Mixing, neutrinos and neutrino oscillations

Particles and Universe

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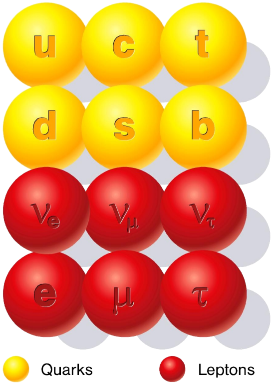
Particle contents of the Standard Model



Fermion flavour always conserved in strong and electromagnetic interactions:

$$\begin{array}{ccc} e^- & \to & e^- + \gamma \\ u & \to & u + g \end{array}$$

Also conserved in weak interaction via Z° exchange, so called Neutral Current (NC) interactions:

$$\nu_e \to \nu_e + Z^0$$

(allowed vertex)

Flavour Changing Neutral Current processes forbidden in SM!

Particle contents of the Standard Model

Fermion families/generations - SU(2) doublets:

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

Relevant only for processes with $W^{\pm}$ exchange,
so called Charged Current (CC) processes.

In the first approximation, $W^{\pm}$ exchange acts only within SU(2) doublets:

$$\begin{aligned} e^- &\rightarrow \nu_e + W^- \\ u &\rightarrow d + W^+ \end{aligned}$$

Cabibbo angle

Quark mixing had to be introduced to explain $K^{\pm}$ and $\pi^{\pm}$ lifetimes, as compared to $\mu^{\pm}$ lifetime. $K^- = (s\bar{u})$, $\pi^- = (d\bar{u})$

Consider decay channels:

$$K^- \rightarrow \pi^\circ + e^- + \bar{\nu}_e$$
$$\pi^- \rightarrow \pi^\circ + e^- + \bar{\nu}_e$$

compared to

$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$$

On fundamental level:

$$s \rightarrow u + W^-$$
$$d \rightarrow u + W^-$$
$$\mu \rightarrow \nu_\mu + W^-$$

Cabibbo realized that CC interactions couple u quark to combination

$$d' = d\cos\theta_C + s\sin\theta_C, \quad \theta_C - \text{Cabibbo angle}$$

$\Rightarrow \pi^-$ and K^- decays suppressed by $\cos^2\theta_C$ and $\sin^2\theta_C$ relative to μ^-

Cabibbo theory (1963)

With his model Cabibbo was able to explain all observed weak decays

However, assumed quark mixing resulted also in prediction of new processes with change of quark flavour in NC interaction

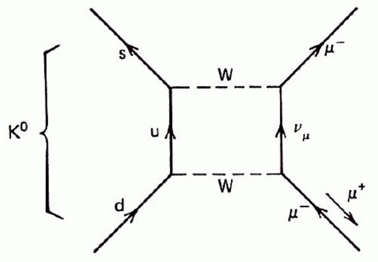
Flavour Changing Neutral Currents

Expected decay channel:

$$K^\circ \rightarrow \mu^+\mu^-$$

On quark level:

$$d\,\bar{s} \rightarrow \mu^+\mu^-$$



Such decays were not observed !?...

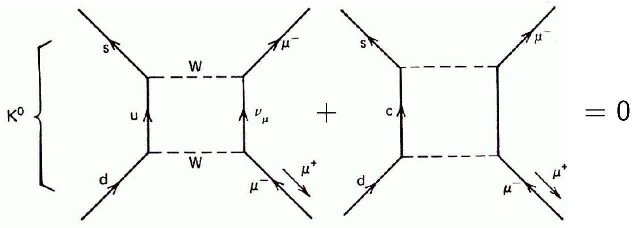
c quark

In 1970 Glashow, Iliopoulos and Maiani (GIM) postulated the existence of the fourth quark c.

Contribution from the c exchange cancels the contribution from the u quark to FCNC decay amplitude

In the GIM model CC interactions couple u with d' and c with s':

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$



$$\sin\theta_C \cos\theta_C \quad + \quad -\sin\theta_C \cos\theta_C \quad = 0$$

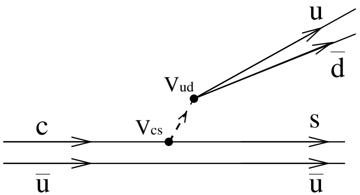
c quark was only discovered in 1974...

