Particles and Universe
Lecture 9 Mixing

Maria Krawczyk, Aleksander F. Żarnecki

Faculty of Physics UW

Mixing of quarks and not only…

Mixing

Mixing is natural in quantum mechanics –

it is due to wave nature of particles

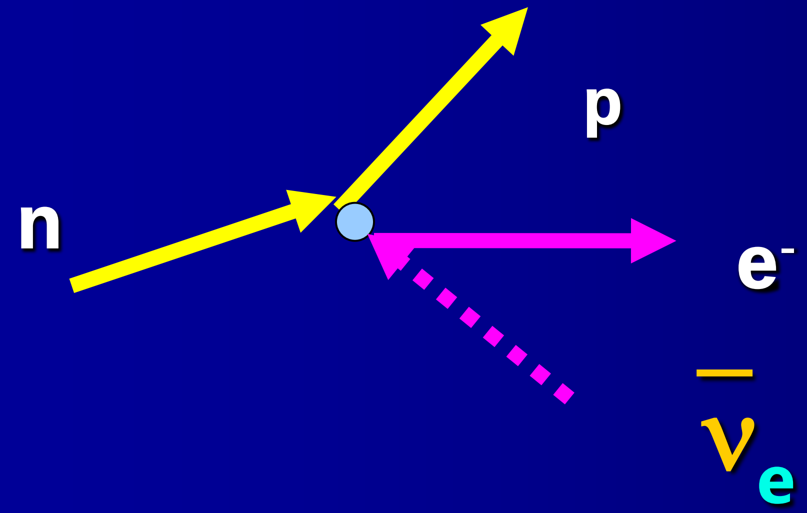
- We have discussed interference of various channels for processes

- Mixing - here one state mixes with another state

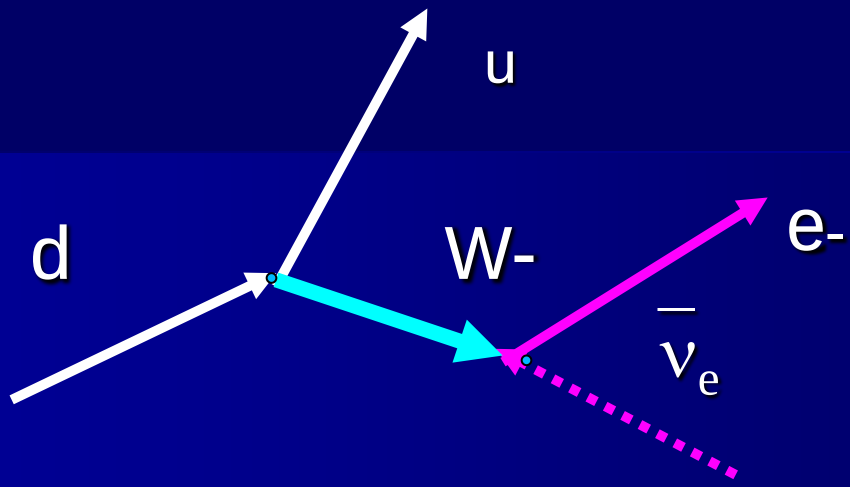
Mixing for weak interaction

Pair of (p,n) and of (electron neutrino, electron) - *doublets* in the neutron decay

β decay



Fundamental beta decay



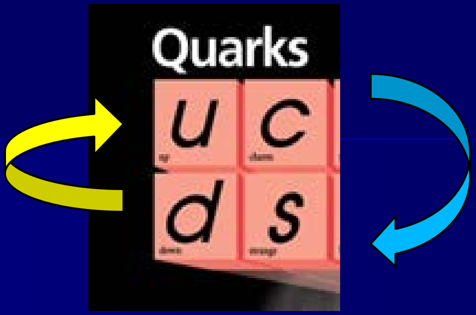
elementary acts of interaction :

