Telescope Resolution Studies

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Status report

- Introduction
- Analysis method
- Simulation Results
- Analytical Results
- Conclusions and Plans

Introduction

Motivation

The main aims of this study

- understand the position measurement in the telescope
- optimize the performance by suggesting the best plane setup

Approach

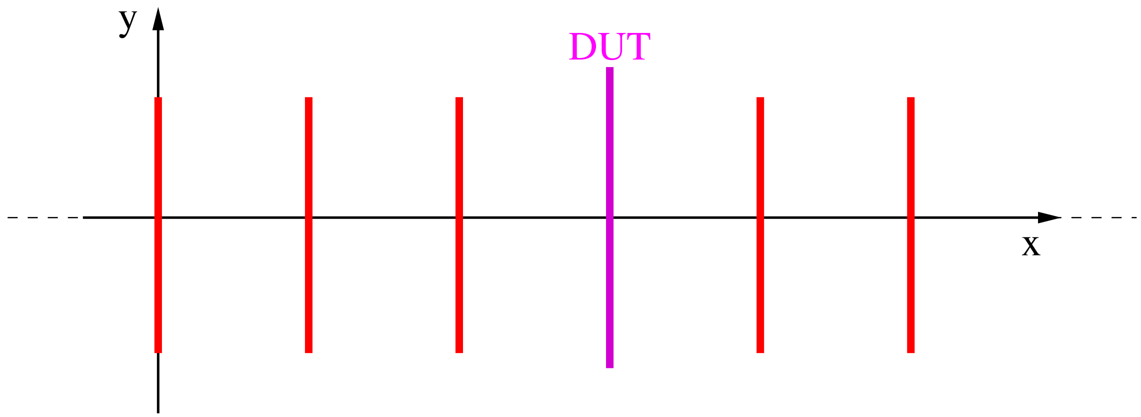
Use analytical method for track fitting including multiple scattering (!!!)

Simplifying assumptions:

- small scattering angles (Gaussian approximation)
- Gaussian position measurement errors
- perfect alignment (could be taken into account !)
- no additional material (windows, etc.) (implemented, but not used)

Analysis method

Geometry description



Geometry can be specified by giving:

- N - number of detector planes (including DUT)
- x_i - position of each plane $(i = 1 \ldots N)$
- σ_i - position resolution in each plane $(i \neq i_{DUT})$
- $\Delta\theta_i$ - average scattering angle in each plane

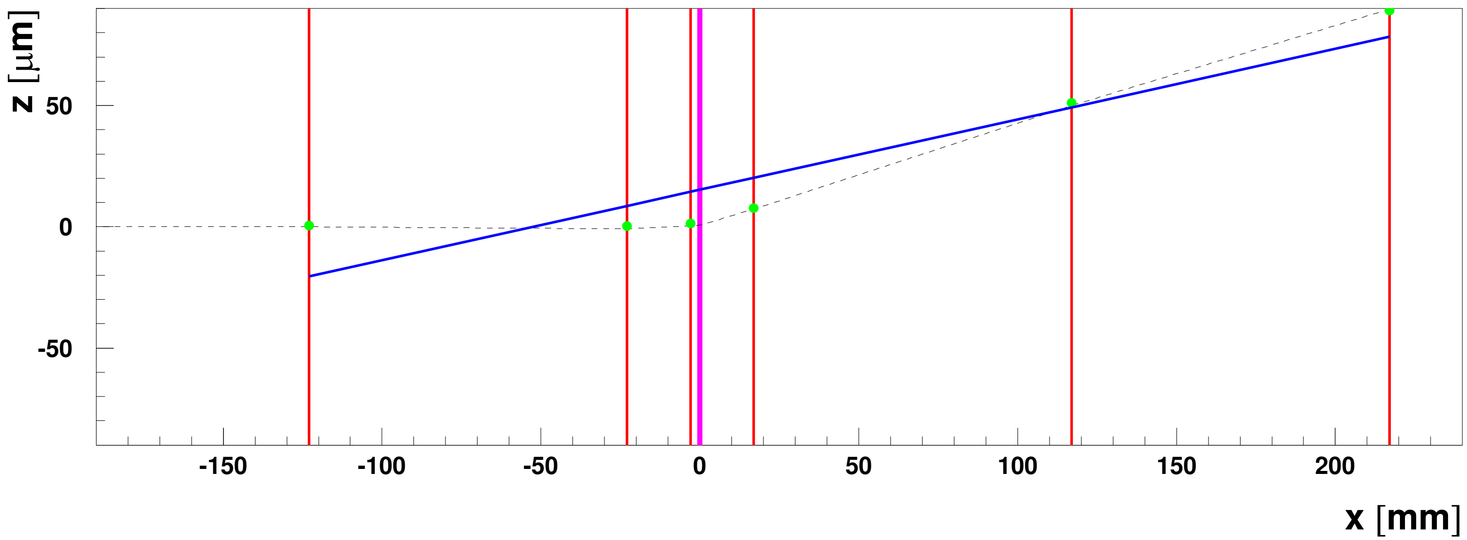
How to find optimum configuration (plane ordering, values of x_i)
for given telescope parameters (N, σ_i, $\Delta\theta_i$)?

Analysis method

Multiple scattering

Distances between planes $\sim \mathcal{O}(100\ mm)$ + scattering angles $\sim \mathcal{O}(0.1\ mrad)$
⇒ track displacement due to scattering $\sim \mathcal{O}(10\ \mu m)$ (for beam energy of few GeV)

GEANT 4 simulation for 6 GeV elctron beam:



Displacement comparable with position resolution $(1 - 2\ \mu m)$!
⇒ significantly influences the measurement, can not be neglected !

Straight line fit is not sufficient...

Analysis method

Multiple scattering

Distances between planes $\sim 0(100\ mm)$ + scattering angles $\sim 0(0.1\ mrad)$

$\Rightarrow$ track displacement due to scattering $\sim 0(10\ \mu m)$ (for beam energy of few GeV)

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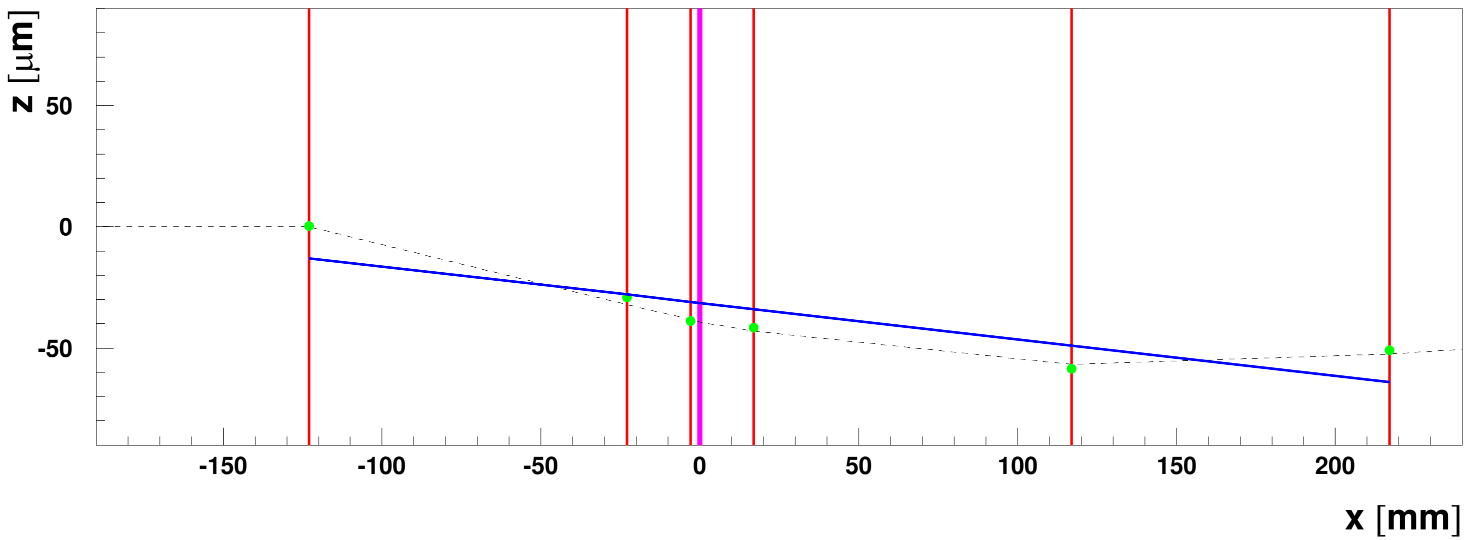


Displacement comparable with position resolution $(1 - 2\ \mu m)$!

$\Rightarrow$ significantly influences the measurement, can not be neglected !

Straight line fit is not sufficient...

Multiple scattering has to be taken into account not only for DUT.

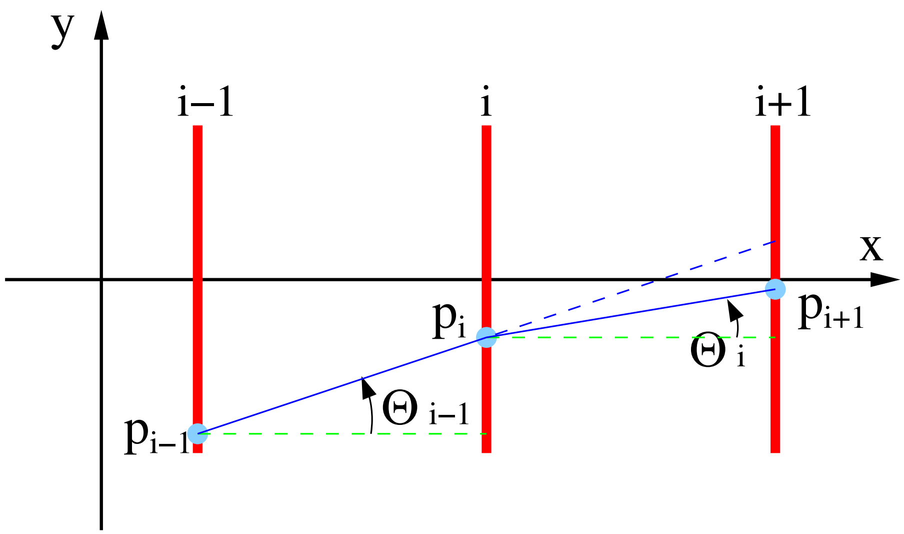
Analysis method

Track fitting

We want to determine track positions in each plane (including DUT), i.e. N parameters ($p_i,\ i = 1 \dots N$), from $N-1$ measured positions in telescope planes ($y_i,\ i \neq i_{DUT}$).

However, we can use constraints on multiple scattering!

Contribution of plane i to χ^2 of the fit



position measurement   multiple scattering

$$\Delta\chi_i^2 = \left(\frac{y_i - p_i}{\sigma_i}\right)^2 + \left(\frac{\Theta_i - \Theta_{i-1}}{\Delta\Theta_i}\right)^2$$

where: $$\Theta_i = \frac{p_{i+1} - p_i}{x_{i+1} - x_i}$$

Both terms present for planes $i \neq 1, i_{DUT}, N$,
first term missing for DUT, second for first and last plane

χ^2 minimum can be found by solving the matrix equation.

As a by-product we get also an expected error on the position reconstructed at DUT.

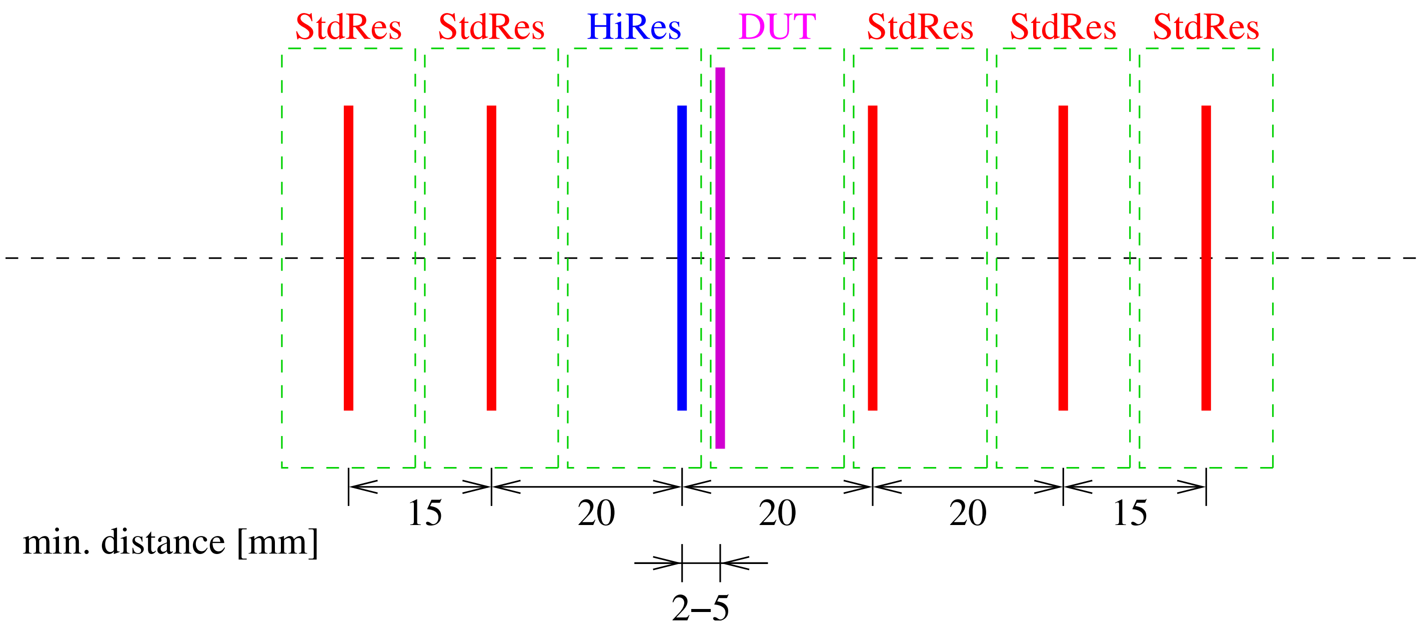
Analysis method

Realistic telescope geometry

thanks to W.Dulinski

The minimum distance between DUT and **one** of the telescope planes, d_{min}, is 5 mm (easy, realistic) or even 2 mm (hard, optimistic).

However, other distances can not be smaller than 15 or 20 mm:



In addition to standard sensor planes with 2 μm resolution we can consider adding one high resolution plane ($\sigma_{HR} \sim 1 \mu m$) in front of DUT

Analysis method

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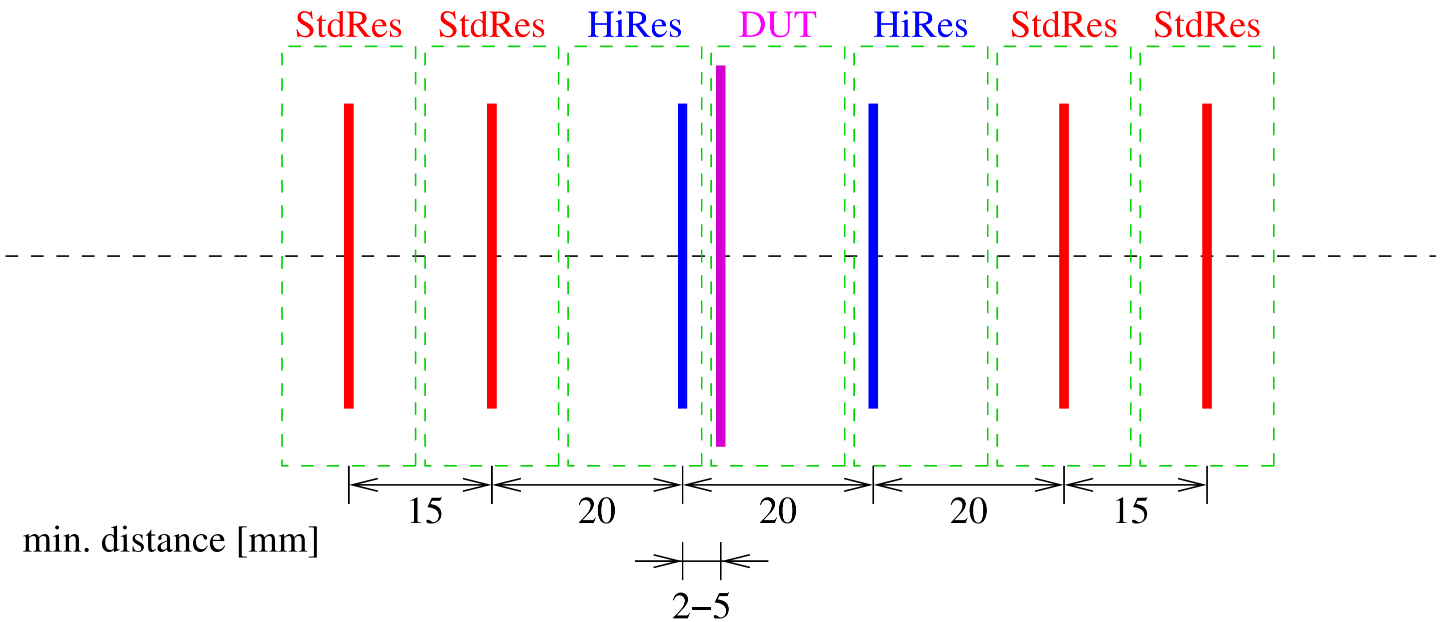


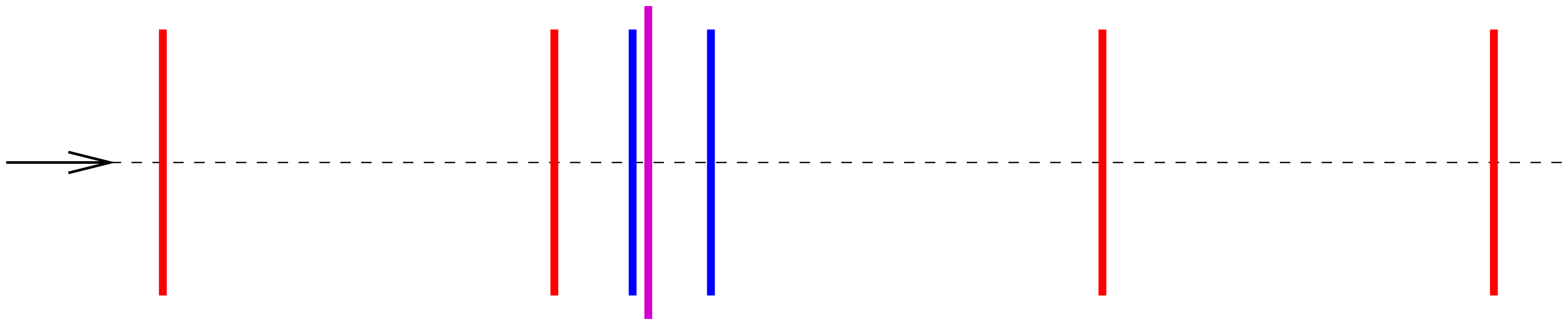
In addition to standard sensor planes with 2 μm resolution we can consider adding one or two high resolution planes ($\sigma_{HR} \sim 1\mu m$) in front of and behind DUT

Simulation Results

Telescope setup

GEANT 4 was used to simulate particle scattering in the telescope for the configuration optimum for the assumed telescope parameters (see later in this talk):

- DUT with 500 μm thickness
- 2 high resolution sensor planes with 120 μm thickness
- 4 standard sensor planes with 120 μm thickness
- minimum distance between DUT and HR plane of 3 mm
- 6 GeV electron beam

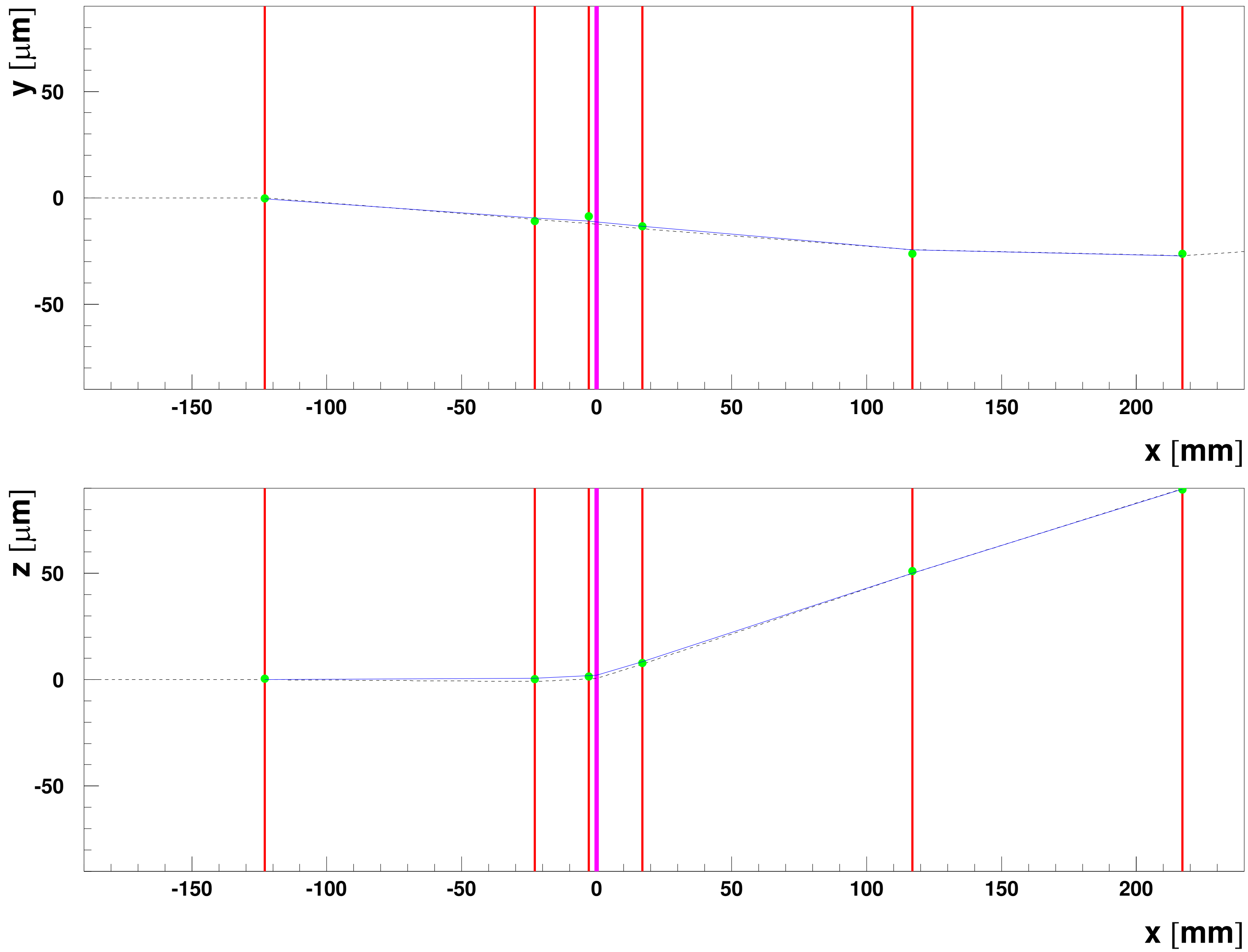


so called **WN–WW** configuration

Simulation Results

Fit results

Example of the GEANT 4 event



Color codes:

sensor planes

DUT

true particle path

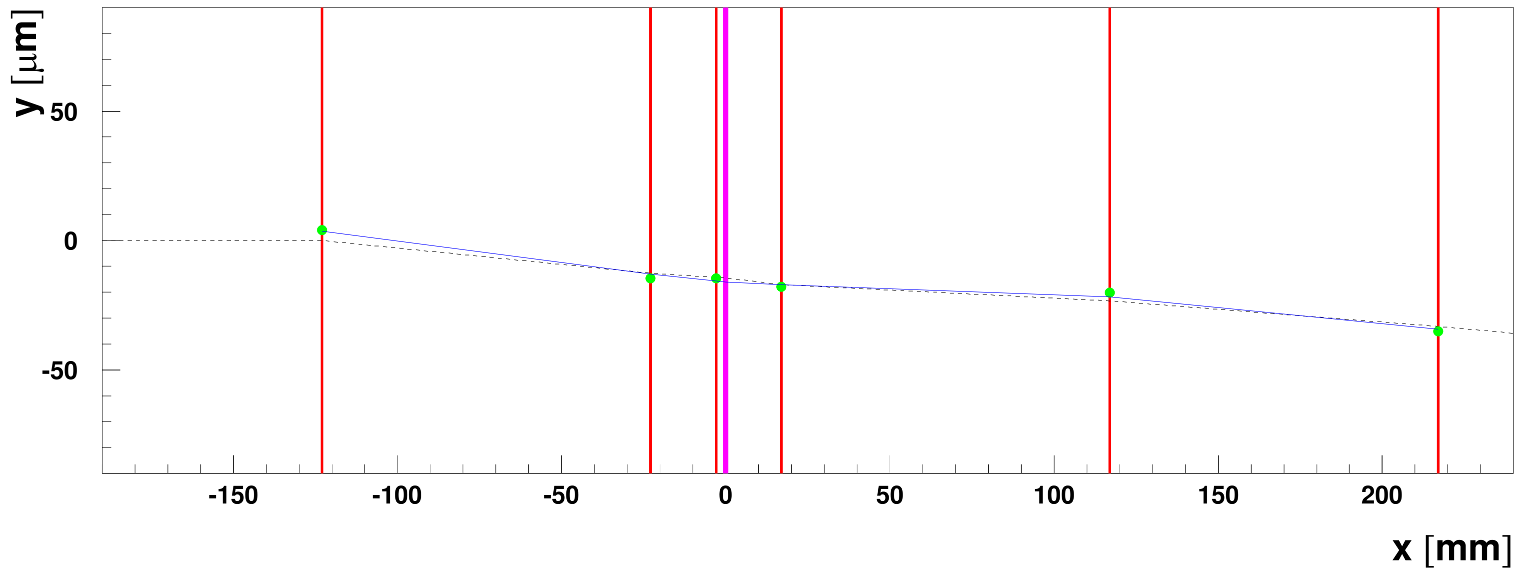
measurements

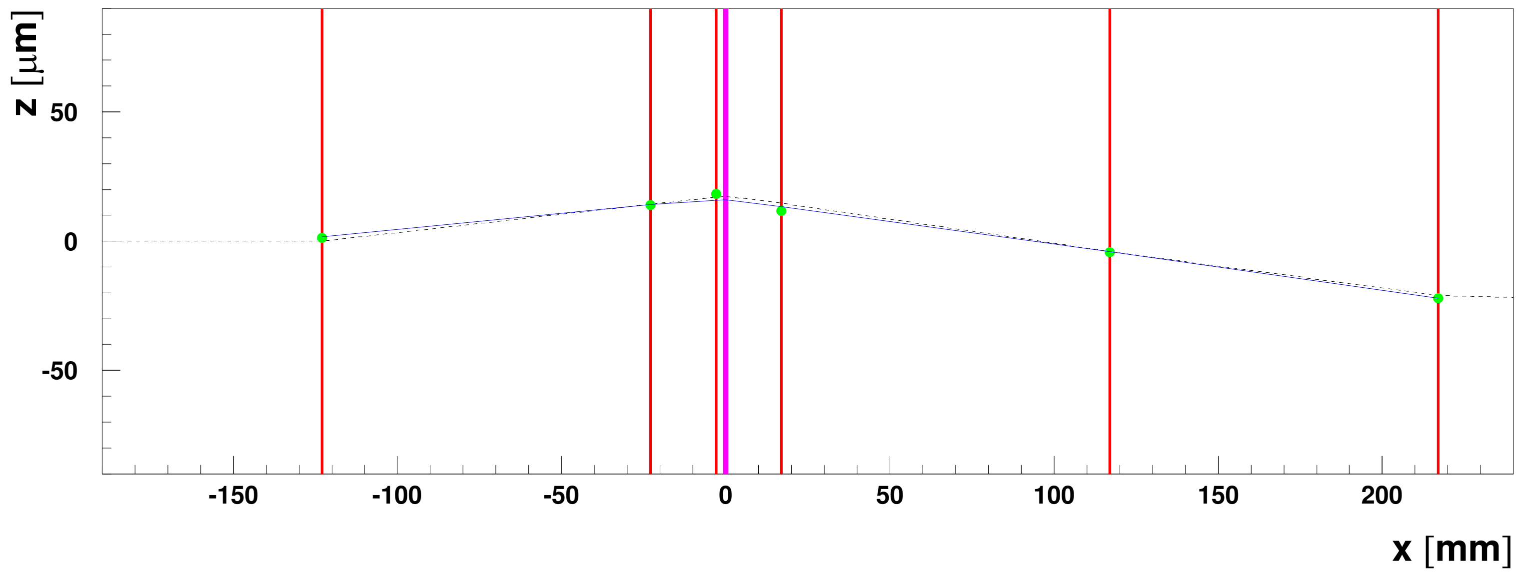
fitted track

Simulation Results

Fit results

Example of the GEANT 4 event





Color codes:

