Particles and Universe

Lecture 7 Coupling constants

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I. Comparison of coupling constants

II. Feynman diagrams

III. Running coupling constants,
asymptotic freedom

Interactions

In macro- and micro scales:

- gravitation – act between all massive particles, only attraction, responsible for Sun system, large astronomical objects, etc.

- electromagnetism (e-m, el-mag) – electric charge of both signs, attraction and repulsion, atoms ...

In microworld in addition interactions:

- strong (nuclear) - bounding nucleons in nuclei (pions exchange) range 10^{-15} m

  strong fundamental (color) - between quarks (gluons exchange), range 10^{-15} m

-- weak (nuclear), eg. neutron decay, range smaller that for strong (pointlike interaction)

  weak fundamental between quarks and leptons (exchange of gauge boson W/Z), range 10^{-18} m

Nuclear forces like van der Waals forces

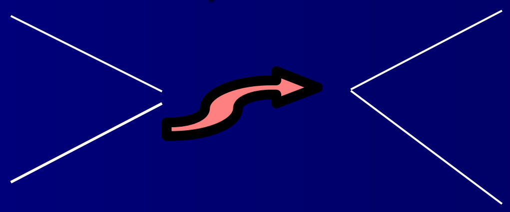
Range of interactions

$c = \hbar = 1$

- Interaction in microworld = emission and absorption of bosons (photon, W/Z, gluons..) → exchange of particles

- Range (Heisenberg, Yukawa) is related to the mass of exchanged particle (carrier of interactions)

$$x \sim 1/M$$



- gravitation and el-mag infinite range → graviton mass? photon mass = 0
- color (strong) int. : range ~ proton radius 10^{-15} m (although mass of gluons zero, confinement!)
- weak int. range 10^{-18} m, related to the mass of bosons W/Z ~80-90 GeV

Strength of interactions

- Long range forces gravitation and el-mag very different - gravitation very weak

  (gravitation between two protons 10^{36} times weaker than el-mag interaction)

- Strength's hierarchy at low* energies:

  strong> electromagn.> weak > gravitation

  ** low energies: 1 GeV up to 100 GeV*

  in the Standard Model – no gravitation!

- Parameter of strength of elementary action

  → coupling constant

Coupling constants

Strength of elementary act of interaction = coupling constant

el-m: e- $\rightarrow$ e- γ , e- $\gamma \rightarrow$ e- e (el. charge)

weak fund.: g ('weak' charge)

e- $\rightarrow \nu_e W^-$, $\nu_e \rightarrow$ e- W^+

d $\rightarrow$ u W^-, t $\rightarrow$ b W^+

d $\rightarrow$ d Z , Z $\rightarrow \nu \; \bar{\nu}$

strong fund., color: g_s ('strong' charge, color charge)

$u_R \rightarrow u_G + g_{R,\text{anty } G}$

Probability of elementary processes*,**

el-m $\alpha=\alpha_{el}= e^2/4\pi \simeq 1/137$

weak fund. $\alpha_w= g^2/4\pi \simeq 1/32$

strong fund, color $\alpha_s= g_s^2/4\pi \simeq 1$

** called coupling constant as well,** for energy ~1 GeV*

Units... (http://pdg.lbl.gov/)

1. PHYSICAL CONSTANTS

[illegible] Values of the Fundamental Physical Constants: 2014" by P.J. Mohr, D.B. Newell, and B.N. Taylor in arXiv:1507.07956 (2015) and RMP (to be submitted). The last group of constants (beginning with the Fermi coupling constant) comes from the Particle Data Group. The figures in parentheses after the values give the 1-standard-deviation uncertainties in the last digits; the corresponding fractional uncertainties in parts per 10^9 (ppb) are given in the [illegible]

Quantity	Symbol, equation	Value	Uncertainty (ppb)
speed of light in vacuum	c	$299\,792\,458$ m s^{-1}	exact*
Planck constant	h	$6.626\,070\,040(81)\times10^{-34}$ J s	12
Planck constant, reduced	$\hbar \equiv h/2\pi$	$1.054\,571\,800(13)\times10^{-34}$ J s	12
		$= 6.582\,119\,514(40)\times10^{-22}$ MeV s	6.1
electron charge magnitude	e	$1.602\,176\,6208(98)\times10^{-19}$ C $= 4.803\,204\,673(30)\times10^{-10}$ esu	6.1, 6.1
conversion constant	$\hbar c$	$197.326\,9788(12)$ MeV fm	6.1
conversion constant	$(\hbar c)^2$	$0.389\,379\,3656(48)$ GeV^2 mbarn	12
electron mass	m_e	$0.510\,998\,9461(31)$ MeV$/c^2 = 9.109\,383\,56(11)\times10^{-31}$ kg	6.2, 12
proton mass	m_p	$938.272\,0813(58)$ MeV$/c^2 = 1.672\,621\,898(21)\times10^{-27}$ kg	6.2, 12
		$= 1.007\,276\,466\,879(91)$ u $= 1836.152\,673\,89(17)\ m_e$	0.090, 0.095
deuteron mass	m_d	$1875.612\,928(12)$ MeV$/c^2$	6.2
unified atomic mass unit (u)	(mass ^{12}C atom)/12 = (1 g)/(N_A mol)	$931.494\,0954(57)$ MeV$/c^2 = 1.660\,539\,040(20)\times10^{-27}$ kg	6.2, 12
permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\,187\,817\ldots\times10^{-12}$ F m^{-1}	exact
permeability of free space	μ_0	$4\pi\times10^{-7}$ N A^{-2} $= 12.566\,370\,614\ldots\times10^{-7}$ N A^{-2}	exact
fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297\,352\,5664(17)\times10^{-3} = 1/137.035\,999\,139(31)^\dagger$	0.23, 0.23
classical electron radius	$r_e = e^2/4\pi\epsilon_0 m_e c^2$	$2.817\,940\,3227(19)\times10^{-15}$ m	At $Q^2=0$. At $Q^2 \approx m_W^2$
(e^- Compton wavelength)/2π	$\lambda\!\!\!^{-}_e = \hbar/m_e c = r_e\alpha^{-1}$	$3.861\,592\,6764(18)\times10^{-13}$ m	0.45
Bohr radius ($m_{\text{nucleus}} = \infty$)	$a_\infty = 4\pi\epsilon_0\hbar^2/m_e e^2 = r_e\alpha^{-2}$	$0.529\,177\,210\,67(12)\times10^{-10}$ m	0.23
wavelength of 1 eV/c particle	$hc/(1\text{ eV})$	$1.239\,841\,9739(76)\times10^{-6}$ m	6.1
Rydberg energy	$hcR_\infty = m_e e^4/2(4\pi\epsilon_0)^2\hbar^2 = m_e c^2\alpha^2/2$	$13.605\,693\,009(84)$ eV	6.1
Thomson cross section	$\sigma_T = 8\pi r_e^2/3$	$0.665\,245\,871\,58(91)$ barn	1.4

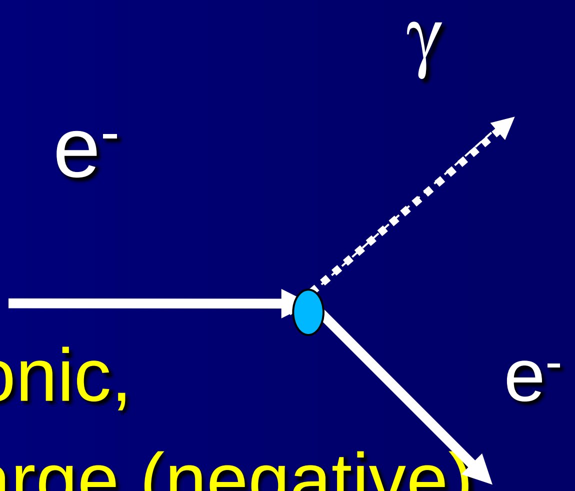
Feynman diagrams

Feynman diagrams –

particles are represented by different lines,

act of elementary interaction - by a vertex

eg. emission of the photon by electron

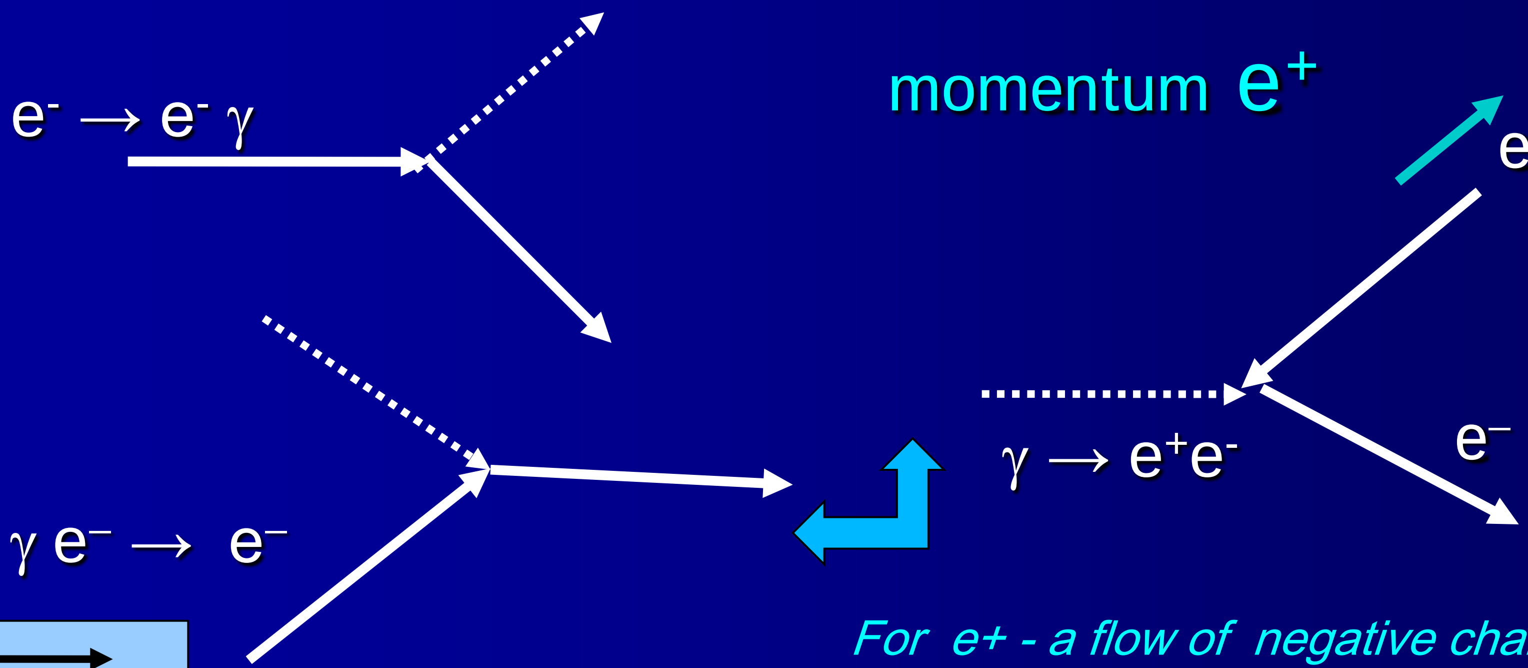


Arrows on a continuous line (fermionic, here for e^-) → flow of electric charge (negative) <u>and</u> momentum, while arrow on the photonic line (here dashed) → only momentum