$d \rightarrow u\,W^-$ and $W^- \rightarrow e^-\,\bar{\nu}_e$

g (g -„weak charge”) $\alpha_w = g^2/4\pi = 1/32$

Two light families – transition in doublets

	el. charge		
Quarks	2/3	u	c
	-1/3	d	s
Leptons	0	ν_e	ν_μ
	-1	e	μ

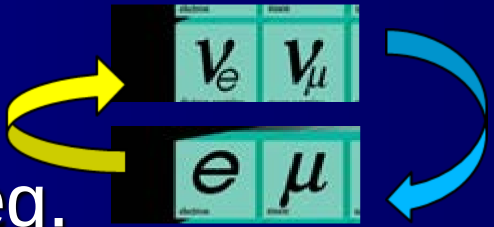


Formaly, transition between quarks IN a given doublet

$u \leftrightarrow d$, $c \leftrightarrow s$

due to exchange of gauge boson W-, eg.

$d \rightarrow u\, W^-$, $c \rightarrow s\, W^+$



Transition in the leptonic doublet, eg.

$e- \rightarrow \nu_e\, W^-$, $\nu_e \rightarrow e-\, W^+$

Probability for transition

EW theory:

Absolute value of weak coupling = g the same for all vertices (eg. $u \to d\, W+$, $W^- \to e- \, \nu_e$), but the coupling itself can have positive and negative sign (as in the el-m interaction - two signs of el. charge)

Let L- an absolute value of the transition amplitude

L = | transition amplitude |

for quarks or leptons in the considered doublet

Since L is proportional to g,

a probability for transition process $L^2 \sim g^2$

Transition for quarks and for leptons - via exchange of W+/-

$u \updownarrow d$ — L
$c \updownarrow s$ — L
$\nu_e \updownarrow e$ — L

$W^{\pm}$

- Theory SU(2) → transition probability($\mathcal{P}$) = L^2
- Experiment (1963)→ difference between quarks and leptons..

$$\mathcal{P}[u \to d\, W^+] + \mathcal{P}[u \to s\, W^+] = \mathcal{P}[\nu_e \to e\, W^+]$$

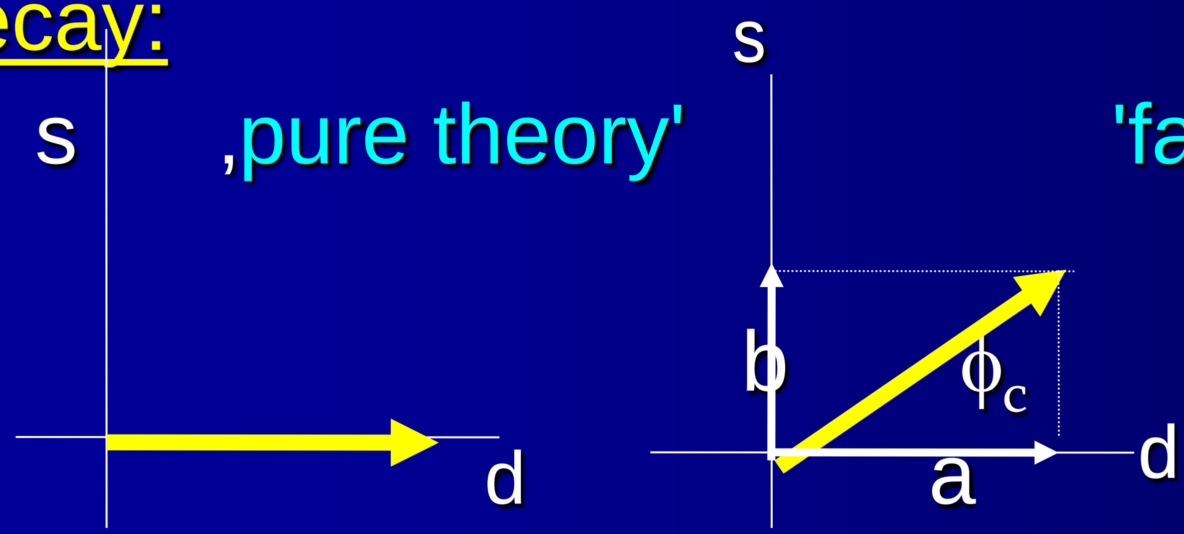
Mixing of s quark to the emission of W^+ by u quark !