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Simulation Results

Fit results

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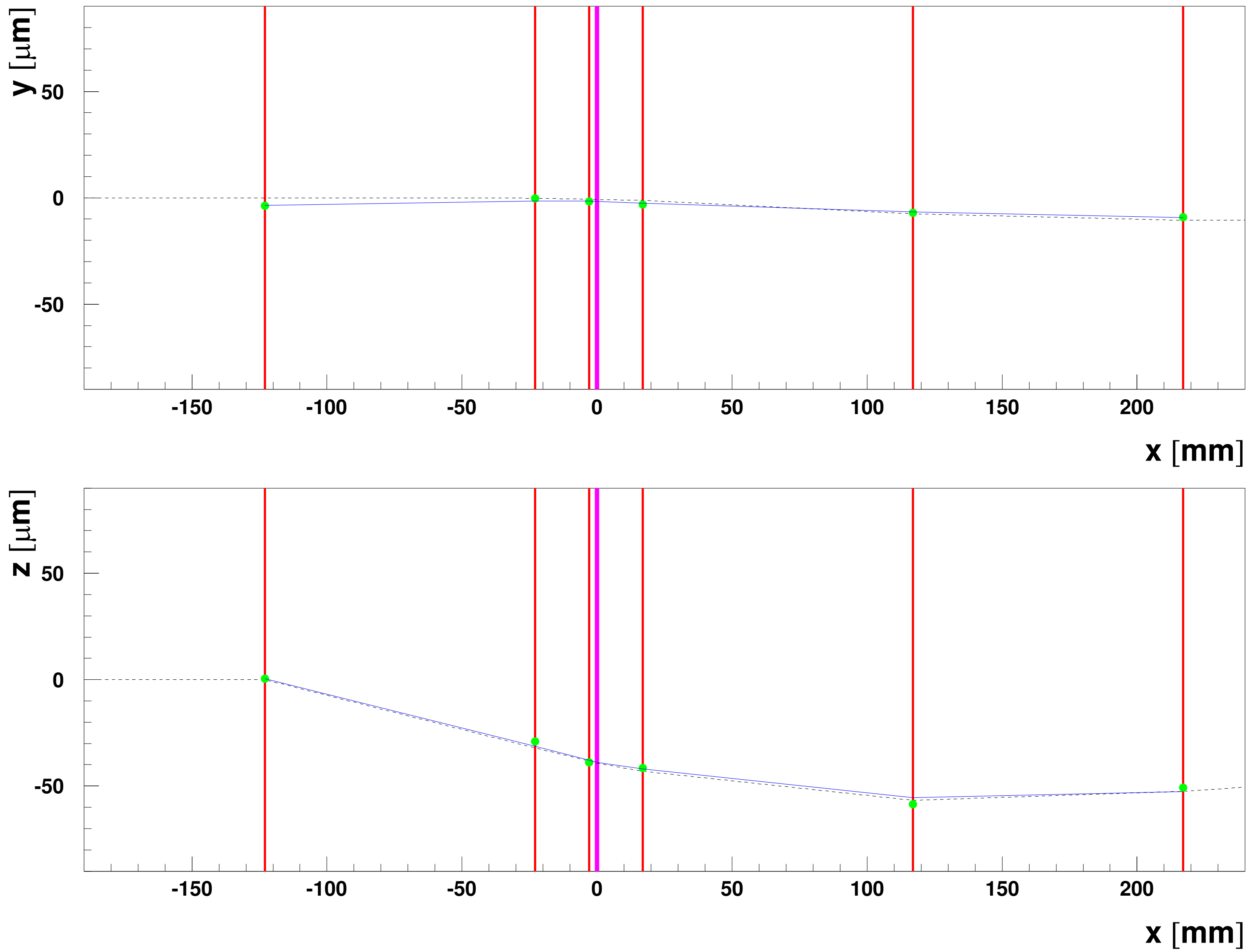


Color codes:

sensor planes

DUT

true particle path

measurements

fitted track

Simulation Results

Fit quality

We have 12 measuremens
(6 planes $\times$ 2 position measurements)

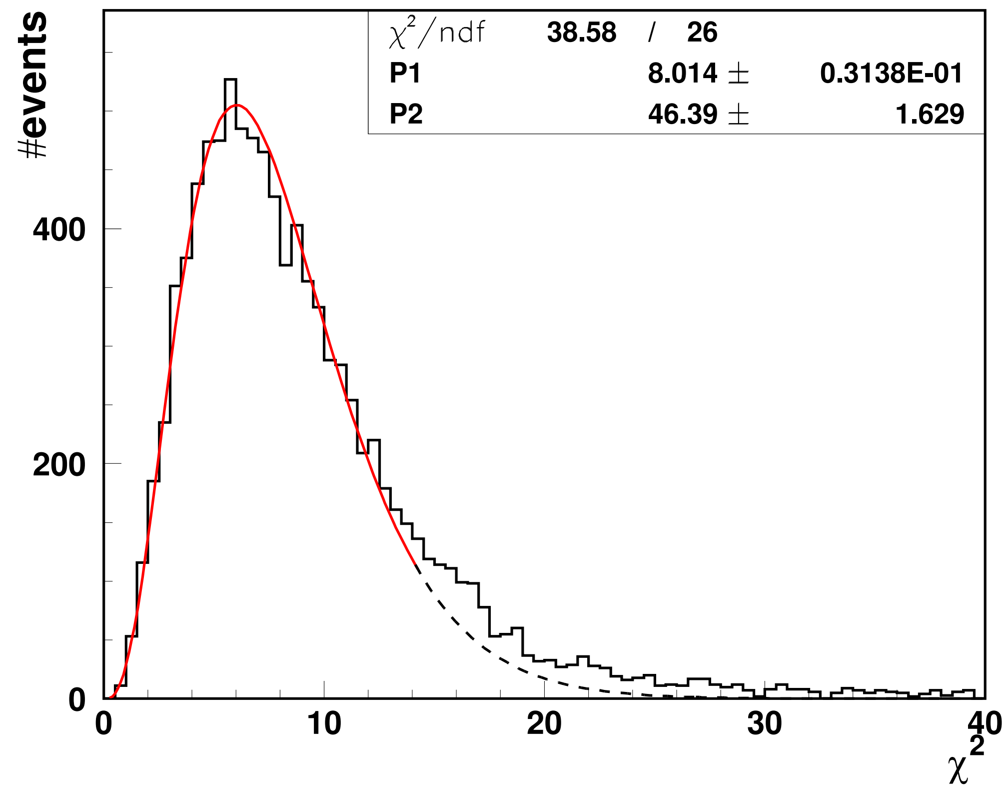
and fit 14 parameters
(2 position coordinates for 7 planes)

However, we also impose
10 constraints on scattering angles.

$\Rightarrow$ Number of degrees of freedom:

$$N_{df} = 12 + 10 - 14 = 8$$

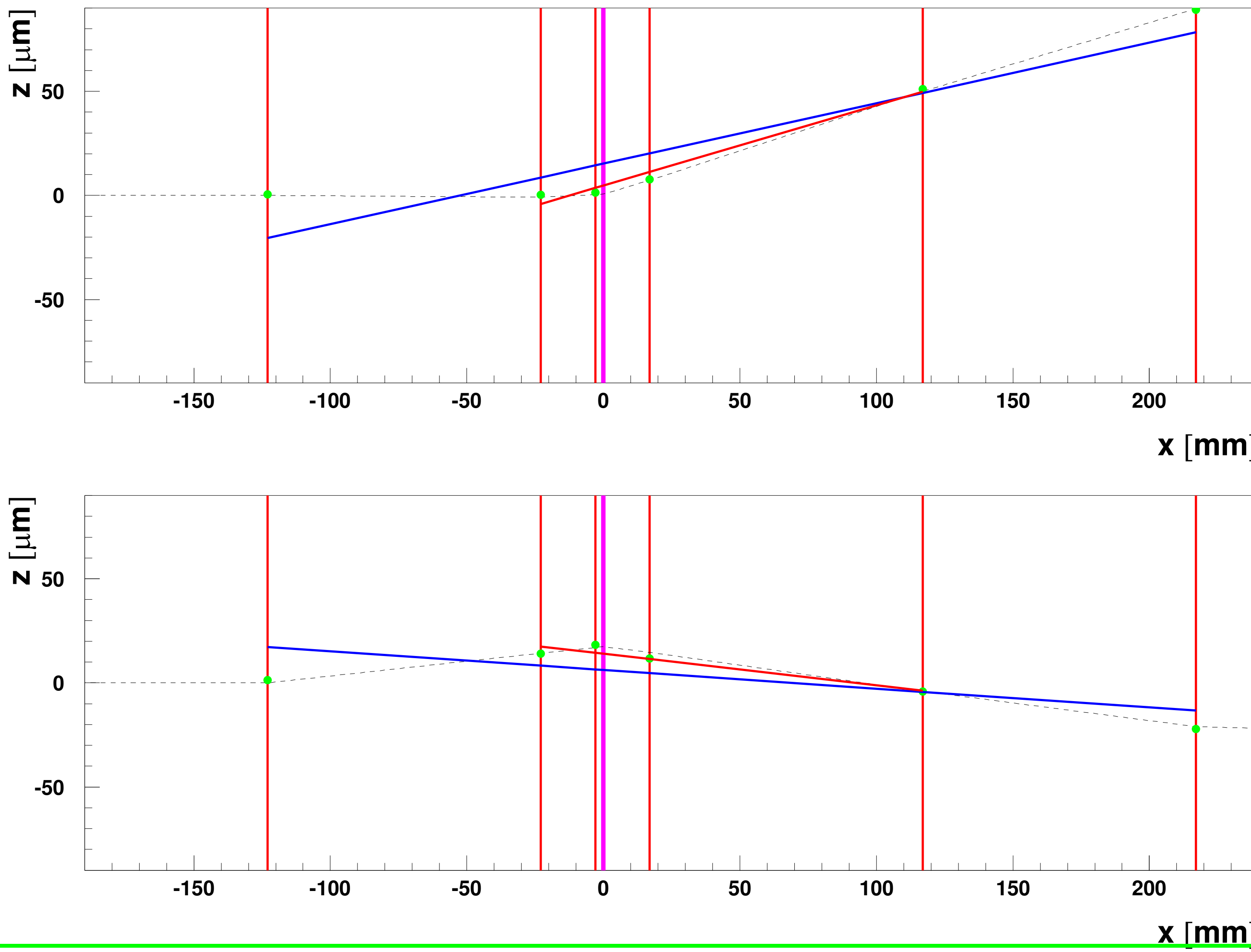
χ^2 distribution for GEANT 4 events



Simulation Results

Comparison with line fits

Example of the GEANT 4 event



Color codes:

measurements

true particle path

line fit to 6 planes

line fit to 4 planes

Simulation Results

Comparison with line fits

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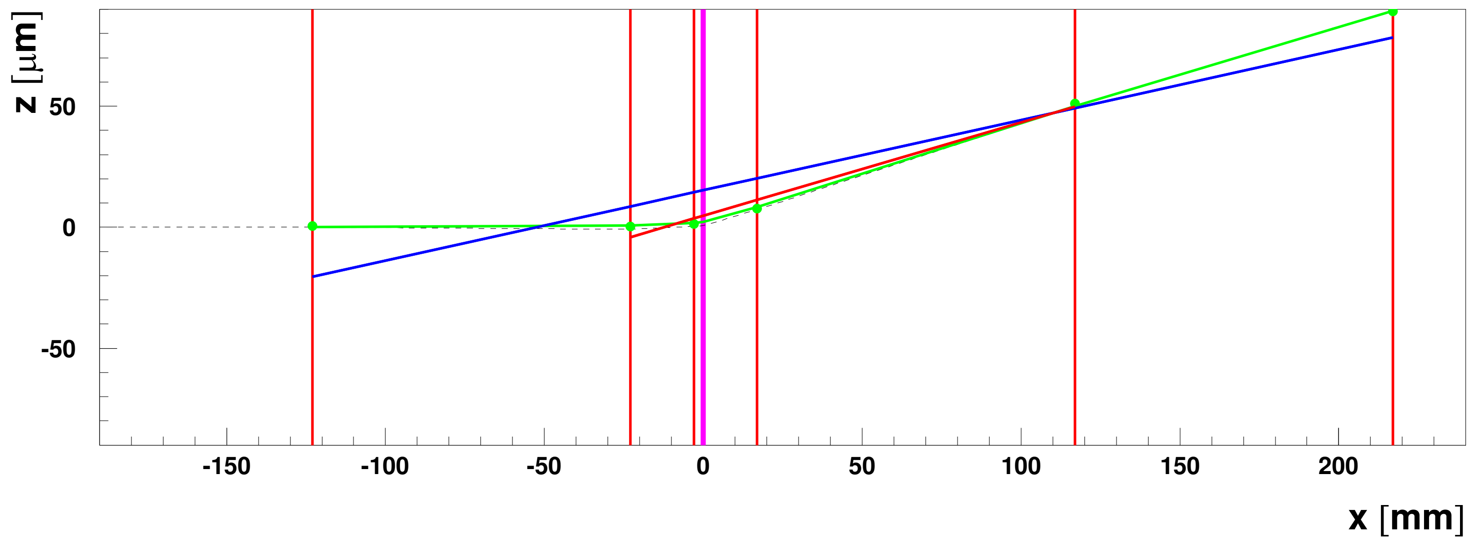


Color codes:

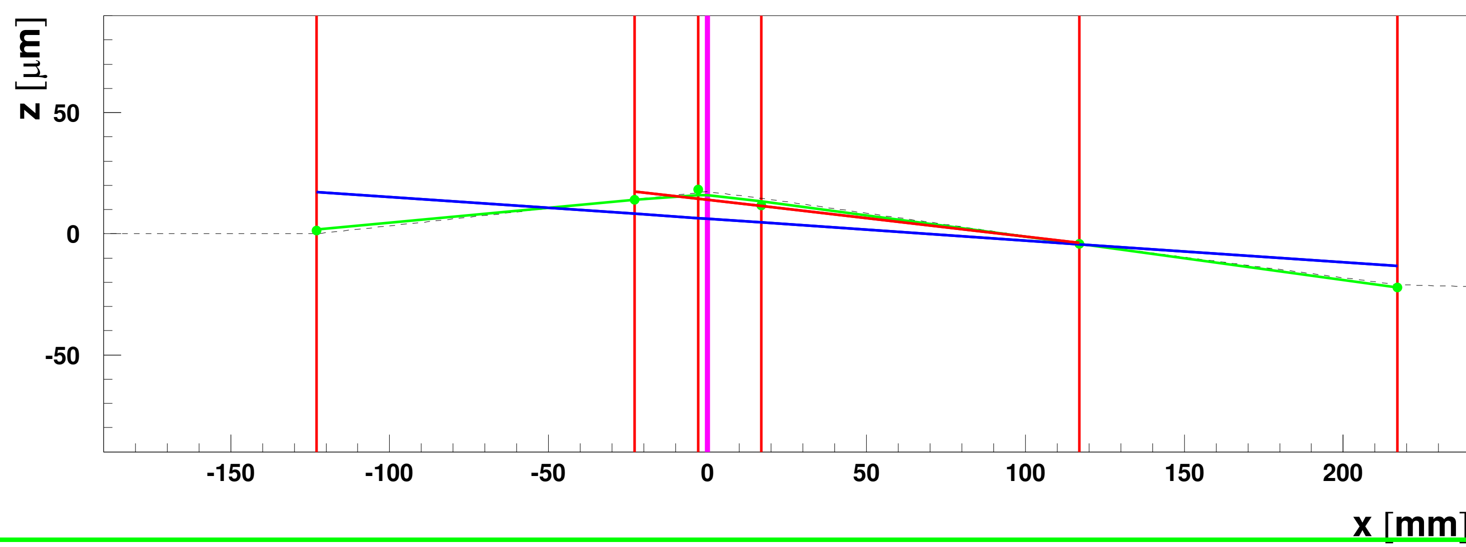
measurements

true particle path

line fit to 6 planes

line fit to 4 planes

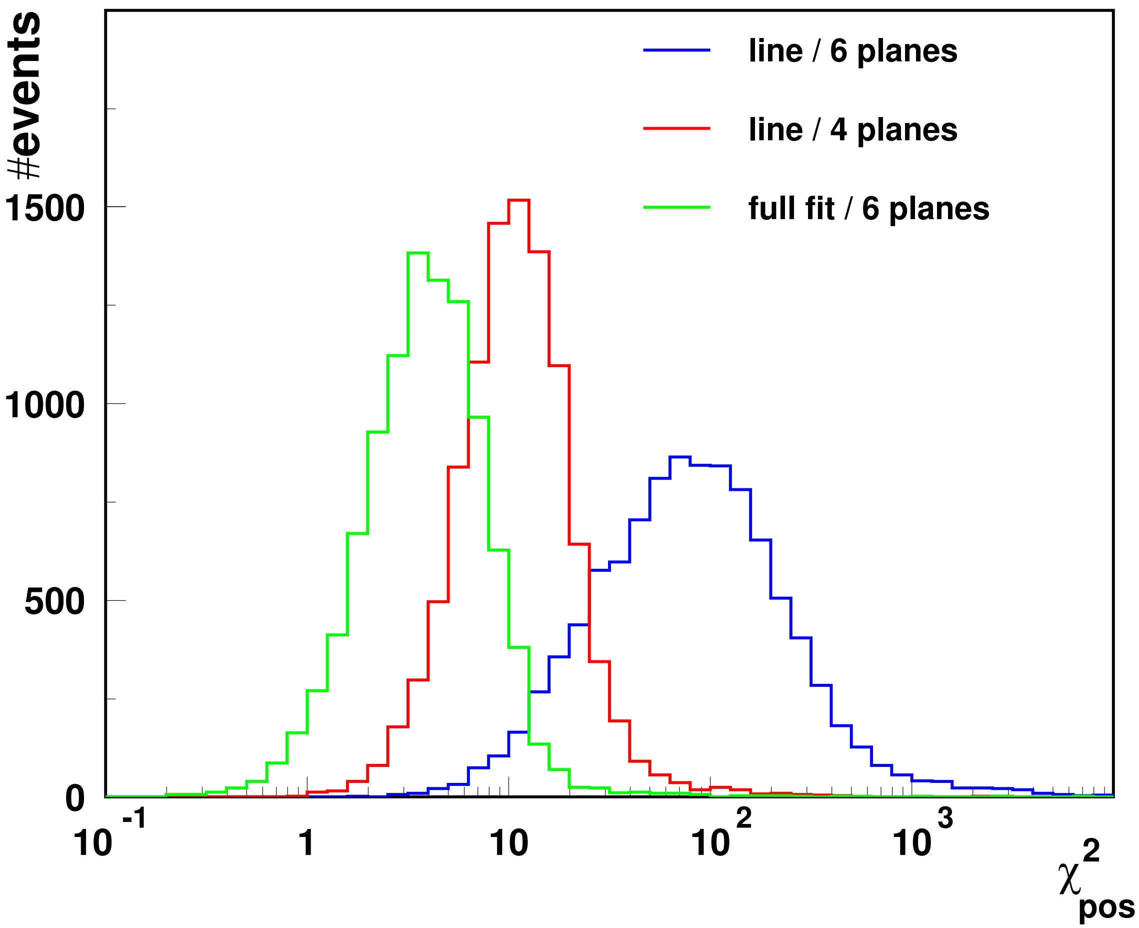
fit with multiple scattering



Simulation Results

Comparison with line fits

χ^2_{pos} distributions for GEANT 4 events



Fit taking into account multiple scattering results in a qualitative improvement in the description of the fitted data.

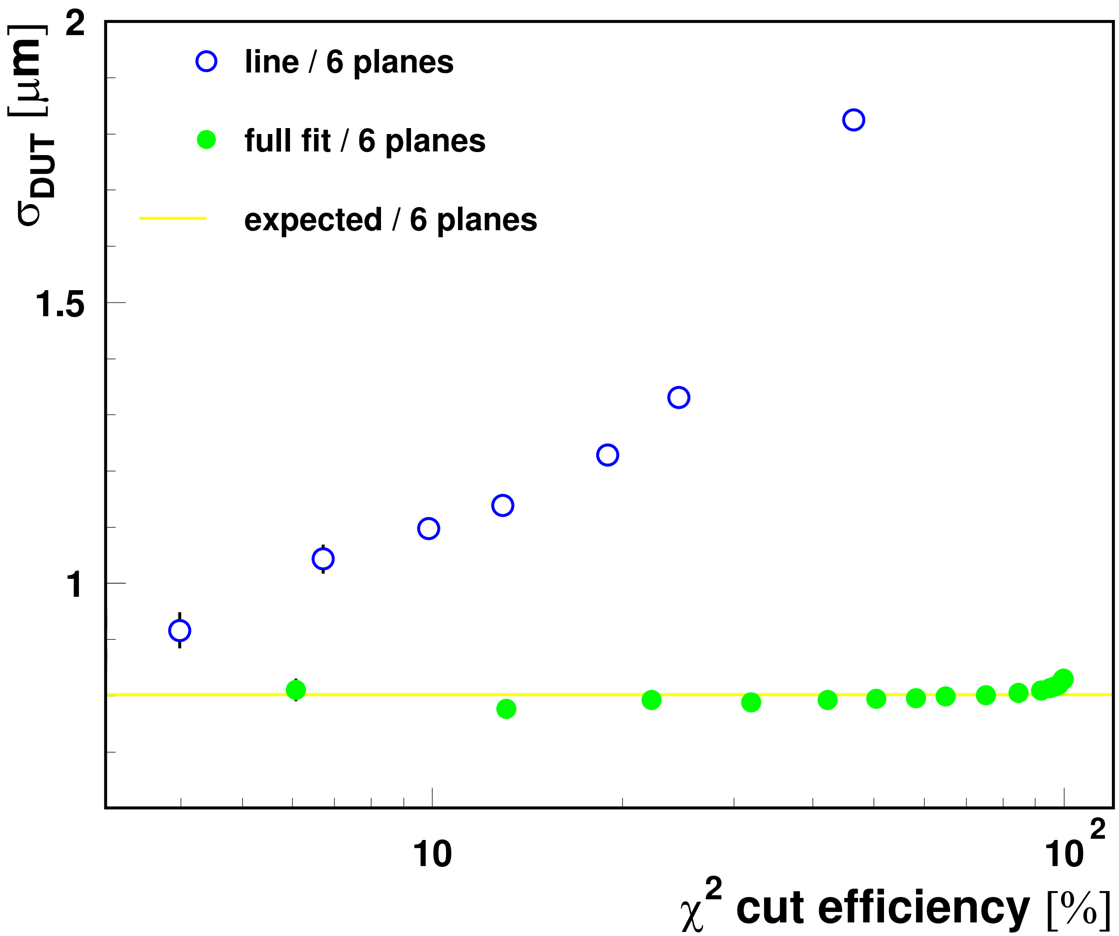
$$\chi^2_{pos} = \sum_{i \neq DUT} \left(\frac{y_i - p_i}{\sigma_i} \right)^2$$

y_i - measured positions
p_i - fitted positions
σ_i - position resolutions

Simulation Results

Comparison with line fits

Position resolution at DUT vs χ^2 cut efficiency



Full fit allows for precise position determination for >90% of events

Stright line fit to 6 planes always gives much worse results.