Feynman diagrams for crossing processes

Crossing processes involving e e γ

(e represents e+ and e-)



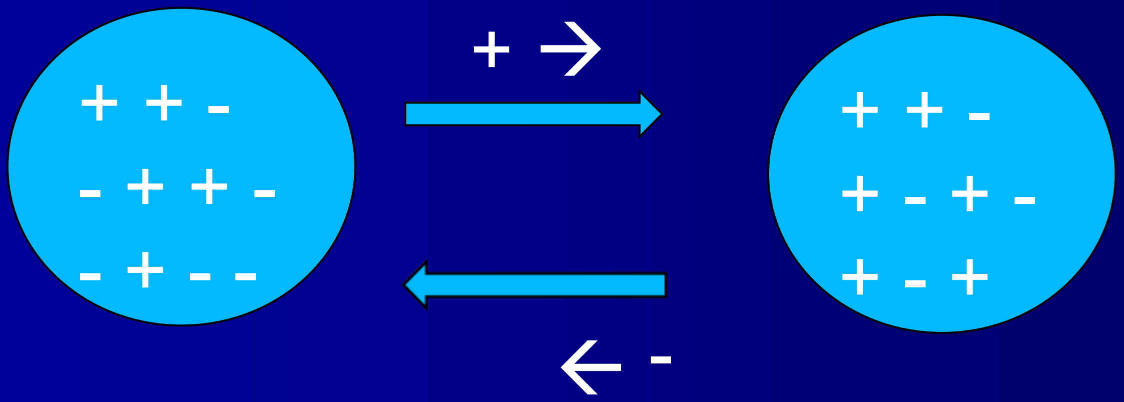
For e+ - a flow of negative charge in oppositve direction than momentum

Flow of charges

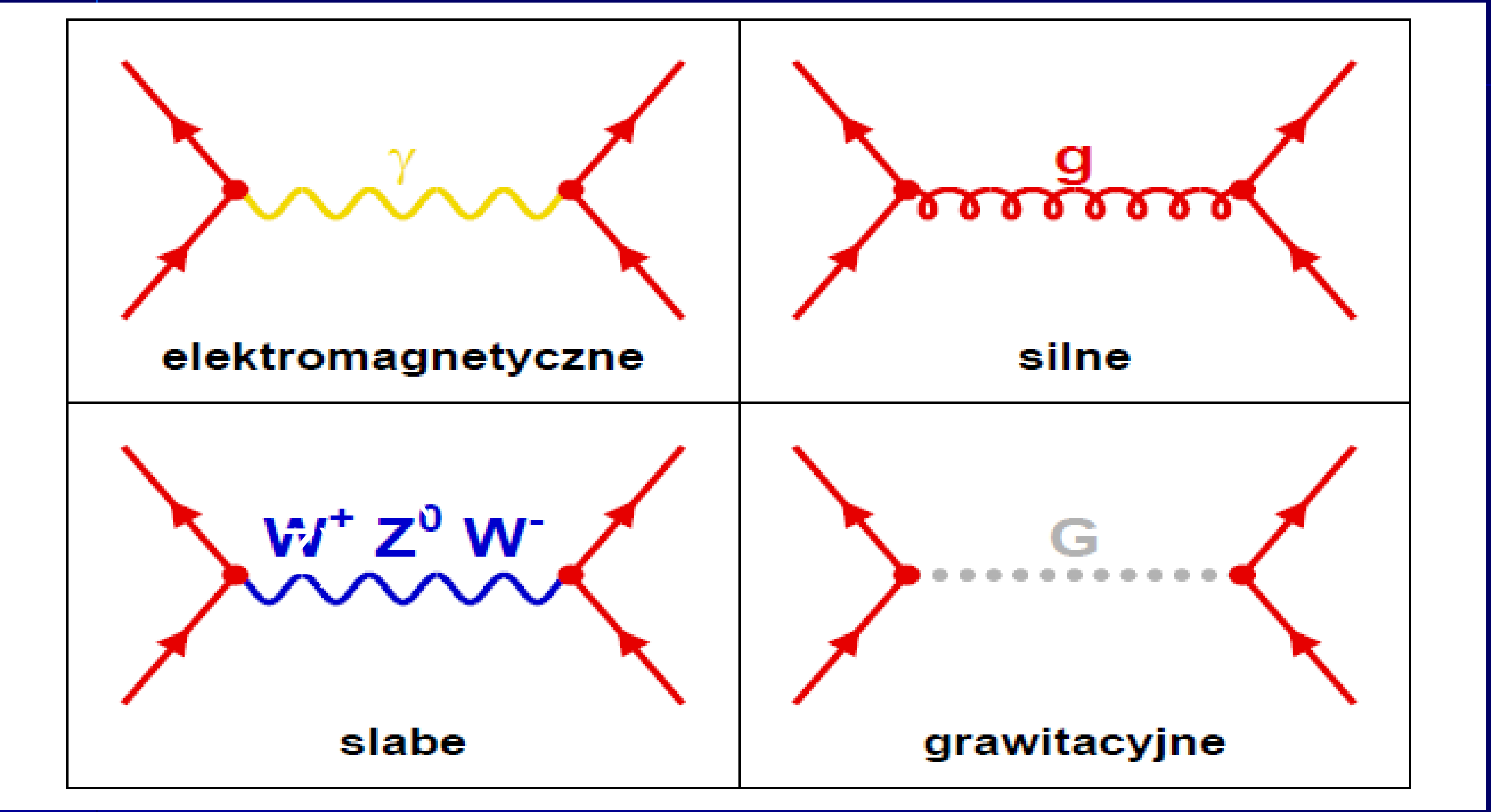
charge of
a system

$N_+ = n_+ - n_-$

$N_- = n_- - n_+$



Feynman diagrams



Electromagnetism and gravity

- Why gravity, so weak compared to electromagnetism was known first?
- Gravity only added while el-magn interaction canceled out for big

- The force for electron (with mass m) and proton (mass M) in the hydrogen atom H

$$F_{el} = e^2/r^2 \quad F_{gr} = GMm/r^2$$

- Ratio $GMm/e^2 = 10^{-40}$

Fundamental constants

Relation to physical phenomena

c – relativistic physics

velocity of light

ħ – quantum physics

Planck constant $\hbar = h/2\pi$

G – gravitation

gravitational constant (Newton)

Units…more

gravitational constant[‡]	G_N	$6.674\,08(31)\times10^{-11}$ m^3 kg^{-1} s^{-2}	4.7×10^4
		$=6.708\,61(31)\times10^{-39}\ \hbar c\ (\mathrm{GeV}/c^2)^{-2}$	4.7×10^4
standard gravitational accel.	g_N	$9.806\,65$ m s^{-2}	exact
Avogadro constant	N_A	$6.022\,140\,857(74)\times10^{23}$ mol^{-1}	12
Boltzmann constant	k	$1.380\,648\,52(79)\times10^{-23}$ J K^{-1}	570
		$=8.617\,3303(50)\times10^{-5}$ eV K^{-1}	570
molar volume, ideal gas at STP	$N_A k(273.15\ \mathrm{K})/(101\,325\ \mathrm{Pa})$	$22.413\,962(13)\times10^{-3}$ m^3 mol^{-1}	570
Wien displacement law constant	$b=\lambda_{\max}T$	$2.897\,7729(17)\times10^{-3}$ m K	570
Stefan-Boltzmann constant	$\sigma=\pi^2k^4/60\hbar^3c^2$	$5.670\,367(13)\times10^{-8}$ W m^{-2} K^{-4}	2300
Fermi coupling constant**	$G_F/(\hbar c)^3$	$1.166\,378\,7(6)\times10^{-5}$ GeV^{-2}	500
weak-mixing angle	$\sin^2\widehat{\theta}(M_Z)\ (\overline{\mathrm{MS}})$	$0.231\,29(5)$[††]	2.2×10^5
$W^\pm$ boson mass	m_W	$80.385(15)$ GeV$/c^2$	1.9×10^5
Z^0 boson mass	m_Z	$91.1876(21)$ GeV$/c^2$	2.3×10^4
strong coupling constant	$\alpha_s(m_Z)$	$0.1181(11)$	1.0×10^7

$\pi = 3.141\,592\,653\,589\,793\,238$ $\quad$ $e = 2.718\,281\,828\,459\,045\,235$ $\quad$ $\gamma = 0.577\,215\,664\,901\,532\,861$

$1\ \mathrm{in}\equiv0.0254$ m	$1\ \mathrm{G}\equiv10^{-4}$ T	$1\ \mathrm{eV}=1.602\ 176\ 6208(98)\times10^{-19}$ J	kT at 300 K $=[38.681\ 740(22)]^{-1}$ eV
$1\ \mathrm{Å}\equiv0.1$ nm	$1\ \mathrm{dyne}\equiv10^{-5}$ N	$1\ \mathrm{eV}/c^2=1.782\ 661\ 907(11)\times10^{-36}$ kg	$0\ °\mathrm{C}\equiv273.15$ K
$1\ \mathrm{barn}\equiv10^{-28}$ m^2	$1\ \mathrm{erg}\equiv10^{-7}$ J	$2.997\ 924\ 58\times10^9$ esu $=1$ C	1 atmosphere $\equiv760$ Torr $\equiv101\ 325$ Pa

Subtle coupling constant

Electric charge e

$\alpha = e^2/4\pi\hbar c \sim 1/137$ – a subtle coupling constant, introduced to describe interaction of electrons with photons by Sommerfeld in 1916 (*in the subtle emission spectrum of hydrogen and silver – relativistic corrections*)

→ important in relativistic (c), quantum (ħ) theory of electric charge (e)

Quantum Electrodynamics (QED funded in 20-30 XX)

where α (or α_{em}, α_{el}) – measures „strength" of el-mag interaction of electrons and photons (→ coupling constant)

Note: formally we often take ħc =1 , eg.. on page 5 in definition of various coupling constants

Gravitation – PLANCK scales

- We neclect gravitation for individual particles at current energies
- When gravitation important in microworld?

