angle describe mixing between d and s quarks or between u and d quarks ?
- Value of Cabibbo angle is equal to ?
- Write a coupling of the c quark to d quark assuming mixing only between two lighter families. What is its sign?

- What discribes the Cabibbo-Kobayashi-Maskawa matrix?
- Do we observe a mixing between b and d quarks?
- How many parameters has the Cabibbo-Kobayashi-Maskawa matrix?
- Did N. Cabibbo got Noble prize in 2008r?
- Is a transition of the u quark to c quark: $u \to c\, Z$ possible ?
- How one can determine the Cabibbo angle ?
- Does the Weinberg angle describe quark mixing?
- Do we have FCNC?
- Is the probability of transition $c \to s\, W+$ equal to probability of transition $\nu_e \text{–>} e^- W^+$?
- Does photon couple to neutrinos?

# Neutrinos

Introduced by Pauli to preserve energy conservation in $\beta$ decays

eg.: $${}^{60}_{28}Co \rightarrow {}^{60}_{29}Ni^{\star} + e^{-} + \bar{\nu}_e$$

on particle level:

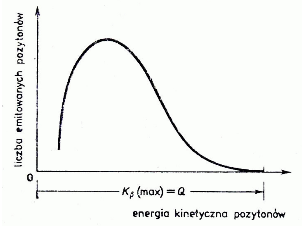
$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Required properties:

- very weak interactions
- negligible mass

We assumed that neutrinos are massless...

Continuous $\beta$ energy spectra:



Direct mass constraints (95% CL):

$$m_{\nu_e} < 2.2\ eV \approx 4.3 \cdot 10^{-6}\ m_e$$
$$m_{\nu_\mu} < 170\ keV \approx 0.0018\ m_\mu$$
$$m_{\nu_\tau} < 15.5\ MeV \approx 0.01\ m_\tau$$

Much stronger mass constraints from astrophysics and cosmology...

# Neutrinos

## Standard Model

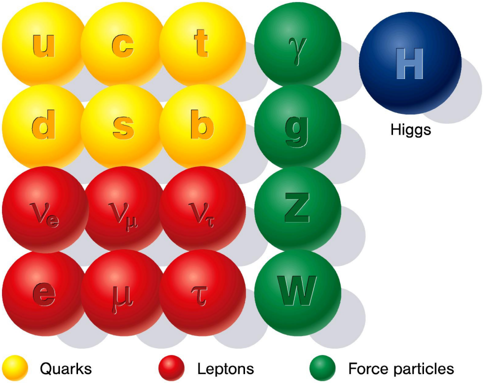
Laboratory measurements have shown that each charged lepton couples to distinct neutrino

$\Rightarrow$ three neutrinos flavours

Until recently we believed that

- all neutrinos are massless
- lepton flavour is conserved

But conservation of the lepton number is not imposed by theory - it was assumed based on experimental evidence...



# Neutrinos

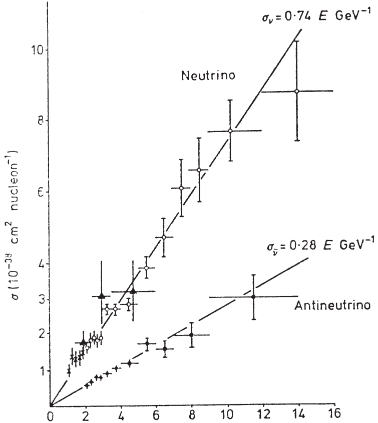
## Interactions

Cross section for neutrino interaction in matter is incredibly small.
For energy of the order of 1 MeV

$$\sigma_{\nu N} \sim 10^{-43}\ cm^2$$

$\Rightarrow$ average free path in matter (!) of the order of light years !!!
For comparison $\sigma_{NN} \sim 10^{-24}\ cm^2$

Cross section grows with energy, but only linearly...



We need very strong sources and very large detectors to study neutrinos...