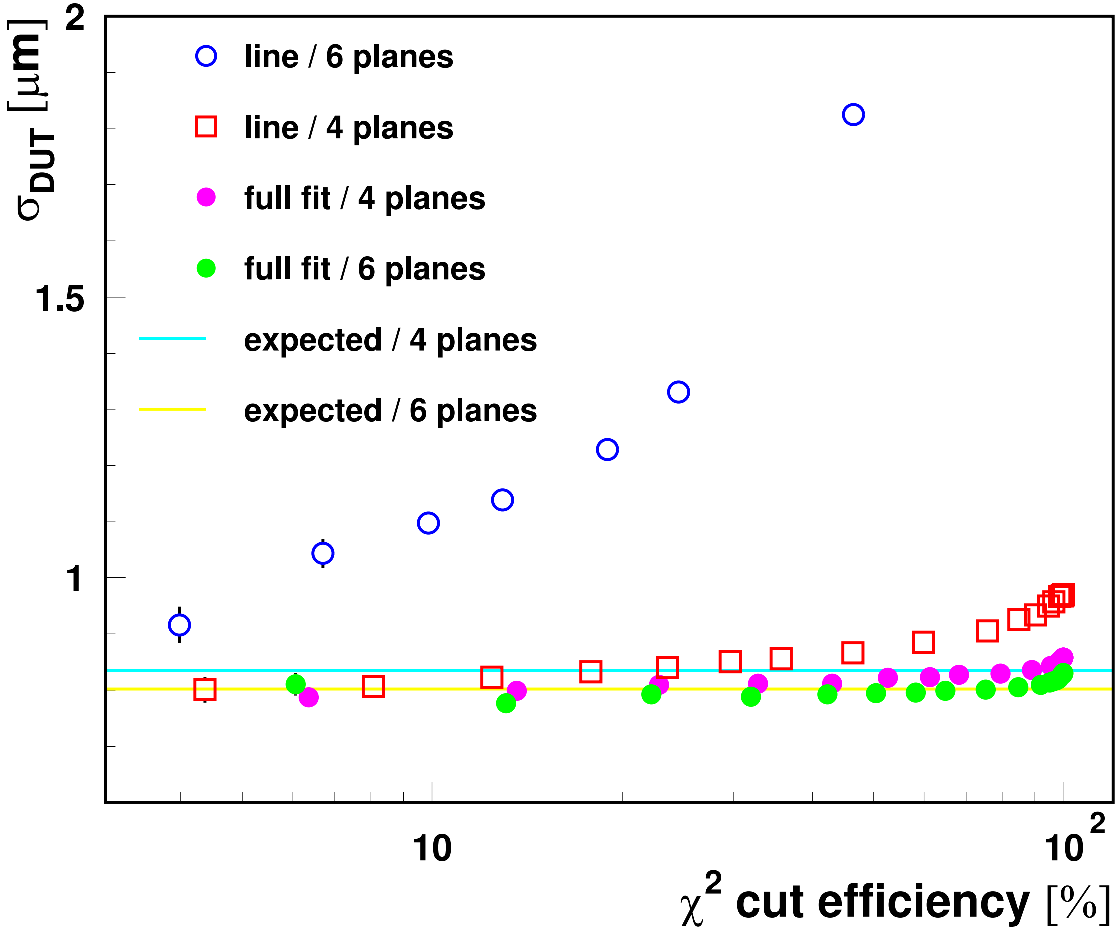
WN–WW configuration

- 500 μm DUT
- 2 HR planes 120 μm, $\sigma = 1\mu m$
- 4 Std planes 120 μm, $\sigma = 2\mu m$
- 3 mm between DUT and first HR
- 6 GeV electron beam

Simulation Results

Comparison with line fits

Position resolution at DUT vs χ^2 cut efficiency



Full fit allows for precise position determination for >90% of events

Stright line fit to 6 planes always gives much worse results.

Fit to 4 planes gets comparable results only for about 10% of events with smallest scattering (best χ^2).

WN–WW configuration

- 500 μm DUT
- 2 HR planes 120 $\mu m, \sigma = 1 \mu m$
- 4 Std planes 120 $\mu m, \sigma = 2 \mu m$
- 3 mm between DUT and first HR
- 6 GeV electron beam

Analytical Results

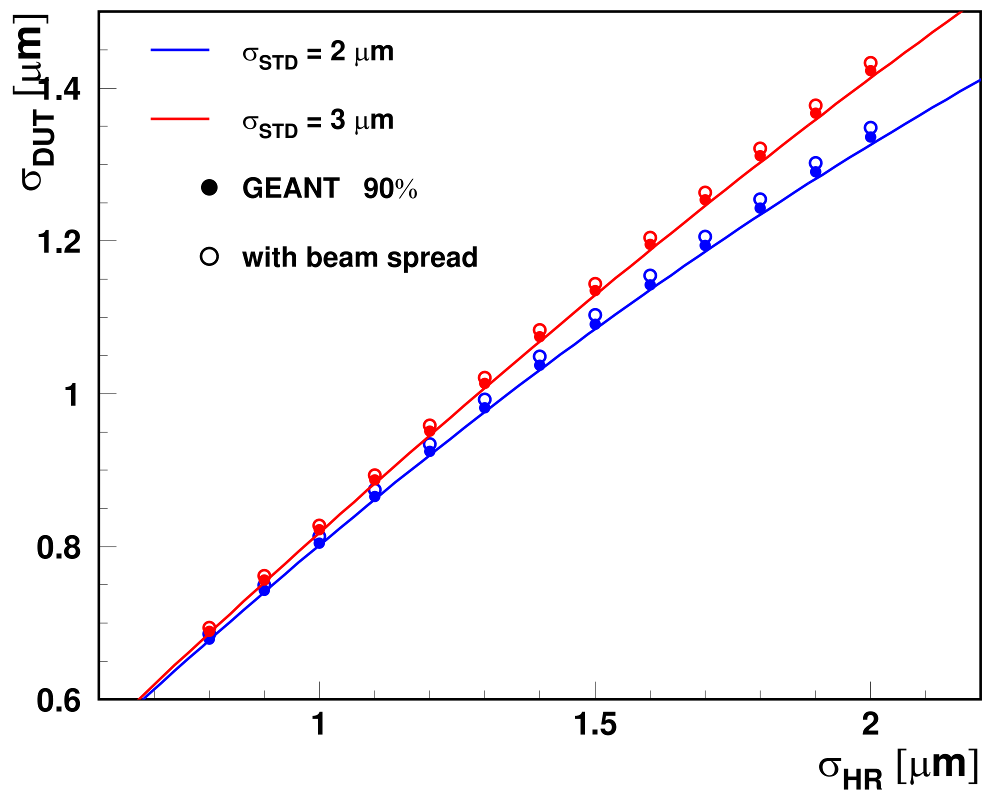
Analytical method

When solving the matrix equation for χ^2 minimum we can also calculate the expected position error at DUT.

It depends only on the assumed telescope geometry and sensor resolution!

Can be calculated analytically without any input (simulation) data.

Comparison of analytical method with GEANT 4 simulation results:



Analytical Results

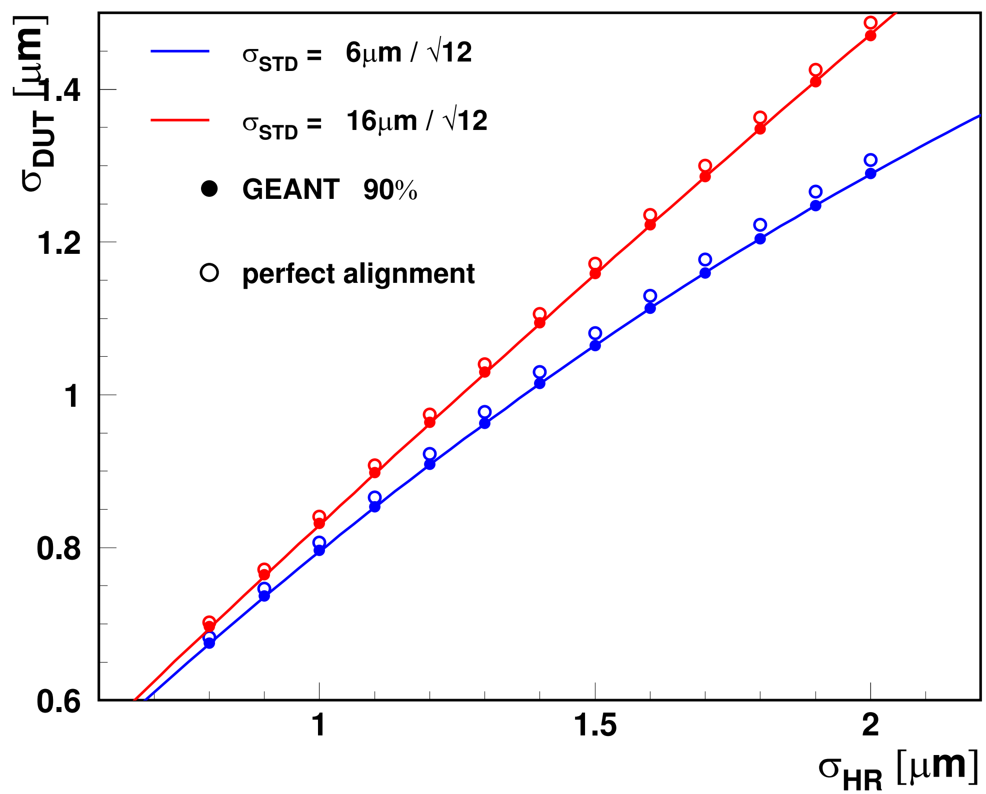
Analytical method

Assumption of the Gaussian position measurement error is not crucial.

For sensor planes with 16 μm or 6 μm pitch and binary readout the obtained position resolution at DUT is very close to that expected for

$$\sigma_{STD} = \frac{\text{pitch}}{\sqrt{12}}$$

GEANT 4 simulation results



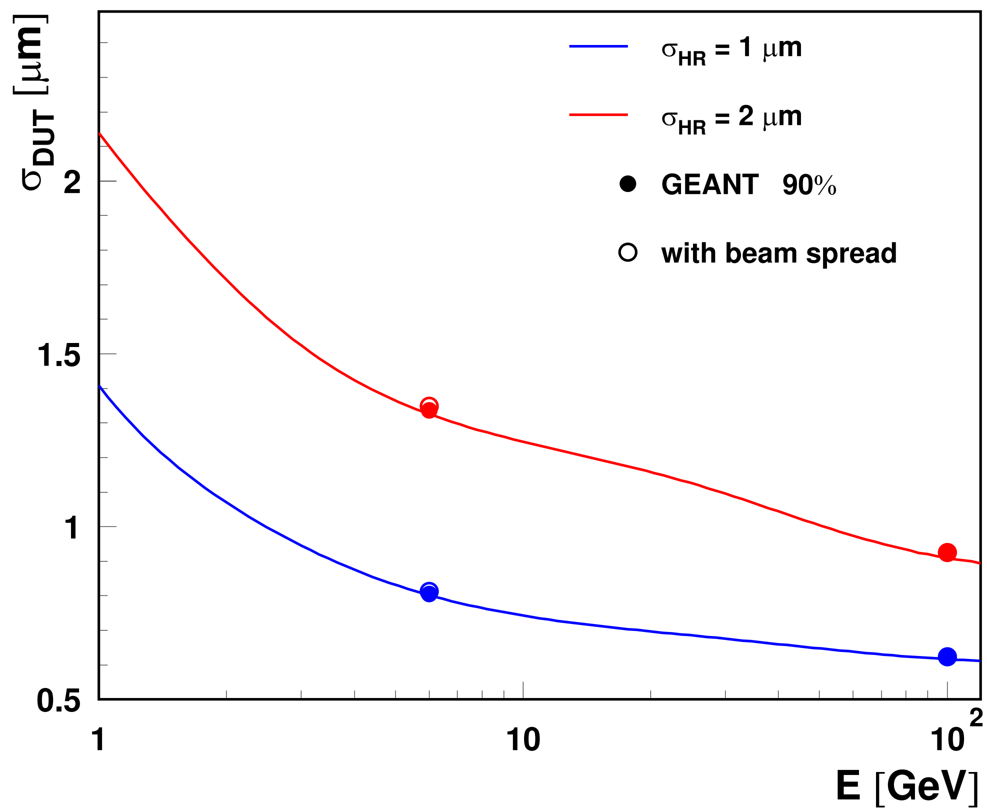
Analytical Results

Analytical method

We can calculate the expected position error for arbitrary telescope geometry (distances between planes, sensor and DUT thicknesses), sensor resolutions and beam energy.

We use this approach to find the optimum telescope setup, i.e. the one giving the best position resolution at DUT.

Comparison of analytical method with GEANT 4 simulation results:

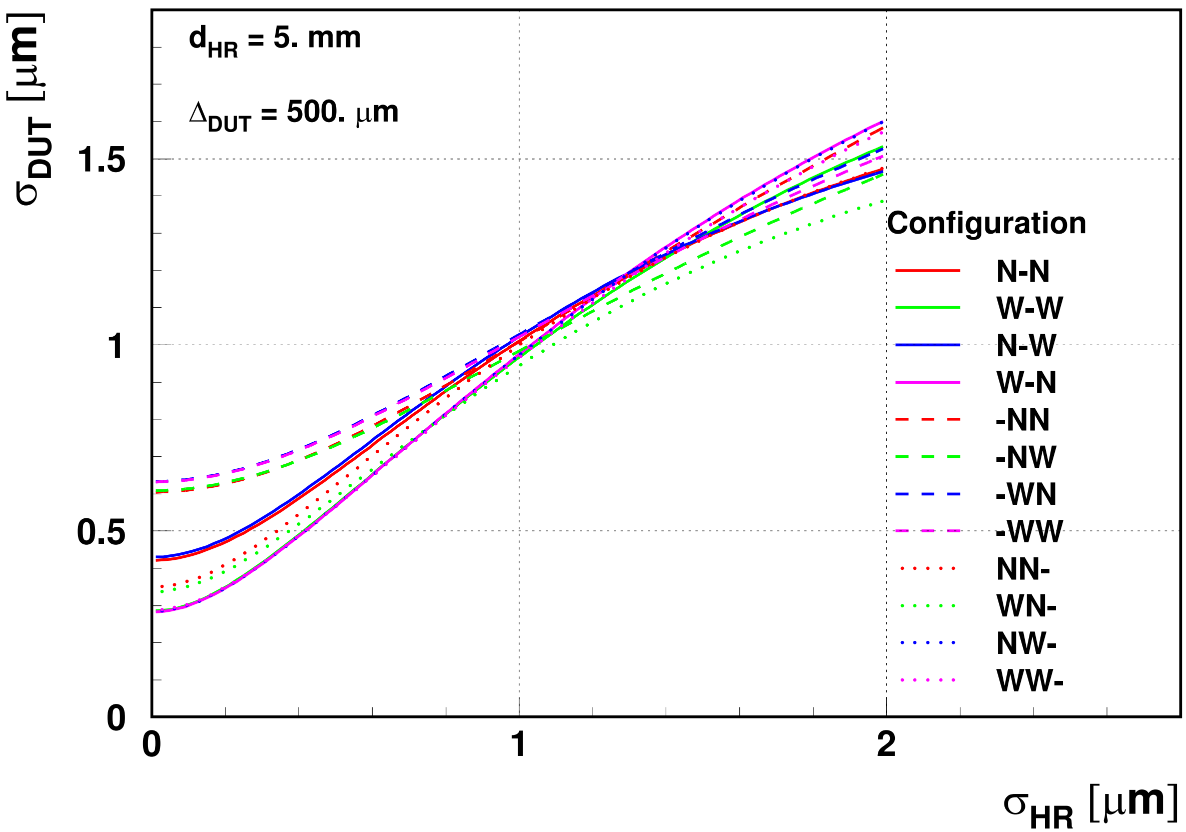


Analytical Results

4 (1+3) telescope planes

Simplest case: 1 high resolution (HR) and 3 standard sensor planes (120 μm each)

Expected position error at DUT, σ_{DUT}, as a function of the HR plane resolution, σ_{HR}, for different telescope configurations: 6 GeV e^- beam, DUT thickness of 500 μm

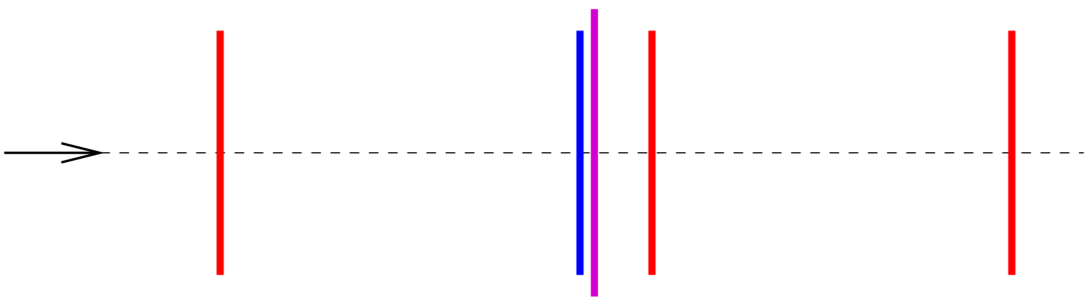


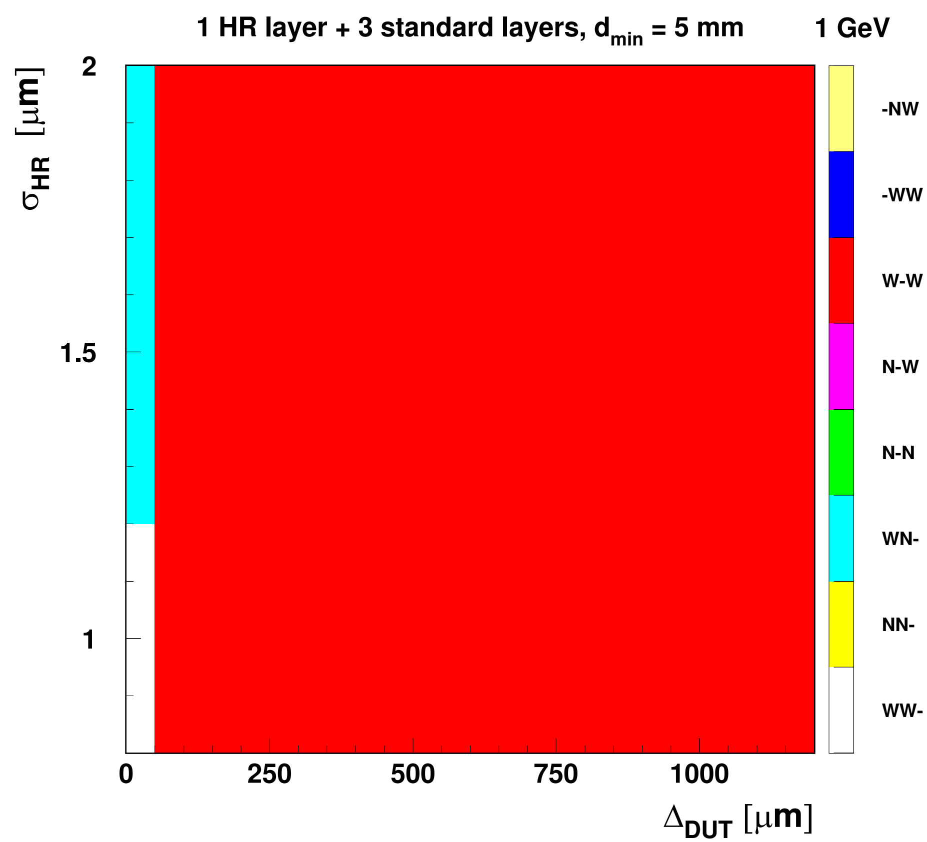
Analytical Results

4 (1+3) telescope planes

Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **1 GeV**:

For lowest beam energies best measurement is obtained in **W-W** configuration:



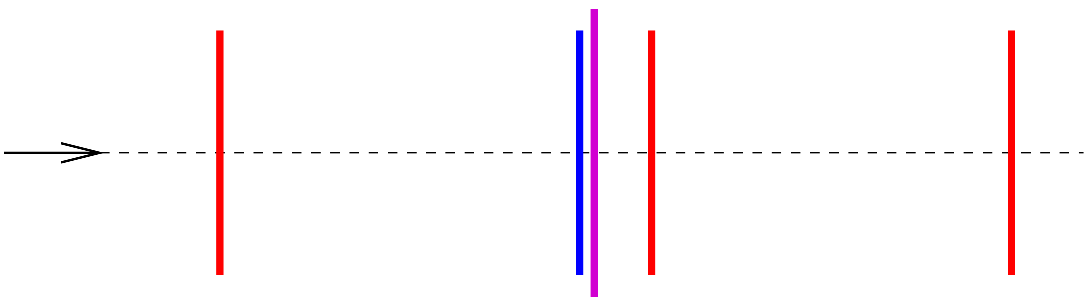


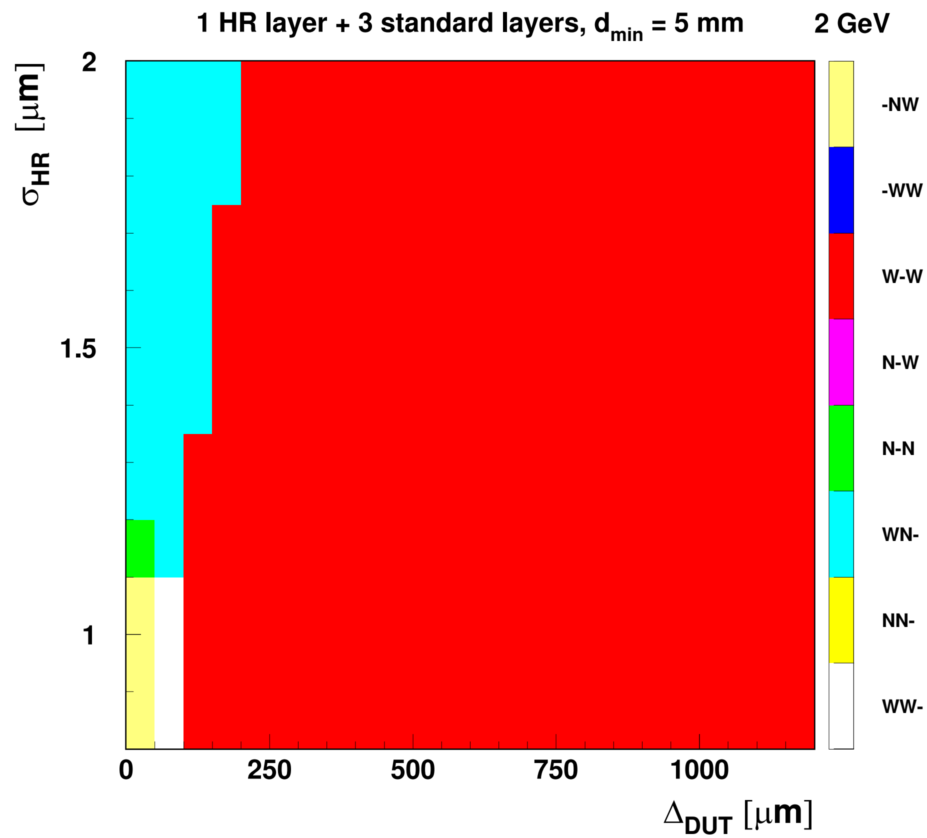
Analytical Results

4 (1+3) telescope planes

Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **2 GeV**:

For lowest beam energies best measurement is obtained in **W-W** configuration:

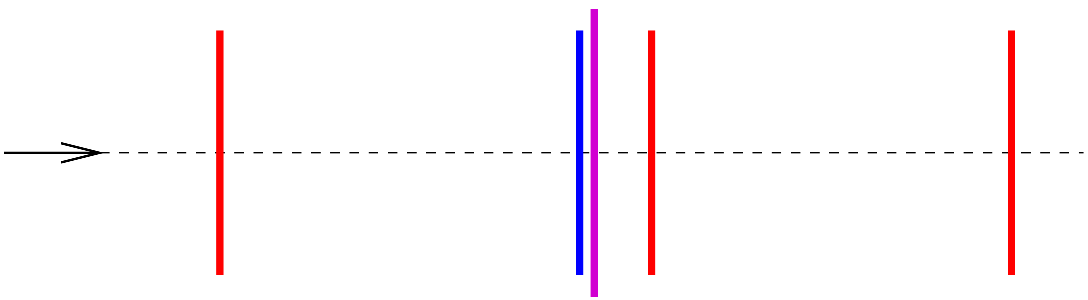




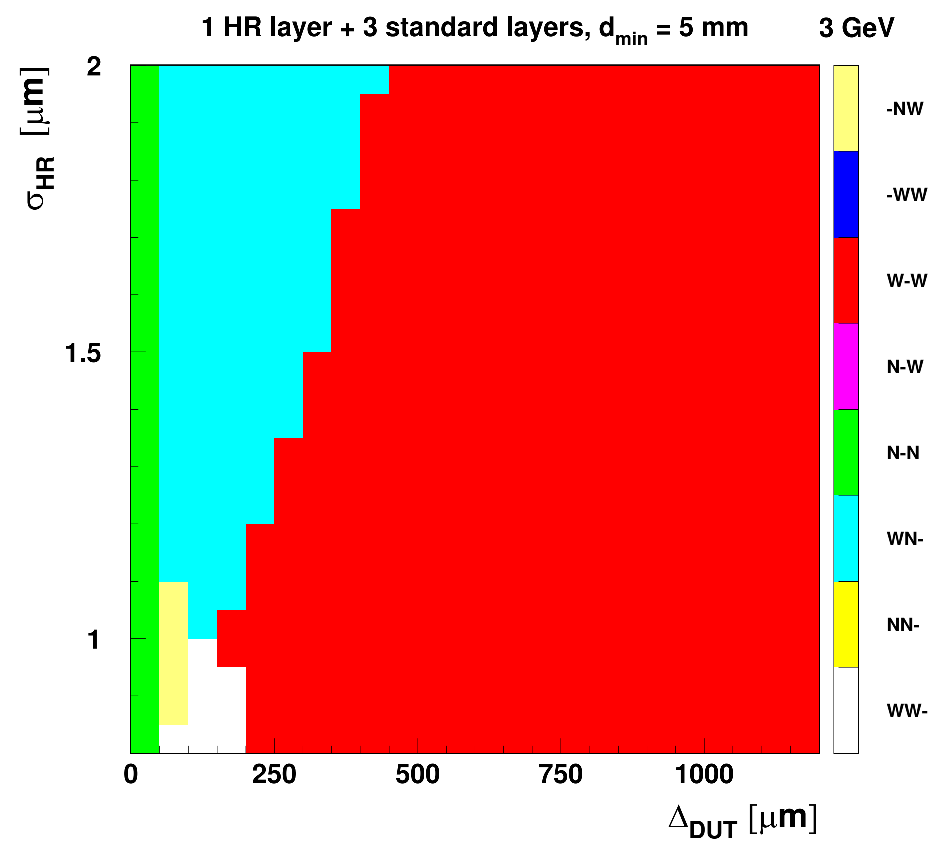
Analytical Results

4 (1+3) telescope planes

For lowest beam energies best measurement is obtained in **W-W** configuration:



Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **3 GeV**:

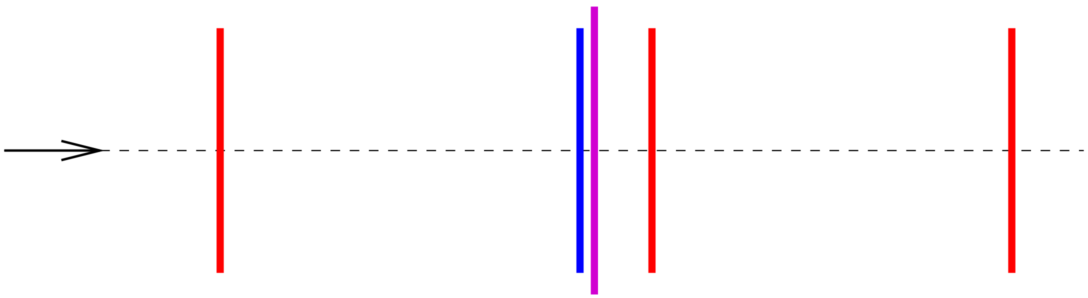


Analytical Results

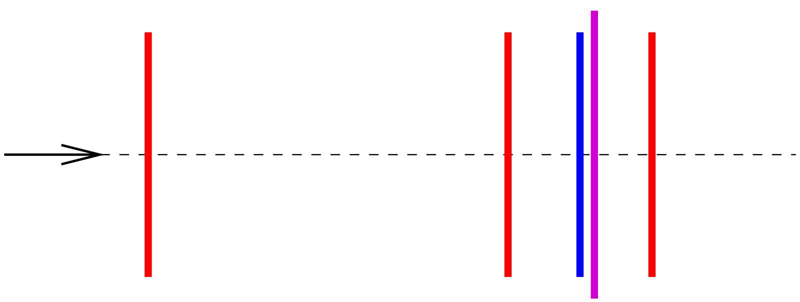
4 (1+3) telescope planes

For intermediate energies:

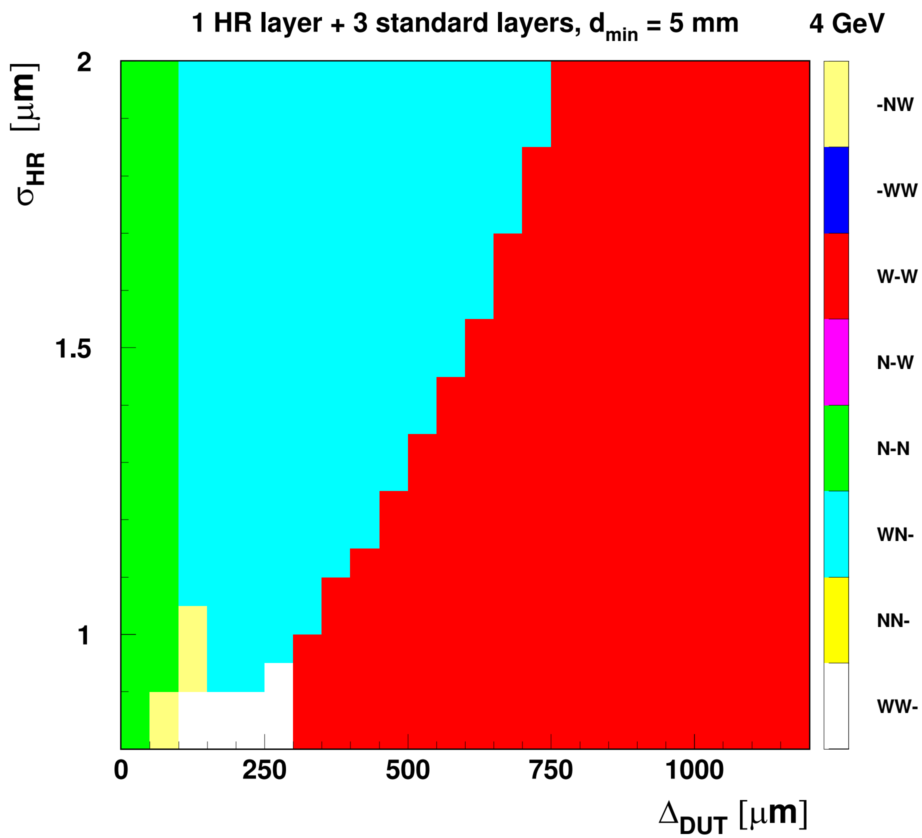
W-W configuration



or **WN-** configuration



Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **4 GeV**:

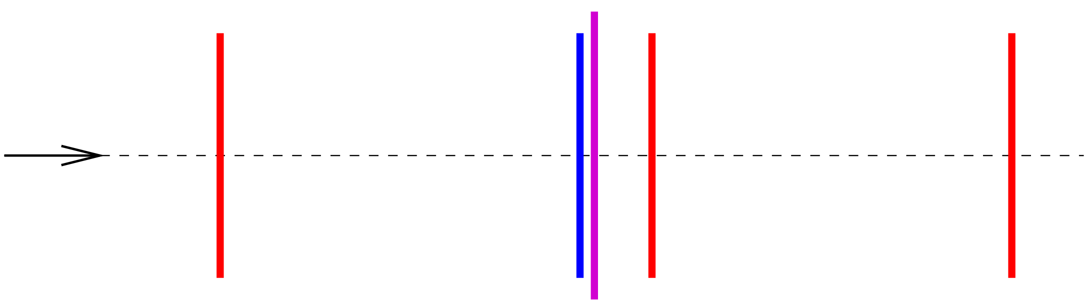


Analytical Results

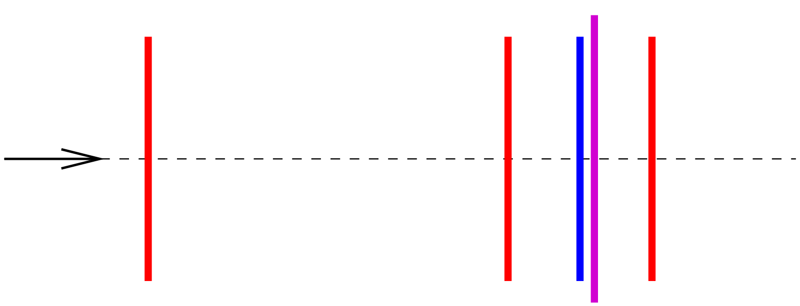
4 (1+3) telescope planes

For intermediate energies:

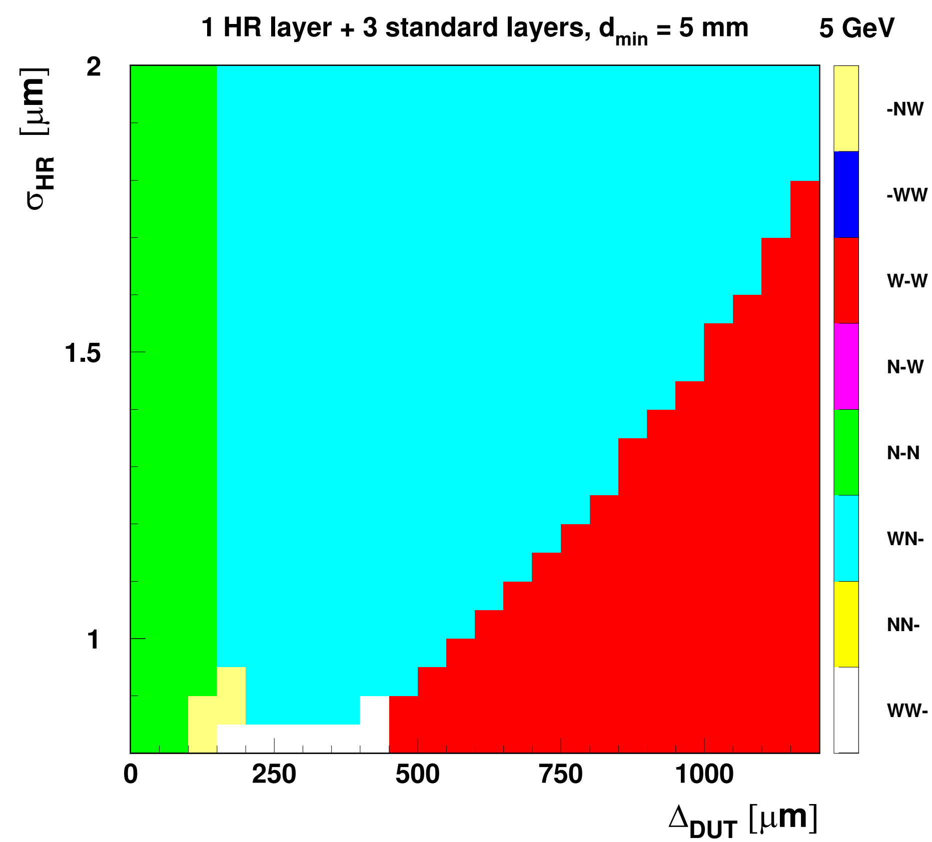
W-W configuration



or **WN-** configuration



Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **5 GeV**:

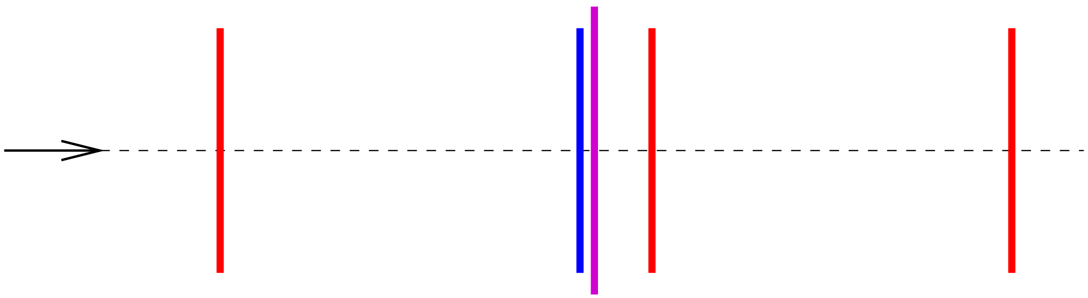


Analytical Results

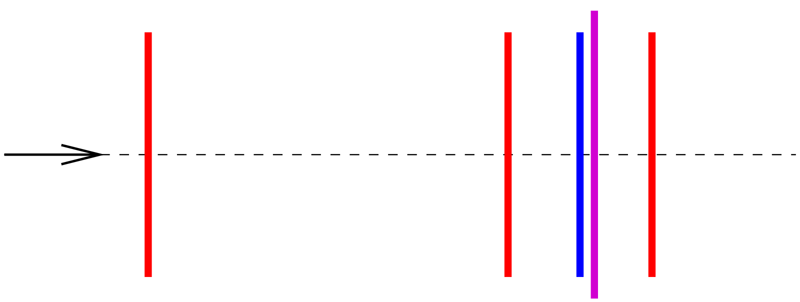
4 (1+3) telescope planes

For intermediate energies:

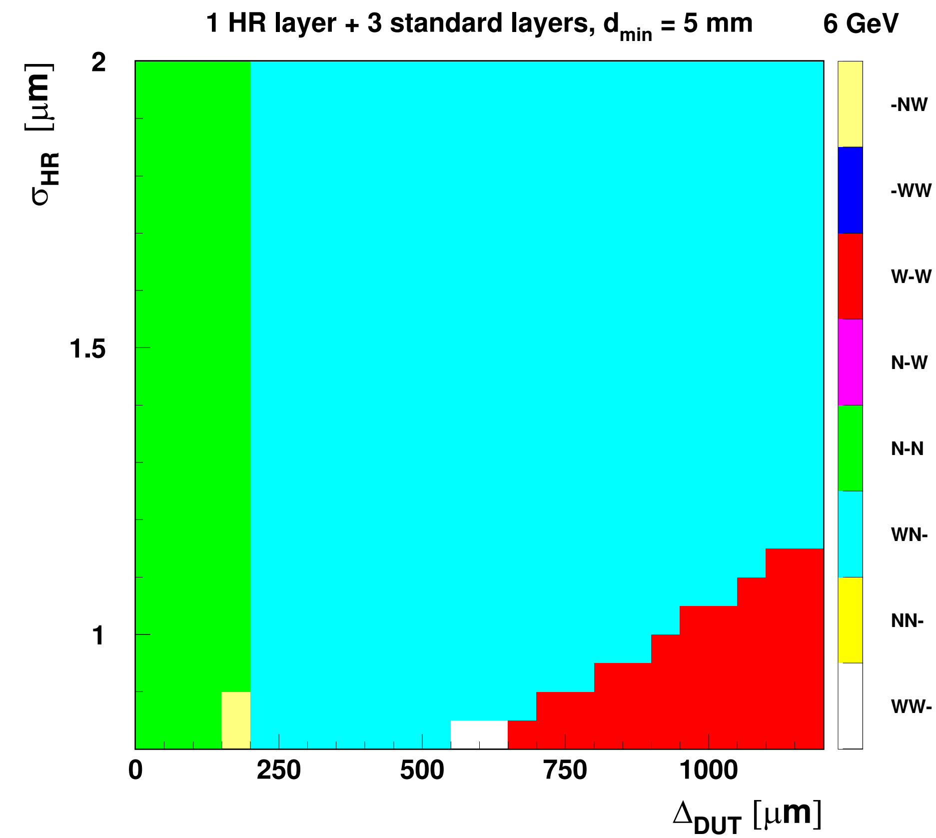
W-W configuration



or **WN-** configuration



Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **6 GeV**:

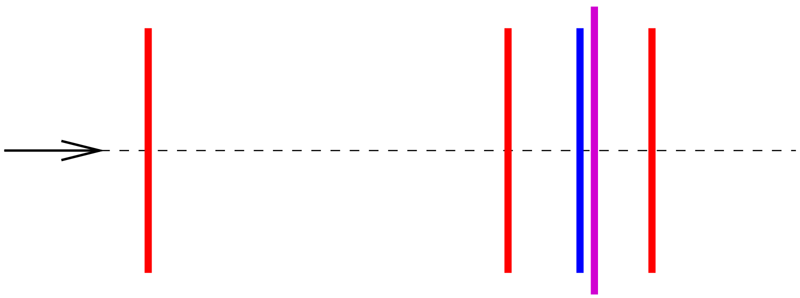


Analytical Results

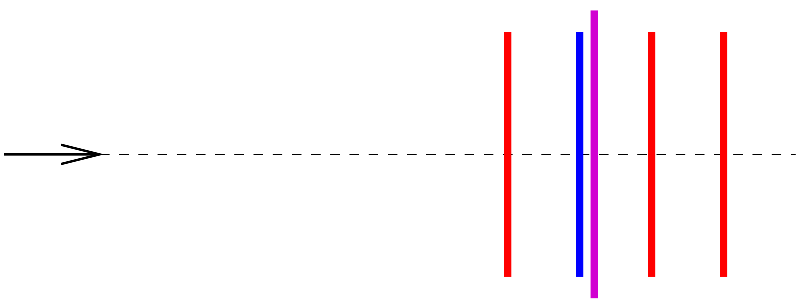
4 (1+3) telescope planes

For higher beam energies:

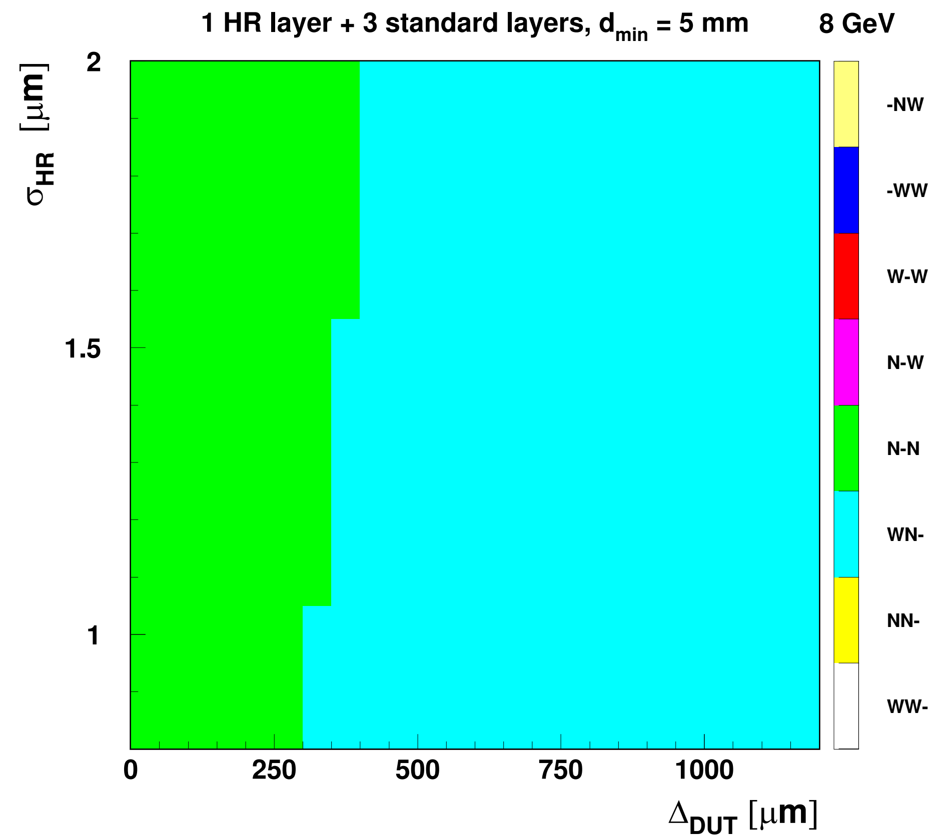
WN- configuration:



or **N-N** configuration



Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **8 GeV**:

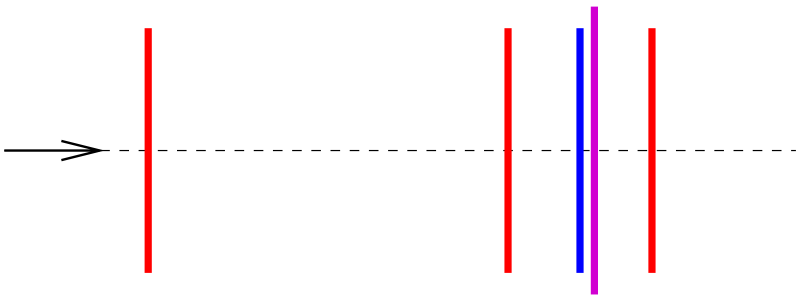


Analytical Results

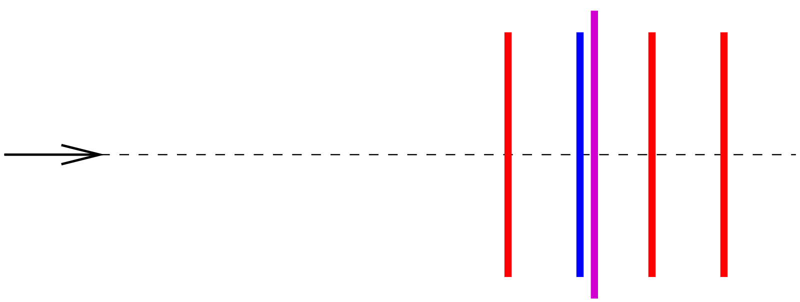
4 (1+3) telescope planes

For higher beam energies:

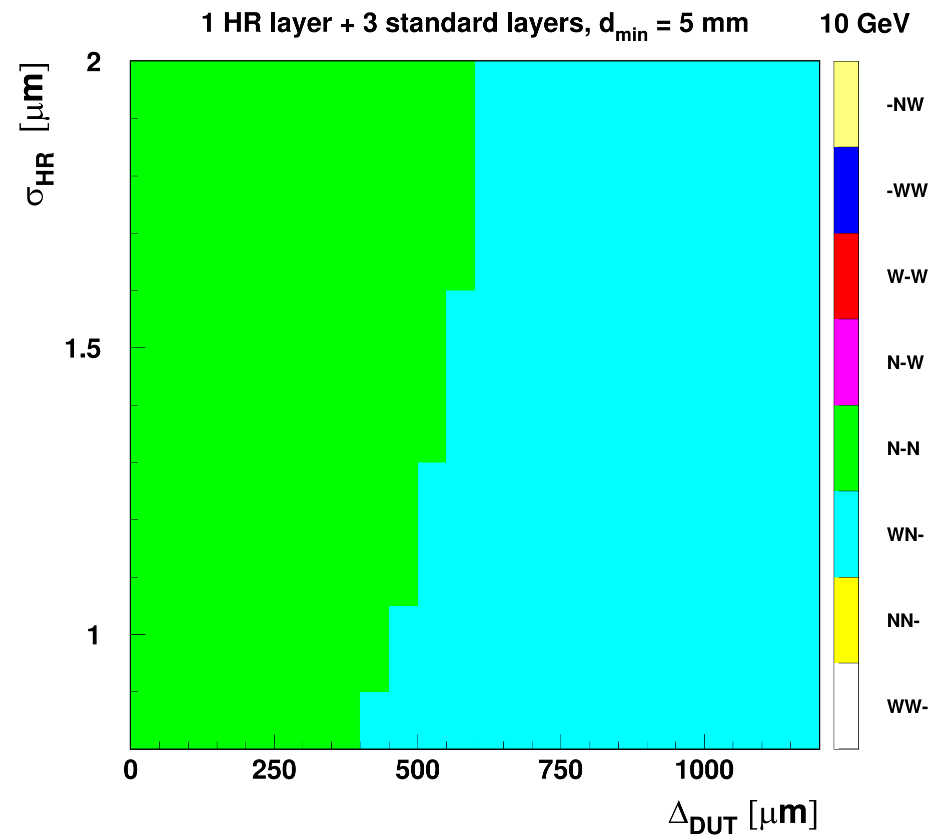
WN- configuration:



or **N-N** configuration



Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **10 GeV**:

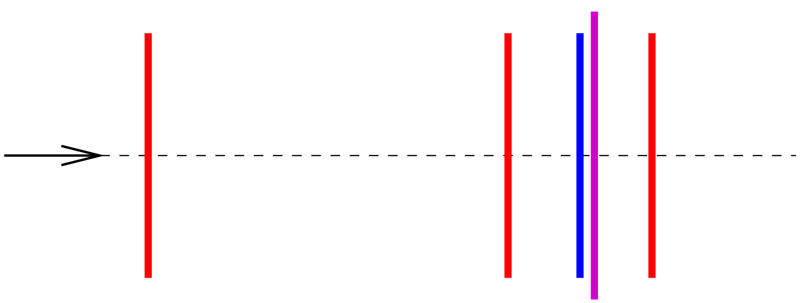


Analytical Results

4 (1+3) telescope planes

For higher beam energies:

WN- configuration:



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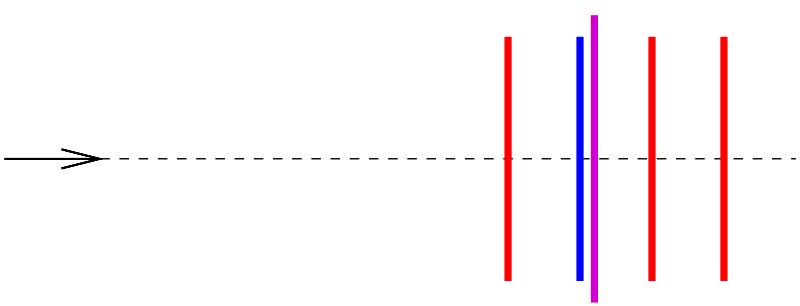
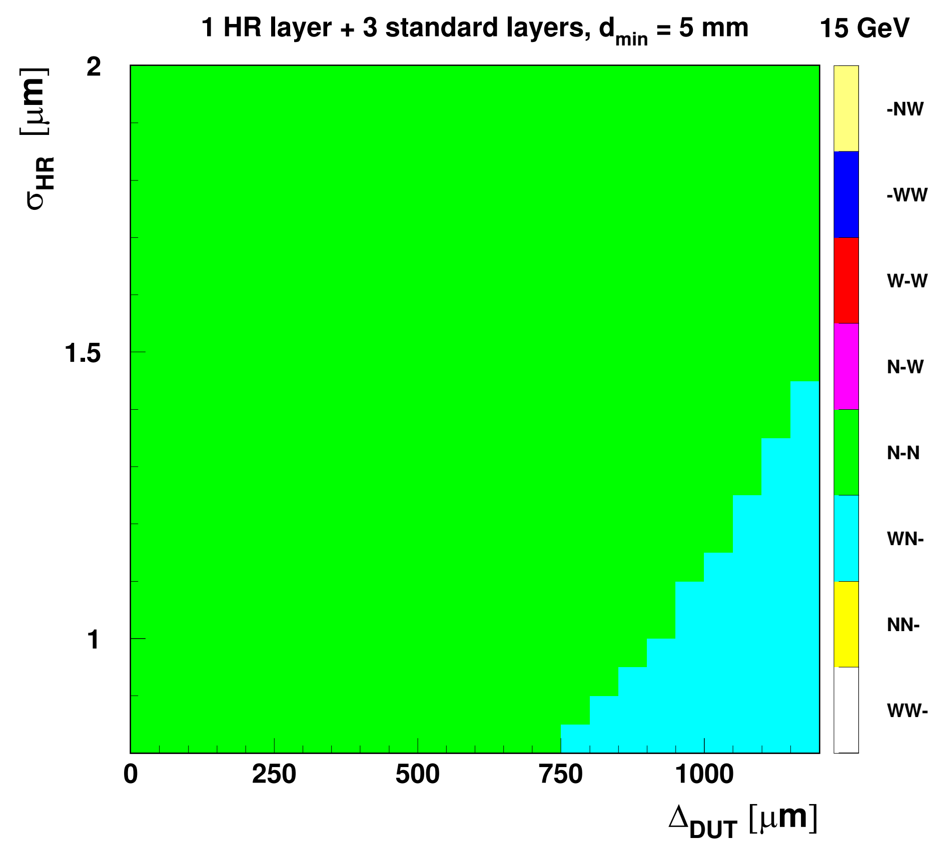


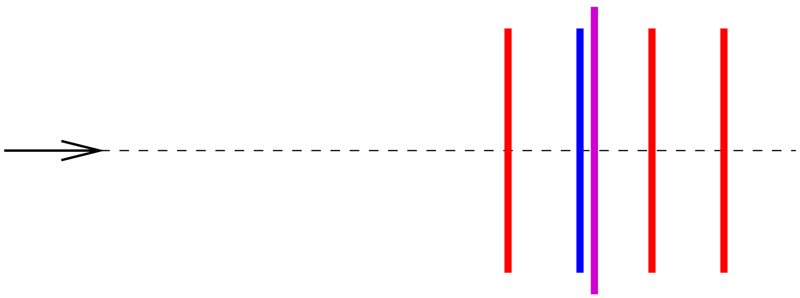
Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **15 GeV**:



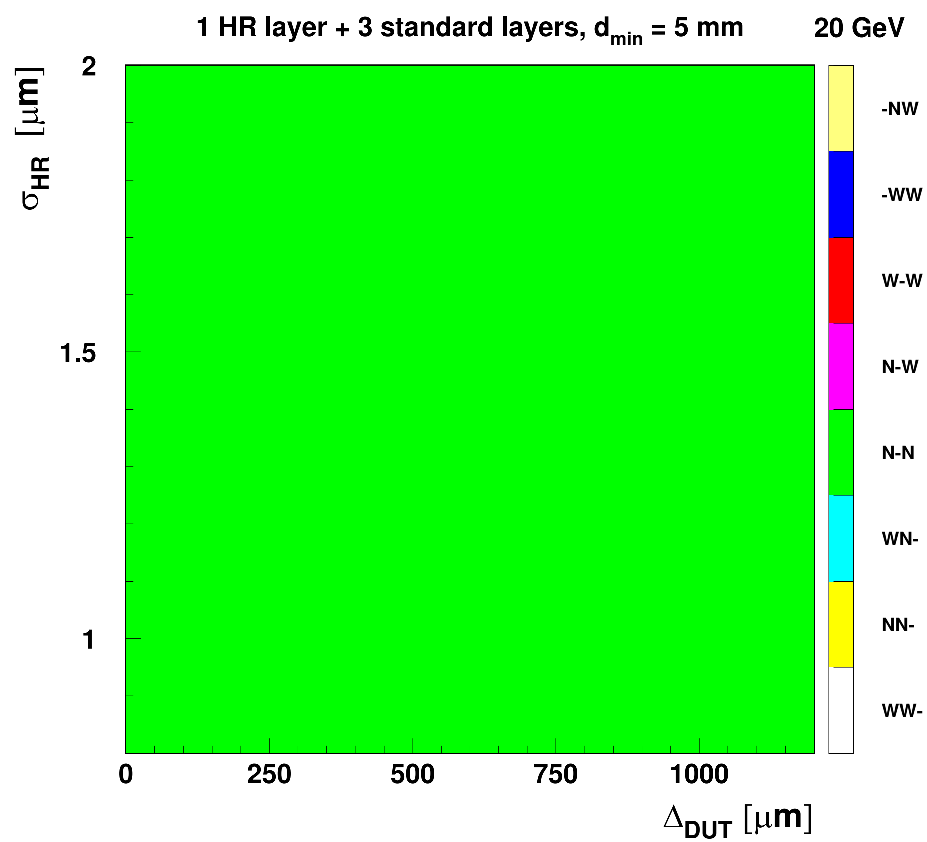
Analytical Results

4 (1+3) telescope planes

For highest beam energies **N-N** configuration



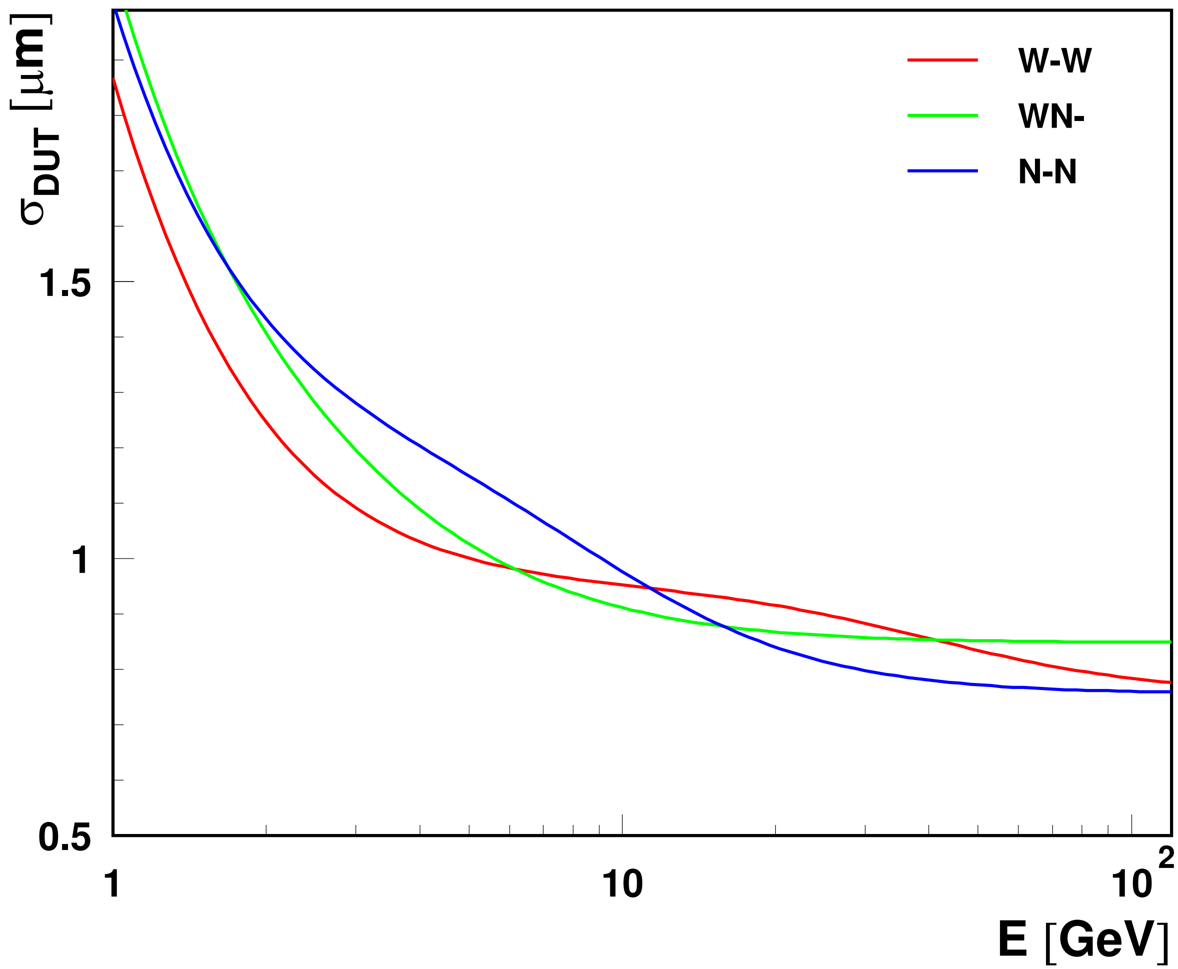
Configuration choice as a function of DUT thickness and HR plane resolution, beam energy of **20 GeV**:



Analytical Results

4 (1+3) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



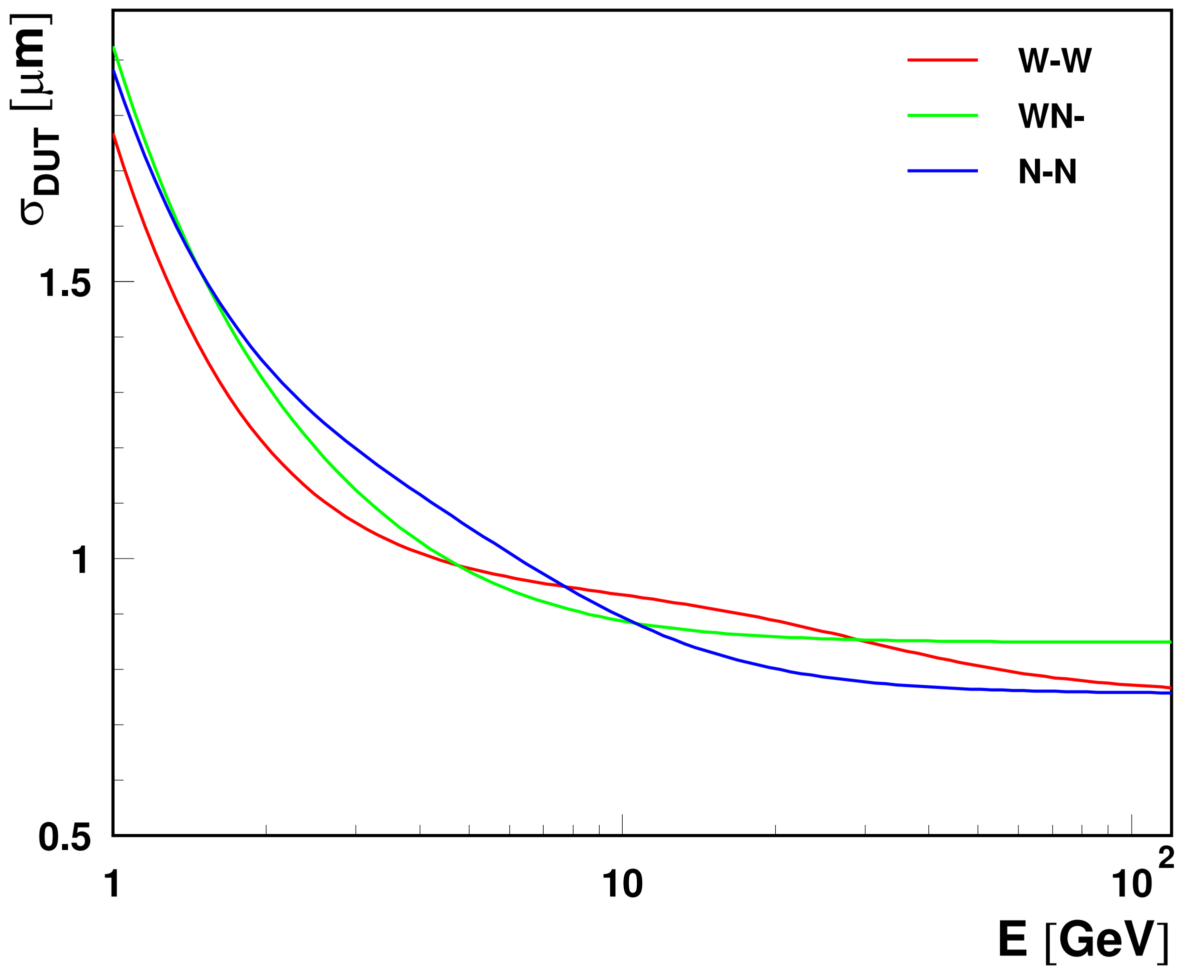
Assumed telescope parameters:

- 1000 μm DUT
- 1 HR plane 120 $\mu m, \sigma = 1 \mu m$
- 3 Std planes 120 $\mu m, \sigma = 2 \mu m$
- 5 mm between DUT and HR plane

Analytical Results

4 (1+3) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



Assumed telescope parameters:

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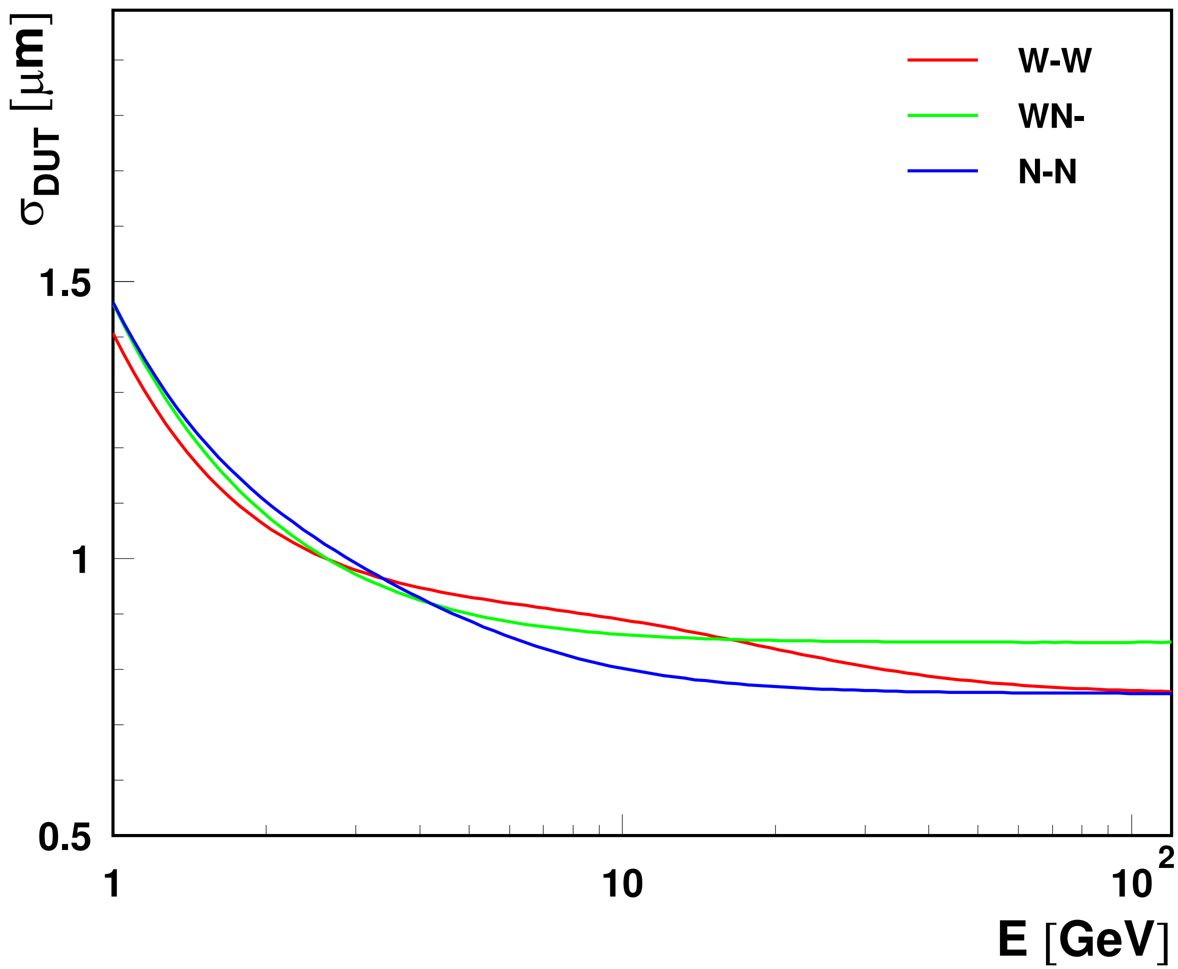
Analytical Results

4 (1+3) telescope planes

Comparison of expected precision,
for different telescope setups,
as a function of beam energy



Assumed telescope parameters:

- 120 μm DUT
- 1 HR plane 120 μm, $\sigma = 1\mu m$
- 3 Std planes 120 μm, $\sigma = 2\mu m$
- 5 mm between DUT and HR plane

Analytical Results

4 (2+2) telescope planes

Two high resolution + two standard planes: more possibilities!

Configuration choice as a function of DUT thickness and HR plane resolution:

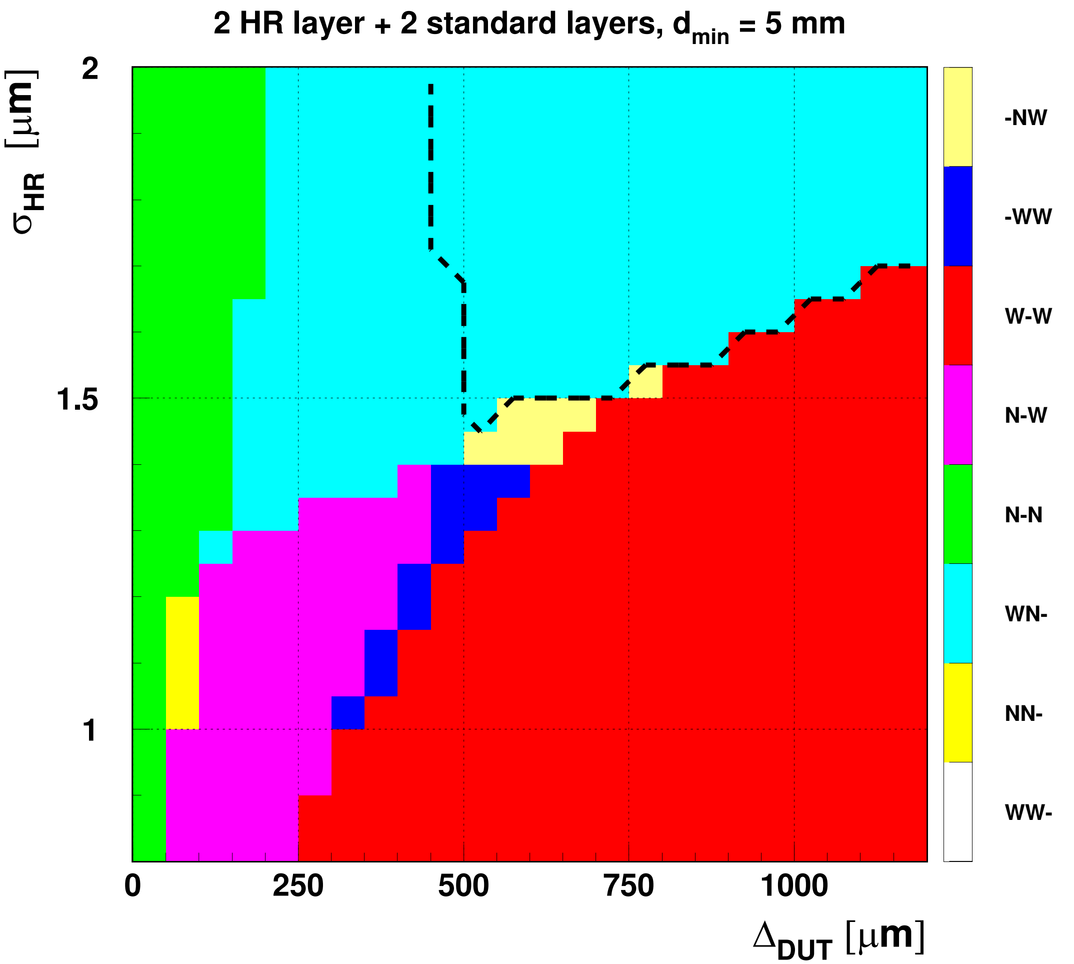
$d_{min} = 5$ mm

Above dashed line:

better performance if both HR planes in front of DUT

large σ_{HR} & Δ_{DUT}

6 GeV e^- beam

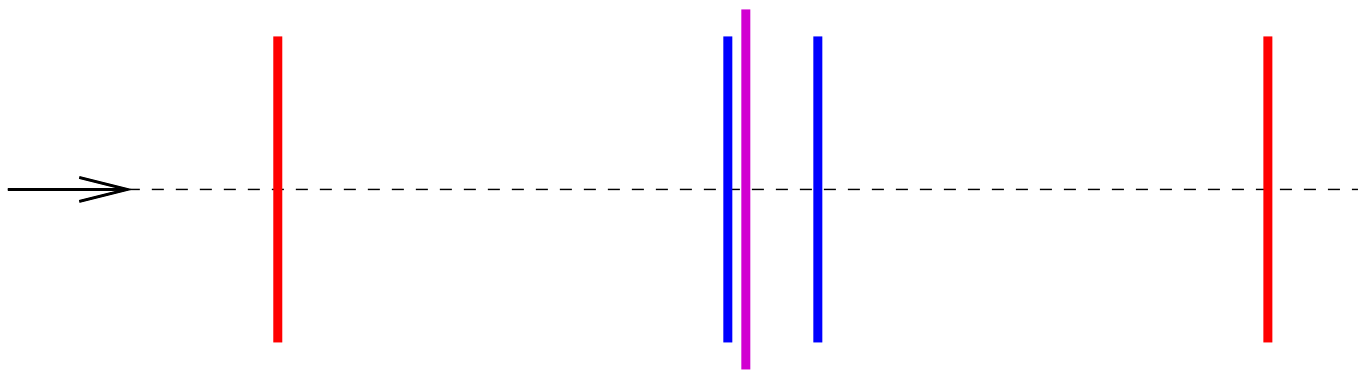


Analytical Results

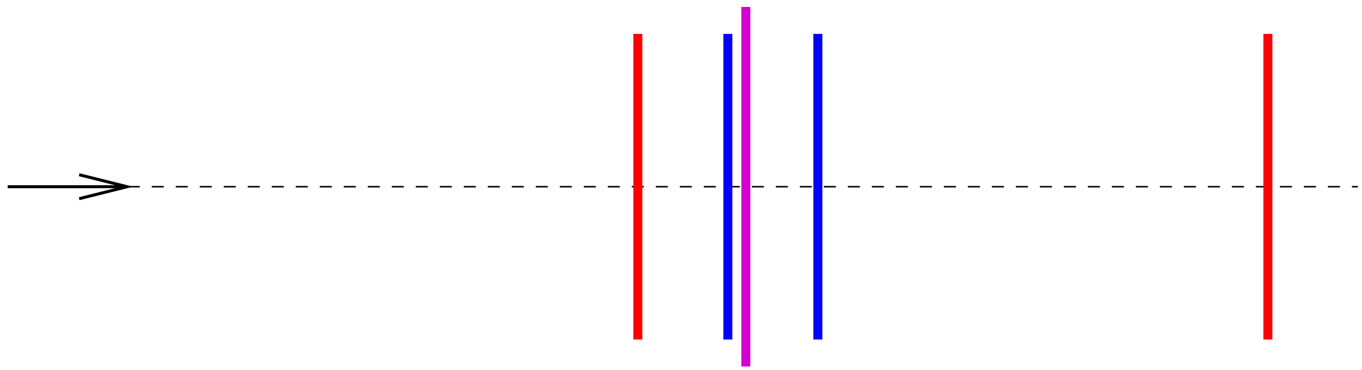
4 (2+2) telescope planes

Assuming HR plane resolution is of the order of 1 μm

W–W configuration gives best precision for lowest beam energies



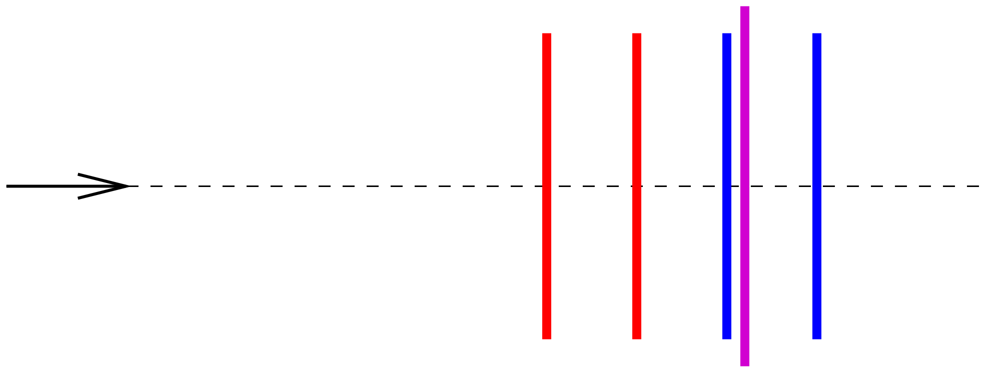
N–W configuration can be better for intermediate energies



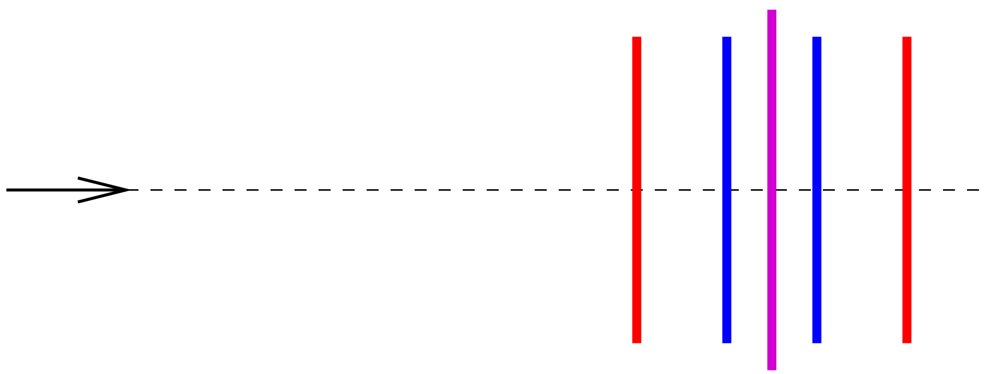
Analytical Results

4 (2+2) telescope planes

NN– configuration gives best precision for high energies and and $\sigma_{HR} \sim 1\mu m$



N–N configuration gives can be better for slightly higher σ_{HR} (symmetric!)



