Beam telescope geometry study

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Outline

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- Track fitting and error estimate
- Simplest example
- Results

Motivation

Beam telescope design

How to obtain the best possible performance ?

The aim of this study

- understand the position measurement

$\Rightarrow$ track fitting with multiple scattering

- identify main factors determining the measurement error
- compare different telescope setups

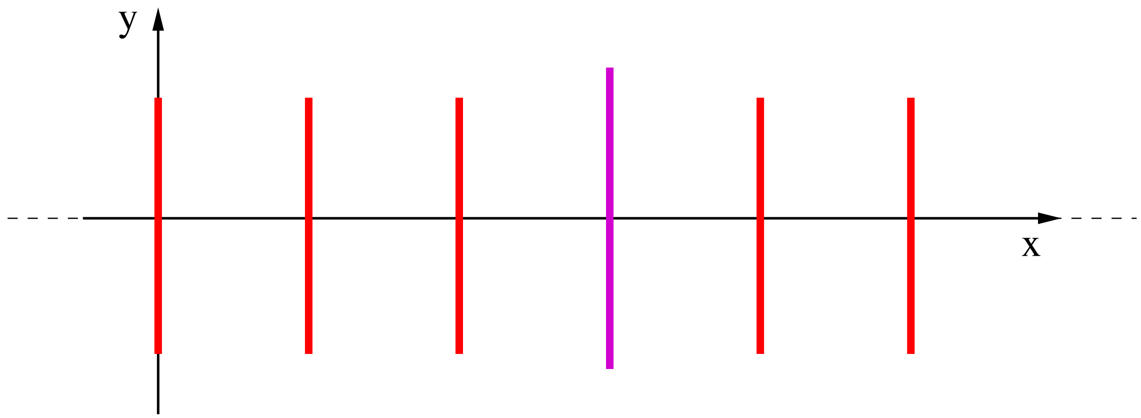
$\Rightarrow$ find the best geometry

Try to use analytical description $\Rightarrow$ avoid time consuming MC simulations.

Geometry description

“Ideal” telescope

No additional material (windows, etc.), perfect alignment, all angles small:



Geometry can be specified by giving:

- N - number of detector planes (including DUT)
- x_i - position of each plane $(i = 1 \ldots N)$
- σ_i - position resolution in each plane $(i \neq i_{DUT})$
- $\Delta\theta_i$ - average scattering angle in each plane

Geometry description

Multiple scattering

Distribution of the scattering angle is approximately Gaussian and the distribution width can be estimated from the formula:

$$\Delta\Theta^{plane} = \frac{13.6\ MeV}{\beta c p}\, z \sqrt{\frac{dx}{X_0}} \left[1 + 0.038 \ln\left(\frac{x}{X_0}\right)\right]$$

where p is particle momentum and dx is plane thickness.

Following configurations were considered:

Beam	dx [μm Si]	$\Delta\Theta$ [mrad]
6 GeV (e^-)	120	0.0606
	500	0.1327
100 GeV (π^-)	120	0.00364
	500	0.00796

