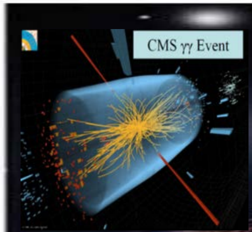

Particles and Universe

Lecture 11 Perturbative calculation

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I.Theory of elementary particles
- description of events;
- perturbative calculation
- renormalization

II. Probability of processes at high energy –
a need for Higgs particle

Description of events

- Relativistic and quantum effects. To describe creation and annihilation of particles and mixing as well as decays → formalism of

  Quantum Field Theory,QFT

  (applied first to the QED)

- Technique of QFT

  \- perturbative calculation:

  the lowest order of pert. series

  quantum corrections: trees and loops

  \- Feynman diagrams

Theory of elementary particles

- 1948 – new phase of quantum mechanics (QFT); precise measurements → a need of more precise calculations
- Feynman method: diagrams and Feynman's rules for calculation – today the universal tool of particle physics

first application in QED

- QED describes interaction of electron and photons. Difficulties since quantum corrections infinite!

Method of removing them → renormalization procedure.

- Weak interaction – even more difficult situation. Proposal: new interaction and new particles → EW interaction with gauge bosons W/Z with Higgs boson - renormalizable !

Nobel prize: Glashow, Salam, Weinberg 1979 (W/Z)
t'Hooft, Veltman 1999 (renormalizability)

Relativistic effects
- free and virtual particles

Relativistic effects (special theory of relativity)

- Einstein postulates:

  - Invariance of laws with respect to change of reference system (for the inertial systems)

  - Constant velocity of light c in the inertial systems

- Lorentz transformation relates measurements of two inertial reference frames

- The most „visible" relativistic effects
  – lifetime for particles moving with velocity close to c longer (*time dilatation*)

  Muons born in the upper atmosphere arrive at the surface of Earth only due this effect (660 m)

  -- similar effect for space (distance) (*length (Lorentz) contraction*)

Lorentz and Poincare invariance

- Invariance with respect to rotations (in space-time) → Lorentz invariance

  If in addition invariance with respect to translation in space and time → Poincare invariance

Not everything is relative !!

If decay (process) is forbidden in some reference frame it is forbidden in any frame

- However – for a description of a particular process

  → a particular frame maybe more convenient

Forbidden processes - examples

- For a *free* electron process $e \rightarrow e\,\gamma$ is forbbiden *Why?*

  Electron at rest has the lowest possible energy ($E = mc^2$), and cannot have lower energy after emission of photon. If it is not possible in one frame then it is not possible in any

  It is possible in matter

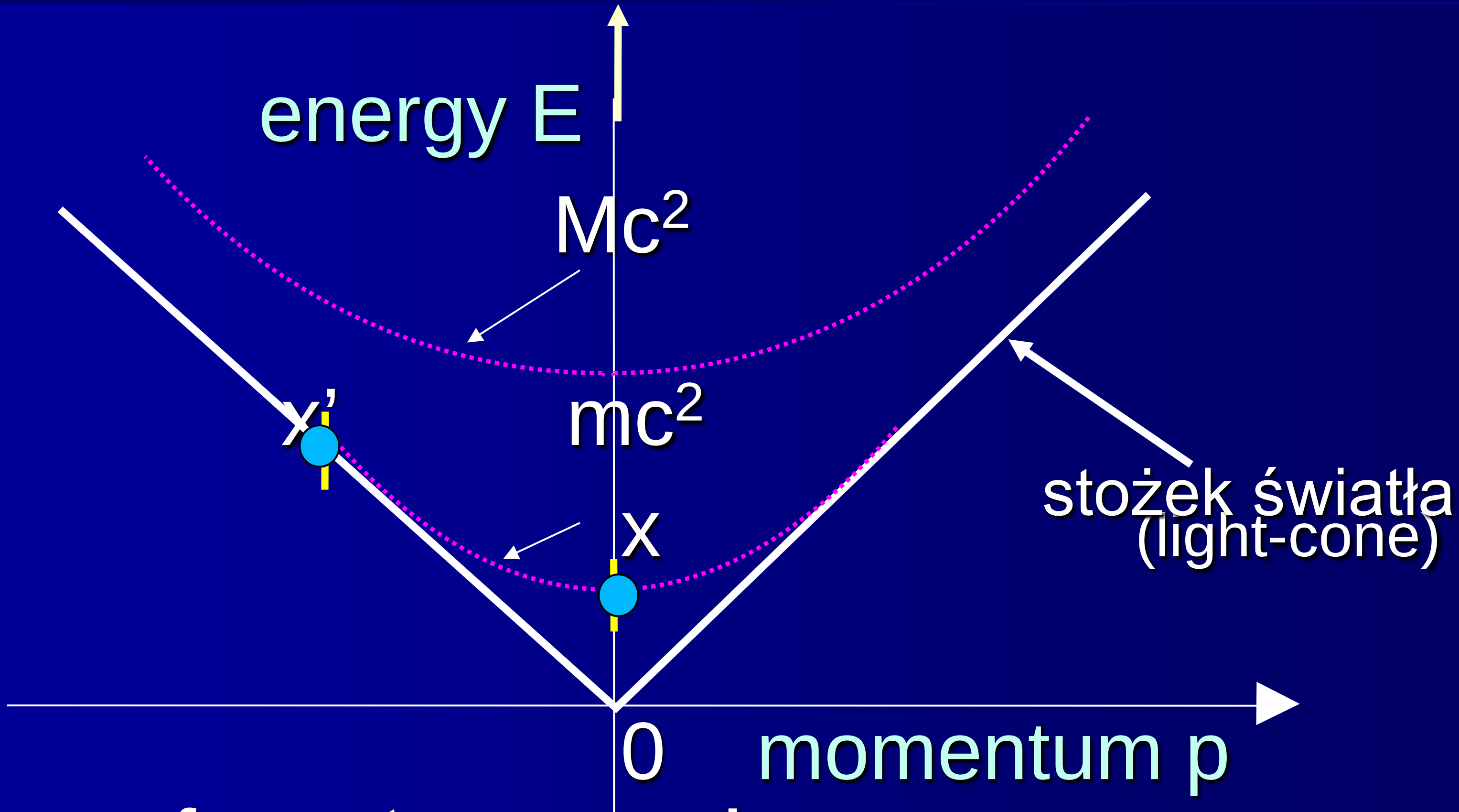
- For a *free* photon the decay $\gamma \rightarrow e^+e^-$ is not possible - observer moving towards a photon see it with a smaller momentum/energy ($E = pc$)...