  From G, h and c we can construct quantity

  $(\hbar c/G)^{1/2}$ - Planck mass

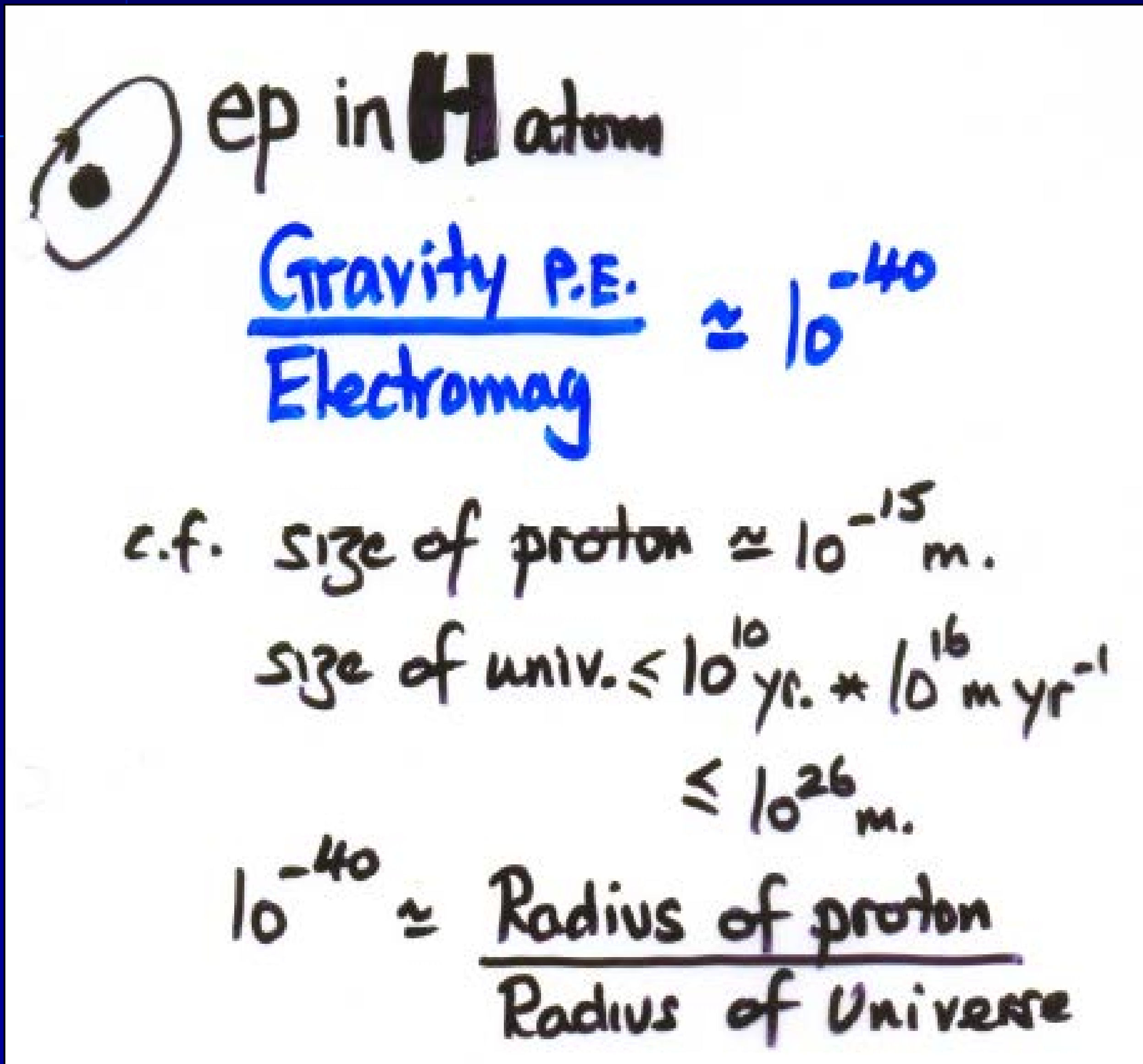
G ~ 6.7 ħc/GeV^2

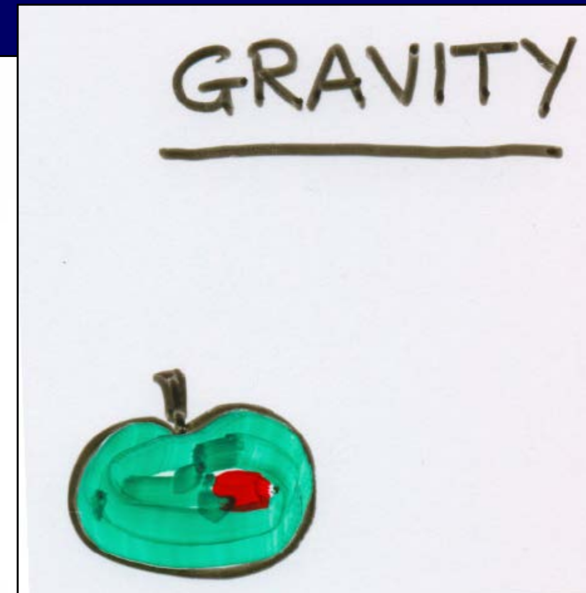
- Planck's scales :

  Planck's mass (energy) ~ 10^{19} GeV

  Planck's lenght ~ 10^{-35} m
- For these scales → relativistic quantum gravity. We are still looking for such theory...

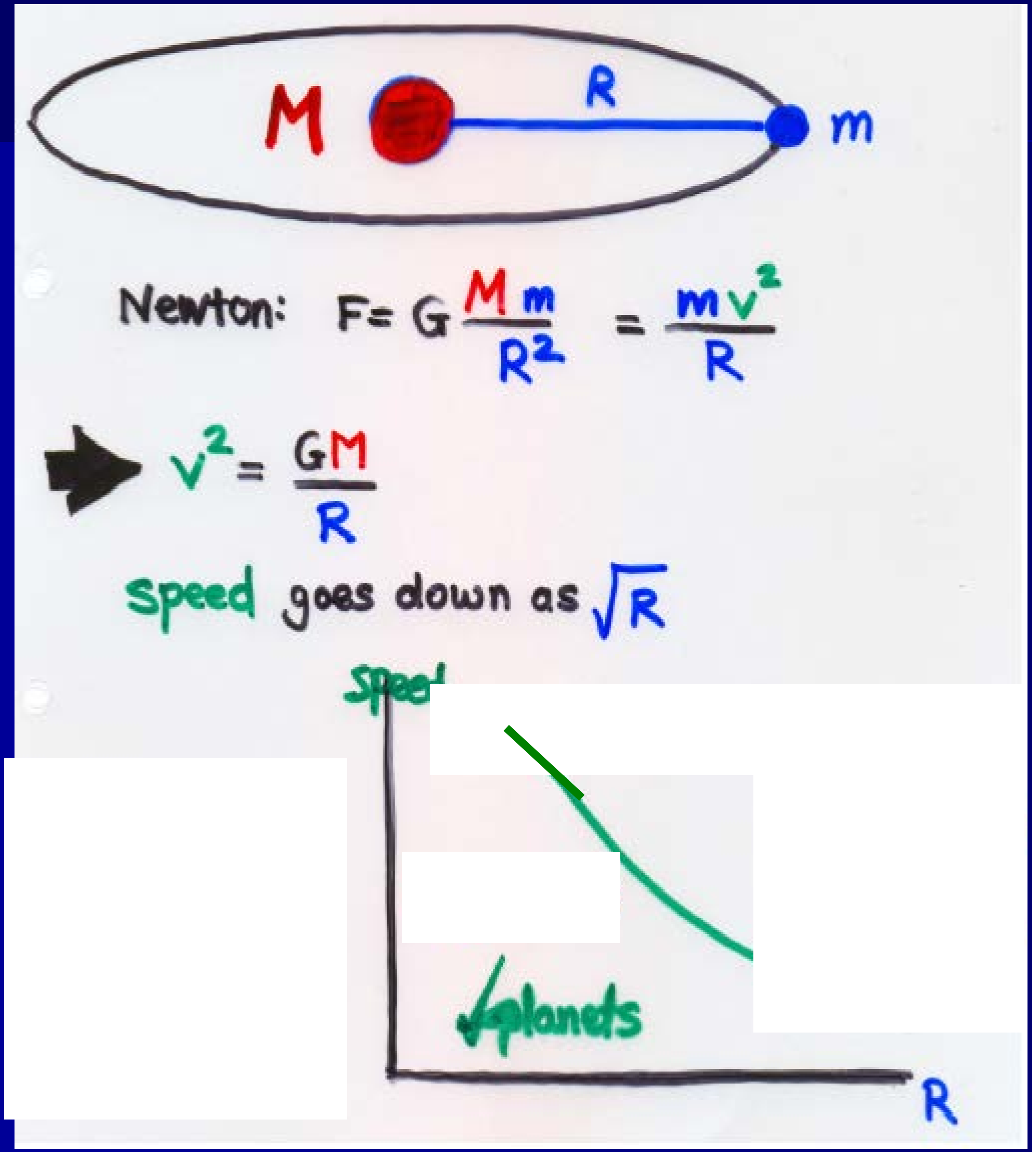
Gravity contra electromagnetism





Lecture by
F. Close

Velocity of object with mass m in the movement due to the gravitational attraction by mass M