# Neutrino sources

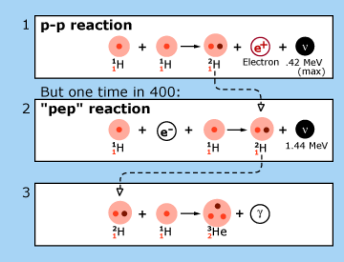
## Sun

Sun is an extremely intense neutrino source.
Most neutrinos come from **p–p** reaction:

$$p + p \quad \rightarrow \quad D + e^{+} + \nu_e \; (E_\nu \leq 0.42 \; MeV)$$

Slightly higher energies in **“pep”** process:

$$p + e^{-} + p \quad \rightarrow \quad D + \nu_e \; (E_\nu \approx 1.44 \; MeV)$$



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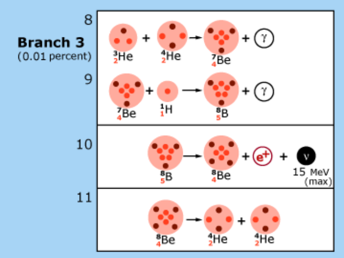
Highest energies result from $^{8}B$ decay

$$^{8}_{5}B \quad \rightarrow \quad ^{8}_{4}Be \ + \ e^{+} \ + \ \nu_e$$

where neutrino energy can reach 15 MeV

Only these neutrinos can be observed in particle detectors...



# Neutrino sources

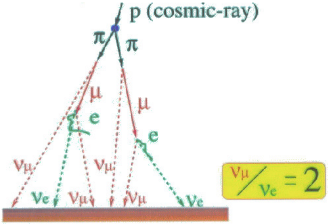
## Cosmic rays

Primary cosmic rays consist mainly of high energy protons and light nuclei, with energies reaching $10^{12}$ GeV ($10^{21}$ eV).

They interact with O and N nuclei in the atmosphere creating a cascade of secondary particles, most of them are pions $\pi^{\pm}$.

Neutrinos result from decay chain:

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ \mu^+ &\rightarrow e^+ + \bar{\nu}_\mu + \nu_e \end{aligned}$$



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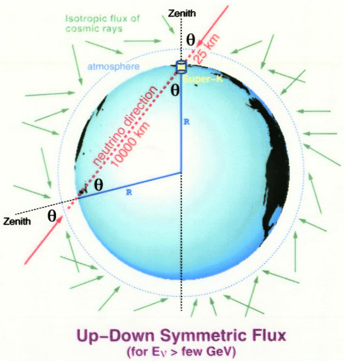

Up–Down Symmetric Flux
(for $E_\nu$ > few GeV)

As the primary cosmic radiation is isotropic, we expect that atmospheric neutrino flux will also be isotropic !

# Neutrino sources

## Nuclear reactors

Fission of single $^{238}U$ nucleus produces energy of about 200 MeV
On average, six electron anti-neutrinos are produced (per fission),
mainly in neutron decay: $n \to p + e^- + \bar{\nu}_e$

Reactor with 1GW power $\Rightarrow \sim 10^{20}$ fissions per second $\Rightarrow \sim 6 \cdot 10^{20}\ \nu/s$

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## Accelerators

We can produce high energy, high current proton beams.
When interacting with dense target, protons produce secondary pions and kaons (as in the cosmic ray interactions). Muon neutrinos are produced in:

$$\begin{aligned} \pi^+ &\rightarrow \mu^+ + \nu_\mu \\ K^+ &\rightarrow \mu^+ + \nu_\mu \end{aligned}$$

By focusing produced pions and kaons prior to their decay, we can obtain high energy neutrino (or anti-neutrino) beams...

# Neutrino sources

## Comparison

| source | flavour | energies | flux |
|---|---|---|---|
| Sun | $\nu_e$ | $\leq$ 15 MeV | $6 \cdot 10^{10}\ \frac{1}{s\ cm^2}$<br>(on Earth) |
| Cosmic rays | $\nu_e$, $\nu_\mu$, $\bar{\nu}_e$, $\bar{\nu}_\mu$ | 100 MeV - TeV | $3 \cdot 10^{7}\ \frac{1}{s\ cm^2}$<br>(above 10 MeV) |
| Reactors | $\bar{\nu}_e$ | MeV range | $5 \cdot 10^{9}\ \frac{1}{s\ cm^2}$<br>(1 GW from 1 km) |
| Beams | $\nu_\mu$, $\bar{\nu}_\mu$ | GeV range | $10^{20}$<br>(total sample) |

All these sources are used nowadays to study neutrino properties

# Super Kamiokande

## Detector

Neutrino detector built in an old mine, under Mt Kamioka in Japan.