  Possible in the matter

Energy and momentum for a *free* particle

Relation defines the **mass shell** for a particle with mass m

$$E=c\sqrt{p^2+m^2c^2}$$



Change of a system : x → x'

Particle on a mass shell = free particle

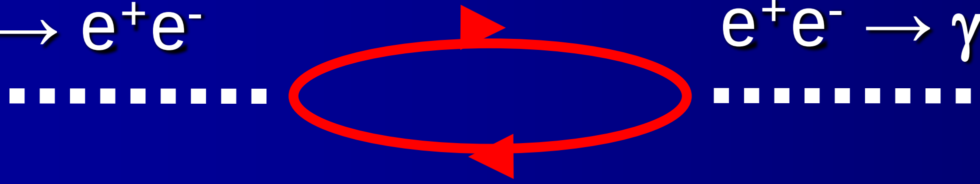
Inner particles in diagrams = virtual particles

Quantum mechanics allows for virtual particles - for whem

$$E^2 \neq p^2 + m^2 \ (c=1)$$

(particles *off* mass-shell)

They exist for a while and are not observed directly eg. they appear in the loops



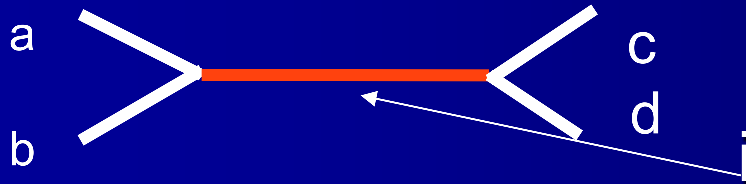
$\gamma \to e^+e^-$

$e^+e^- \to \gamma$

Examples- virtual particles

- In a collision of two particles a,b

two particles are produced c, d: a+b → c+d



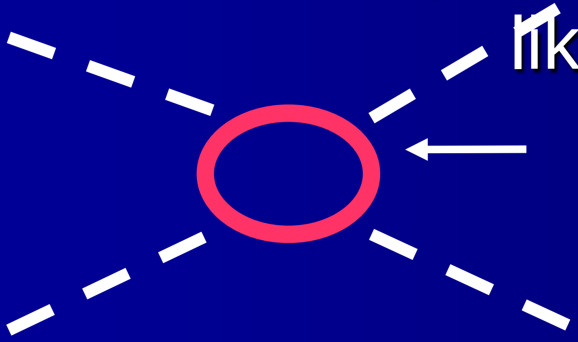
inner = virtual particle

There are various channels

(= various virtual exchange) *like in two split experiments*

- Some processes are possible only due to them– like light-on-light scattering

$\gamma \quad \gamma \to \gamma\ \gamma$



el. charged virtual particle

photons are neutral and can not interact with themseves

Probability of processes

Probability amplitude and probability of processes

- Knowing properties of particles we can calculate probability of processes they are involved in
- Initial particles we produce or prepare, but the final particles we can only observe – but there are many ways from the initial to the final particles

  (*channels of processes*)
- We calculate *probability amplitudes for each channel* → interference of the amplitudes
- Feynman diagrams for all channels of process → we sum probability amplitudes <u>not</u> probabilities!

$$|A|^2 = |\Sigma A_i|^2$$

Question which channel is like a question - which slit a photon used

Infinities

- Description of the process - all channels should be incuded, with possible virtual particles
- Virtual particles may have various energies, in loops even arbitrary energies and all these cases should be included.
- If contributions are not damped – a problem
  -> infinite probabilities for a given process!
- ... → problems with spin 1 particles, damping smaller for larger spin

(problem with graviton even worst)

QED:
infinities and renormalization

- Problems with photon (spin 1)? –
  In QED infinite contributions from various diagrams cancel - as was shown in 1948 by Feynman, Tomonaga, Schwinger (Nobel 1964) → renormalization procedure

  (original idea - Kramers in 1938)

- QED gives finite predictions - very precise, eg. for anomalous magnetic moment *(see below)*

Perturbative calculation

- Prediction for a process – we need to sum various contributions - with many vertices
- In principle -> series of infinite number of terms…
  - it is OK if next terms are small

  (small perturbation, perturbation series)

- Eg. in QED small coupling constant

  $\alpha = e^2/(4\ \pi)$ ~1/137

  so next term in the perturbative expansion ~ α^2 - 1%

- More terms in the pertubation series –

  including higher orders in the expansion in the coupling

  constant (higher powers of the coupling constant)

  –> higher precision of theoretical prediction

Anomalous magnetic moment for muon

Anomalous magnetic moment for muon (or $g\text{-}2|_\mu$)

The magnetic moment μ proportional to the spin *s*

spin and magn. moment

- vector quantities

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c}\,\vec{s} \;; \quad g_\mu = 2\left(1 + a_\mu\right)$$

For the fundamental particle with spin ½ the simplest act of the el-(magnetic) interaction → g = 2 (Dirac 1925) so deviation (or g-2) is called the anomalous magnetic moment

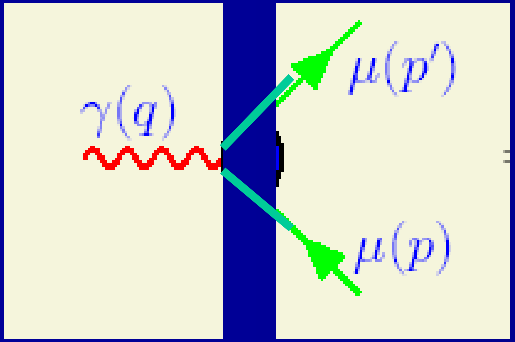
First measurement for electron in 1922! Next in 1948r

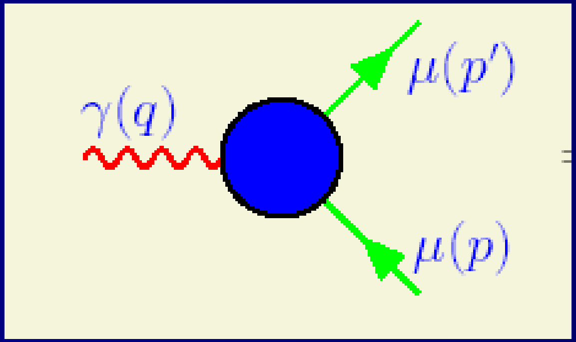
Stern, Gerlach 22: $g_e = 2$; **Kusch, Foley 48:** $g_e = 2\left(1.00119 \pm 0.00005\right)$

Origin of a_μ

(for muon)