CKM matrix (1972)

For six quarks (three families) mixing matrix is 3×3.
CKM - Cabibbo, Kobayashi , Maskawa

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Example: decay of D° meson ($c\bar{u}$)



$$3.8\% \quad D^\circ \rightarrow K^-\pi^+ \sim |V_{cs}V_{ud}|^2 \sim \cos^4\theta_C$$
$$0.015\% \quad D^\circ \rightarrow \pi^-K^+ \sim |V_{cd}V_{us}|^2 \sim \sin^4\theta_C$$

Current experimental values (PDG'2016):

$$V_{\rm CKM} = \begin{pmatrix} 0.97434^{+0.00011}_{-0.00012} & 0.22506 \pm 0.00050 & 0.00357 \pm 0.00015 \\ 0.22492 \pm 0.00050 & 0.97351 \pm 0.00013 & 0.0411 \pm 0.0013 \\ 0.00875^{+0.00032}_{-0.00033} & 0.0403 \pm 0.0013 & 0.99915 \pm 0.00005 \end{pmatrix}$$

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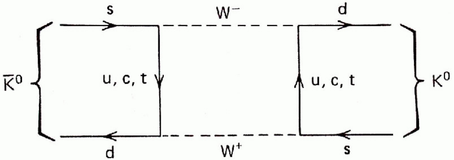
Particle-antiparticle mixing

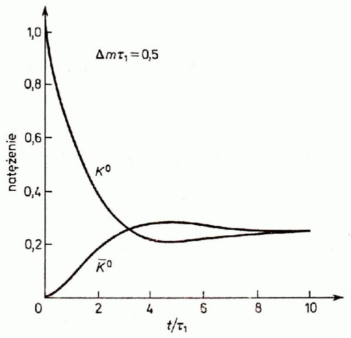
Neutral mesons produced in strong interactions are quark-antiquark pairs, eg. K° ($\bar{s}d$) or $\bar{K}^\circ$ ($s\bar{d}$)

These two states differ only in strangeness, which is not conserved in weak interactions

⇒ possible transition $K^\circ \leftrightarrow \bar{K}^\circ$



After K° is produced, it can either decay or oscillate to $\bar{K}^\circ$:



It turns out that K° and $\bar{K}^\circ$ are not "well defined" physical states !
We can not define masses or lifetimes for them !...

Mass eigenstates

Physical states with well defined mass and lifetime are

$$\text{short-lived} \quad |K_S\rangle \approx \frac{1}{\sqrt{2}}\left(|K^\circ\rangle + |\bar{K}^\circ\rangle\right) = \frac{1}{\sqrt{2}}\left(|d\bar{s}\rangle + |s\bar{d}\rangle\right) \quad CP = +1$$

$$\text{long-lived} \quad |K_L\rangle \approx \frac{1}{\sqrt{2}}\left(|K^\circ\rangle - |\bar{K}^\circ\rangle\right) = \frac{1}{\sqrt{2}}\left(|d\bar{s}\rangle - |s\bar{d}\rangle\right) \quad CP = -1$$

for simplicity we neglect CP violation effects...

If we produce a strong-eigenstate (K° or $\bar{K}^\circ$), we produce a mixture:

$$|K^\circ\rangle = \frac{1}{\sqrt{2}}\left(|K_S\rangle + |K_L\rangle\right)$$

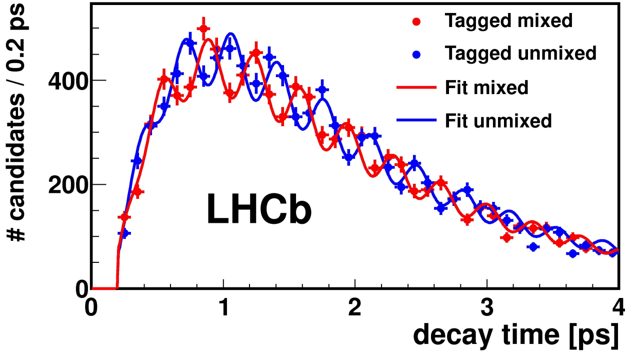
But there is a very tiny difference in mass between K_S and K_L !

They propagate in space with slightly different velocities and their relative phase changes! That is why we observe oscillations...

$B_s^\circ - \bar{B}_s^\circ$ oscillations

Oscillations are also observed for other heavy neutral mesons: D°, B°, B_s°.

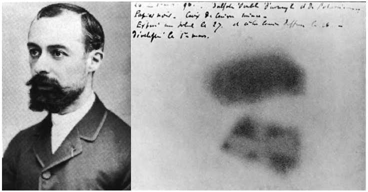
LHCb results showing $B_s^\circ - \bar{B}_s^\circ$ oscillations (2013)



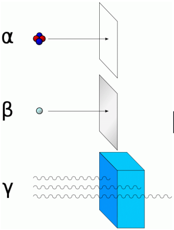
Natural radioactivity

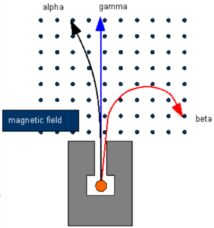
Discovered by Henri Becquerel in 1896.