Mixing of s with d → Cabibbo angle ϕ_c

So, u quark couples both to d and s

definition: $L^2 = \mathcal{P}[\nu_e \rightarrow e^- W^+]$, vector L

u decay:



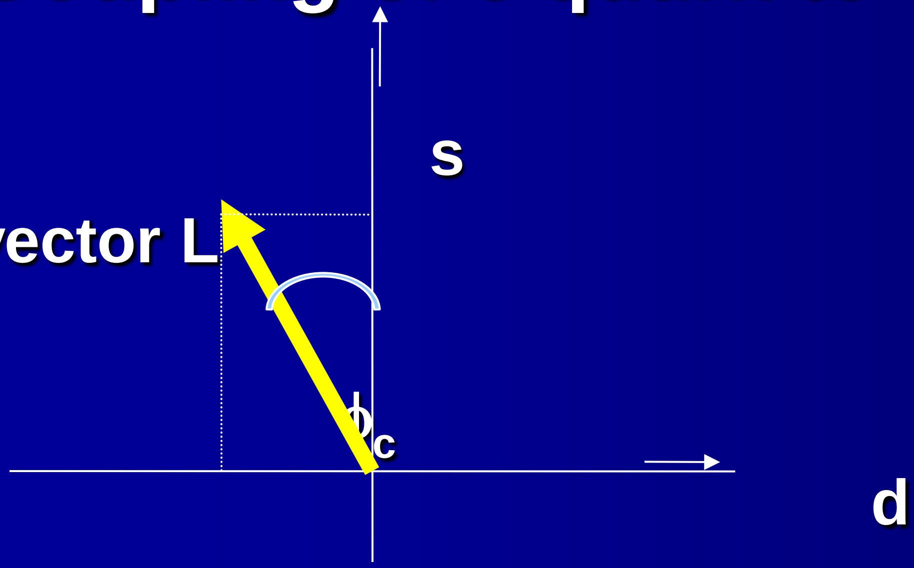
$a^2+b^2=L$

-> couplings usW ~b=g sin (ϕ_c) and udW ~a=g cos (ϕ_c)

Exp: ϕ_c=12.7° *(Nicola Cabibbo 1963)*

Decay of c quark to s and d

Coupling of c quark to s and d (facts)



So, couplings cdW ~ - g sin (ϕ_c) and csW ~ g cos (ϕ_c)

minus!

Cabibbo angle – mixing between the 1st & 2nd family of quarks

- Cabibbo angle needed to describe data – Theory ? – no prediction

- In fact a mixing is among 3 families.. 3 x 3 matrix (unitary)
  → Cabibbo–Kobayashi–Maskawa matrix (CKM)
  4 parameters:
  3 angles (including Cabibbo angle) and a phase

Kobayashi and Maskawa postulate such matrix in 1973 r before discovery of the 3d family.

Data needed a phase.. (it signals CP violation)

Matrices for mixing of quarks

- 2x2 matrix (rotation of vektor L)

$$\begin{pmatrix} d \\ s \end{pmatrix} \rightarrow \begin{pmatrix} \cos\phi_c & \sin\phi_c \\ -\sin\phi_c & \cos\phi_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- 3 x 3 matrix (CKM matrix)

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} \rightarrow \begin{pmatrix} . & . & . \\ . & . & . \\ . & . & . \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Similar mixing for up quarks: u, c, t

Kobayashi & Maskawa

Nobel' 2008

.