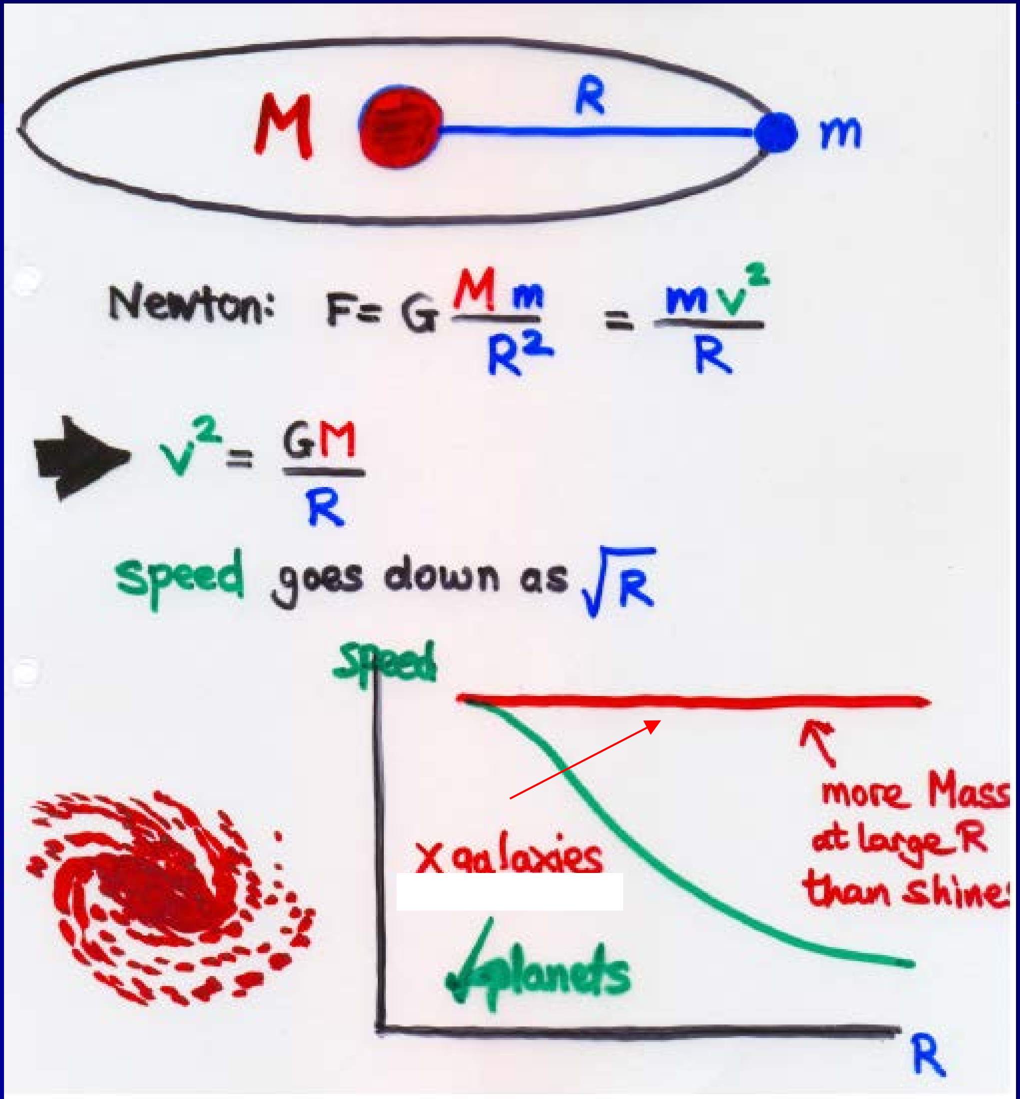


Velocity decreases for larger radius R

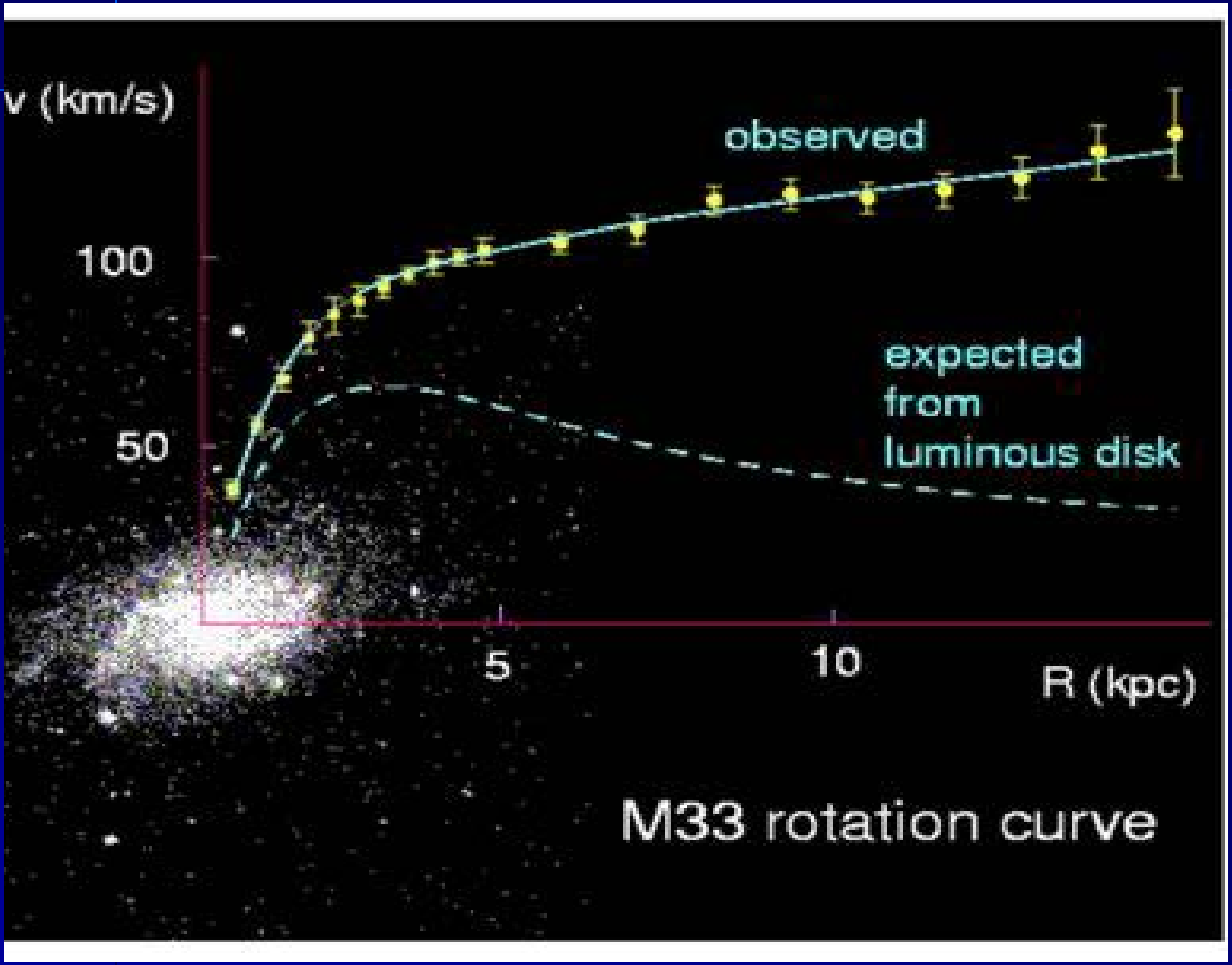
For planetes !

Velocity of particles in galaxies?

Dark Matter



Rotation curve



Dark matter?

- We do not know what it is, but it must be neutral and:
- cold dark matter – heavy dark matter (small kinetic energy)

  or

- hot dark matter – light dark matter (large kinetic energy)

more – next lectures

Electromagnetic interaction contra strong (color) interaction

Electrostatics

- Two types of electric charges positive (+) and negative (–)

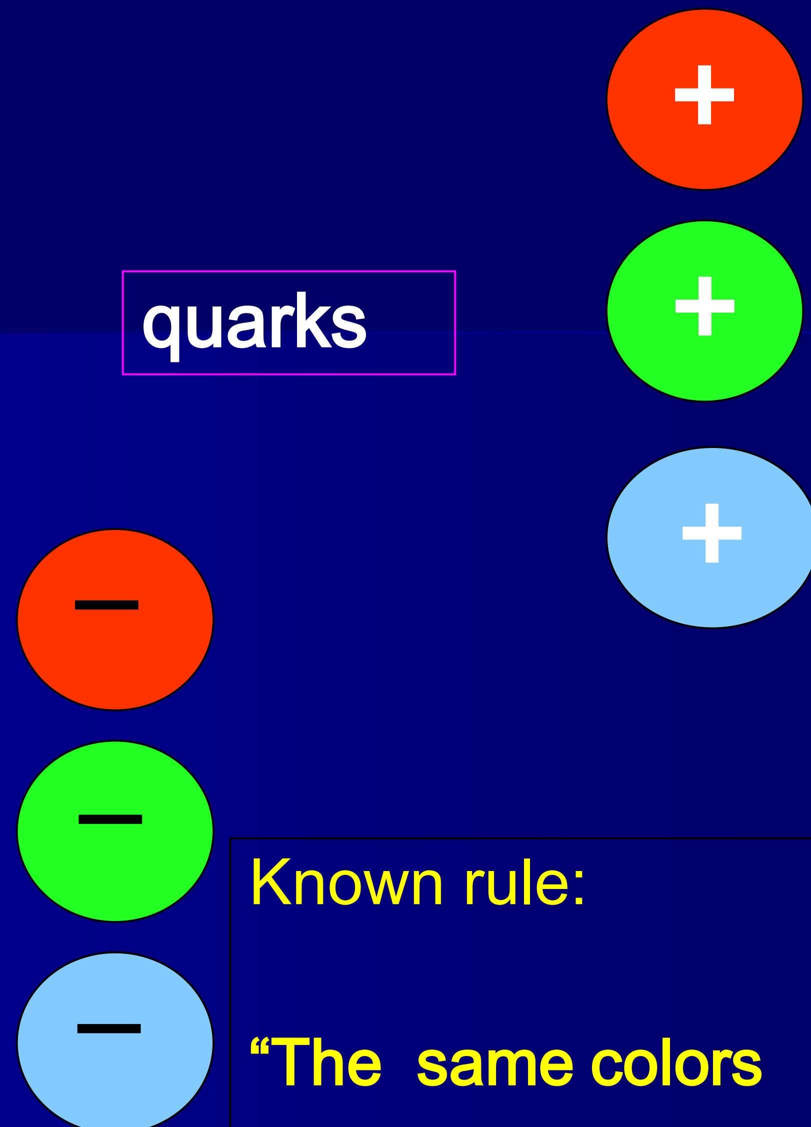
CHROMOSTATICS

- Three types of color charges (colors), each „positive” (+) and „negative” (–) → means color and anticolor

3 colors

quarks

antiquarks

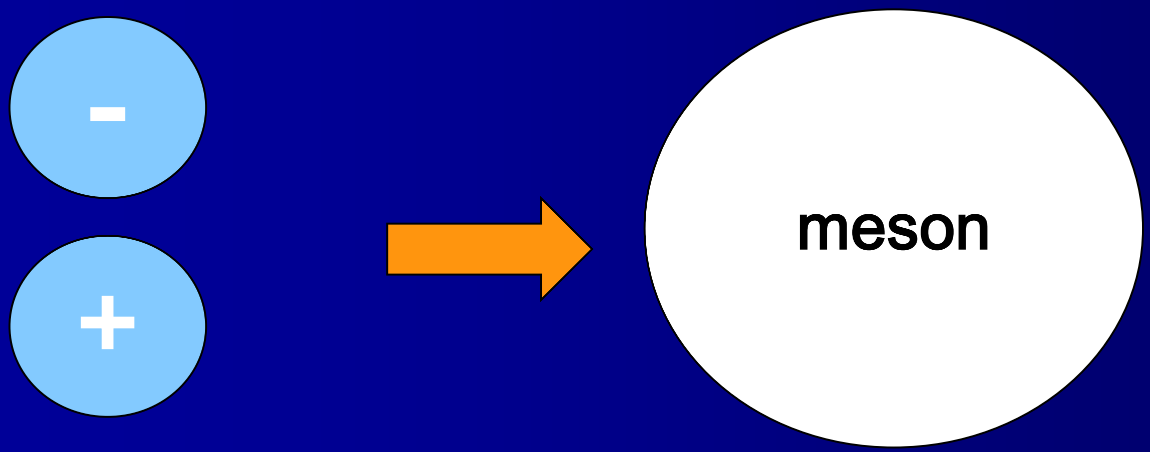


Known rule:

“The same colors repulsive, opposite colors- attractive”

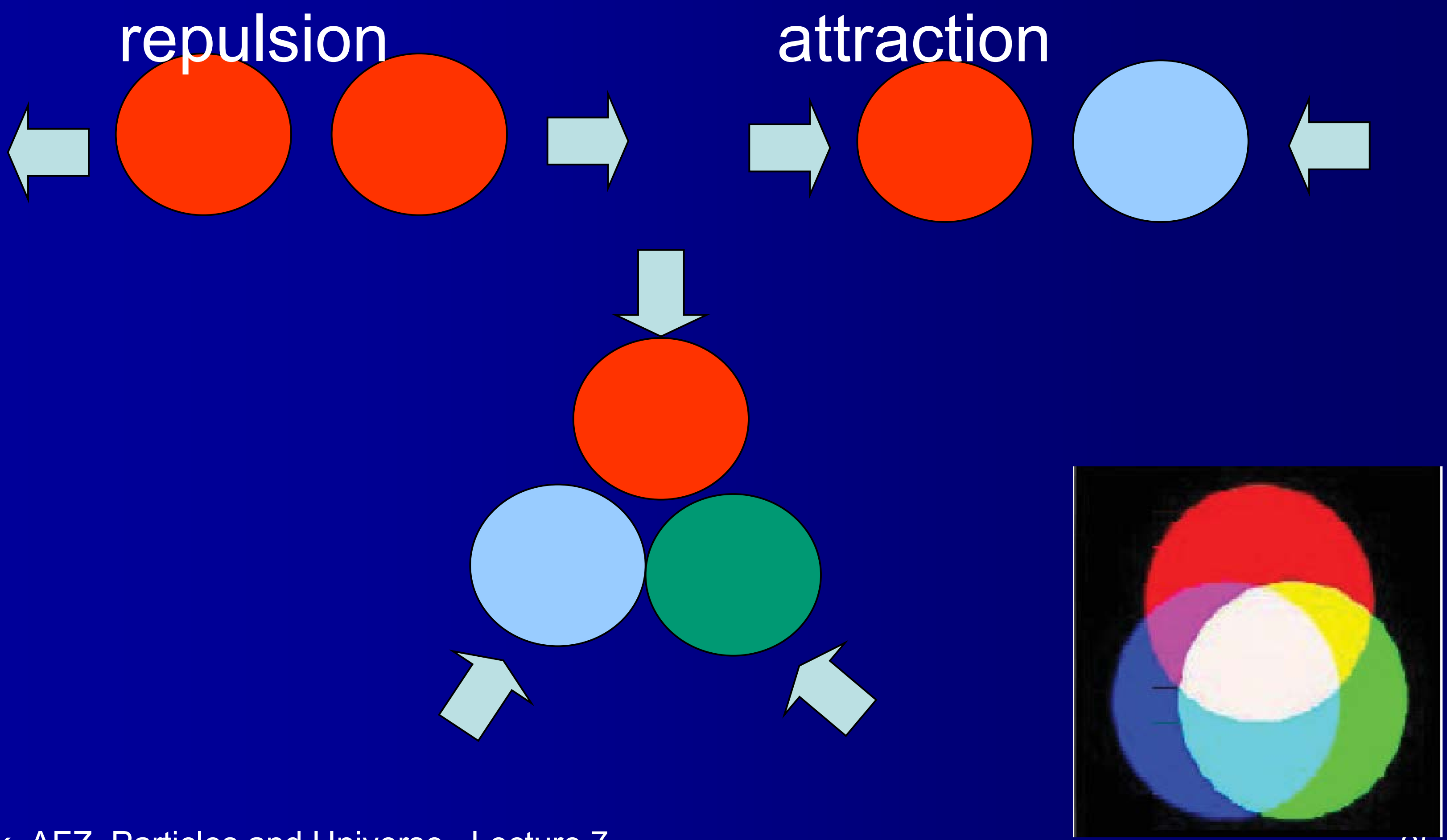
The simplest system: meson= quark+antiquark