Nobel prize for H.Becquerel, M.Sklodowska-Curie and P.Curie in 1903.



Three types of radiation were recognized, based on penetration of matter and bending in the magnetic field: α, β, γ.





Radioactivity described as transformation of elements. Atoms considered elementary at the begining...

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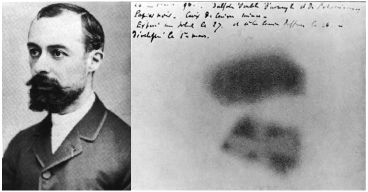

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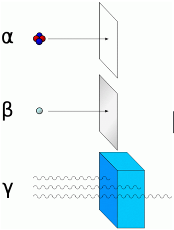
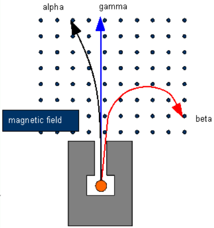




Radioactivity described as transformation of elements. Atoms considered elementary at the begining...

Successful description of α and γ decays.

Problem with β decays:

energy and momentum conservation seemed violated...

Introduced by Pauli to preserve energy conservation in β decays

eg.: $^{60}_{28}Co \rightarrow {}^{60}_{29}Ni^{\star} + e^{-} + \bar{\nu}_e$

on particle level:

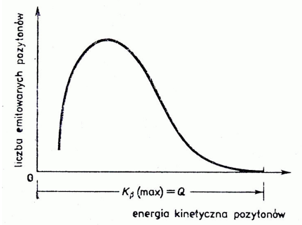
$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

Required properties:

- very weak interactions
- negligible mass

We assumed that neutrinos are massless...

Continuous β energy spectra:



Direct mass constraints (95% CL):

$$m_{\nu_e} < 2.2\ eV \approx 4.3 \cdot 10^{-6}\ m_e$$
$$m_{\nu_\mu} < 170\ keV \approx 0.0018\ m_\mu$$
$$m_{\nu_\tau} < 15.5\ MeV \approx 0.01\ m_\tau$$

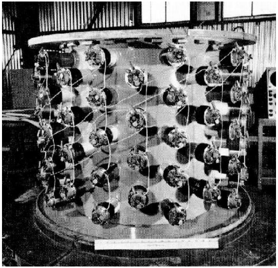
Much stronger mass constraints from astrophysics and cosmology...

Reines-Cowan experiment (1953-1959)

First observed were neutrinos coming from the nuclear reactor.
Neutrino production in neutron decays:

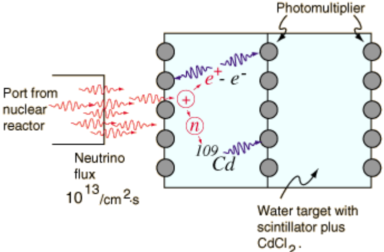
$$n \rightarrow p\, e^- \,\bar{\nu}_e$$

Detector:



Detection process:

$$\bar{\nu}_e\, p \rightarrow n\, e^+$$



- positrons annihilate emitting photons
- neutrons are captured by cadmium atoms ⇒ delayed photon emission
- coincidence of two signals allows for sufficient background suppression

Standard Model

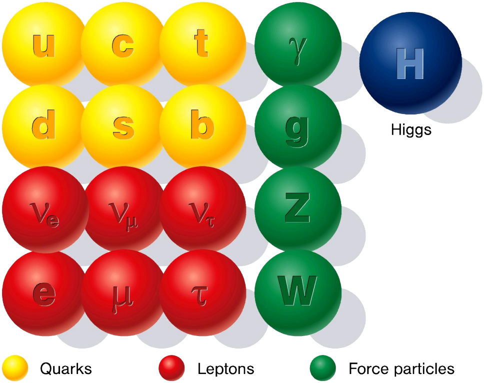
Laboratory measurements have shown that each charged lepton couples to distinct neutrino

$\Rightarrow$ three neutrinos flavours

Until recently we believed that

- all neutrinos are massless
- lepton flavour is conserved

But conservation of the lepton number is not imposed by theory - it was assumed based on experimental evidence...



Neutrinos

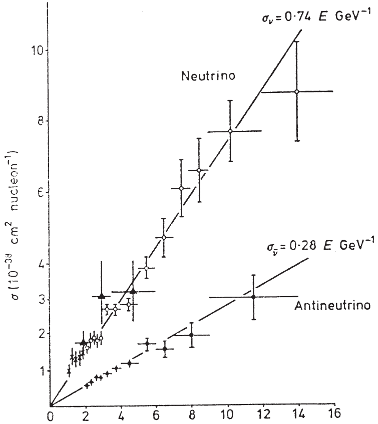
Interactions

Cross section for neutrino interaction in matter is incredibly small.
For energy of the order of 1 MeV

$$\sigma_{\nu N} \quad \sim \quad 10^{-43} \; cm^2$$

$\Rightarrow$ average free path in matter (!) of the order of light years !!!
For comparison $\sigma_{NN} \sim 10^{-24} \; cm^2$

Cross section grows with energy, but only linearly...