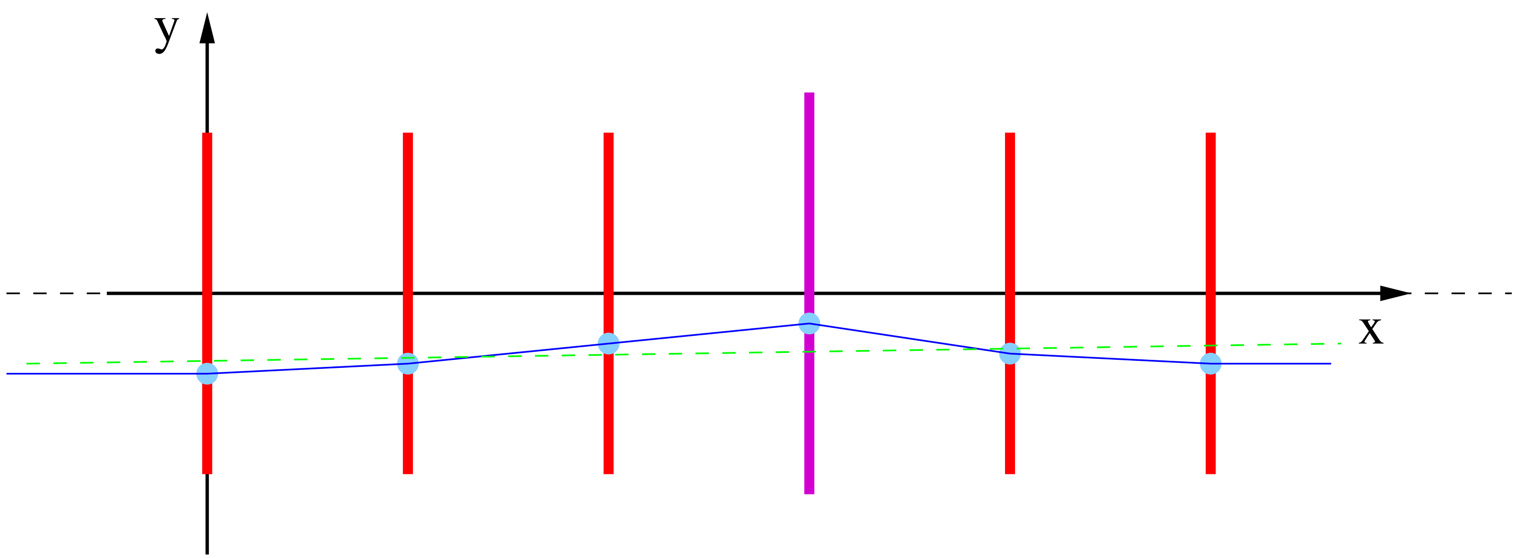
Track fitting

Multiple scattering

Important for low energies !

Distances between planes $\sim \mathcal{O}(10\ mm)$ + scattering angles $\sim \mathcal{O}(0.1\ mrad)$

$\Rightarrow$ track displacement due to scattering $\sim \mathcal{O}(1\ \mu m)$



Displacement comparable with position resolution ($\sim 2\ \mu m$) !

$\Rightarrow$ can not be neglected !

Straight line fit is not sufficient...

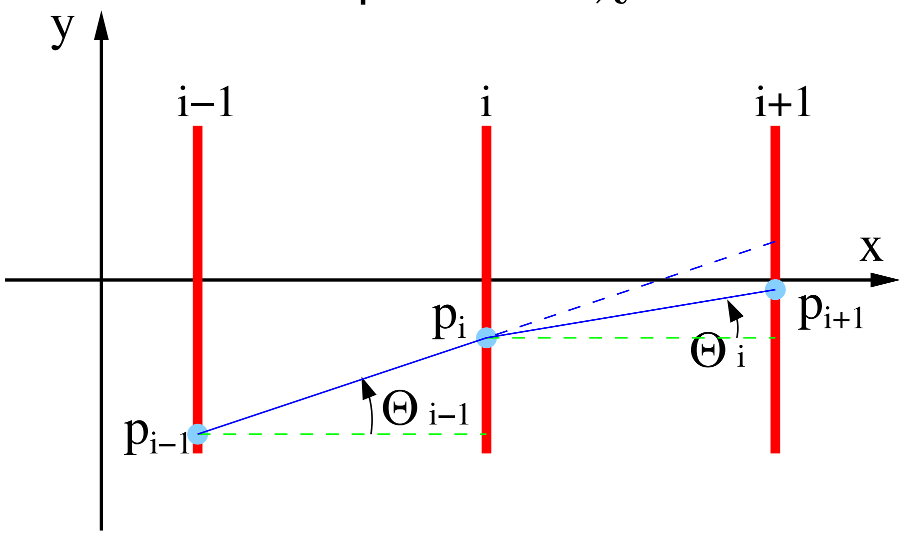
Track fitting

Method

We want to determine track positions in each plane (including DUT), i.e. N parameters ($p_i,\ i = 1 \ldots N$), from $N-1$ measured positions in telescope planes ($y_i,\ i \neq i_{DUT}$).

However, we can use constraints on multiple scattering!

Contribution of plane i to χ^2 of the fit



$$\Delta\chi_i^2 = \underbrace{\left(\frac{y_i - p_i}{\sigma_i}\right)^2}_{\text{position measurement}} + \underbrace{\left(\frac{\Theta_i - \Theta_{i-1}}{\Delta\Theta_i}\right)^2}_{\text{multiple scattering}}$$

$$\text{where: } \Theta_i = \frac{p_{i+1} - p_i}{x_{i+1} - x_i}$$

Both terms present for planes $i \neq 1, i_{DUT}, N$, first term missing for DUT, second for first and last planes.

Track fitting

Method

We get general formula for χ^2:

$$\chi^2 \;=\; \sum_{i=1}^{N} \varepsilon_i (y_i - p_i)^2 \;+\; \sum_{i=2}^{N-1} \left(\frac{(a_i + a_{i-1})p_i - a_{i-1}p_{i-1} - a_i p_{i+1}}{\Delta\Theta_i} \right)^2$$

where $\varepsilon_i = \frac{1}{\sigma_i^2}$ for sensor planes, ${\varepsilon_i}_{DUT} = 0$ (no measurement) and $a_i = \frac{1}{x_{i+1} - x_i}$

Fitting a track, i.e. finding minimum of χ^2 is equivalent to solving the set of N equations:

$$\frac{\partial \chi^2}{\partial p_i} \;=\; 0, \qquad i = 1 \ldots N$$

We can transform it to **matrix equation**:

$$\sum_j A_{ij}\, p_j \;=\; \varepsilon_i\, y_i$$

$$\text{where: } A_{ij} \;=\; \frac{1}{2}\, \frac{\partial^2 \chi^2}{\partial p_i\, \partial p_j}$$

Error estimate

Method

Final formula for A is complicated, but can be calculated analytically.