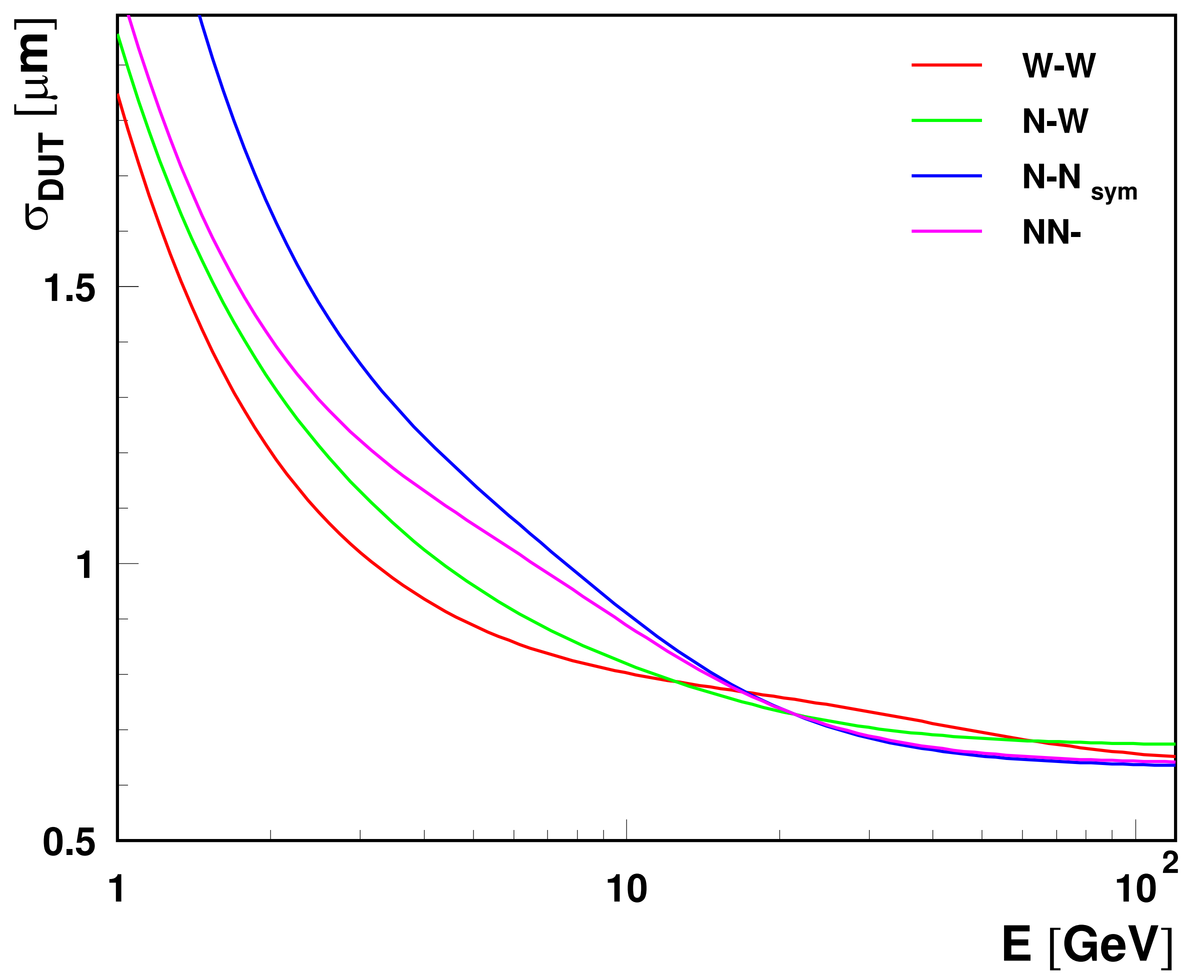
Analytical Results

4 (2+2) telescope planes

Comparison of expected precision,
for different telescope setups,
as a function of beam energy

Assumed telescope parameters:

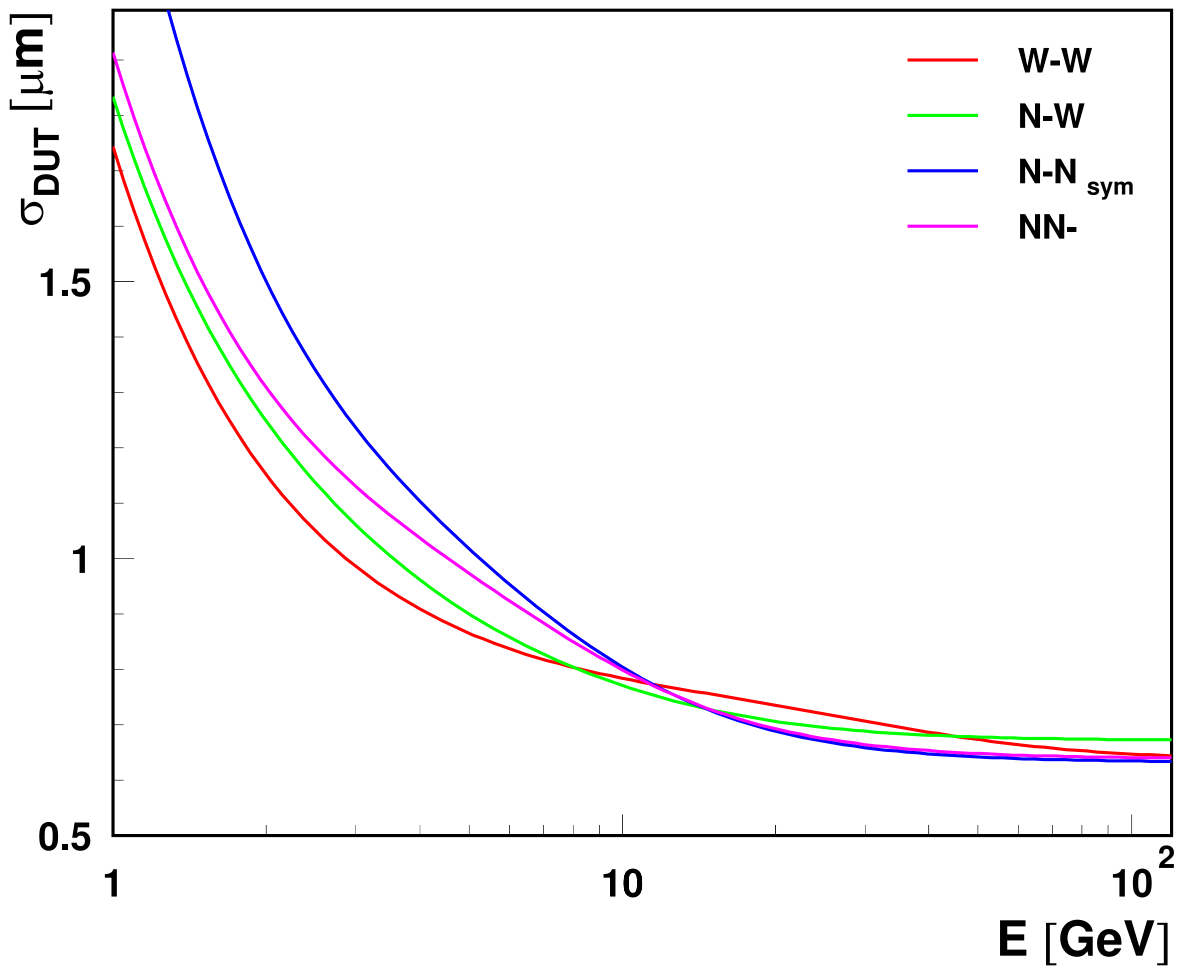
- 1000 μm DUT
- 2 HR planes 120 μm, $\sigma = 1\mu m$
- 2 Std planes 120 μm, $\sigma = 2\mu m$
- 5 mm between DUT and HR plane



Analytical Results

4 (2+2) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



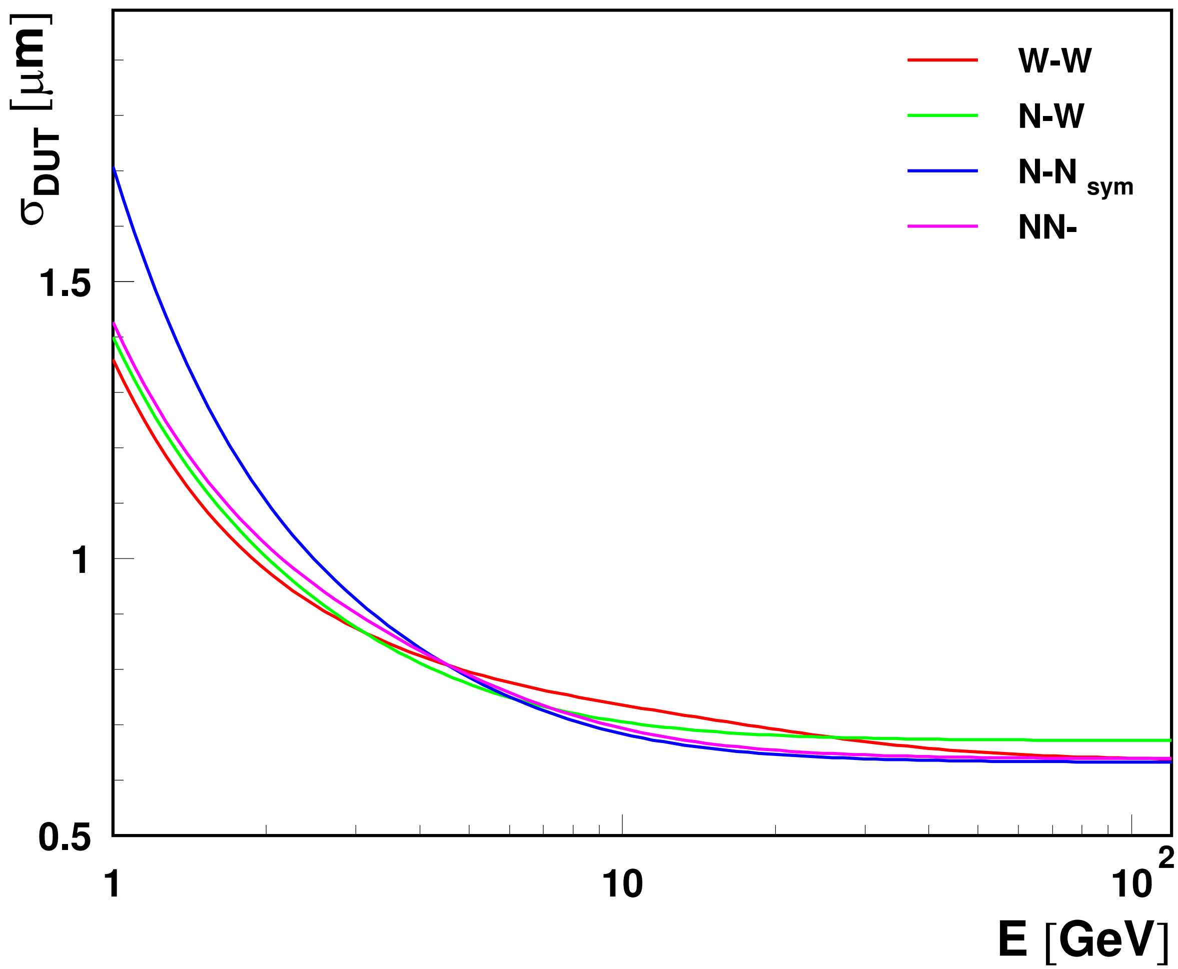
Assumed telescope parameters:

- 500 μm DUT
- 2 HR planes 120 μm, $\sigma = 1 \mu m$
- 2 Std planes 120 μm, $\sigma = 2 \mu m$
- 5 mm between DUT and HR plane

Analytical Results

4 (2+2) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



Assumed telescope parameters:

- 120 μm DUT
- 2 HR planes 120 μm, $\sigma = 1\mu m$
- 2 Std planes 120 μm, $\sigma = 2\mu m$
- 5 mm between DUT and HR plane

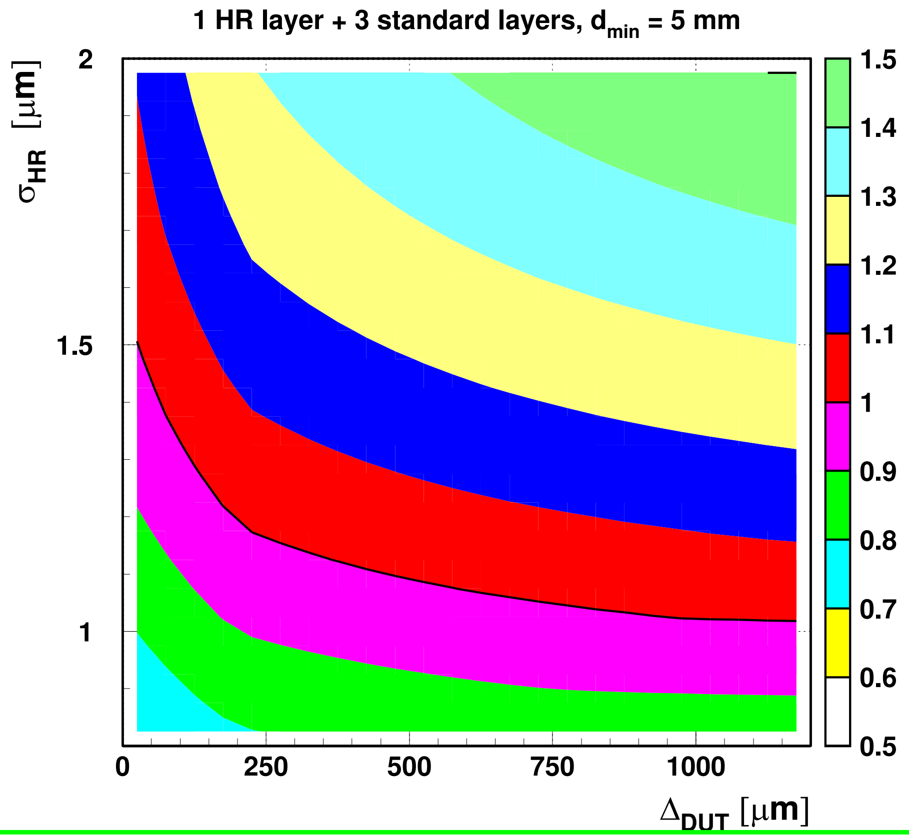
Analytical Results

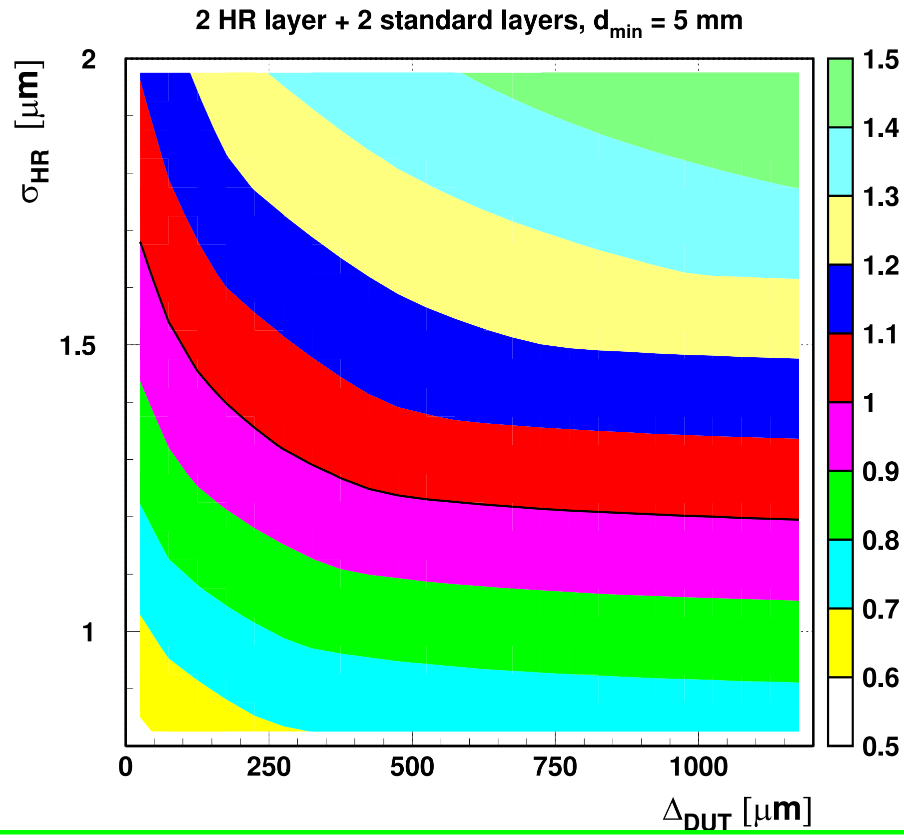
4 telescope planes

Configuration with two HR planes always gives better precision than with one HR plane.

Expected statistical precision of position reconstruction at DUT [μm]:

1 HR plane   6 GeV e^- beam   2 HR planes d_{min} = 5 mm



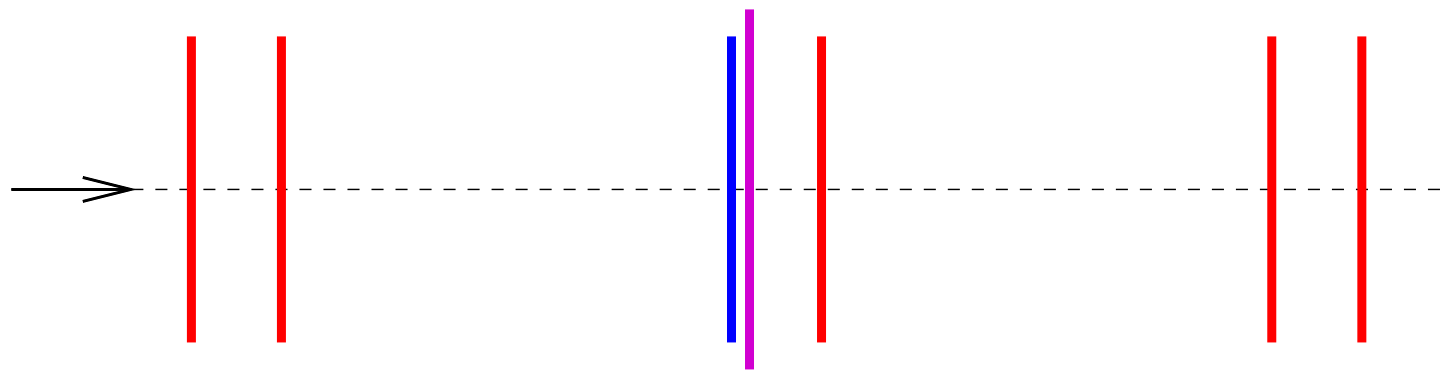


Analytical Results

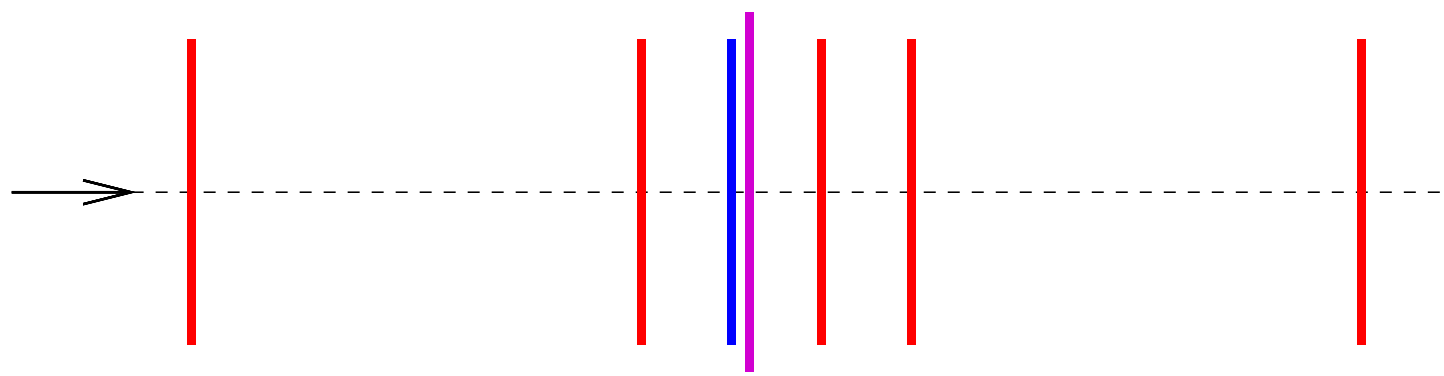
6 (1+5) telescope planes

Assuming HR plane resolution is of the order of 1 μm

NW–WN configuration gives best precision for lowest beam energies



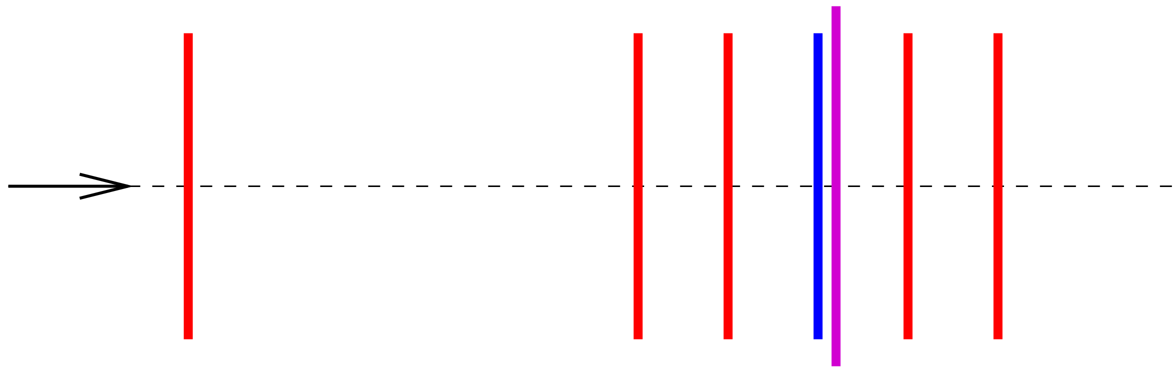
WN–NW configuration can be better for intermediate energies



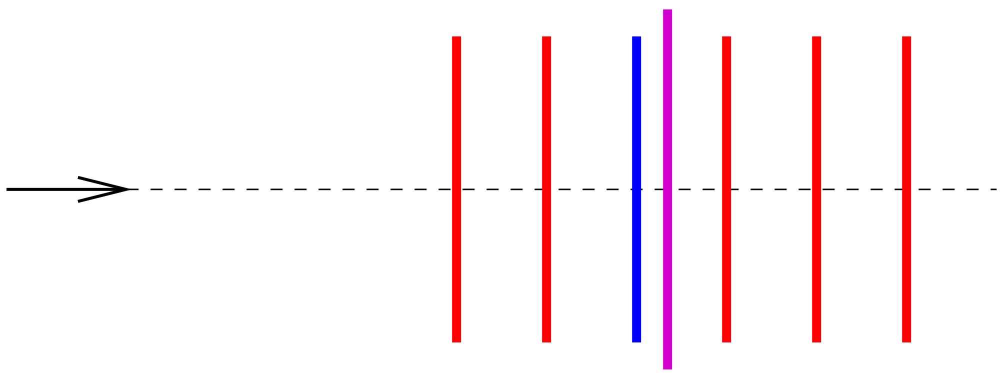
Analytical Results

6 (1+5) telescope planes

WNN–N configuration gives best precision for still higher energies



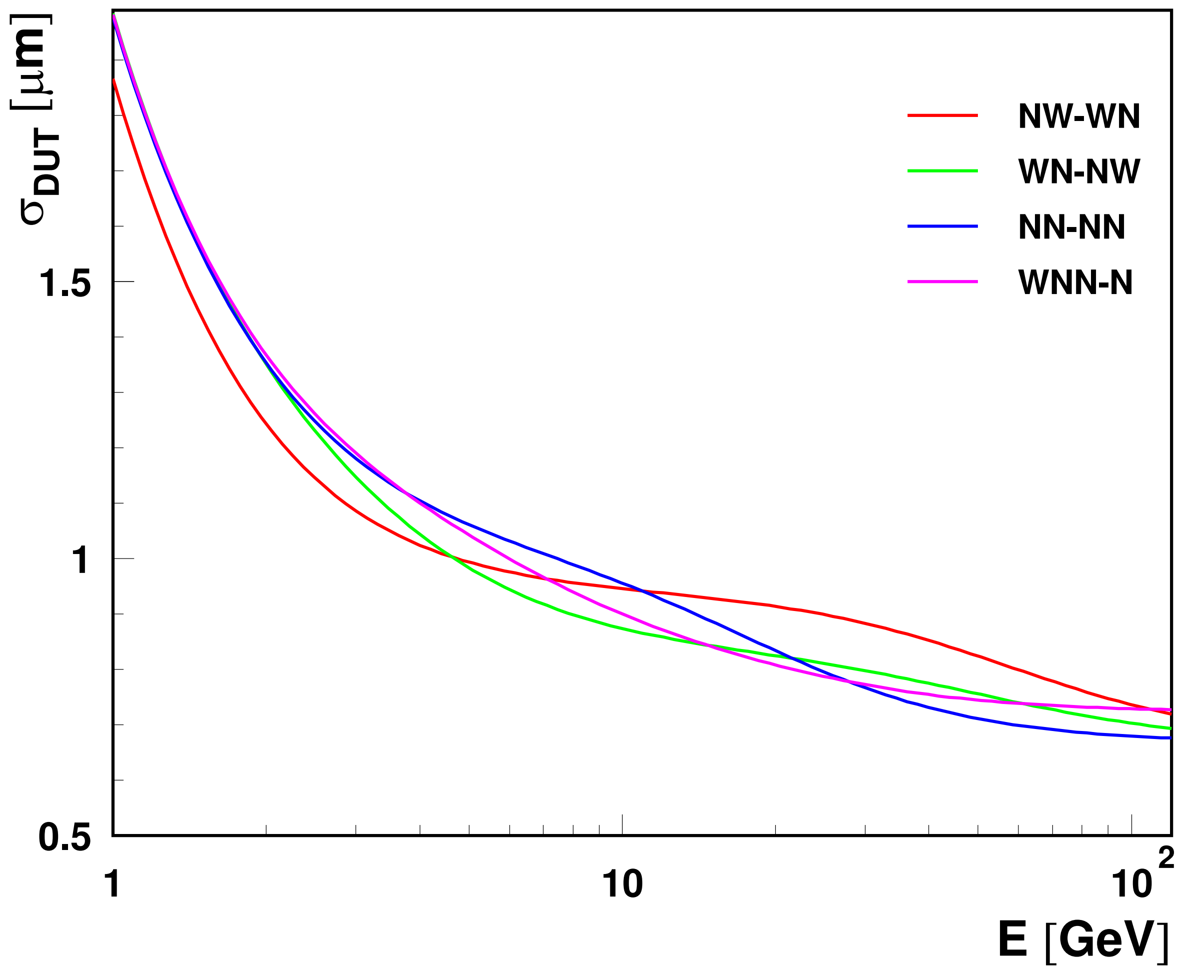
and **NN–NN** is optimal for highest beam energies