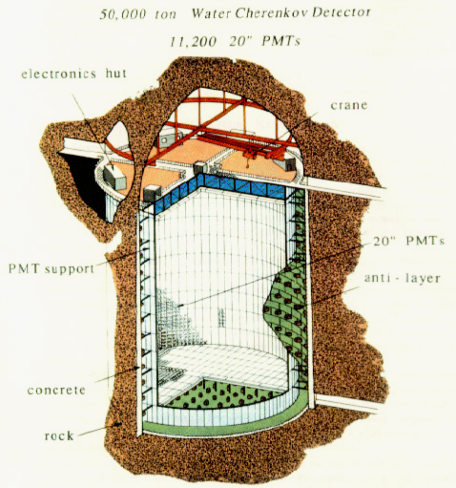
Steel tank with 50,000 tons of ultra-pure water.

Look for neutrino interactions:

$$\nu_l \, n \quad \rightarrow \quad l^- p$$

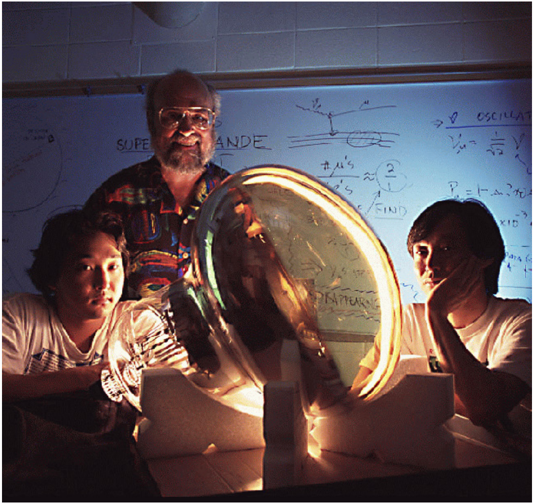
$\sim$11'000 photomultipliers installed to measure Cherenkov radiation

Radiation of charged particles traveling with velocity greater than speed of light (in water).