We need very strong sources and very large detectors to study neutrinos...

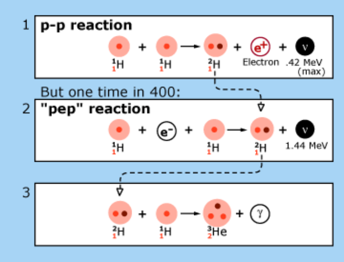
Sun

Sun is an extremely intense neutrino source.
Most neutrinos come from **p–p** reaction:

$$p + p \rightarrow D + e^{+} + \nu_e \; (E_\nu \leq 0.42 \; MeV)$$

Slightly higher energies in **“pep”** process:

$$p + e^{-} + p \rightarrow D + \nu_e \; (E_\nu \approx 1.44 \; MeV)$$



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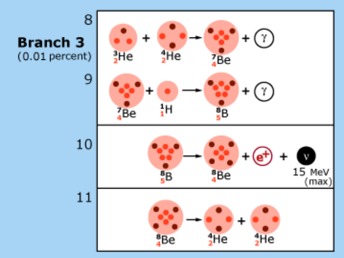
Highest energies result from ^{8}B decay

$$^{8}_{5}B \quad \rightarrow \quad ^{8}_{4}Be \; + \; e^{+} \; + \; \nu_e$$

where neutrino energy can reach 15 MeV

Only these neutrinos can be observed in particle detectors...



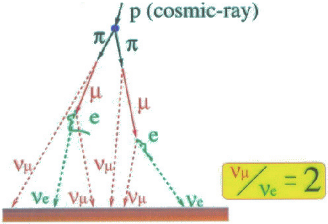
Cosmic rays

Primary cosmic rays consist mainly of high energy protons and light nuclei, with energies reaching 10^{12} GeV (10^{21} eV).

They interact with O and N nuclei in the atmosphere creating a cascade of secondary particles, most of them are pions $\pi^{\pm}$.

Neutrinos result from decay chain:

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$
$$\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$$



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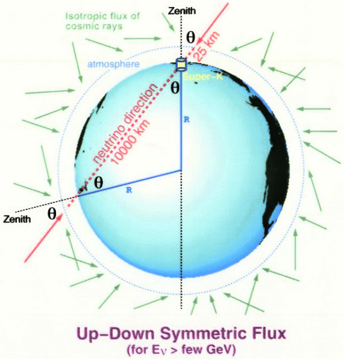

As the primary cosmic radiation is isotropic, we expect that atmospheric neutrino flux will also be isotropic !

Nuclear reactors

Fission of single ^{238}U nucleus produces energy of about 200 MeV
On average, six electron anti-neutrinos are produced (per fission), mainly in neutron decay: $n \rightarrow p + e^- + \bar{\nu}_e$

Reactor with 1GW power $\Rightarrow \sim 10^{20}$ fissions per second $\Rightarrow \sim 6 \cdot 10^{20}\ \nu/s$

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Accelerators

We can produce high energy, high current proton beams.
When interacting with dense target, protons produce secondary pions and kaons (as in the cosmic ray interactions). Muon neutrinos are produced in:

$$\begin{aligned} \pi^+ &\to \mu^+ + \nu_\mu \\ K^+ &\to \mu^+ + \nu_\mu \end{aligned}$$

By focusing produced pions and kaons prior to their decay, we can obtain high energy neutrino (or anti-neutrino) beams...

Comparison

source	flavour	energies	flux
Sun	ν_e	$\leq$ 15 MeV	$6 \cdot 10^{10} \frac{1}{s\ cm^2}$ (on Earth)
Cosmic rays	ν_e, ν_μ, $\bar{\nu}_e$, $\bar{\nu}_\mu$	100 MeV - TeV	$3 \cdot 10^7 \frac{1}{s\ cm^2}$ (above 10 MeV)
Reactors	$\bar{\nu}_e$	MeV range	$5 \cdot 10^9 \frac{1}{s\ cm^2}$ (1 GW from 1 km)
Beams	ν_μ, $\bar{\nu}_\mu$	GeV range	10^{20} (total sample)

All these sources are used nowadays to study neutrino properties

Detector

Neutrino detector built in an old mine, under Mt Kamioka in Japan.