From extra interactions …



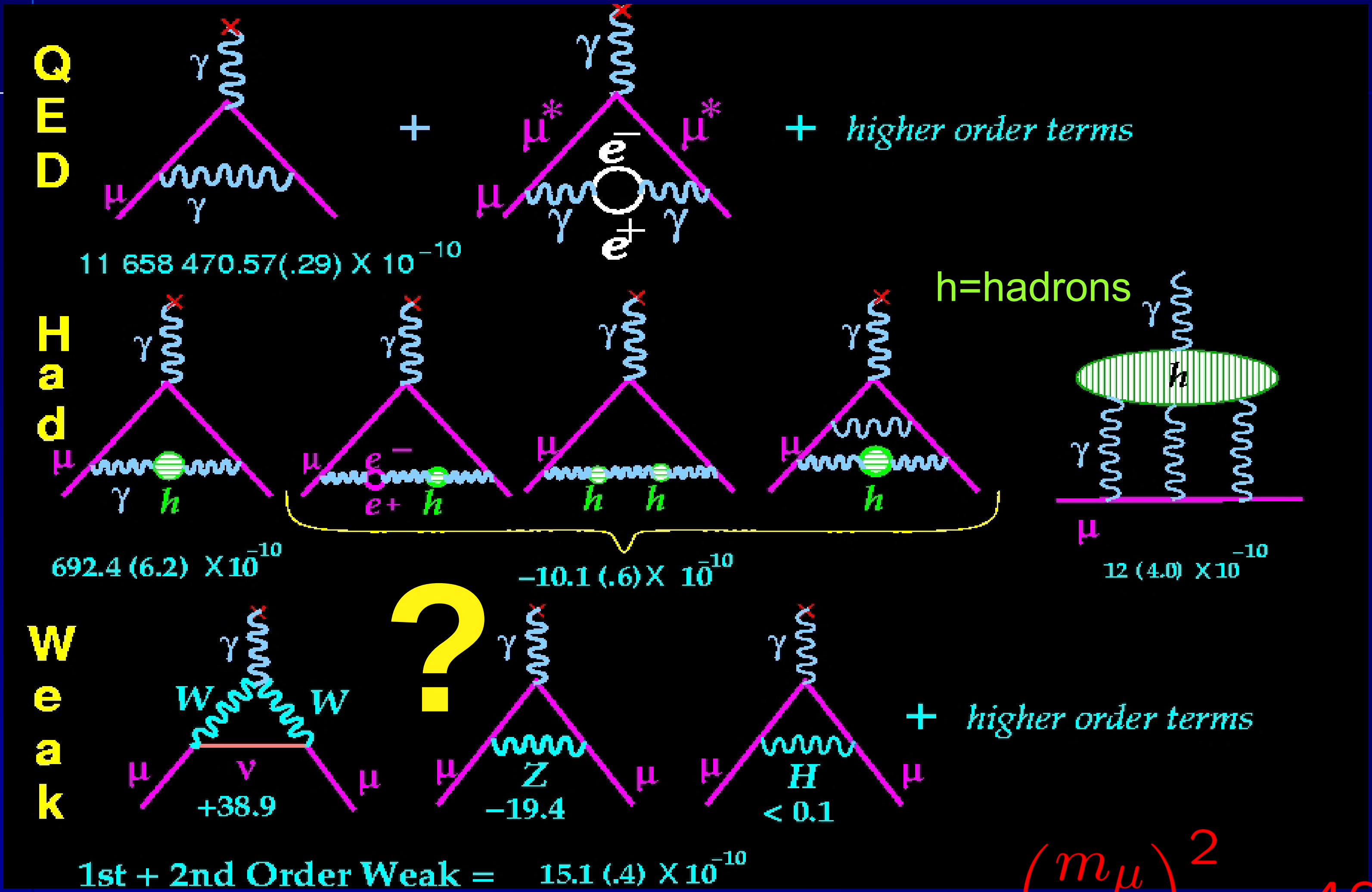


$g = 2$ if only this act

$a = g-2 \neq 0$ if more actions

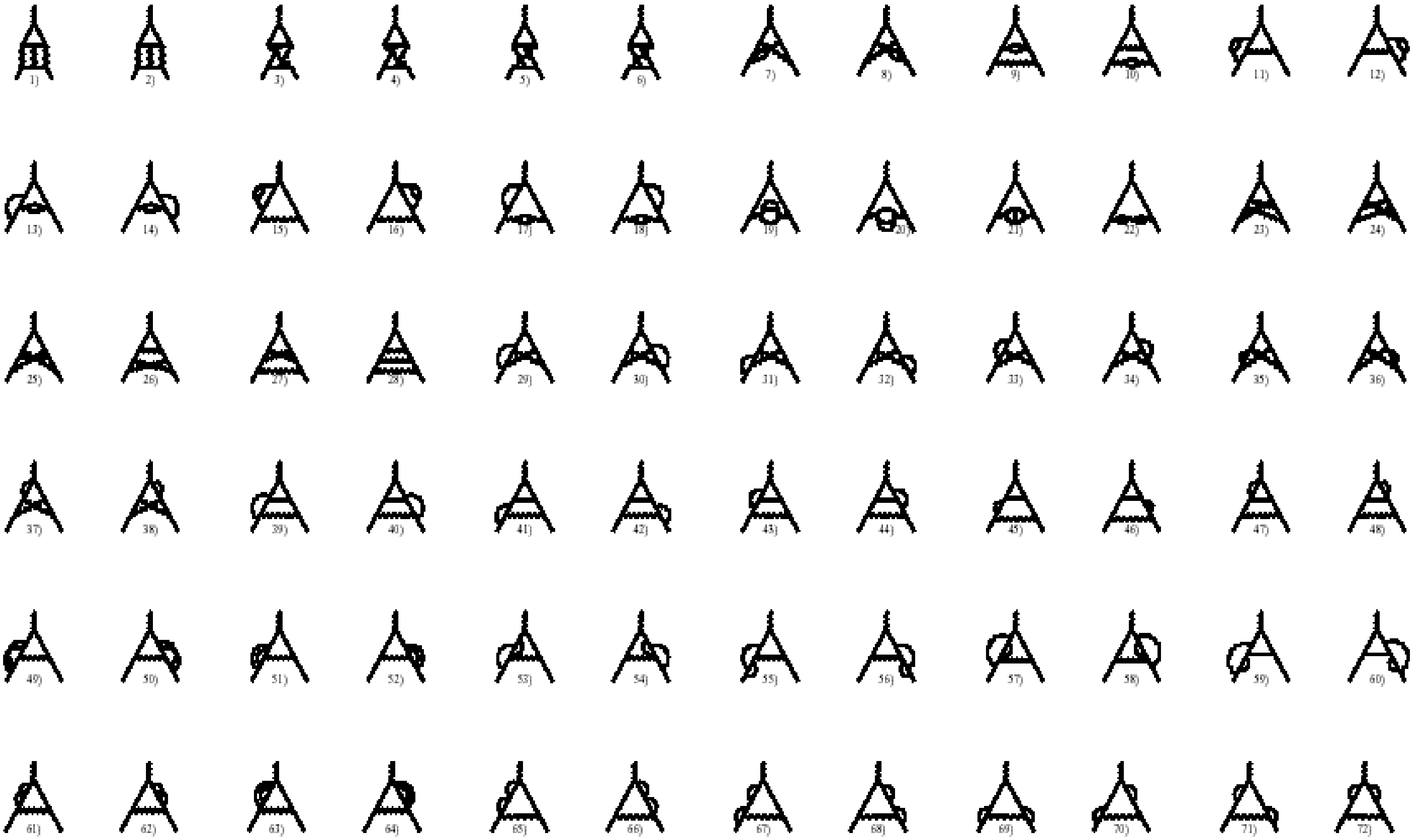
four- momenta: q, p, p'

Standard Model : QED, hadronic (h) and EW (W/Z i H) contributions



e vs. μ : relative contribution of heavier things $\left(\frac{m_\mu}{m_e}\right)^2 \simeq 40,000$

3- order of pert. calculation (QED)



QED Contribution a_μ^{QED}

$$\begin{aligned} a_\mu^{\mathrm{QED}} \cdot 10^{10} = \Sigma C_i (\tfrac{\alpha}{\pi})^i = & \quad 11614097.3 \ (\text{1-loop}) \\ + & \quad 41321.8 \ (\text{2-loop}) \\ + & \quad 3014.2 \ (\text{3-loop}) \\ + & \quad 38.1 \ (\text{4-loop}) \\ + & \quad 0.4 \ (\text{5-loop}) \end{aligned}$$

Terms up to α^3 are known analytically, a recent more accurate numerical calculation of the α^4 terms and the leading log α^5 terms gave (T. Kinoshita and M. Nio, 2005; A.L. Kataev, 2006):

$$a_\mu^{\mathrm{QED}} \ = \ (116584719.4 \pm 1.4) \cdot 10^{-11}.$$

From the latest value of a_e (G. Gabrielse et al., 2006; M. Passera, 2006):

$\alpha^{-1} = 137.035999710(96)$, $a_\mu^{\mathrm{QED}} \ = \ (116584718.09 \pm 0.14 \pm 0.08) \cdot 10^{-11}$.