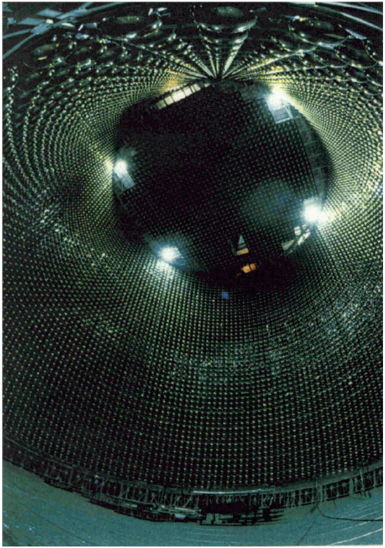


# Super Kamiokande

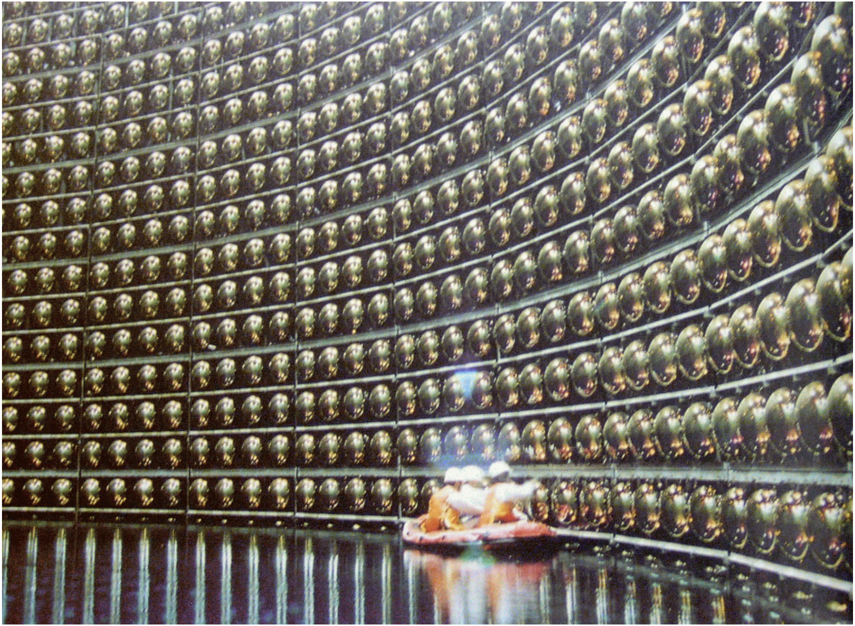
Photomultiplier



Detector



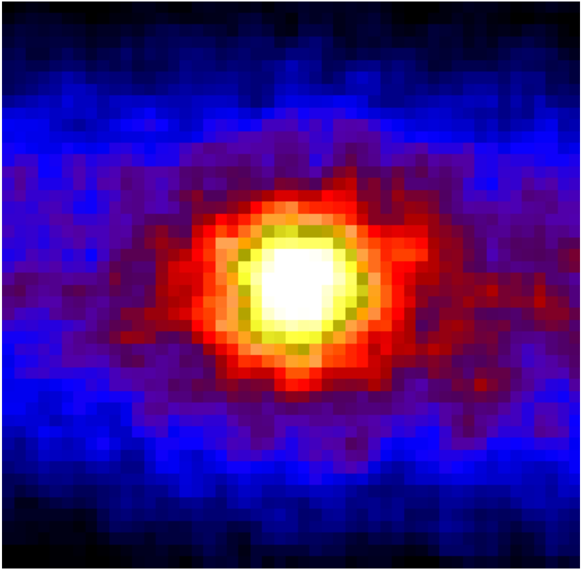
# Super Kamiokande



# Super Kamiokande

Image of Sun reconstructed by Super Kamiokande from the measured neutrinos

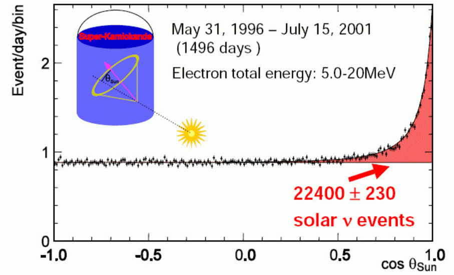
actual Sun size $\sim \frac{1}{2}$ pixel



# Super Kamiokande

## Solar neutrinos observation

Solar neutrinos clearly identified by their direction (w.r.t. Sun position)



Flux measured by SK:

$$\Phi_S^{(B)} = 2.4 \pm 0.1 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

Expected from Solar Model:

$$\Phi_S^{(B)} = 5.3 \pm 0.6 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

Two processes used in neutrino detection: at energies $\sim 10$ MeV

- Charged Current (CC) scattering: $\nu_e + N/e^- \rightarrow e^- + X \Rightarrow$ only $\nu_e$
- Neutral Current (NC) scattering: $\nu_l + N/e^- \rightarrow \nu_l + X \Rightarrow$ all $\nu$'s

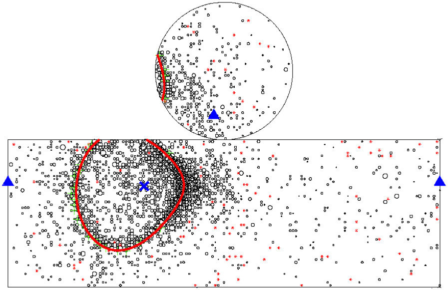
But NC cross section much smaller $\Rightarrow$ Flux measurement restricted to $\nu_e$

# Super Kamiokande

Thanks to very sensitive readout and low background, discrimination between different neutrino flavours possible for high energy neutrinos

**Electron neutrino**

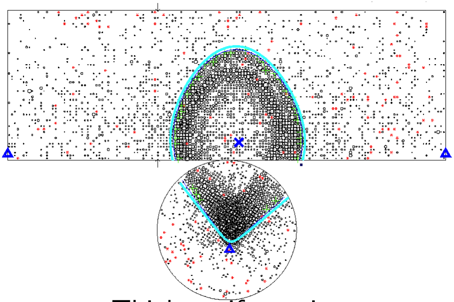
$\nu_e \; n \rightarrow e^- p$ event



Thin, irregular ring

**Muon neutrino**

$\nu_\mu \; n \rightarrow \mu^- p$ event



Thick, uniform ring

Electron is much lighter than muon and scatters/interacts much more

# Super Kamiokande

## Atmospheric neutrino observations

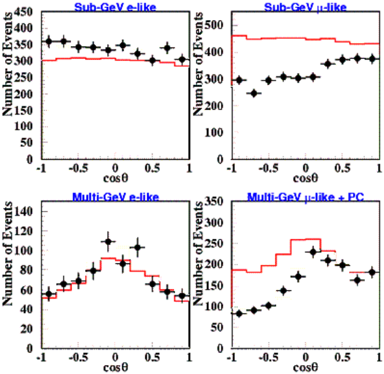
Measured neutrino direction for electron neutrinos (left) and muon neutrinos (right)

Similar number of electron neutrinos going down ($\cos\theta > 0$) and going up ($\cos\theta < 0$) $\Rightarrow$ in agreement with predictions (red)

Clear deficit of muon neutrinos!