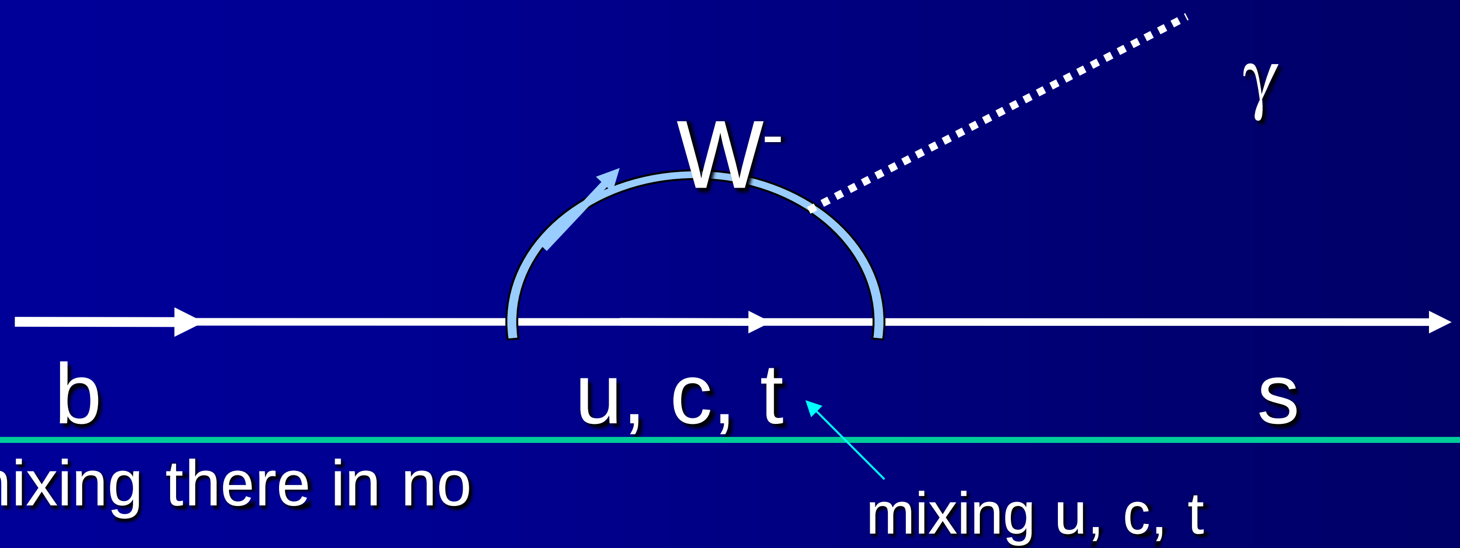
Why Cabibbo did not get Nobel prize ?!

The prize was not for a mixing but for an observation that starting with 3 family of quarks there appears a phase in the mixing matrix needed for violation of CP in kaon decays…

Example: mixing in decay $b \to s\gamma$

Very precise measurement and SM prediction

($Br \sim 10^{-4}$)



Without mixing there in no such process!

Besides b W^- t we have b W^-u and bW^-c (and sW^-c,sW^-u, sW^-t)

V_{CKM}

K.A. Olive *et al.* (PDG), Chin. Phys. **C38**, 090001 (2014

- Matrix (with one phase)

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\bar{\eta} = 0.354 \pm 0.015$$

- For the magnitudes

$$V_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00014 & 0.22536 \pm 0.00061 & 0.00355 \pm 0.00015 \\ 0.22522 \pm 0.00061 & 0.97343 \pm 0.00015 & 0.0414 \pm 0.0012 \\ 0.00886^{+0.00033}_{-0.00032} & 0.0405^{+0.0011}_{-0.0012} & 0.99914 \pm 0.00005 \end{pmatrix}$$

No mixing for transition via Z boson

Transition without changing of el. charge

$d \rightarrow d\,Z$

Mixing ? $d \rightarrow s\,Z$? NO!

Flavour changing neutral current - FCNC

Why? No answer

Mixing of leptons? NO if neutrinos massless

A comment – mixing of gauge bosons?