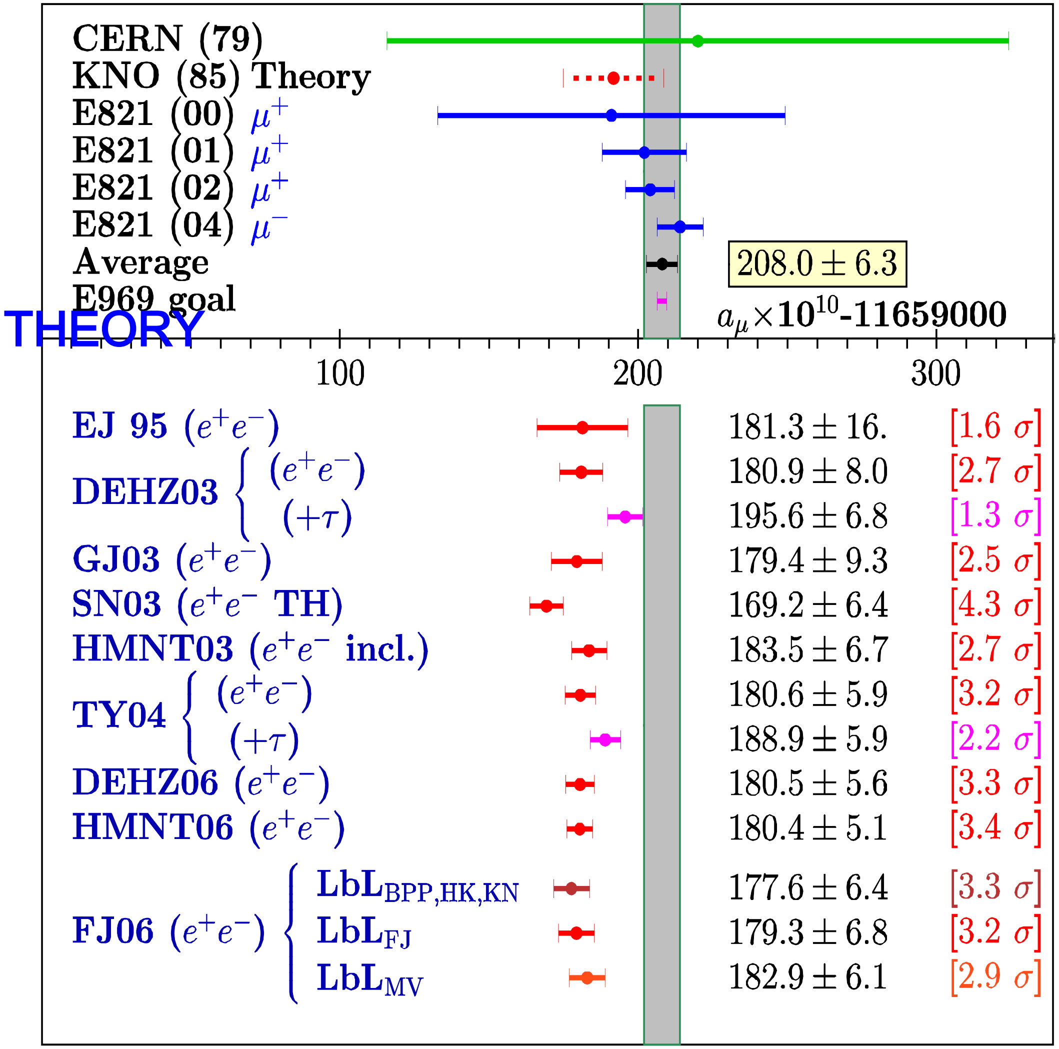
The errors are due to: a/ $\mathcal{O}(\alpha^5)$, b/ α

Jegerlehner'07

$g-2|_\mu$: comparison exp-theory(SM)

EXP

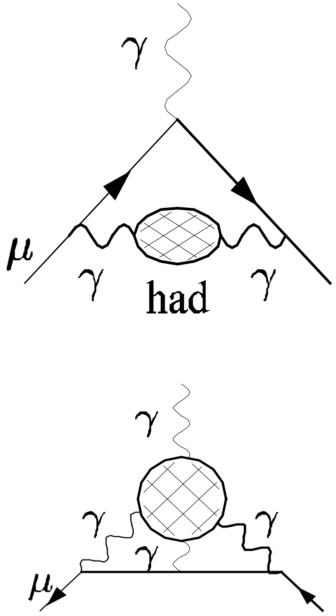
New Physics?

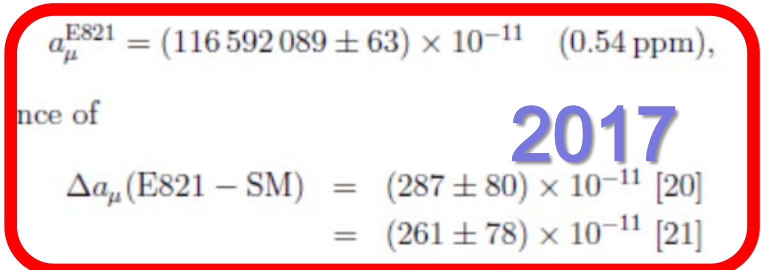


$$\delta a_\mu = (287 \pm 91)\ 10^{-11}$$

Jegerlehner'07

(3.2 σ)





$a_\mu^{E821} = (116\,592\,089 \pm 63) \times 10^{-11}$ (0.54 ppm),

nce of

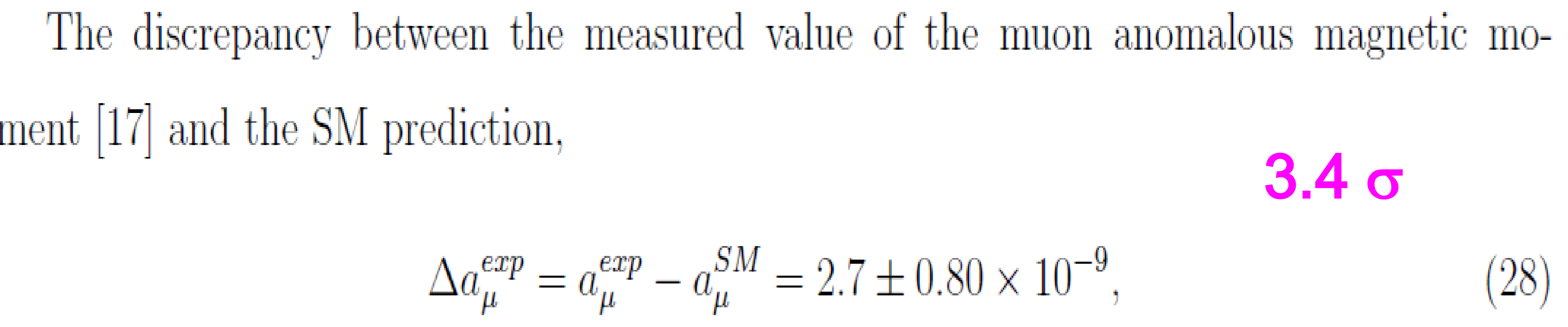
2017

$\Delta a_\mu(E821 - SM) = (287 \pm 80) \times 10^{-11}$ [20]

$= (261 \pm 78) \times 10^{-11}$ [21]

New calculation in SM (2013)