Colors vanish, eg.



3 colors

Needed to get white baryons (3 quarks) (eg. proton)



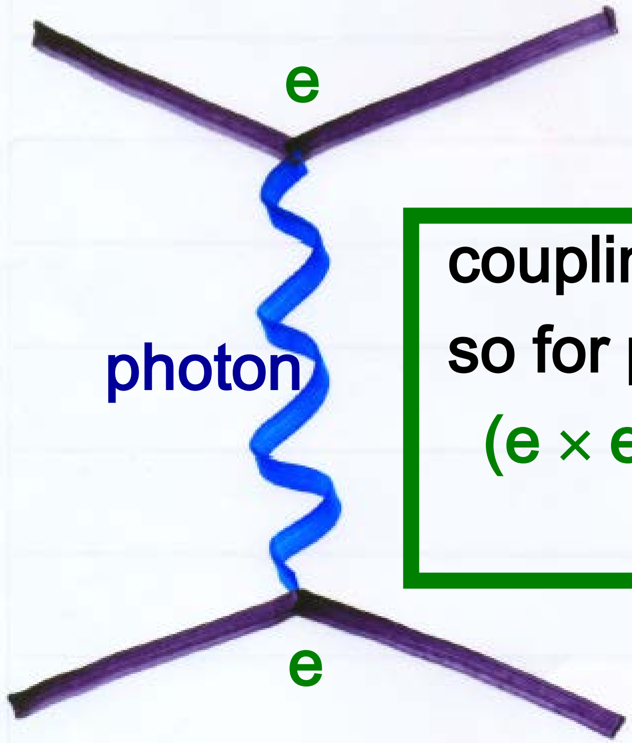
Quantum Electrodynamics: QED

Electric charge → Atom → Molecule

Quantum Chromodynamics: QCD

Color charge → Nucleon → Nucleus

Feynman's diagram for elecromagnetic interaction



coupling constant e
so for process
$(e \times e)/4\pi \sim 1/137$

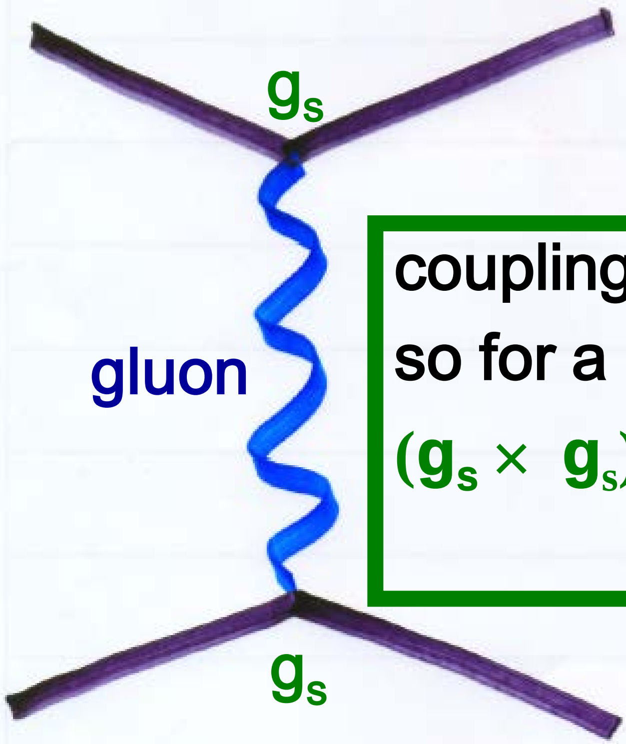
α

Time t →

Scattering process for two electrons

ee → ee

Feynman's diagram for color interaction



coupling constant g_s

so for a process

$(g_s \times g_s)/4\pi \sim 1$

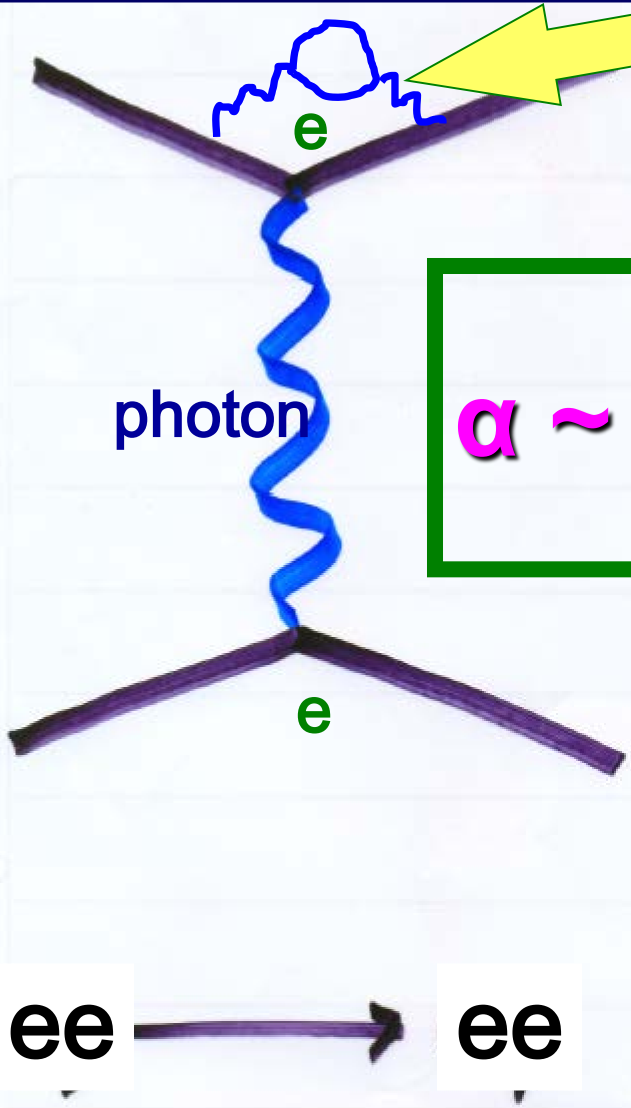
α_s

Scattering process for two quarks

Time t →

qq → qq

Elektromagnetic interaction



Quantum effect
(here e+e- loop)
depends on momentum p

$\alpha \sim 1/137$ ⇔ (for $p \to 0$)

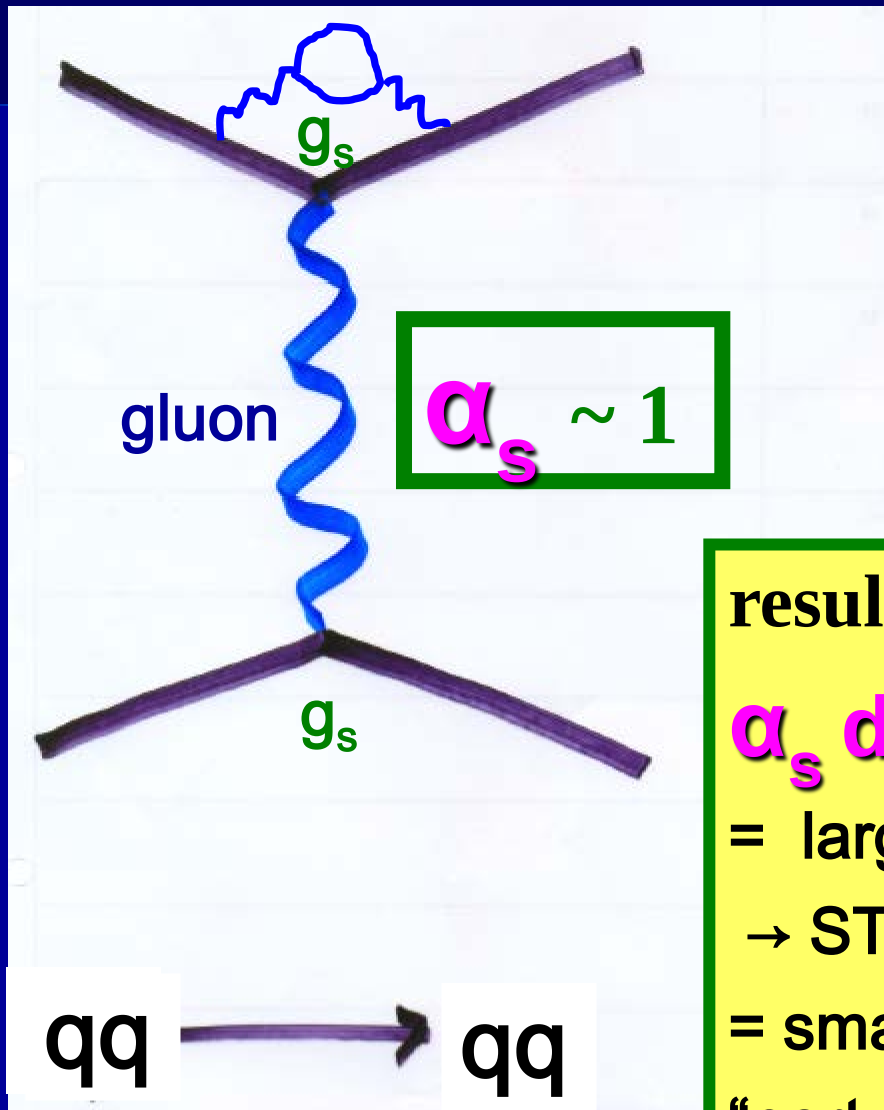
Quantum effect:

α rises with momentum:

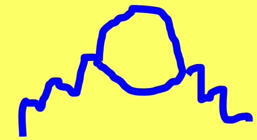
for p~100 GeV ~1/128

$E=pc$, $[p]=GeV/c$

Color interaction

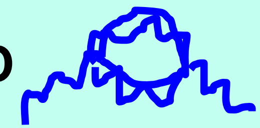


Quantum effect in QCD as in QED:



quark loop

In QCD in addition gluon loop



!

result:

α_s decreases with p !