Analytical Results

6 (1+5) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



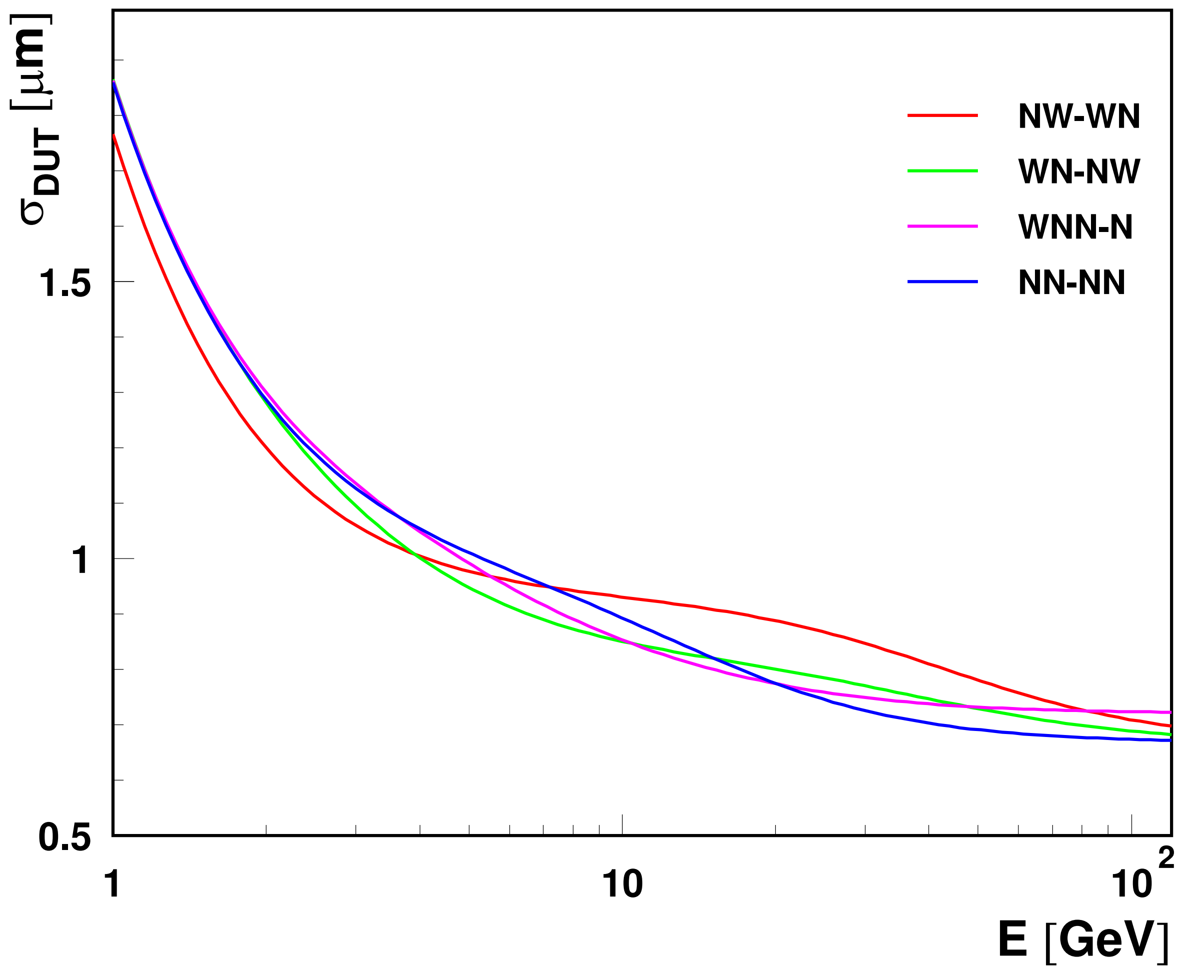
Assumed telescope parameters:

- 1000 μm DUT
- 1 HR plane 120 μm, $\sigma = 1 \mu m$
- 5 Std planes 120 μm, $\sigma = 2 \mu m$
- 5 mm between DUT and HR plane

Analytical Results

6 (1+5) telescope planes

Comparison of expected precision,
for different telescope setups,
as a function of beam energy



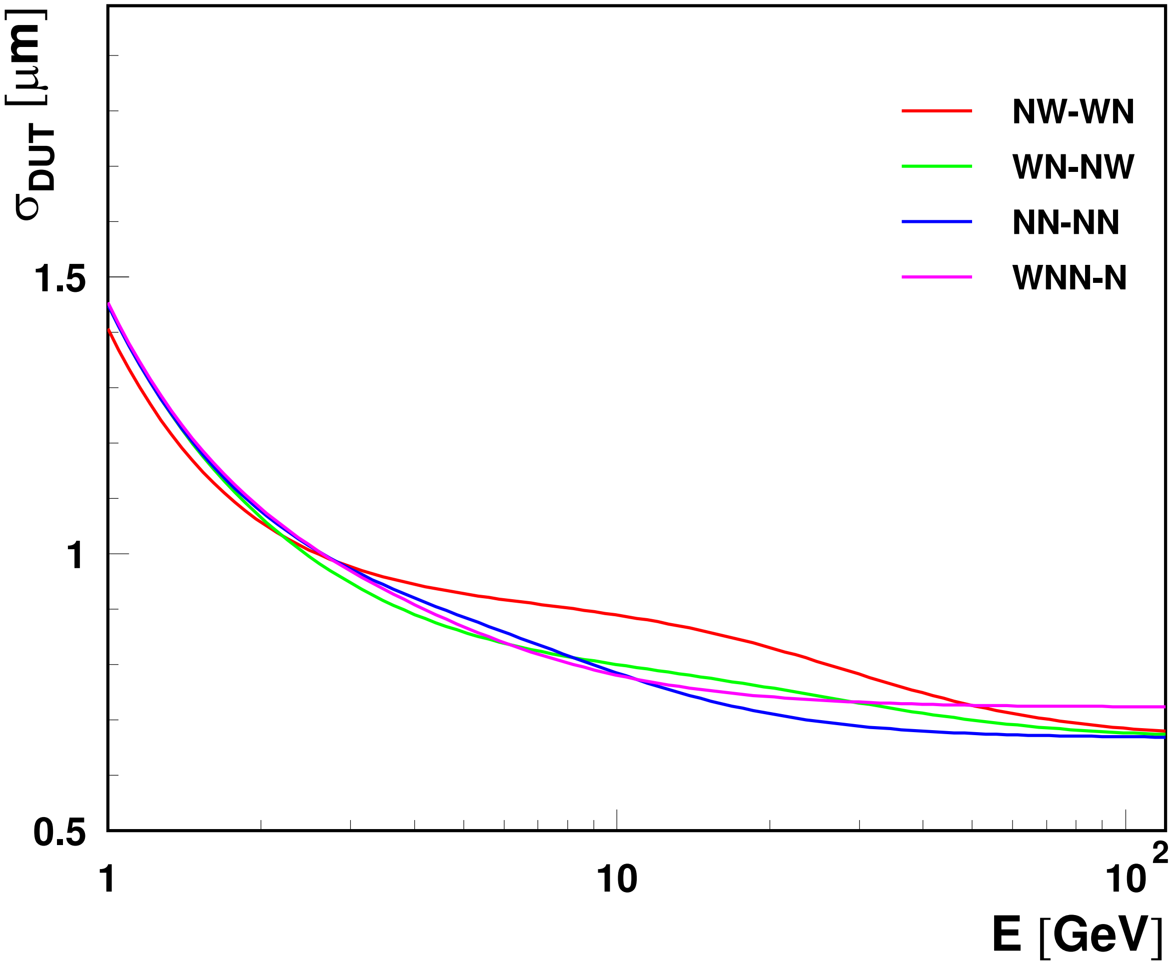
Assumed telescope parameters:

- 500 μm DUT
- 1 HR plane 120 μm, $\sigma = 1 \mu m$
- 5 Std planes 120 μm, $\sigma = 2 \mu m$
- 5 mm between DUT and HR plane

Analytical Results

6 (1+5) telescope planes

Comparison of expected precision,
for different telescope setups,
as a function of beam energy



Assumed telescope parameters:

- 120 μm DUT
- 1 HR plane 120 $\mu m, \sigma = 1 \mu m$
- 5 Std planes 120 $\mu m, \sigma = 2 \mu m$
- 5 mm between DUT and HR plane

Analytical Results

6 (2+4) telescope planes

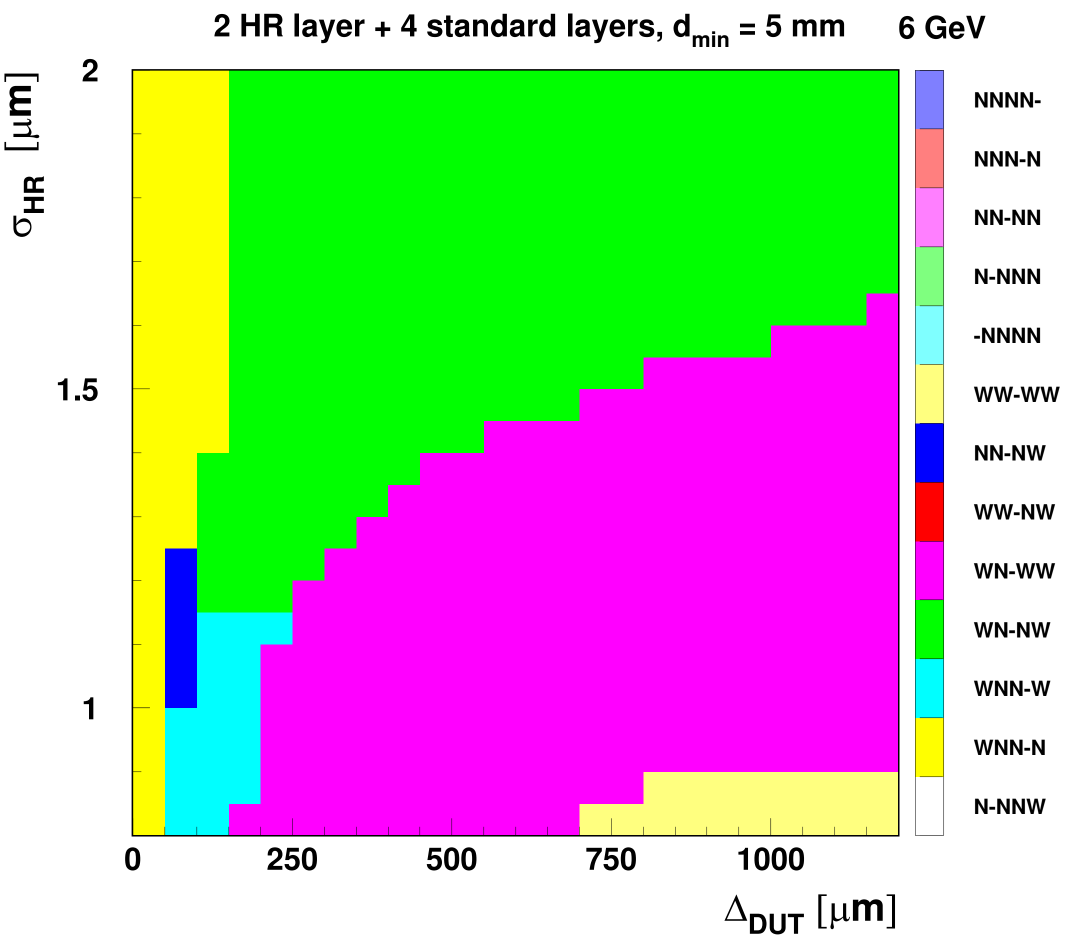
Two high resolution + four standard planes: even more possibilities!

Configuration choice as a function of DUT thickness and HR plane resolution:

$d_{min} = 5$ mm

Best performance with second HR plane always placed behind DUT

6 GeV e^- beam

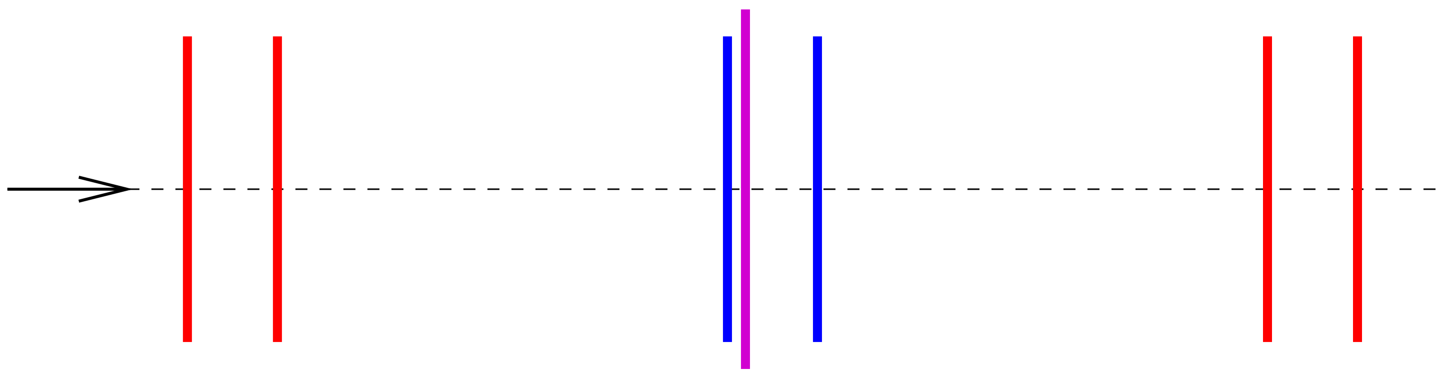


Analytical Results

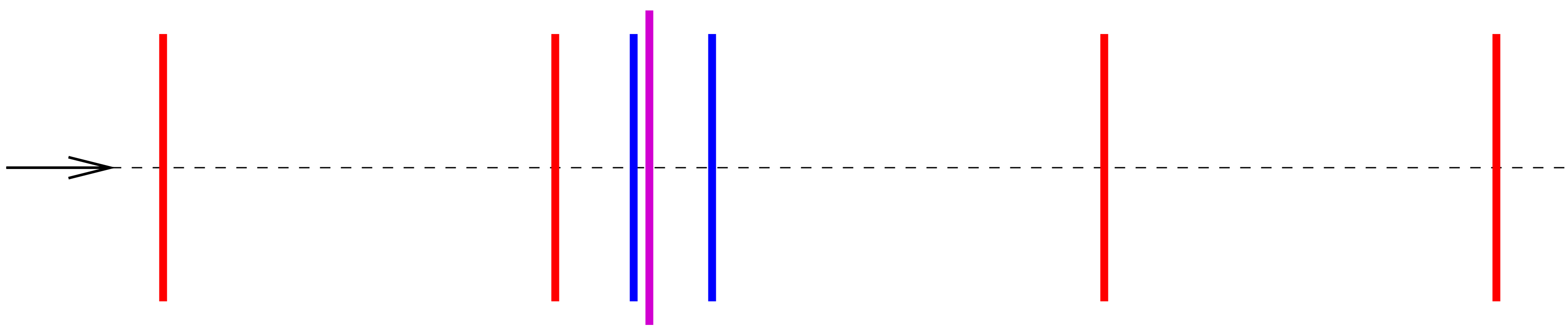
6 (2+4) telescope planes

Assuming HR plane resolution is of the order of 1 μm

NW–WN configuration gives best precision for lowest beam energies



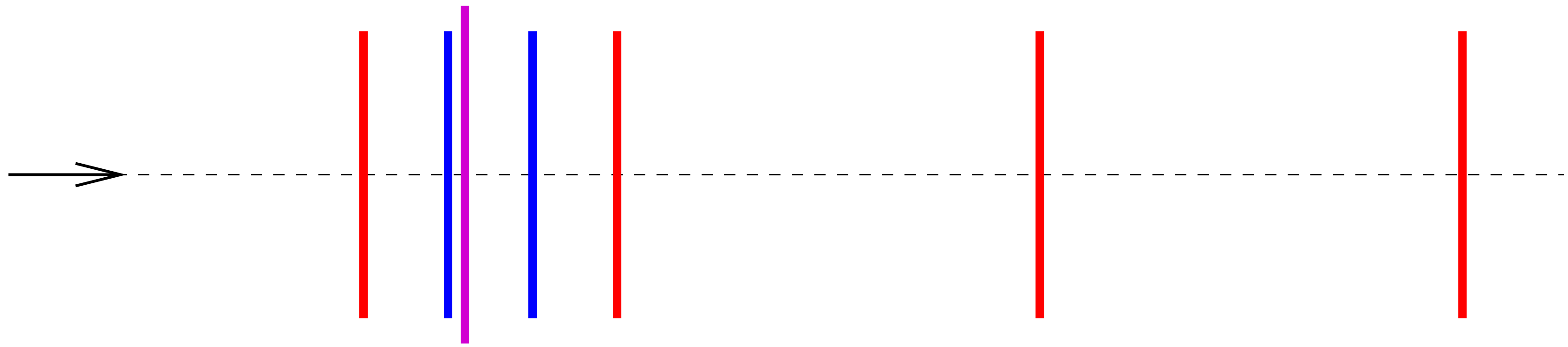
WN–WW configuration can be better for intermediate energies



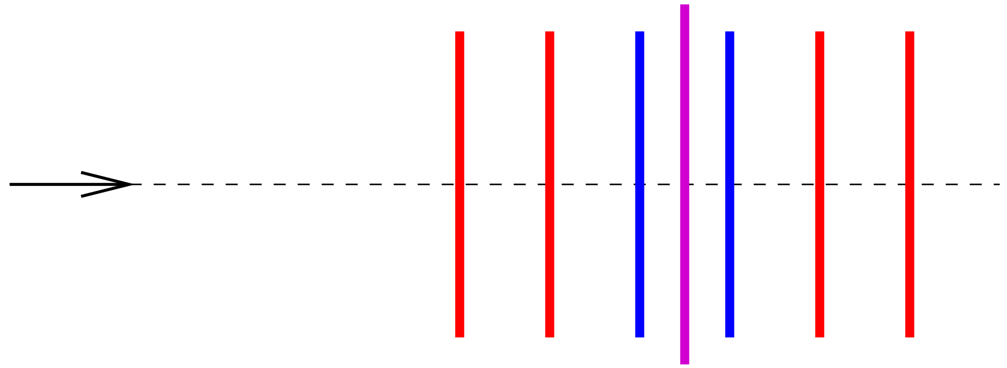
Analytical Results

6 (2+4) telescope planes

N–NWW configuration gives best precision for still higher energies



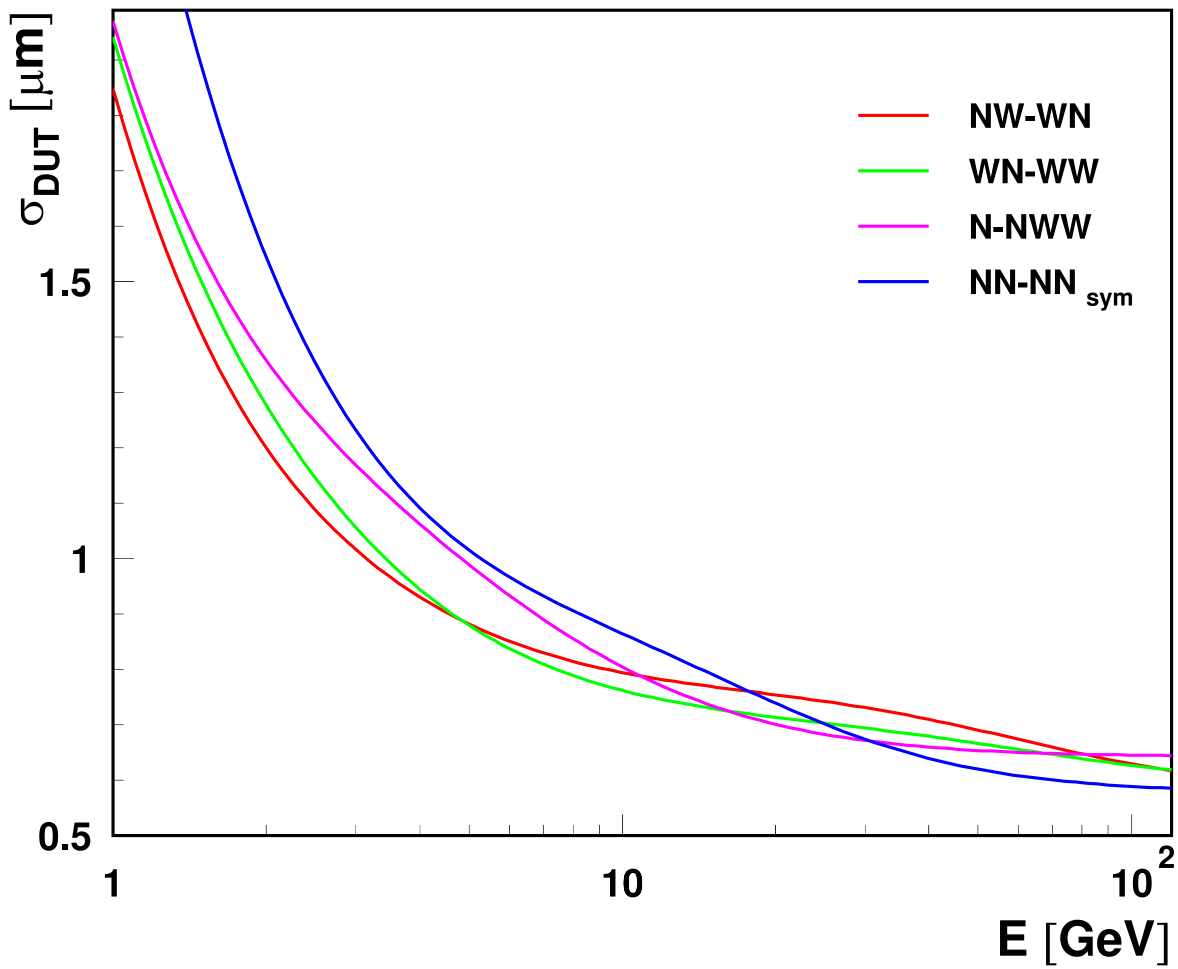
and **NN–NN** is optimal for highest beam energies



Analytical Results

6 (2+4) telescope planes

Comparison of expected precision,
for different telescope setups,
as a function of beam energy



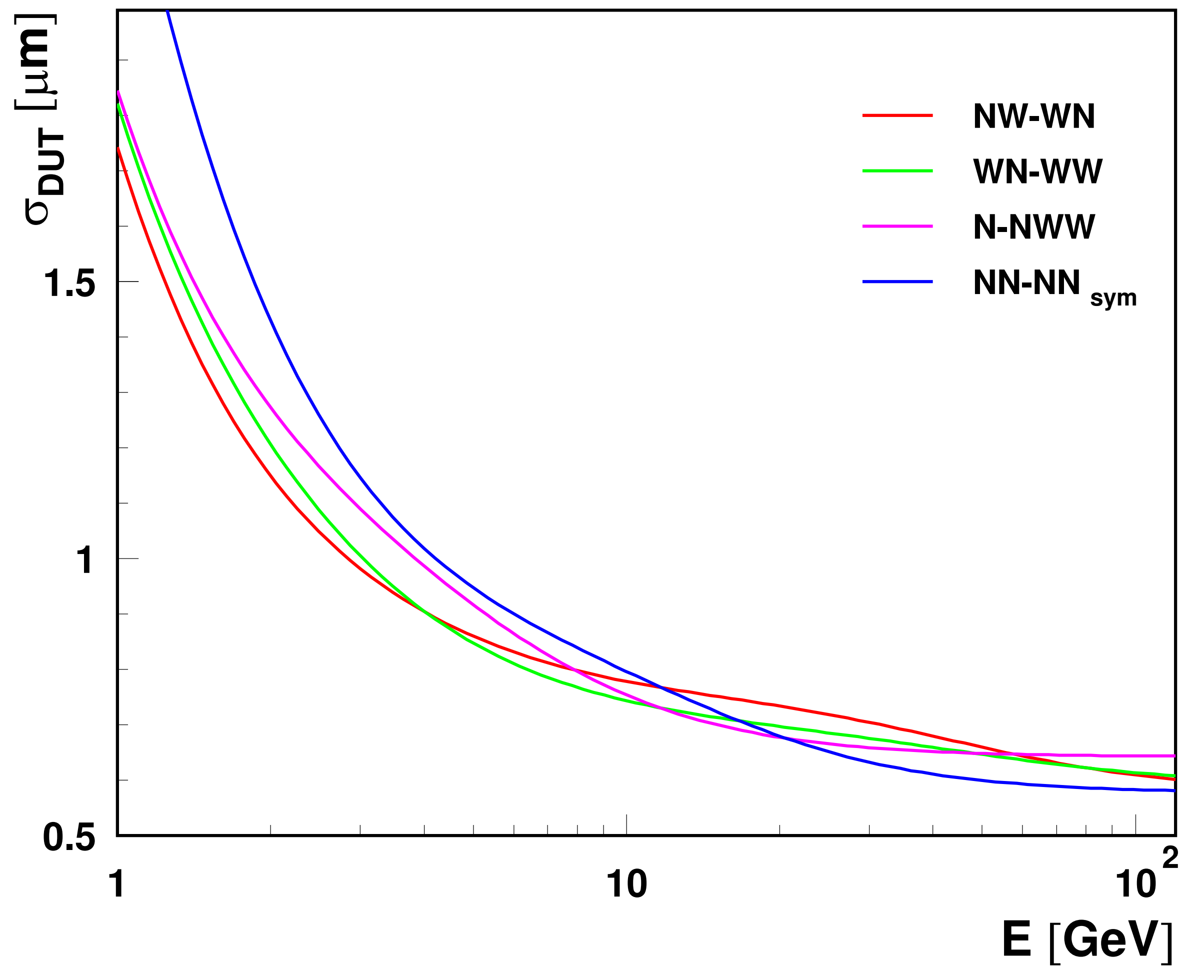
Assumed telescope parameters:

- 1000 μm DUT
- 2 HR planes 120 μm, $\sigma = 1 \mu m$
- 4 Std planes 120 μm, $\sigma = 2 \mu m$
- 5 mm between DUT and HR plane

Analytical Results

6 (2+4) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



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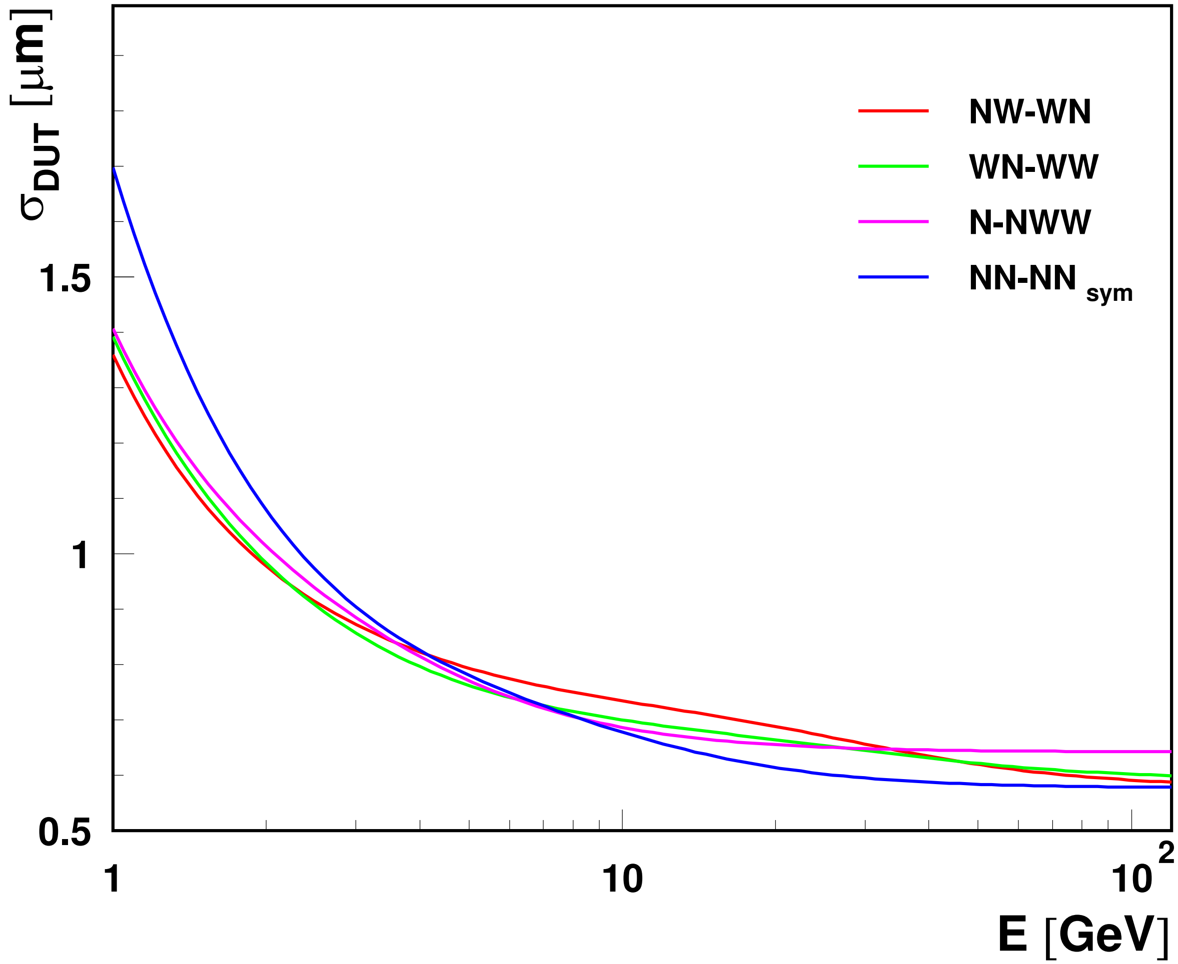
Analytical Results