Dermisek, Rava – May 2013

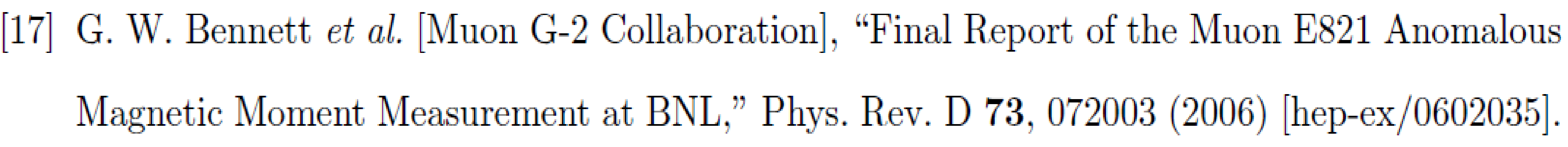


The discrepancy between the measured value of the muon anomalous magnetic moment [17] and the SM prediction,

3.4 σ

$$\Delta a_\mu^{exp} = a_\mu^{exp} - a_\mu^{SM} = 2.7 \pm 0.80 \times 10^{-9}, \tag{28}$$

Experiment Brookhaven 2006



[17] G. W. Bennett *et al.* [Muon G-2 Collaboration], "Final Report of the Muon E821 Anomalous Magnetic Moment Measurement at BNL," Phys. Rev. D **73**, 072003 (2006) [hep-ex/0602035].

The most precise quantity…

Brookhaven,
USA

In 2014

tranport to
Fermilab
(Chicago)
http://muon-g-2.fnal.gov/bigmove/

The BNL muon storage ring

Pomiar g-2
dla mionu

www.g-2.bnl.gov

Small, very precise experiment at low energies - *www.g-2.bnl.gov*

News

arXiv.org > physics > arXiv:1701.02807

$$a_e^{exp} = 1\,159\,652\,180.73\,(28) \times 10^{-12} \quad \pm 0.24\,\text{ppb}$$
$$a_\mu^{exp} = 1\,165\,920\,89\,(63) \times 10^{-11} \quad \pm 0.54\,\text{ppm}$$

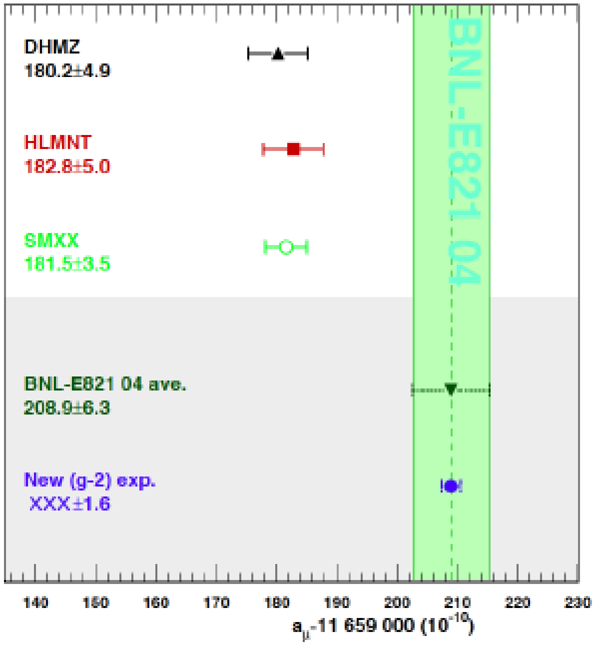
Due to the magnetic field, the muon spin precesses relative to its momentum while the muon undergo a cyclotron motion with a frequency of 149 ns. Our measurement technique directly yields the anomalous spin precession frequency, the difference between the spin precession frequency and the cyclotron frequency, and depends directly on a_μ and the magnetic field strength B via

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = \vec{\omega}_a = -\frac{Q}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

$$a_\mu = \frac{\omega_a}{\omega_p}\frac{\mu_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2}$$

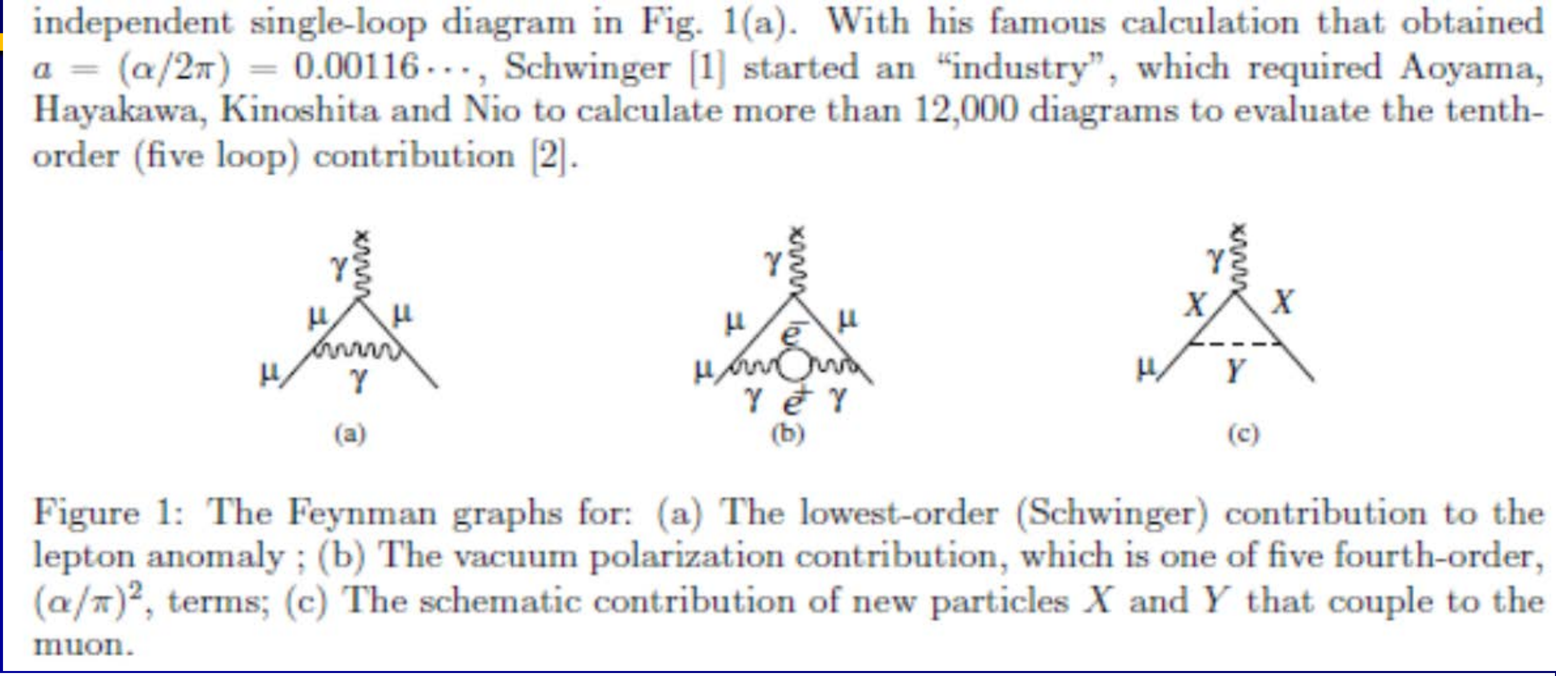
	VALUE ($\times 10^{-11}$) UNITS
QED	116 584 718.95 ± 0.08
HVP	6 850.6 ± 43
HLbL	105 ± 26
EW	153.6 ± 1.0
Total SM	116 591 828 ± 49

$$\Delta a_\mu(\text{E821} - \text{SM}) = (287 \pm 80) \times 10^{-11} \; [20]$$
$$= (261 \pm 78) \times 10^{-11} \; [21]$$



More..