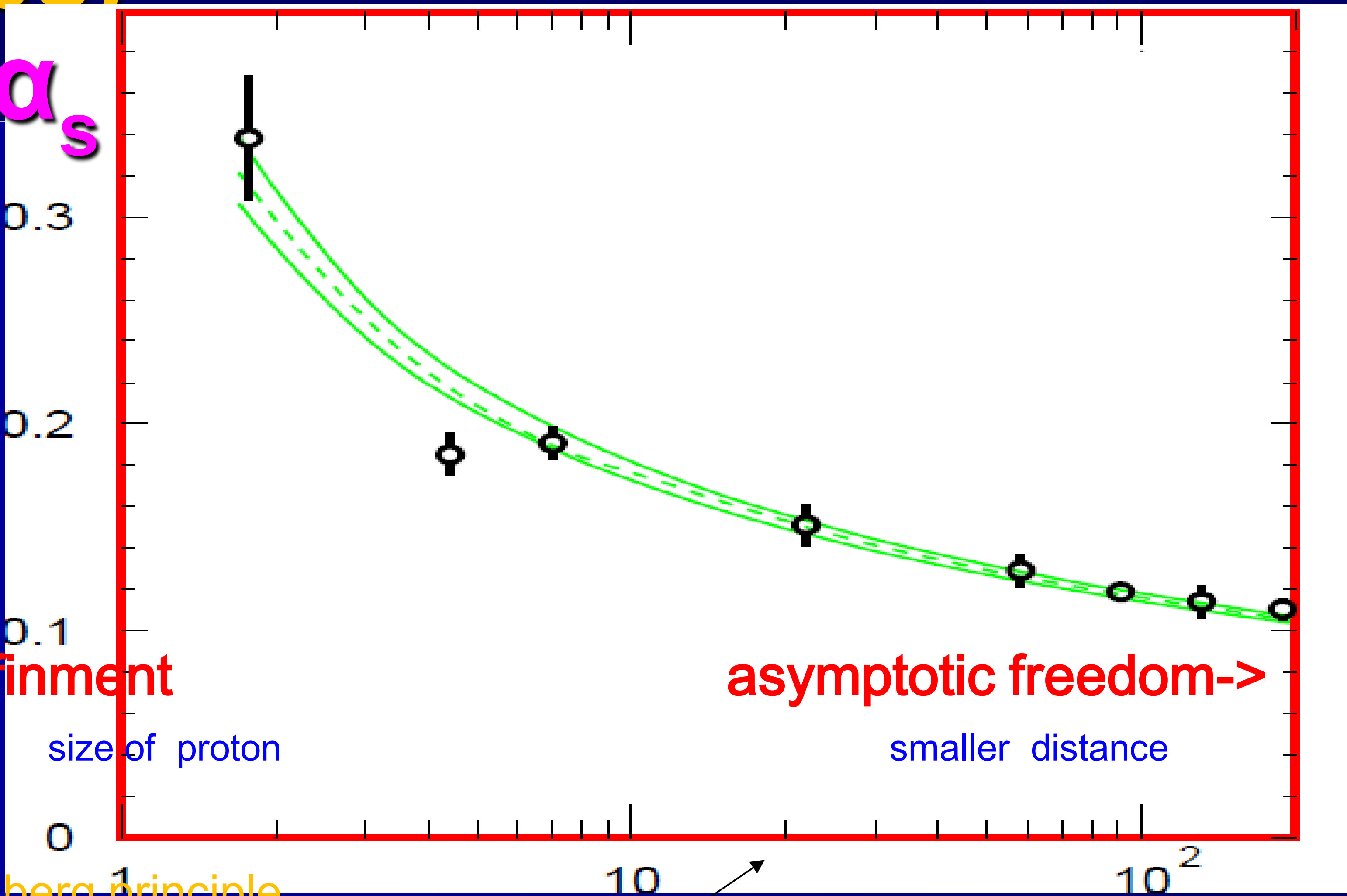
= large for small p

→ STRONG INTERACTION!

= small for large momentum

"perturbative QCD"

Running coupling constant (2008) α_s



← confinment

asymptotic freedom->

momentum (GeV)

Heisenberg principle

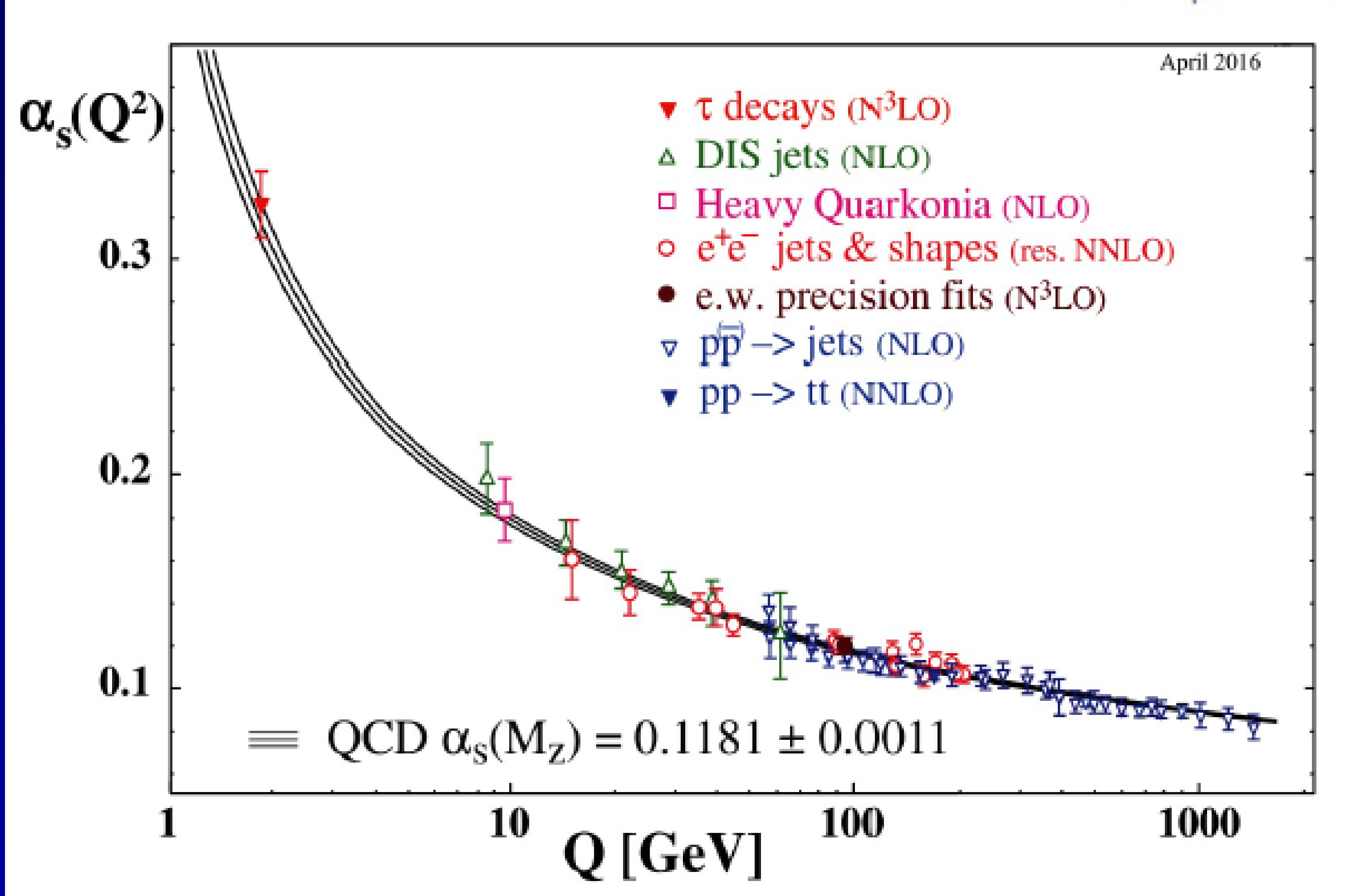
$\Delta x \sim 1/\Delta p$ ($\hbar$ c=1)

Running coupling constant α_s

... QCD coupling α_s

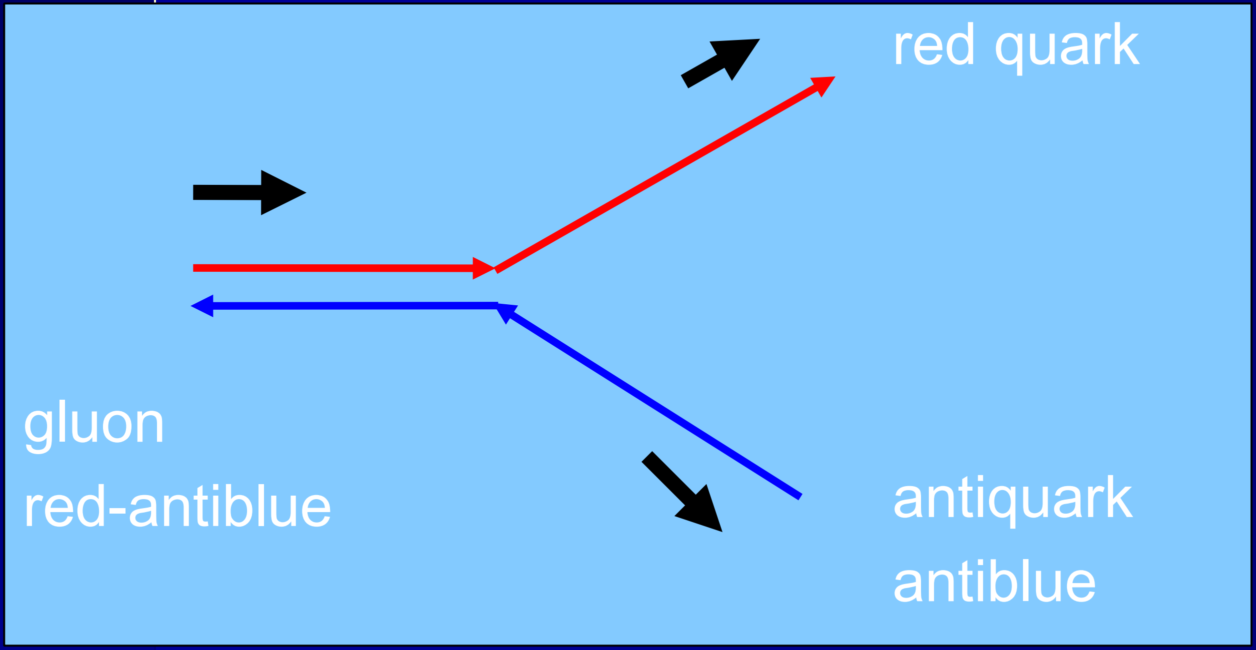
Least precisely known of all couplings:

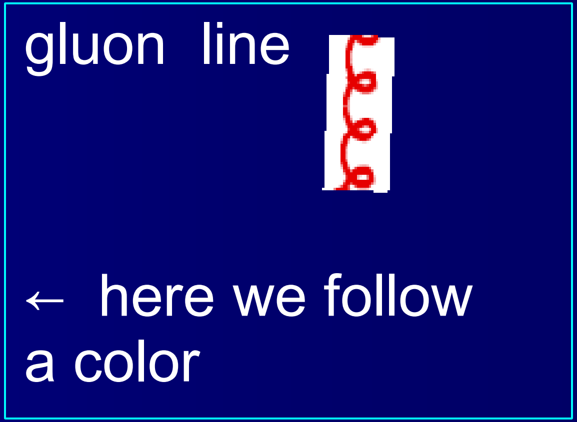
$\delta\alpha \sim 3\cdot10^{-10}$, $\delta G_F \sim 5\cdot10^{-8}$, $\delta G \sim 10^{-5}$, $\delta\alpha_s \sim 5\cdot10^{-3}$



Color interaction

Example gluon-> quark + antiquark

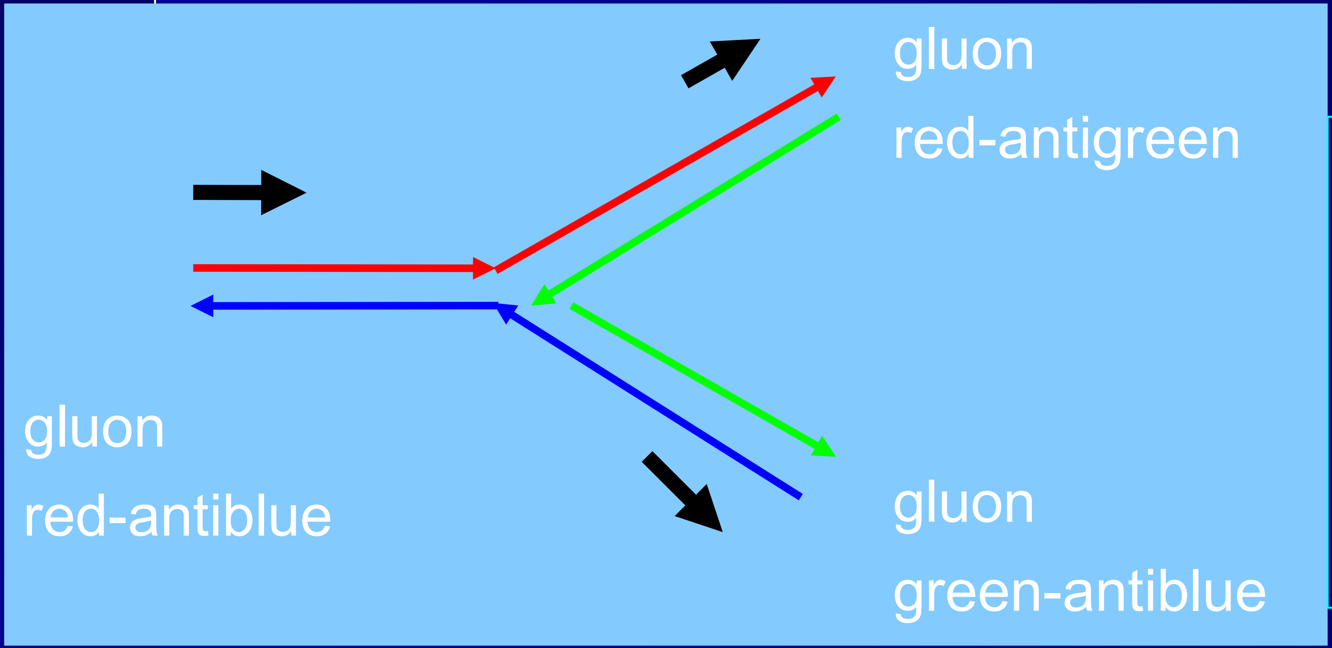




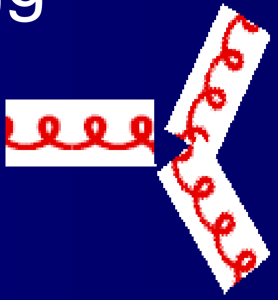
- color lines - flow of color (color conservation)
- black arrows – momenta of particles (momenta conservation)

Color interaction

Example gluon-> gluon+ gluon



ggg

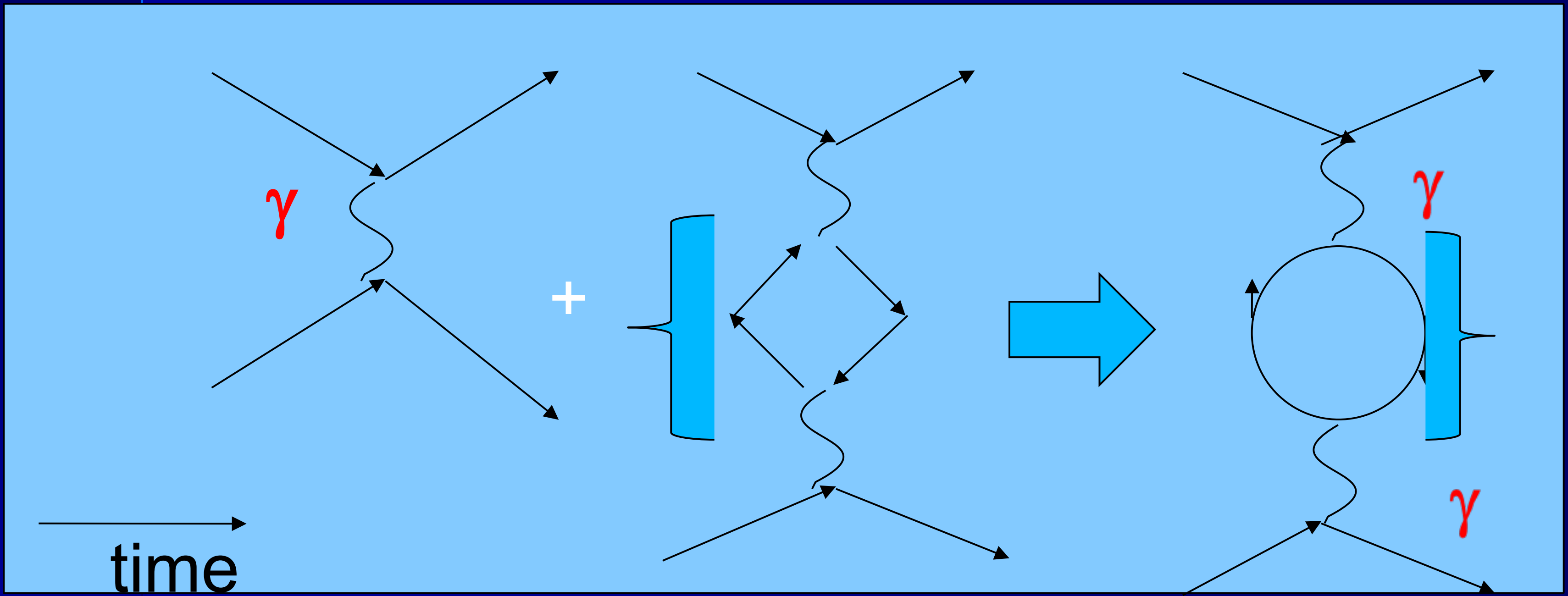


Also gggg

- color lines - flow of color (color conservation)
- black arrows – momenta of particles (momenta conservation)

Extraction of α

Measurement of α in $e^- e^- \to e^- e^-$

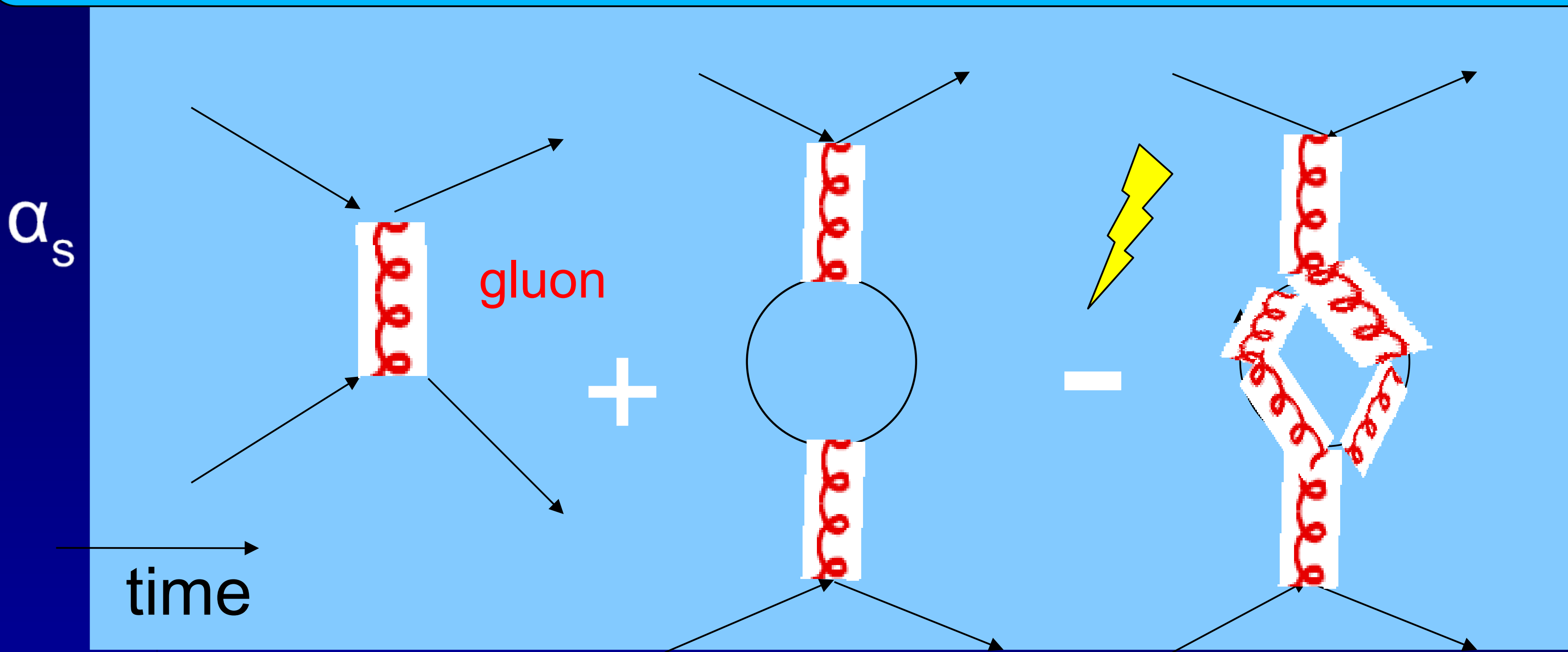


$$\alpha \times [1 + \mathrm{corrections}(p)] = \alpha(p)$$

Electron loop → α depends on momentum p (‚runs'); is rising with energy (momemtum)

Extraction of α_s

Scattering qq -> qq



Pętla kwarkowa - efekt podobny jak dla oddziaływań el-mag (powoduje wzrost stałej α_s). Tu dodatkowo pętla gluonowa, która ma przeciwny znak →

i w efekcie α_s maleje ze wzrostem pędu !!

Running coupling constants - unification?

- Couplings are running with energy (momentum) – this is an effect of quantum corrections
- Structure of interaction decides about rising or decreasing of coupling constants

  key point - are carries of interactions „charged" or not (means do they interact with themselves), eg. photon neutral, while gluons „charged"
- if some couplings are rising and other decreasing – at some energy they can have similar values

→ unified description?