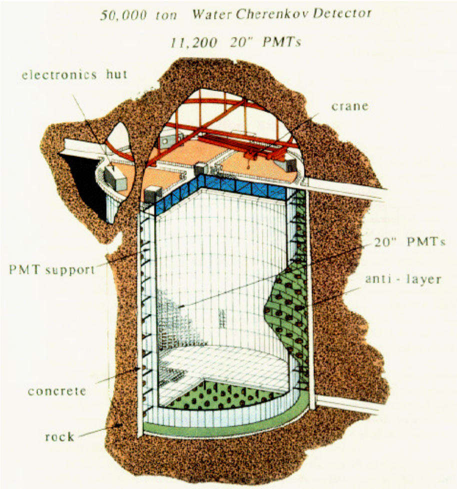
Steel tank with 50,000 tons of ultra-pure water.

Look for neutrino interactions:

$$\nu_l \, n \quad \rightarrow \quad l^- p$$

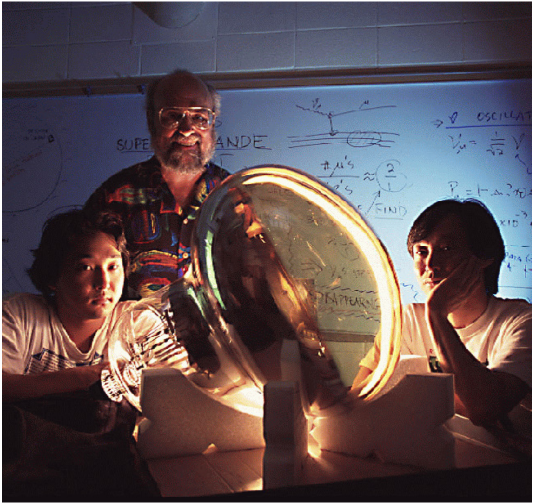
$\sim$11'000 photomultipliers installed to measure Cherenkov radiation

Radiation of charged particles traveling with velocity greater than speed of light (in water).

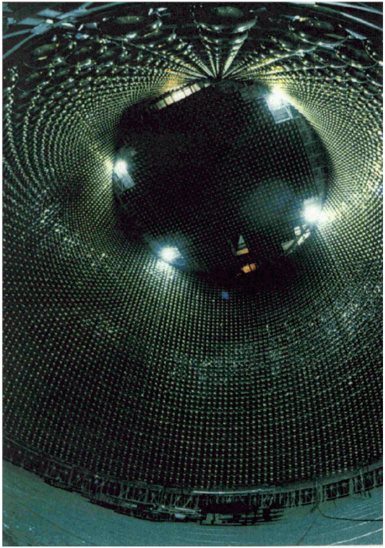


Super Kamiokande

Photomultiplier



Detector



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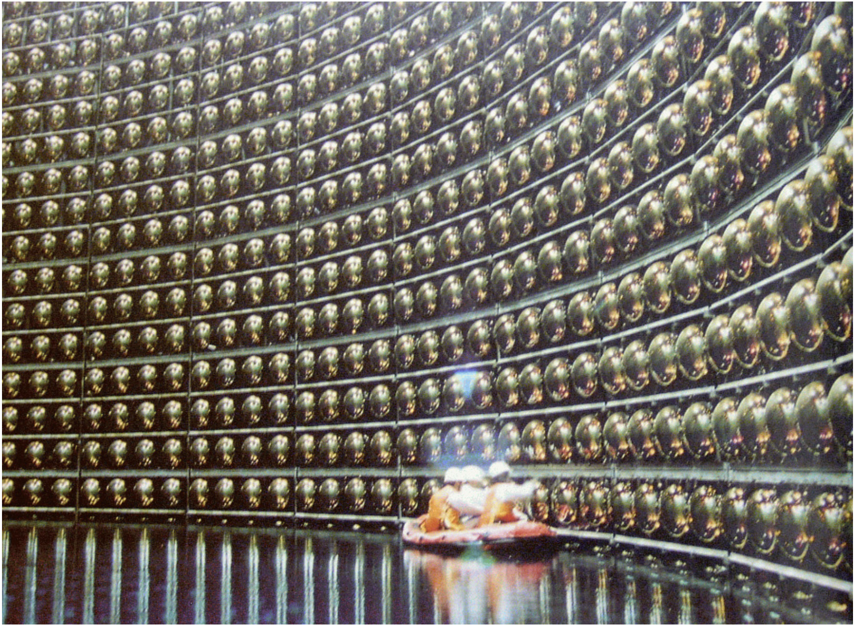
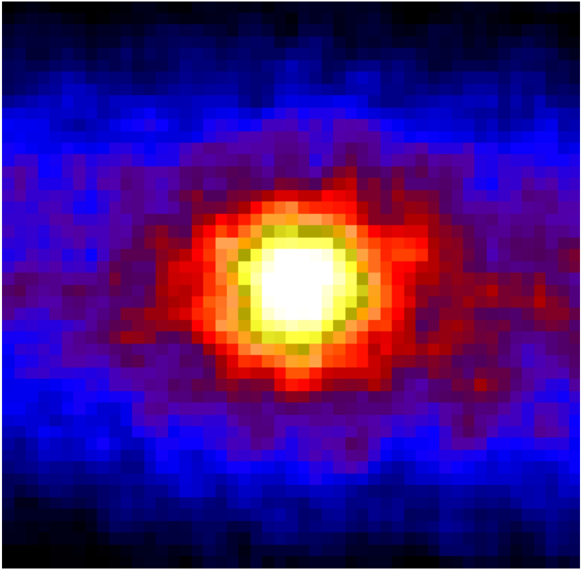