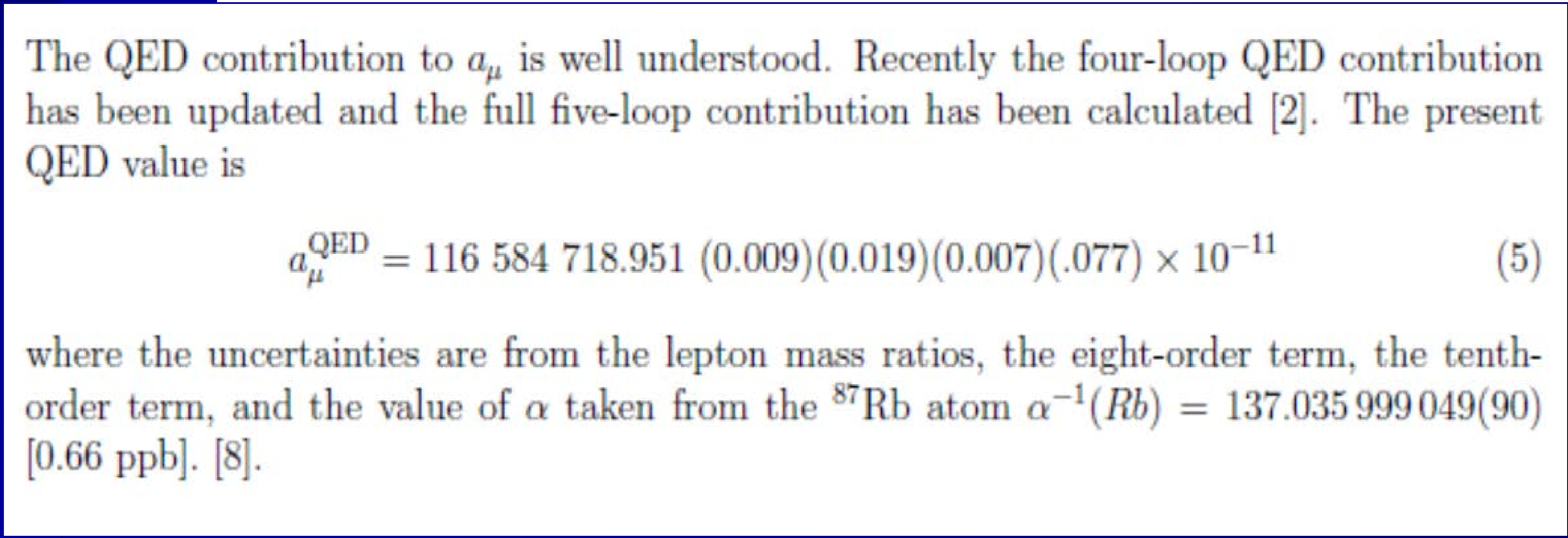
The Muon $(g-2)$ Theory Value



independent single-loop diagram in Fig. 1(a). With his famous calculation that obtained $a = (\alpha/2\pi) = 0.00116\cdots$, Schwinger [1] started an "industry", which required Aoyama, Hayakawa, Kinoshita and Nio to calculate more than 12,000 diagrams to evaluate the tenth-order (five loop) contribution [2].

Figure 1: The Feynman graphs for: (a) The lowest-order (Schwinger) contribution to the lepton anomaly ; (b) The vacuum polarization contribution, which is one of five fourth-order, $(\alpha/\pi)^2$, terms; (c) The schematic contribution of new particles X and Y that couple to the muon.

arXiv:1311.2198v1 [hep-ph] 9 Nov 2013



The QED contribution to a_μ is well understood. Recently the four-loop QED contribution has been updated and the full five-loop contribution has been calculated [2]. The present QED value is

$$a_\mu^{\rm QED} = 116\,584\,718.951\,(0.009)(0.019)(0.007)(.077) \times 10^{-11} \tag{5}$$

where the uncertainties are from the lepton mass ratios, the eight-order term, the tenth-order term, and the value of α taken from the ^{87}Rb atom $\alpha^{-1}(Rb) = 137.035\,999\,049(90)$ [0.66 ppb]. [8].

Renormalization

Idea

- QED – infinities only in expressions containing mass and electric charge. Using quantities measured in experiment we can remove infinities in the following way:

A – expression for physical quantity;

let 1st order correction Δ contains an infinite contribution Δ,

$$A = e_0(1+\Delta) + \ldots = e_0(1+\Delta'+..)(1+\Delta''+..) = e_{fiz}(1+\Delta'' + \ldots),$$

so the prediction for quantity A finite (Δ'' - finite)

(+...higher terms)

For magnetic moments infinities are at the intermediate stages of calculations – but they are the same for electron and muon. We can express them by each other and for electric charge use the exp. value.

Trick very useful

- Electric charge is a *free (initial)* parameter of the QED; the same for the electron mass m_e
- Theory does not predict values of these parameters – we can hide infinities in them
- If infinities only in expressions for free parameters of the theory → theory is renormalizable and provide definite predictions for physical processes
- It is not satisfactory, but works OK

Non-renormalizability

- There are theories with infinities not only in free parameters
- For long time it was considered that theories with spin-1 particles, but different than a photon, are non-renormalizable (eg. Fermi theory for weak int.).
- Today we known that theories with spin-1 particles are renormalizable if they are gauge theories, (moreover – non-abelian Yang-Mills theories)

Gravitation: gauge theory, but not renormalizable

Gauge transformation- a reminder

Schrodinger equation, based on E = ($\bar{p}^2/2m + V$)

- $i\,\partial\,\psi(\bar{x},t)\,/\partial\,t = -\,1/2m\ \ \bar{\nabla}^2\,\psi(\bar{x},t)$

  (for potential V=0 and using $\hbar = 1$)
- $E \to i\,\partial_0$ $\qquad\qquad \partial^\mu = (\partial_0, -\,\bar{\nabla})$

  $\bar{p} \to -\,i\ \ \bar{\nabla}$
- local transformation, phase $\alpha(\bar{x},t)$

  $\psi'(\bar{x},t) = e^{\,i\,\alpha(\bar{x},t)}\,\psi(\bar{x},t)$

$|\psi(\bar{x},t)|^2$ probability does not change, but

invariance of equation only if we add interaction !

Predictions for high energies

Probability of processes for large energies

- Bad, if probability of processes rises with energy (bigger than 100 % !?)
- QED: Compton scattering – at the lowest order of pert. calculation two diagrams, each rising with energy but the sum of diagrams – OK (cancelation)
- Cancelation results from the structure of the theory (gauge symmetry)

Calculation of probability using Feynman rules

- To each line and vertices in Feynman diagram a factor is assigned. Here we track only the energy E.

