Make-up Exam

Statistical Analysis of Experimental Data 2023/2024

Please note:

- each problem should be uploaded to Kampus platform as a separate file, please upload the notebook file when needed or give Google Colab link as a comment;
- solutions have to include description and justification of the approach used, as well as discussion of the presented results;
- solutions should be uploaded until **Friday, March 1, 23:55 CET;**
- by uploading the solutions to Kampus you declare that they resulted from your own work and that you have not shared nor discussed them with anyone.

Problem 1

Electronic scale is used to weigh three gold coins, with masses m_1, m_2 and m_3. Unfortunately, at least two coins need to be put on the scale to trigger the measurement. Four measurements (m_A, m_B, m_C and m_D) were made of the following coin mass combinations:

$$m_A = m_1 + m_2 \qquad m_B = m_1 + m_3$$
$$m_C = m_2 + m_3 \qquad m_D = m_1 + m_2 + m_3$$

The uncertainty of single measurement is σ. Assuming mass measurements are independent:

1. find the optimal estimate of m_1, based on the maximum-likelihood method;

2. find the corresponding uncertainty of the m_1 estimate and compare it with the uncertainty expected from the "wrong" solution, $m_1 = m_D - m_C$.

Problem 2

Invariant mass of the produced muon pairs was reconstructed in the mass range from 2 GeV to 4 GeV for 5000 events (file available for download). There is a clear peak observed on top of the background known to be uniform (flat). Consider two hypothesis describing the resonant contribution:

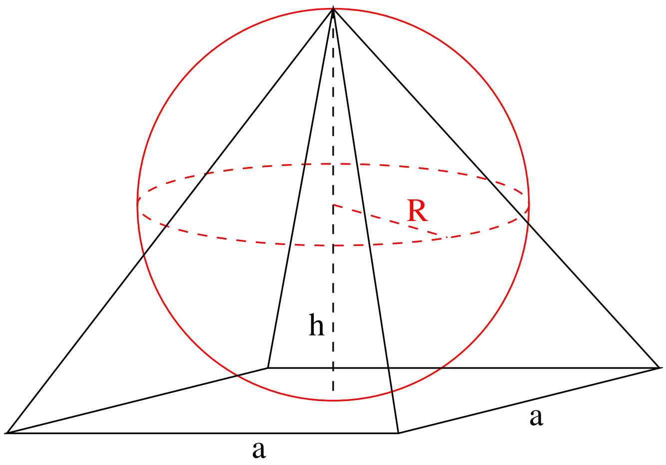
1. signal of resonance production is described by nonrelativistic Breit-Wigner distribution (Cauchy distribution), experimental smearing can be neglected;

2. natural width of the resonance can be neglected and the peak should be described as due to experimental resolution (gausian).

Which hypthesis describes the data better? Can we reject one of them based on the presented data?

Problem 3

Consider rectangular pyramid with square base, with base side a=2 and pyramide height h=2. Using Monte Carlo method, find the volume of the intersection of this pyramid with a sphere of R=1 standing in the middle of its base, see plot below. What is the number of MC generations required to get the value for the volume with 10^{-3} relative precision?



Problem 4

Events with 4 hardonic jets observed in e^+e^- collisions at high energies are expected to come mainly from two processes:

- Z° boson pair-production $e^+e^- \to Z^\circ\, Z^\circ$ contributing to about 5% of events,
- $W^\pm$ boson pair-production $e^+e^- \to W^+\, W^-$ contributing to the remaining 95% of events.

The invariant masses of the produced bosons ($M_Z = 91.2\,\mathrm{GeV}$ and $M_W = 80.4\,\mathrm{GeV}$) can be reconstructed (from their subsequent decays into pairs of jets) with precision $\sigma_M = 4\,\mathrm{GeV}$. Assuming two boson masses are measured in each event (and have to be used in event classification):

- use the Neyman–Pearson Lemma to find optimal selection criteria to separate the two production processes;
- calculate the relative statistical uncertainty on the number of Z° boson pair production events reconsted from the number of selected events (with ;
- by changing the selection cut, find the cut value resulting in the highest measurement precision of the Z boson production rate.

Measurements of the two boson masses considered for each event are independent. Assume the expected rate of W^+W^- events is known and the sample of 200'000 events is collected.
Hint: problem has an analytical solution.

Problem 5

Consider an deep underground experiment looking for proton decay in a huge water tank with 10^{34} of protons (30'000 tons of water). Expected background in the measurement is one false count in 100 days. After one year of data taking only 1 count was registered. Calculate the 95% CL **frequentist** limit on the proton lifetime. Compare it with the expected limit. To simplify the problem, the expected limit can be defined as the limit corresponding to the median of the expected event number distribution.