Direction measured best for high energy $\Rightarrow$ deficit of up-going $\nu_\mu$

Can muon neutrino “disappear” in Earth ?!



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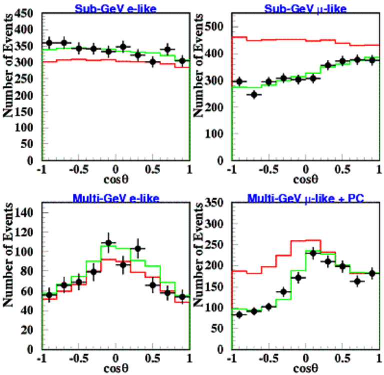

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Direction measured best for high energy
$\Rightarrow$ deficit of up-going $\nu_\mu$

Can muon neutrino "disappear" in Earth ?!

No! We can only explain it assuming neutrinos oscillate!
Predictions of oscillation model indicated by green histogram

# Neutrino oscillations

**Neutrino mixing** for two neutrino flavours

We have to distinguish between two different neutrino state definitions

- flavour eigenstates - neutrinos with defined lepton flavour, as produced in weak interactions: $\nu_e$, $\nu_\mu$
- mass eigenstates - free neutrinos with well defined mass, as propagating in vacuum: $\nu_1$, $\nu_2$

Flavour eingenstates can be described as a mixture of mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Assuming $\nu_e$ is produced at $t = 0$, the neutrino state can be written as:

$$|\nu\rangle_0 = \cos\theta_{12}\, |\nu_1\rangle_0 + \sin\theta_{12}\, |\nu_2\rangle_0 = |\nu_e\rangle$$

If neutrino were massless, $|\nu_1\rangle$ and $|\nu_2\rangle$ would travel with the same velocity, and neutrino state would not change: $|\nu\rangle_t \equiv |\nu_e\rangle$

# Neutrino oscillations

**Neutrino mixing** for two neutrino flavours

Free states $|\nu_i\rangle$ evolve with time:

$$|\nu_i\rangle(t,\vec{x}) = |\nu_i\rangle_0 \cdot \exp\left(-\frac{i}{\hbar}(Et - \vec{p}\,\vec{x})\right)$$

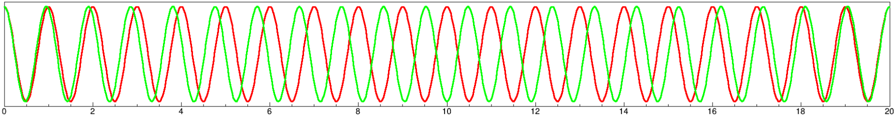
$\Rightarrow$ **If** $m_1 \neq m_2$ then the relative phase of $|\nu_1\rangle$ and $|\nu_2\rangle$ states changes and

$$|\nu\rangle(t,\vec{x}) \quad = \quad a(t)\,|\nu_1\rangle + b(t)\,|\nu_2\rangle \;\neq\; |\nu_e\rangle(t,\vec{x})$$

But we can decompose it into flavour eigenstates

$$|\nu\rangle(t,\vec{x}) \quad = \quad A(t)\,|\nu_e\rangle + B(t)\,|\nu_\mu\rangle$$

Flavour contributions change in time $\Rightarrow$ oscillations



# Neutrino oscillations

**Survival probability** for two neutrino flavours

Probability that $\nu_e$ produced at $t_0 = 0$ interacts as $\nu_e$ after time $t$

$$P_{\nu_e \to \nu_e}(t) = |A(t)|^2$$

Assuming mass differences are small, $|m_1 - m_2| \ll m_1 \sim m_2 \ll E_\nu$

$$P_{\nu_e \to \nu_e}(t) = 1 - \sin^2(2\theta_{12}) \sin^2\left(\frac{(m_2^2 - m_1^2)\, t}{4E} \cdot \frac{c^4}{\hbar}\right)$$