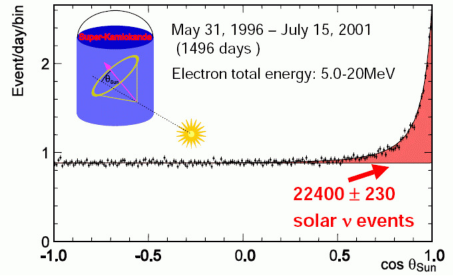
Image of Sun
reconstructed by Super Kamiokande from the measured neutrinos

actual Sun size $\sim \frac{1}{2}$ pixel



Solar neutrinos observation

Solar neutrinos clearly identified by their direction (w.r.t. Sun position)



Flux measured by SK:

$$\Phi_S^{(B)} = 2.4 \pm 0.1 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

Expected from Solar Model:

$$\Phi_S^{(B)} = 5.3 \pm 0.6 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

Two processes used in neutrino detection: at energies $\sim$ 10 MeV

- Charged Current (CC) scattering: $\nu_e + N/e^- \rightarrow e^- + X \Rightarrow$ only ν_e
- Neutral Current (NC) scattering: $\nu_l + N/e^- \rightarrow \nu_l + X \Rightarrow$ all ν's

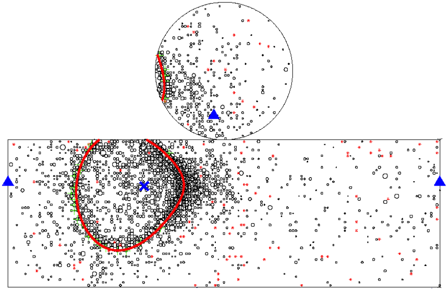
But NC cross section much smaller $\Rightarrow$ Flux measurement restricted to ν_e

Super Kamiokande

Thanks to very sensitive readout and low background, discrimination between different neutrino flavours possible for high energy neutrinos

Electron neutrino

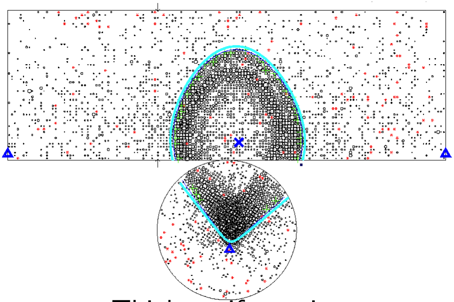
$\nu_e\ n \rightarrow e^- p$ event



Thin, irregular ring

Muon neutrino

$\nu_\mu\ n \rightarrow \mu^- p$ event



Thick, uniform ring

Electron is much lighter than muon and scatters/interacts much more

Atmospheric neutrino observations

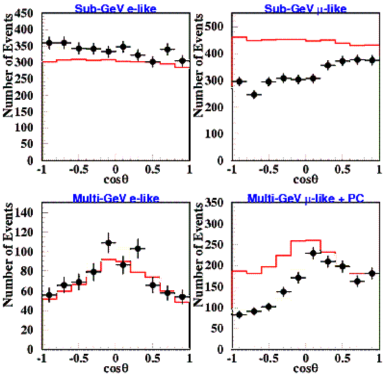
Measured neutrino direction for electron neutrinos (left) and muon neutrinos (right)

Similar number of electron neutrinos going down ($\cos\theta > 0$) and going up ($\cos\theta < 0$) $\Rightarrow$ in agreement with predictions (red)

Clear deficit of muon neutrinos!

Direction measured best for high energy $\Rightarrow$ deficit of up-going ν_μ

Can muon neutrino "disappear" in Earth ?!