- Incoming or outgoing photon
  (and each spin 1 particle) – a factor E
  Virtual photon (spin1 particle) – a factor $1/E^2$

- Incoming or outgoing spin ½ particle - a factor $\sqrt{E}$,
  virtual spin ½ particle – a factor $1/E$

- Incoming or outgoing spin 0 particle – a factor 1,
  virtual spin 0 particle – a factor $1/E^2$

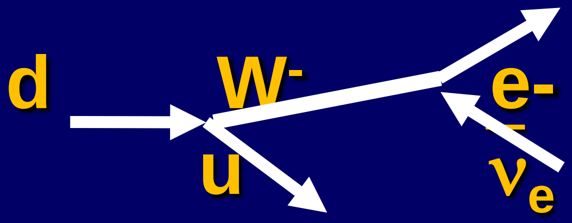
- Additional factors from couplings ….,

Product of factors→probability amplitude A

(probability = $|A|^2$)

Electroweak interaction

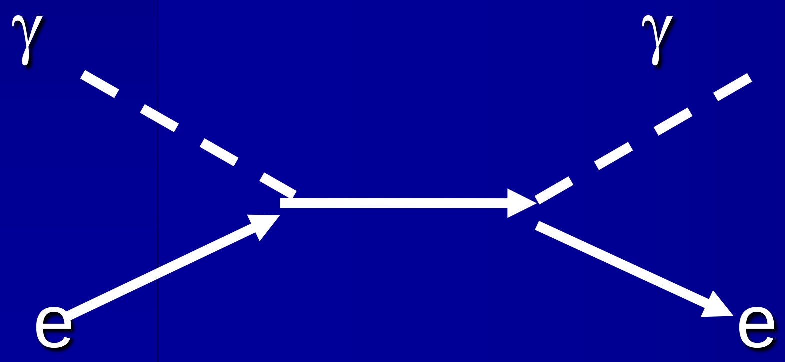
Decay d → u e $\bar{\nu}_e$

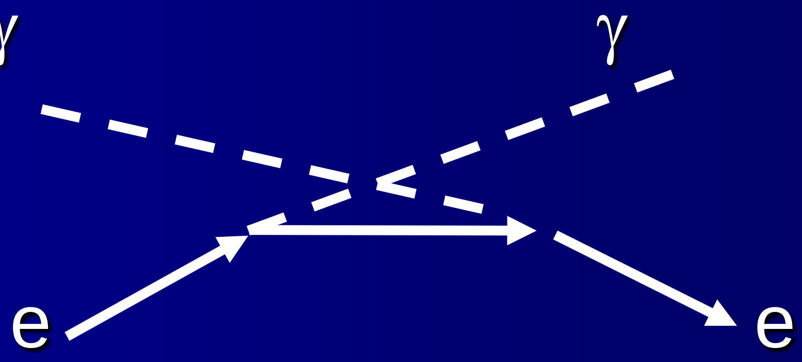


- W boson is very massive (80.4 GeV), so in decay of quark d (mass ~MeV) it is very virtual (far off mass-shell)

- Boson W has spin 1→ problem with renormalizability

- To understand this problem, we analyse first Comptona scattering γ e → γ e (QED), two diagrams:





Calculation of the probability amplitude for the Compton process

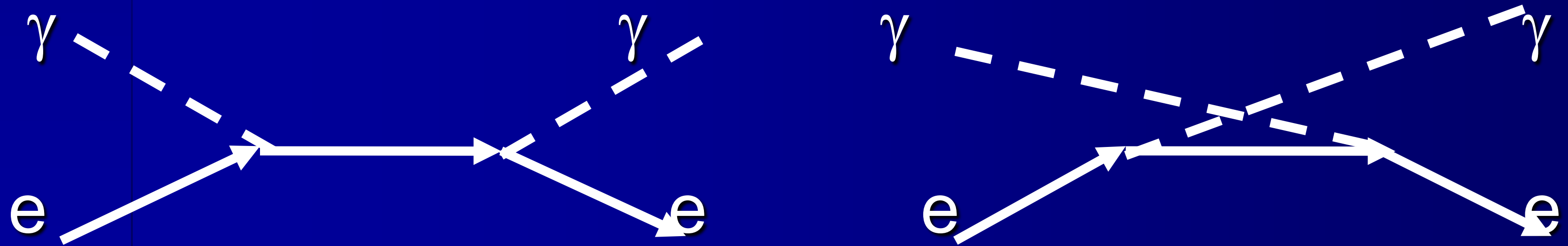
For this process the amplitude rises with the energy as

$$E^2 (\sqrt{E}\,)^2\ 1/E = E^2$$

(for probability E^4)

Bad behaviour for the individual diagrams, but in sum cancelation and the final amplitude does not grow with energy

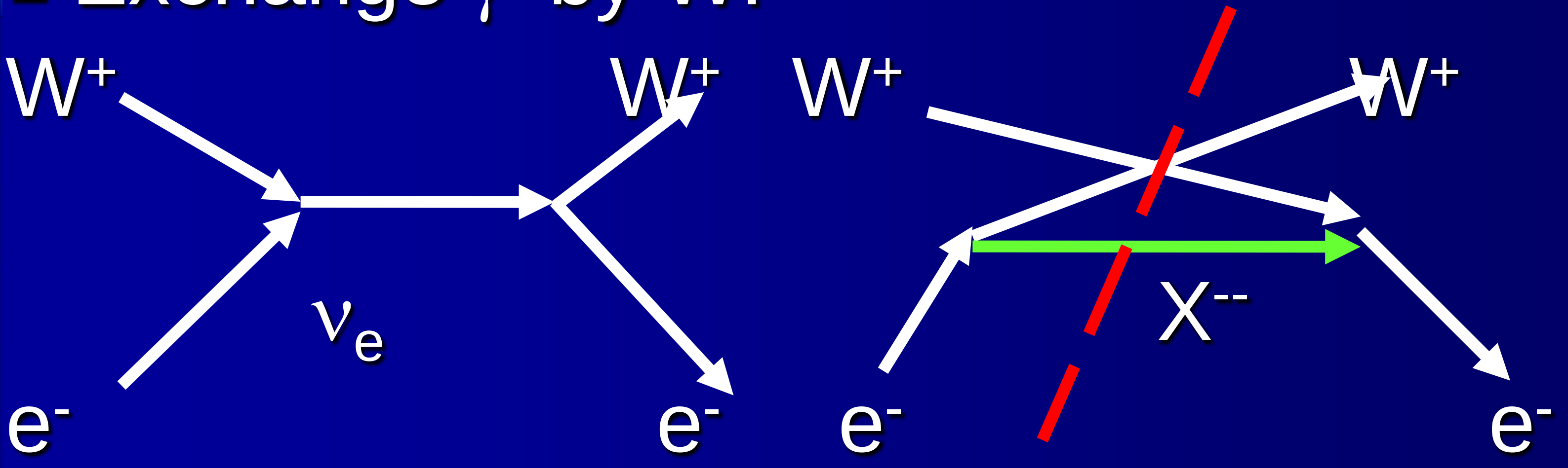
$\gamma\ e \to \gamma\ e$



Scattering W e → eW

Tu strzałki na liniach W oznaczają pędy

- Exchange γ by W:



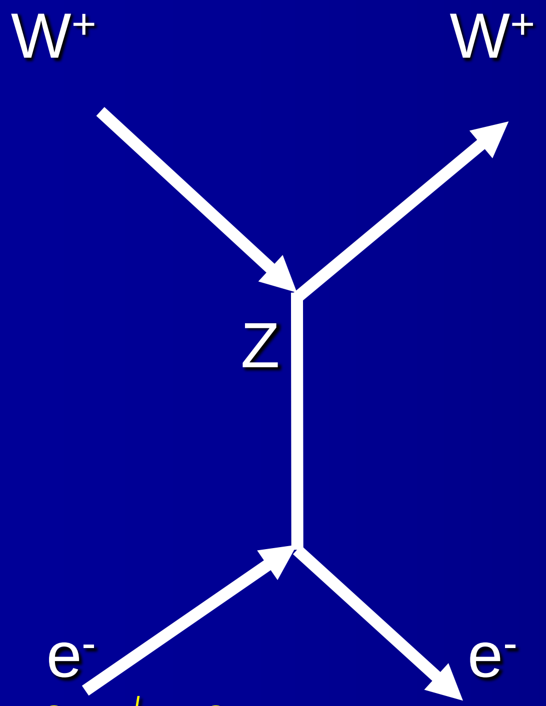
Charge conservation → X^{--} double charged particle – not observed !