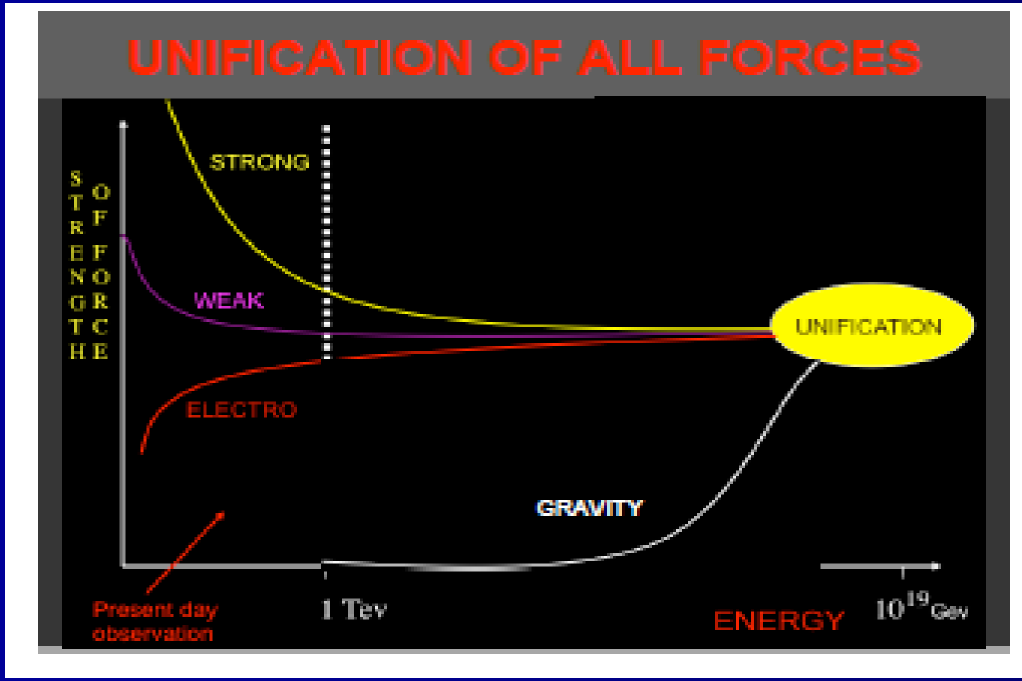
(Running couplings „constants”)!

For larger energy: strong interaction weaker
weak interaction - weaker
el-mag interaction stronger

α_s

α_w

α_{el}



D. Gross, Photon 2005

Gravity ???

Electromagnetic and strong coupling constants

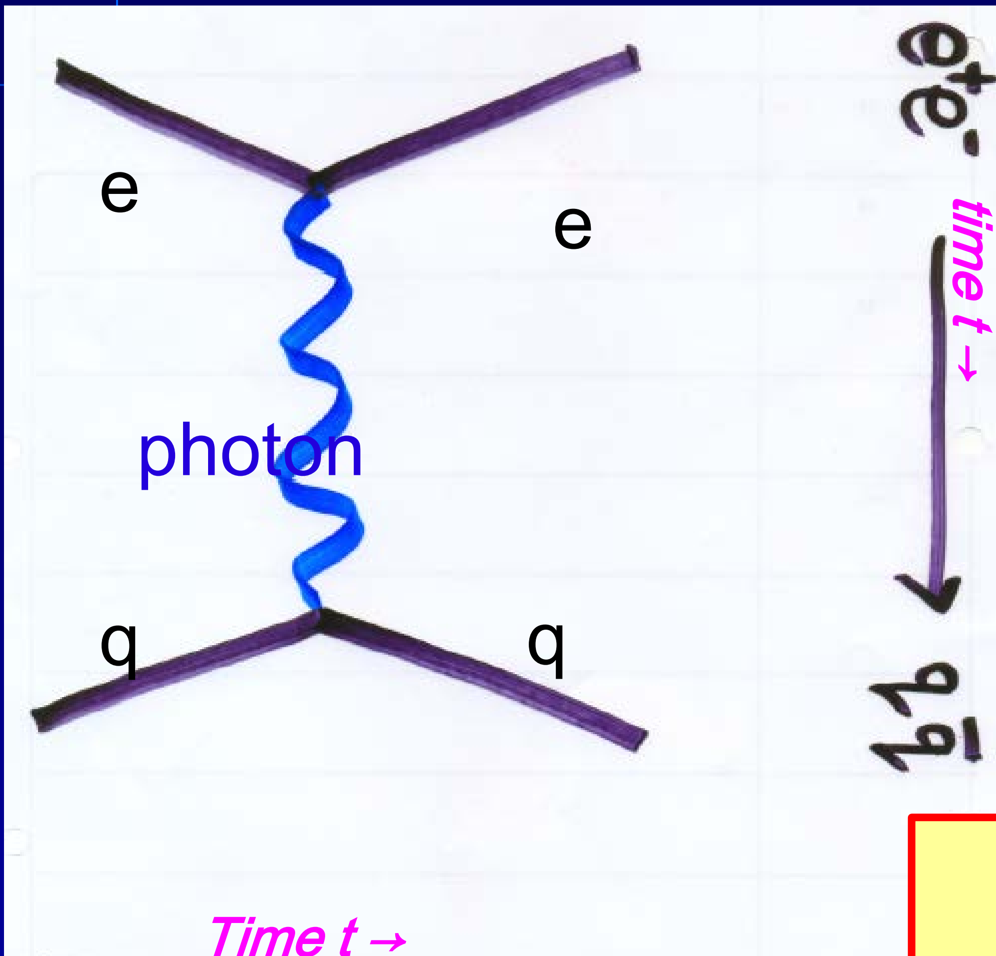
- QED Landau pole ($\rightarrow$ 0 in the denominator)

$$\alpha_{em}(q^2) = \alpha(q^2) = \frac{\alpha(\mu^2)}{\left[1 - \frac{1}{\pi}\alpha(\mu^2)\ln\left(\frac{q^2}{\mu^2}\right)\right]}$$

- QCD (N_f – number of fermions) Lambda_QCD = 0.2 GeV

$$\alpha_s(Q^2) \equiv \frac{g_s^2(Q^2)}{4\pi} = \frac{12\pi}{(33 - 2N_f)\ln(Q^2/\Lambda_{QCD}^2)}$$

Crossing for fixed external particles



$e^- \, q \rightarrow e^- \, q$

Here 2 e

(e- e- lub e+ e+, lub e-e+)

And 2 quarks q

(q q , q anty-q,

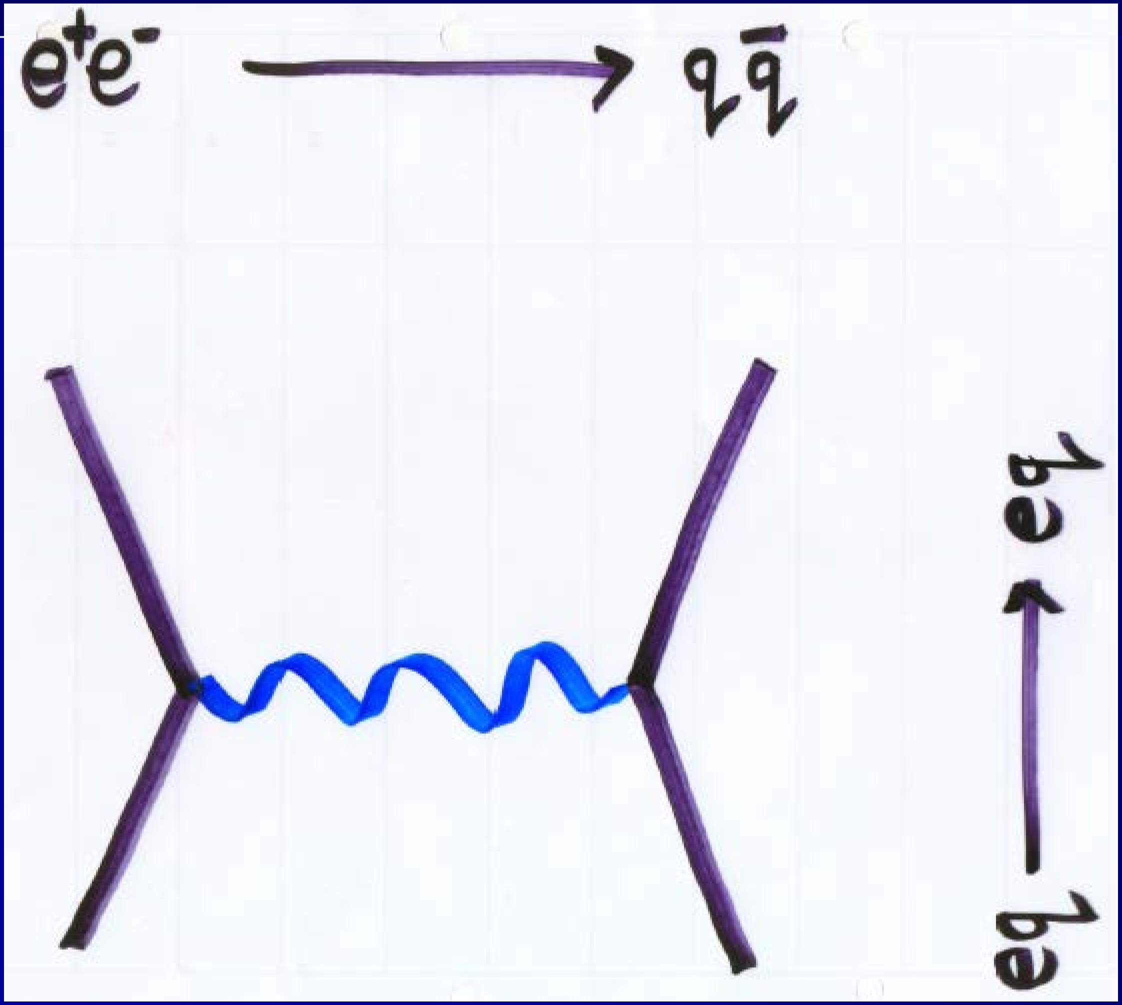
anty-q anty-q)

Crossing processes

positron in <-> electron out

anti-q out <-> q in

Feynman diagram



Crossing processes – example: e-e- → e-e-

*(**time** – from left to right)*

- Other processes (crossing processes) we got exchage replacing initial particles with the final particles simultanous replacing particles with antiparticles

- Yellow particle -> to be transfered to the future (as antiparticle) and pink particle – to be transfered to the past (as an antipariticle):

- e-e- → e-e- ⟹ e- e+ → e- e+, and next
  - e- e+ → e- e+ ⟹ e+e+ → e+ e+

Question to lecture 7

- Is the range of weak forces larger or smaller then the range of strong interactions?
- Which particles interact using nuclear forces, which particles using the color forces .
- Is gravitation important in the microworld for low energies?
- What is the value of the Planck length? What is value of the Planck mass?
- Write 3 elementary acts of interactions between partcicles from the first family
- What is the value of subtle coupling constant for momentum $p \to 0$?, for momentum p= 100 GeV ?
- Do two electrons interact stronger or weaker for larger energies (momenta)?
- What is value of strong coupling constant for momentum (energy) about 1 GeV? For which momentum α_s is equal to 1/10?
- When quarks are more free: for small or large energies?
- Write 2 processes obtained by crossing of the scattering process $u\, d \to u\, d$
- To what quarks does the green-antired gluon decay ?
- What is the reason of a running of coupling constants? Why the subtle coupling constant is growing while the strong coupling constant decreases with a grow of energy?