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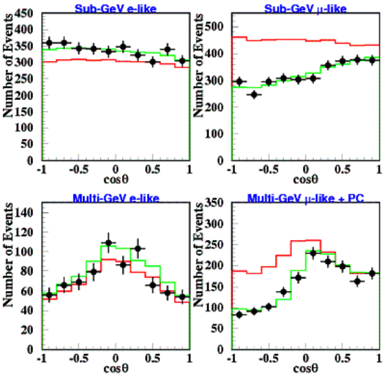


No! We can only explain it assuming neutrinos oscillate!
Predictions of oscillation model indicated by green histogram

Neutrino mixing   for two neutrino flavours

We have to distinguish between two different neutrino state definitions

- flavour eigenstates - neutrinos with defined lepton flavour, as produced in weak interactions: ν_e, ν_μ
- mass eigenstates - free neutrinos with well defined mass, as propagating in vacuum: ν_1, ν_2

Flavour eingenstates can be described as a mixture of mass eigenstates:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Assuming ν_e is produced at $t = 0$, the neutrino state can be written as:

$$|\nu\rangle_0 = \cos\theta_{12} |\nu_1\rangle_0 + \sin\theta_{12} |\nu_2\rangle_0 = |\nu_e\rangle$$

If neutrino were massless, $|\nu_1\rangle$ and $|\nu_2\rangle$ would travel with the same velocity, and neutrino state would not change: $|\nu\rangle_t \equiv |\nu_e\rangle$

Neutrino mixing for two neutrino flavours

Free states $|\nu_i\rangle$ evolve with time:

$$|\nu_i\rangle(t,\vec{x}) = |\nu_i\rangle_0 \cdot \exp\left(-\frac{i}{\hbar}(Et - \vec{p}\,\vec{x})\right)$$

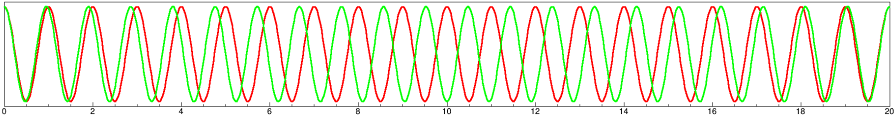
$\Rightarrow$ **If** $m_1 \neq m_2$ then the relative phase of $|\nu_1\rangle$ and $|\nu_2\rangle$ states changes and

$$|\nu\rangle(t,\vec{x}) = a(t)\,|\nu_1\rangle + b(t)\,|\nu_2\rangle \neq |\nu_e\rangle(t,\vec{x})$$

But we can decompose it into flavour eigenstates

$$|\nu\rangle(t,\vec{x}) = A(t)\,|\nu_e\rangle + B(t)\,|\nu_\mu\rangle$$

Flavour contributions change in time $\Rightarrow$ oscillations



Survival probability for two neutrino flavours

Probability that ν_e produced at $t_0 = 0$ interacts as ν_e after time t

$$P_{\nu_e \to \nu_e}(t) \quad = \quad |A(t)|^2$$

Assuming mass differences are small, $|m_1 - m_2| \ll m_1 \sim m_2 \ll E_\nu$

$$P_{\nu_e \to \nu_e}(t) \quad = \quad 1 - \sin^2(2\theta_{12}) \, \sin^2 \left(\frac{(m_2^2 - m_1^2) \, t}{4E} \cdot \frac{c^4}{\hbar} \right)$$

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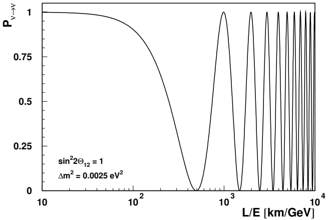
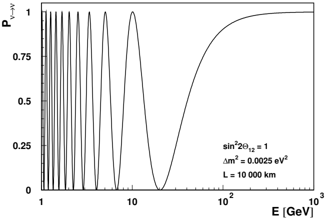
$$P_{\nu_e \to \nu_e}(t) \quad = \quad 1 - \sin^2(2\theta_{12}) \, \sin^2\left(1.27 \cdot \Delta m^2 \, \frac{L}{E}\right)$$

$L = ct$ [km], Δm^2[eV^2] i E [GeV].

As a function of distance:



As a function of energy: $(L \sim 2R_Z)$



For very large L/E $(E/L \ll \Delta m^2)$ we get: $P_{\nu_e \to \nu_e} \approx 0.5$

Neutrino mixing

For three flavours:

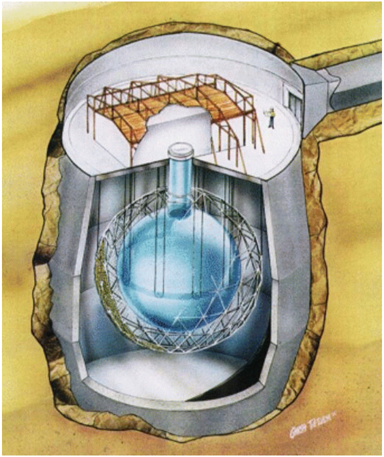
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} V_{11} & V_{12} & V_{13} \\ V_{21} & V_{22} & V_{23} \\ V_{31} & V_{32} & V_{33} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

PMNS - Pontecorvo-Maki-Nakagawa-Sakata matrix
corresponding to CKM matrix for quarks

$$U = \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix}}$$

$\nu_e \leftrightarrow \nu_\mu$	$\nu_\mu \leftrightarrow \nu_\tau$	$\nu_e \leftrightarrow \nu_\tau$
solar neutrinos	atmospheric	reactor, $\theta_{13} \sim 8^\circ$
$\theta_{12} \approx 35^\circ$	$\theta_{23} \approx 45^\circ$	established only in 2012!