# Neutrino oscillations

**Survival probability** for two neutrino flavours

Probability that $\nu_e$ produced at $t_0 = 0$ interacts as $\nu_e$ after time $t$

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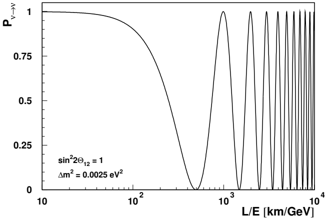
Assuming mass differences are small, $|m_1 - m_2| \ll m_1 \sim m_2 \ll E_\nu$

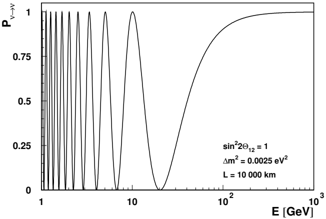
$$P_{\nu_e \to \nu_e}(t) \;=\; 1 - \sin^2(2\theta_{12})\,\sin^2\left(1.27 \cdot \Delta m^2\,\frac{L}{E}\right)$$

$L = ct$ [km], $\Delta m^2$[eV$^2$] i $E$ [GeV].

As a function of distance:



As a function of energy: $(L \sim 2R_Z)$



For very large $L/E$ $(E/L \ll \Delta m^2)$ we get: $P_{\nu_e \to \nu_e} \approx 0.5$

# Neutrino oscillations

## Neutrino mixing

For three flavours:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS - Pontecorvo-Maki-Nakagawa-Sakata matrix

corresponding to CKM matrix for quarks

$$U = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}$$

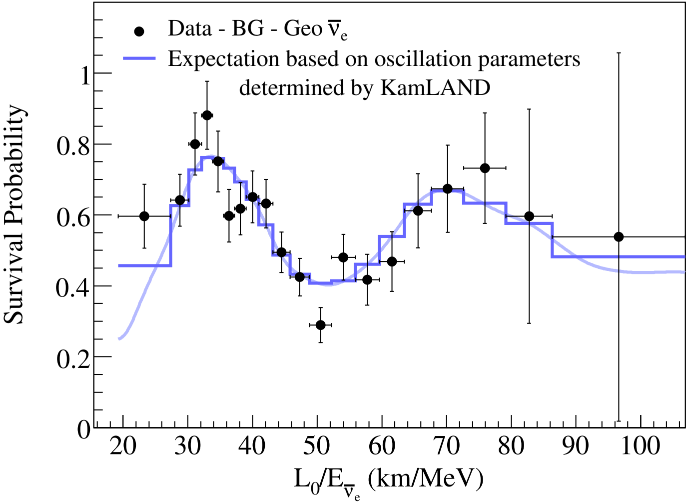
$\nu_e \leftrightarrow \nu_\mu$
solar neutrinos
$\theta_{12} \approx 35^\circ$

$\nu_\mu \leftrightarrow \nu_\tau$
atmospheric
$\theta_{23} \approx 45^\circ$

$\nu_e \leftrightarrow \nu_\tau$
reactor, $\theta_{13} \sim 8^\circ$
established only in 2012!

## KamLAND

First direct observation of neutrino “regeneration” for reactor neutrinos



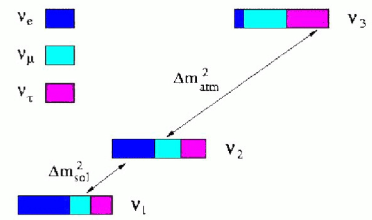
# Neutrino oscillations

## Summary

Neutrinos are produced as $\nu_e$, $\nu_\mu$ or $\nu_\tau$

But they propagate as superposition of mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$

$\Rightarrow$ lepton flavour is not conserved, neutrinos oscillate



Oscillations are described by 3 mixing angles, 2 mass squared differences and a phase, which could be a source of CP violation in lepton sector.

Oscillations needed to describe a wide range of experimental results:

- deficit of solar electron neutrinos
- deficit of up-going muon atmospheric neutrinos
- oscillations of electron anti-neutrino flux in reactor experiments
- muon neutrino disappearance in accelerator experiments
- electron and taon neutrino appearance in accelerator experiments