6 (2+4) telescope planes

Comparison of expected precision, for different telescope setups, as a function of beam energy



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- 5 mm between DUT and HR plane

Analytical Results

6 vs 4 telescope planes

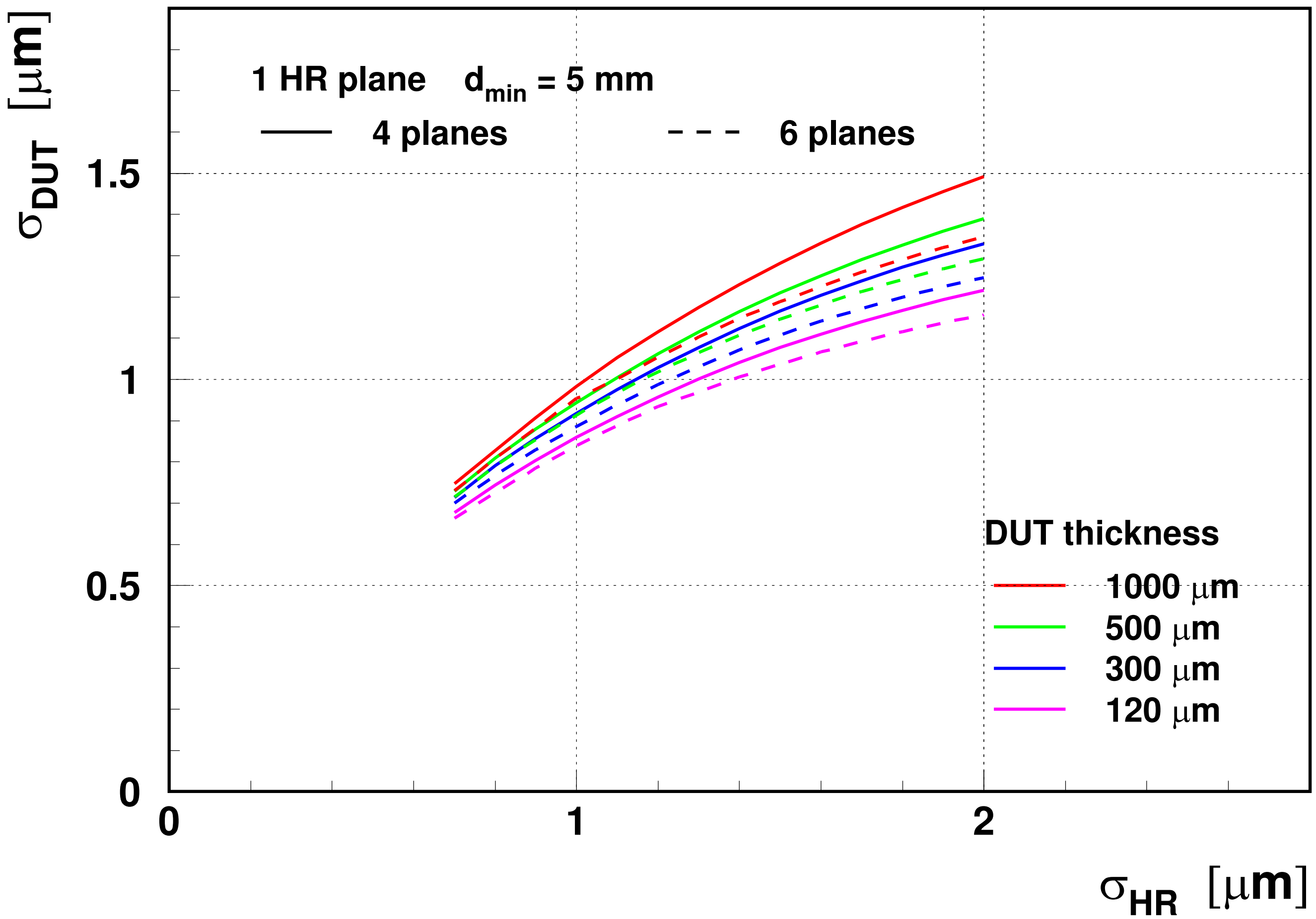
Configuration with 6 planes planes always gives better precision than 4 planes.

Expected position error at DUT, σ_{DUT}, as a function of σ_{HR}

1 HR plane

d_{min}= 5 mm

6 GeV



Analytical Results

6 vs 4 telescope planes

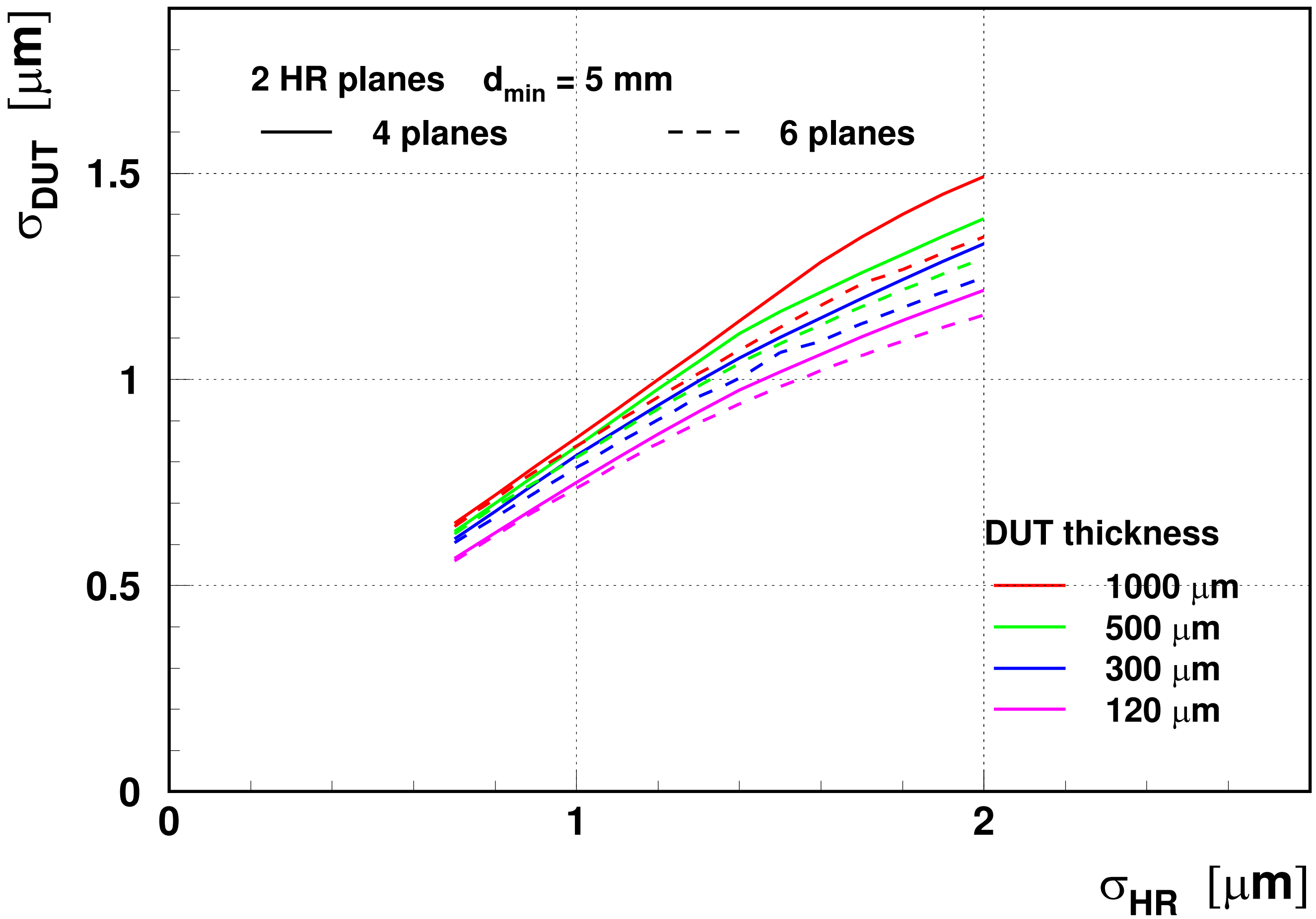
Configuration with 6 planes planes always gives better precision than 4 planes.

Expected position error at DUT, σ_{DUT}, as a function of σ_{HR}

2 HR planes

d_{min}= 5 mm

6 GeV



Analytical Results

6 vs 4 telescope planes

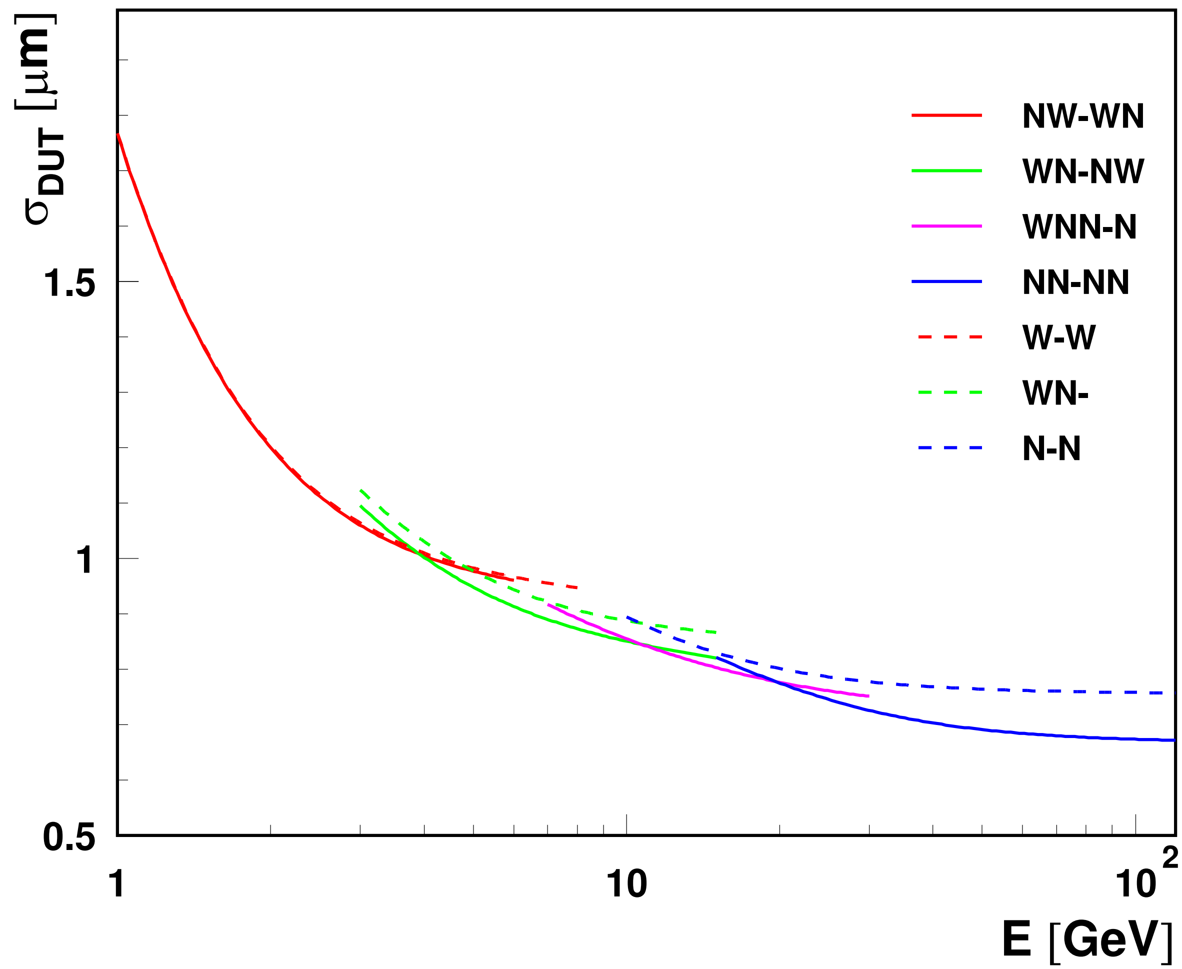
Expected position error at DUT, σ_{DUT}, as a function of beam energy:

Significant improvement at high energies.

For lowest energies influence of additional planes very small.

Assumed telescope parameters:

- 500 μm DUT
- 1 HR plane 120 $\mu m, \sigma = 1 \mu m$
- 3 or 5 Std planes $\sigma = 2 \mu m$
- 5 mm between DUT and HR plane



Analytical Results

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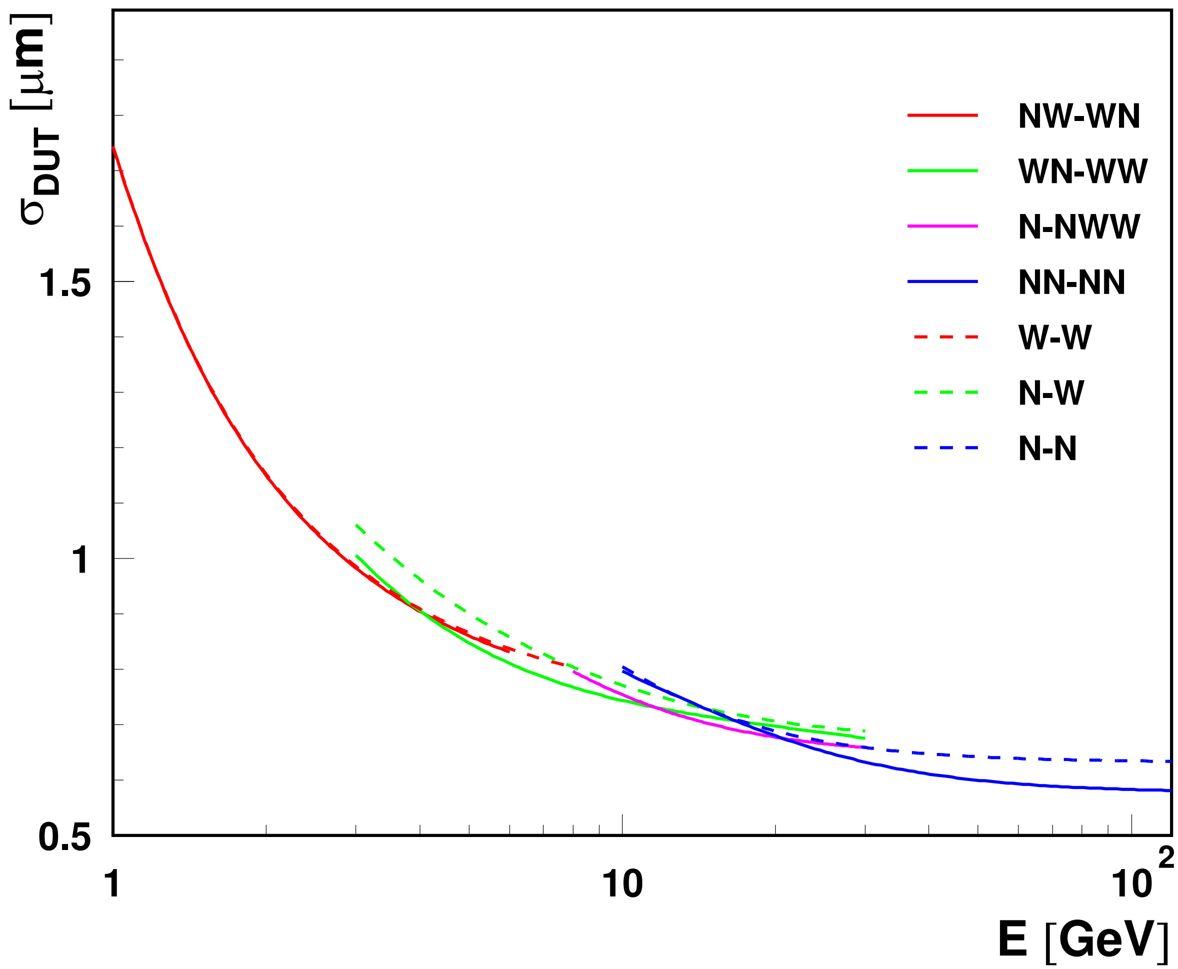


Analytical Results

Influence of additional material layers

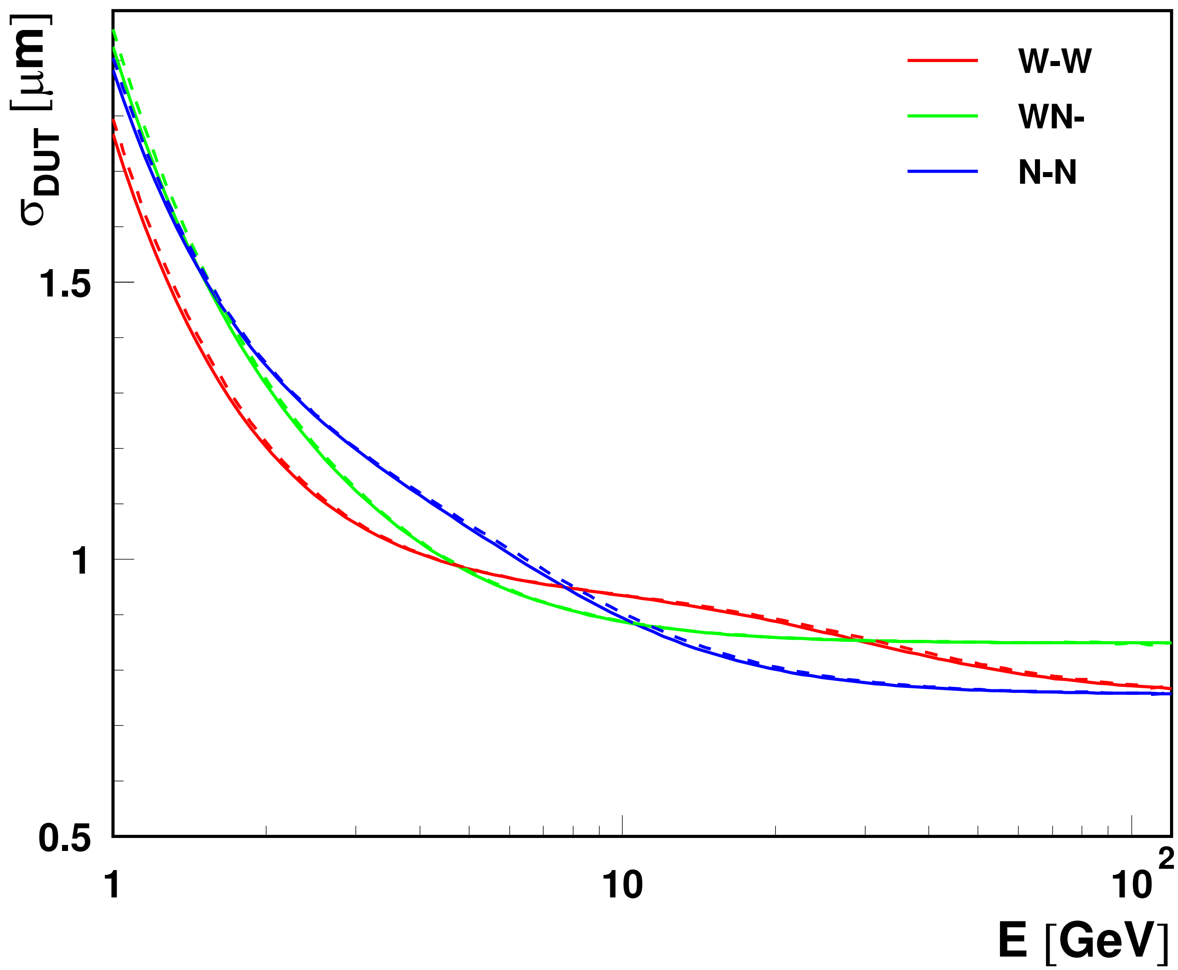
Expected position error at DUT, σ_{DUT}, as a function of beam energy, 6 telescope planes:

Results without (solid) and with (dashed) additional 60 μm of Al placed before and after DUT

Negligible influence

Assumed telescope parameters:

- 500 μm DUT
- 1 HR plane 120 μm, $\sigma = 1 \mu m$
- 3 Std planes 120 μm, $\sigma = 2 \mu m$
- 5 mm between DUT and HR plane



Analytical Results

Influence of additional material layers

Expected position error at DUT, σ_{DUT}, as a function of beam energy, 6 telescope planes:

Results without (solid) and with (dashed) additional 60 μm of Al placed before and after DUT

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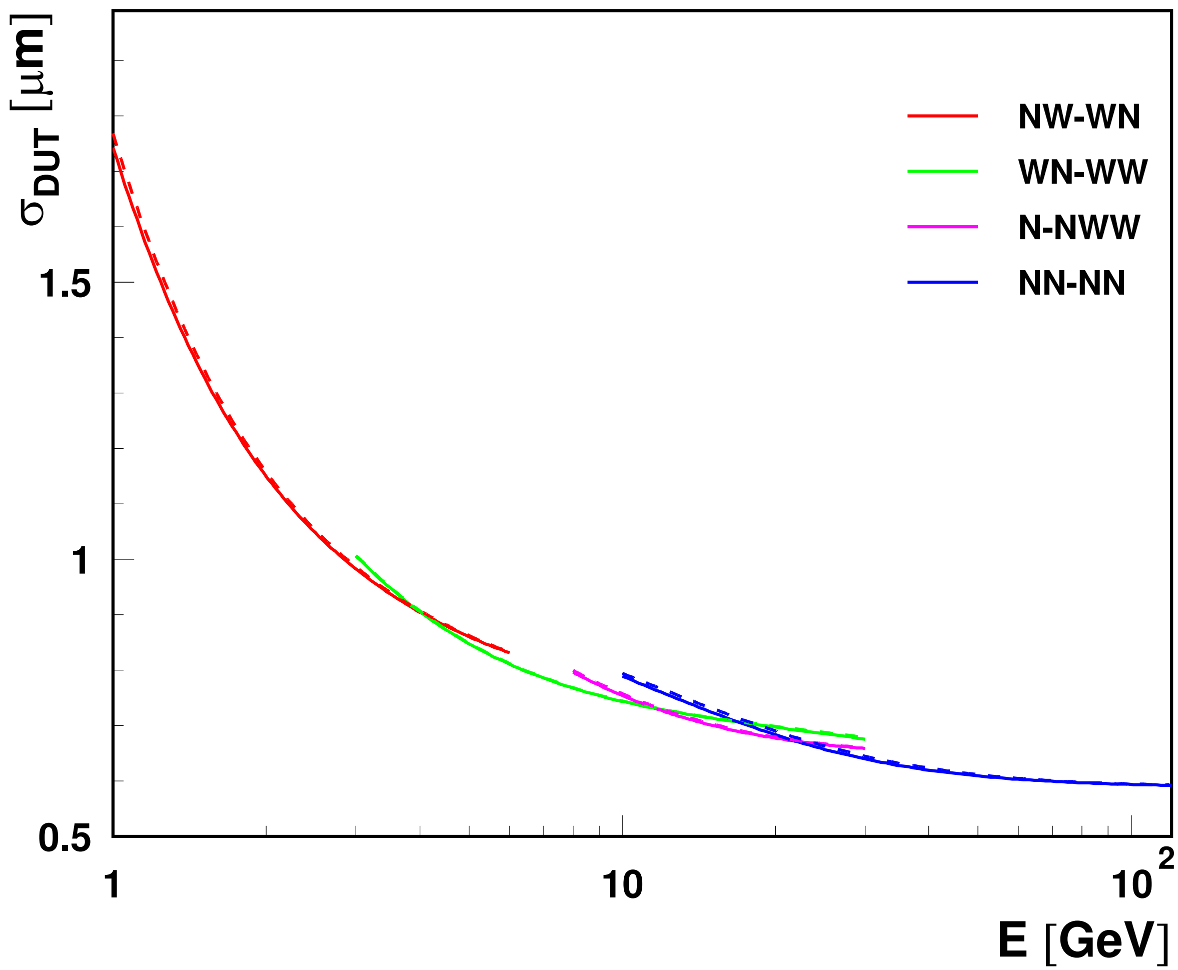

Conclusions

- Analytical method for track fitting with multiple scattering has been developed and verified using GEANT 4 simulation.
- Qualitative improvement as compared to stright line fits, whole sample of events can be used for analysis
- Expected performance of the telescope, with realistic geometry assumptions, can be studied without time-consuming MC simulation.
- The optimum telescope setup is not uniquely defined, many possibilities $\Rightarrow$ best configurations, depending on energy and telescope parameters, suggested.
- If one configuration has to be chosen, **W–W** or **NW–WN** should be used gain at low energies much bigger than the loss at higher energies
- 6 sensor planes always give better position resolution than 4 planes but the difference is significant only at high energies

Plans

- Try to include sensor alignment uncertainty

  Is it possible to include sensor alignment in the event fitting procedure?
- Prepare “public” version of the fitting procedure

For detailed description of the analysis and previous results see:

http://hep.fuw.edu.pl/u/zarnecki/talks/afz_jra1_sep06.pdf

http://hep.fuw.edu.pl/u/zarnecki/talks/afz_jra1_apr06.pdf

http://hep.fuw.edu.pl/u/zarnecki/talks/afz_jra1_jul06.pdf