SNO (Sudbury Neutrino Observatory)
confirmed the oscillation of solar neutrinos



Large tank with 7000 t of water ($H_2 0$)
Sphere with 1000 t of **heavy water** ($D_2 0$)

Cherenkov radiation measured by
9500 photomultipliers

By using heavy water detector is sensitive not only to electron neutrinos, but also to muon and taon neutrinos.

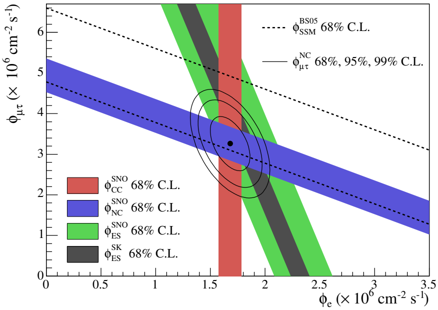
Flavour "blind" neutrino interaction (NC):

$$\nu_x + D \rightarrow \nu_x + p + n$$

Detector placed at depth of over 2000 m

SNO

Measured neutrino fluxes:



Total neutrino flux measured (blue band):

$$\Phi_{tot}^{SNO} = 5.1 \pm 0.5 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

in good agreement with solar model (dashed lines).

Independent measurement of ν_e flux (red band):

$$\Phi_{e}^{SNO} = 1.7 \pm 0.1 \cdot 10^6 \frac{1}{s \cdot cm^2}$$

$\Rightarrow$ only about $\frac{1}{3}$ of neutrinos coming from the Sun is detected as electron neutrinos $\Rightarrow$ ideal mixing of 3 flavours!

KamLAND

Build at the location of the Kamiokande experiment, predecessor of Super-Kamiokande.

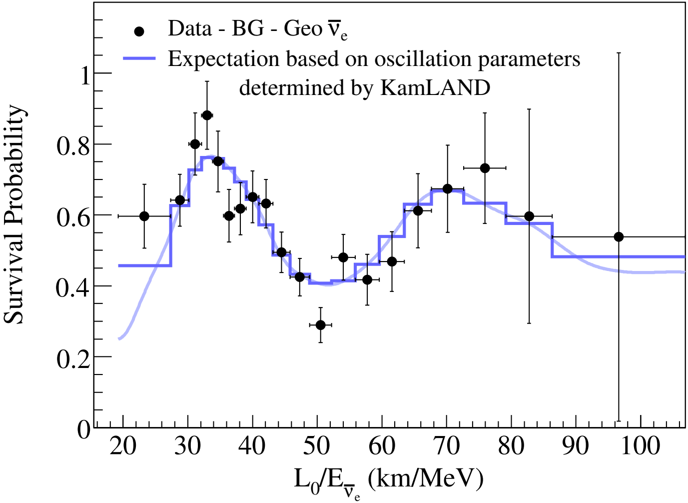
Focus on the measurement of neutrinos coming from surrounding (140-210 km) nuclear power plants

Detector similar to SNO:

- outer tank with 3200 t of water
- inner tank with 2000 t of oil
- central sphere with 1000 t of liquid scintillator
- signal detected by 2100 photomultipliers

KamLAND

First direct observation of neutrino “regeneration”

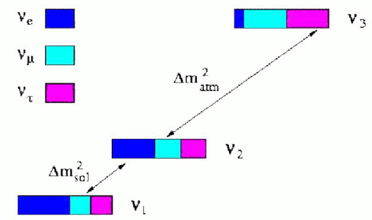


Summary

Neutrinos are produced as ν_e, ν_μ or ν_τ

But they propagate as superposition of mass eigenstates ν_1, ν_2 and ν_3

$\Rightarrow$ lepton flavour is not conserved, neutrinos oscillate



Oscillations are described by 3 mixing angles, 2 mass squared differences and a phase, which could be a source of CP violation in lepton sector.

Oscillations needed to describe a wide range of experimental results:

- deficit of solar electron neutrinos
- deficit of up-going muon atmospheric neutrinos
- oscillations of electron anti-neutrino flux in reactor experiments
- muon neutrino disappearance in accelerator experiments
- electron and taon neutrino appearance in accelerator experiments