Yes,

Z boson and photon are combinations of the initial gauge bosons of two groups: SU(2) and U(1) $\rightarrow$

Weinberg angle to describe this mixing

„Mixture called Z" couples to neutrins,
while „mixture called photon" does not
(destruction of the corresponding prob. amplitude !)

Cabibbo angle: u $\rightarrow$ d W+ versus $\nu_e \rightarrow$ e W+
Weinberg angle: u $\rightarrow$ d W+ versus u $\rightarrow$ u Z

Quark mixing and high energy behaviour

Calculation of probability using Feynman rules

- To each line and vertices in Feynman diagram a factor is assigned. Below we track only the energy E.
  Note, that probability can not grow ~ E: excess over 1 possible!?

- Incoming or outgoing photon
  (and each spin 1 particle) – a factor E
  Virtual photon (spin1 particle) – a factor $1/E^2$

- Incoming or outgoing spin ½ particle - a factor $\sqrt{E}$,
  virtual spin ½ particle – a factor $1/E$

- Incoming or outgoing spin 0 particle – a factor 1,
  virtual spin 0 particle – a factor $1/E^2$

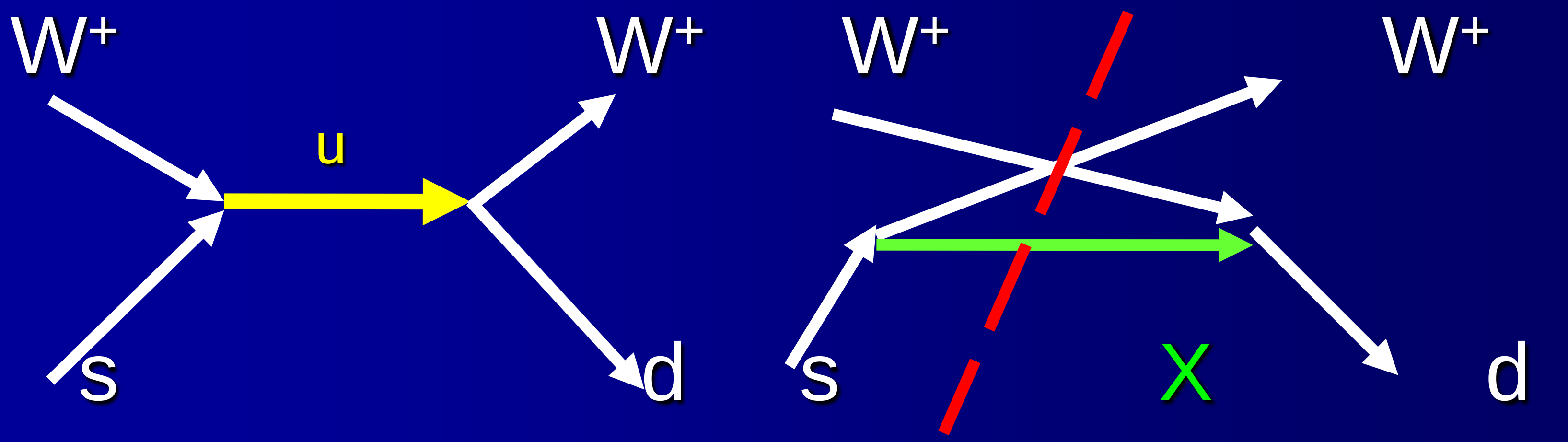
- Additional factors from couplings ….,

Product of factors→probability amplitude A

(probability = $|A|^2$)