Bad high energy behaviour

$E^2 (\sqrt{E})^2 1/E = E^2$

Z boson needed!

- Using diagram with Z boson



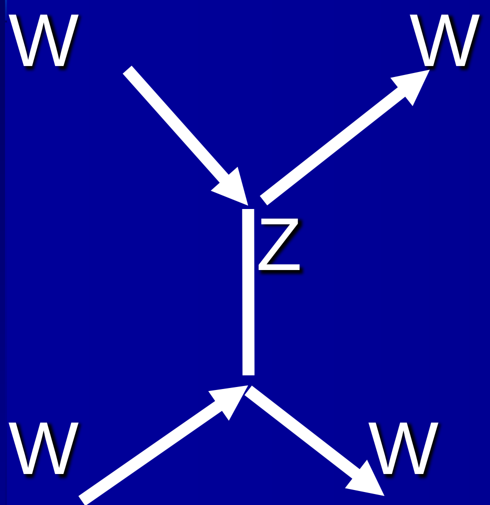
Behaviour E^2 $(\sqrt{E})^2$ ($\underline{\text{vertex WWZ}\sim E}$)/$E^2 = E^2$, and a proper sign of coupling WWZ – cancelation like for the Compton process!

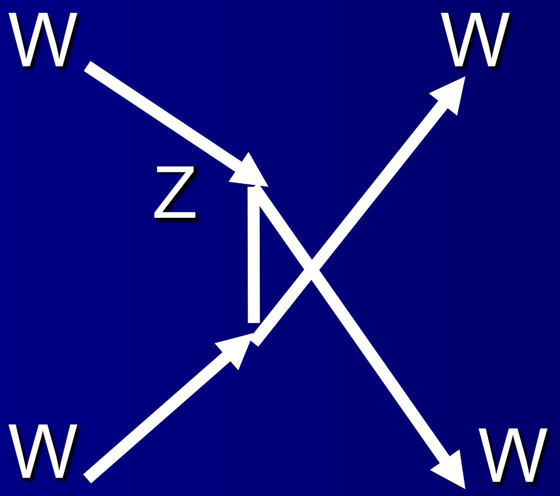
Price for a good behaviour for E: it must exist neutral particle Z with a define interaction (coupling) !

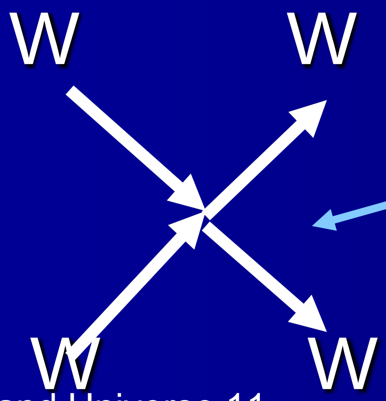
It was found – this is a Z boson !

→ A success of theory !

Scattering WW -> WW
~ E^4 (since $E^4\ E^2/E^2$) - even worst…







new type of interaction

(selfinteraction of W)

-> E^4 cancels but not E^2

E^2 term, in fact this is E^2 M^2

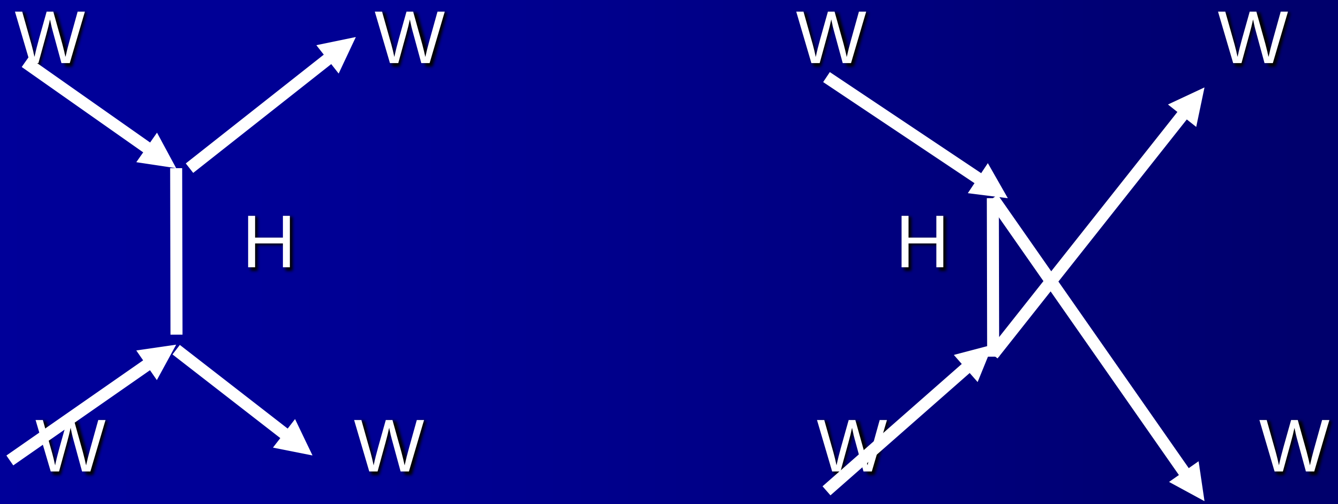
- In the amplitude - wrong term E^2 M^2 ,

  where M- mass of W or Z

- New contribution needed to cancel this bad term → the simplest diagram with exchange of spinie-0 particle, which couples to W/Z proportionally to mass

Higgs boson needed !

Scattering of W on W: H contribution



Couplings proportional to mass of particles to which H couples, and as a result →

a good high energy behaviour WW → WW !

Mass generation and consistent description of processes at high energies

- Good high energy behaviour - if H exists and couples to W/Z proportionally to their masses
- Higgs particle is related to the mass generation in SSB

→properties of H particle related to the mass generation and those needed for a good high energy behaviour WW → WW

THE SAME !!!

- Expected relation to gravity (mass..)

The theory ends here - Veltman

Facts and Mysteries in Elementary Particle Physics, 2003

- *„The theory ends here. We need help. Experiments must clear up this mess.”*
  →LHC pp, ILC e^+e^- (PLC $\gamma\,\gamma$, $e\gamma$)

- **July 2012 :**
  LHC - Higgs particle with mass 125 GeV

Questions to lecture 11

- Does fact of decaying of a particle depend on the reference frame?
- Do *a free particle* and *a particle on the mass shell* mean the same?
- Why a free positron can not emit a photon and remain on the mass shell?
- What does it mean – *a virtual particle*?
- Does QED have infinite expressions at intermediate stages of calculation?
- Compare the one- and two –loop QED corrections for the anomalous magnetic moments for muon?
- What is a precision we know (in 2013) the anomalous magnetic moments for muon (experiment minus theory (SM))?
- When the theory is renormalizable?
- Proper high energy behaviour of the probability of EW processes demads existence of spin 0 particle. How such a particle couples to gauge bozon Z?
- Is the Higgs boson discovered?