Scattering of boson W on quark- behaviour at large energies

W+ collides with s quark – two diagrams

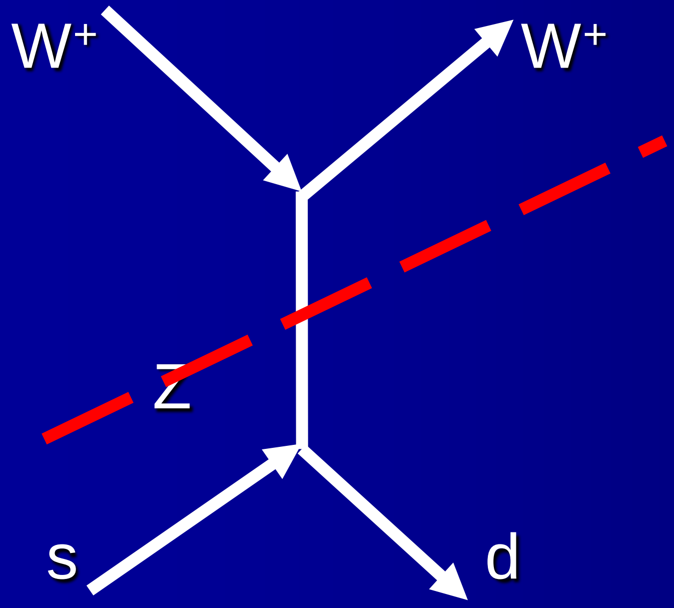


El. charge conservation: particle X with el. charge - 4/3 does not exist !

So, bad behaviour (amplitude): $E^2 (\sqrt{E})^2 \, 1/E = E^2$

Diagram with Z boson – no help here!

Diagram with Z boson Z?

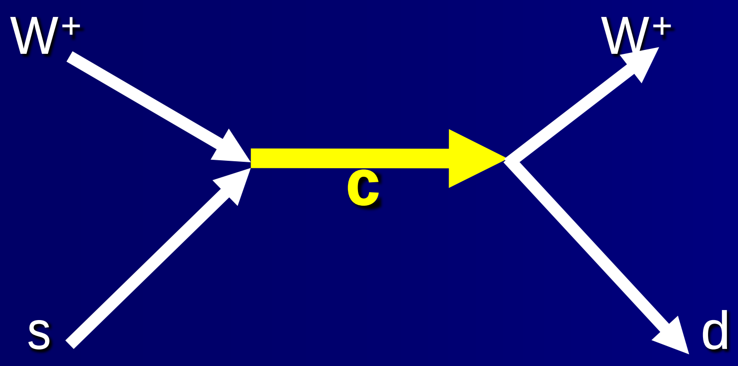


This process does not exist –

„absence of FCNC"!

New diagram with c quark

(possible only if mixing of s and d quarks)



Couplings:

cs W	$\cos(\phi_c)$
cd W	$-\sin(\phi_c)$

due to minus sign a cancelation of bad high energy behaviour for a process

$W+\ s \rightarrow u \rightarrow W+\ d$

Comment on discovery of c quark

- So to the process $W^+ s \rightarrow u \rightarrow W^+ d$

we add process with c - quark

$W^+ s \rightarrow c \rightarrow W^+ d$

(these are various channels of the process $W^+s \rightarrow W^+d$)

- In fact in 1964 this was only a hypothesis about existence of c quark with fixed properties, (including Cabibbo angle) – so that terms $\sim E^2$ cancel.
- c quark discovered in 1974 has these properties

success of theory !!!

Quark mixing

- Exists
- Important
- Described – but not understand

→

- Flavour problem !

Questions to lecture 11

- Does Cabibbo angle describe mixing between d and s quarks or between u and d quarks ?
- Value of Cabibbo angle is equal to ?
- Write a coupling of the c quark to d quark assuming mixing only between two lighter families. What is its sign?

- What discribes the Cabibbo-Kobayashi-Maskawa matrix?
- Do we observe a mixing between b and d quarks?
- How many parameters has the Cabibbo-Kobayashi-Maskawa matrix?
- Did N. Cabibbo got Noble prize in 2008r?
- Is a transition of the u quark to c quark: $u \rightarrow c\, Z$ possible ?
- How one can determine the Cabibbo angle ?
- Does the Weinberg angle describe quark mixing?
- Do we have FCNC?
- Is the probability of transition $c \rightarrow s\, W+$ equal to probability of the transition $\nu_e \to e^- W^+$?
- Does photon couple to neutrinos?