To solve the equation (i.e. fit the track) we need to find inverse matrix:

$$p_i \;=\; \sum_j \left(A^{-1}\right)_{ij} \;\; \varepsilon_j \, y_j$$

Inverse matrix has to be calculated (numerically) only once (for given geometry) and can then be used to calculate tracks for all collected events...

What we get "for free" are the errors on fitted positions.
The error on the particle position at plane i is given by:

$$\tilde{\sigma}_i \;=\; \sqrt{\left(A^{-1}\right)_{i\,i}}$$

Error on the position reconstructed at DUT: $\sigma_{DUT} \equiv \tilde{\sigma}_{i_{DUT}}$

Example

Simplest configuration

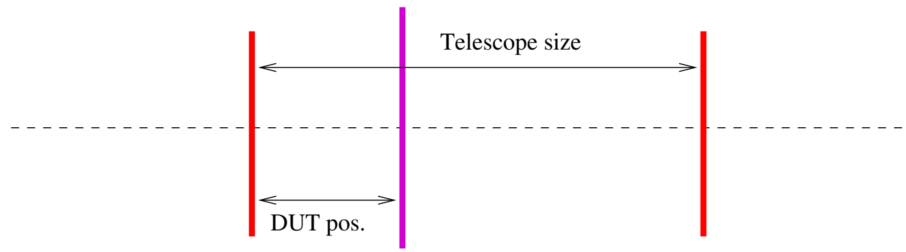
Consider setup consisting of 2 detector planes + DUT ($N = 3$)

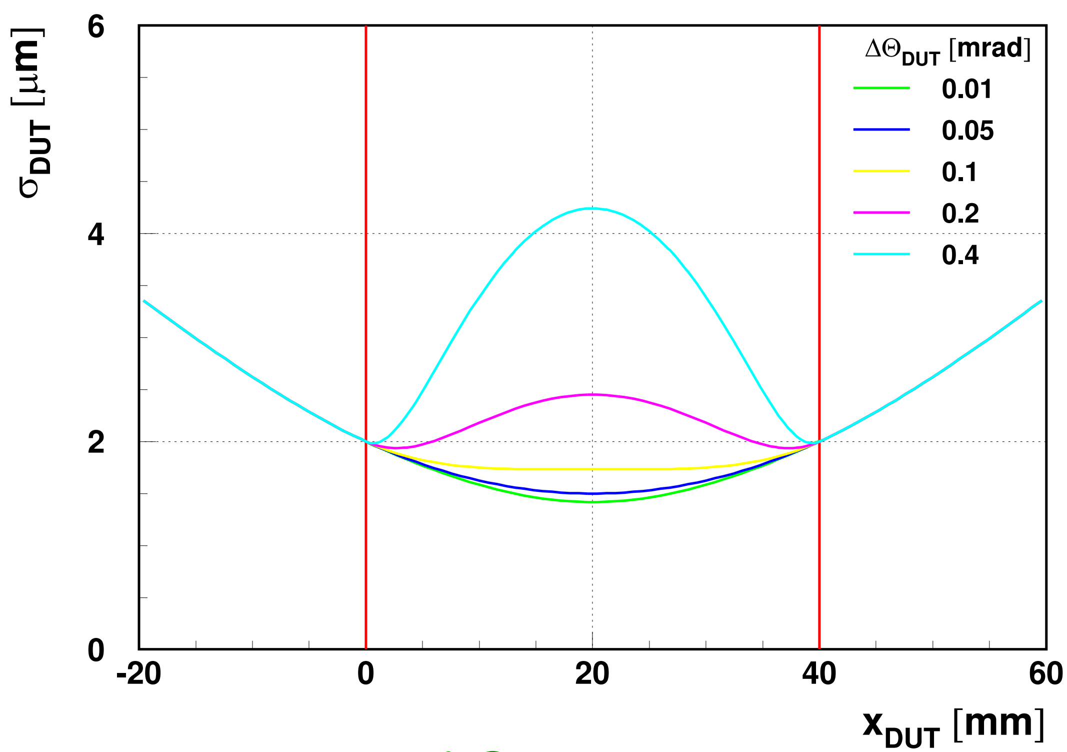
Error of position reconstruction at DUT depends on: telescope size, DUT position (inside or outside telescope) and multiple scattering in the middle plane (DUT or sensor).

For DUT in the center:

$$l = x_2 - x_1 = x_3 - x_2$$

$$\sigma_{DUT} = \sqrt{\frac{1}{2}\sigma^2_{pixel} + \frac{1}{4}\Delta\theta^2_{DUT}\ l^2}$$



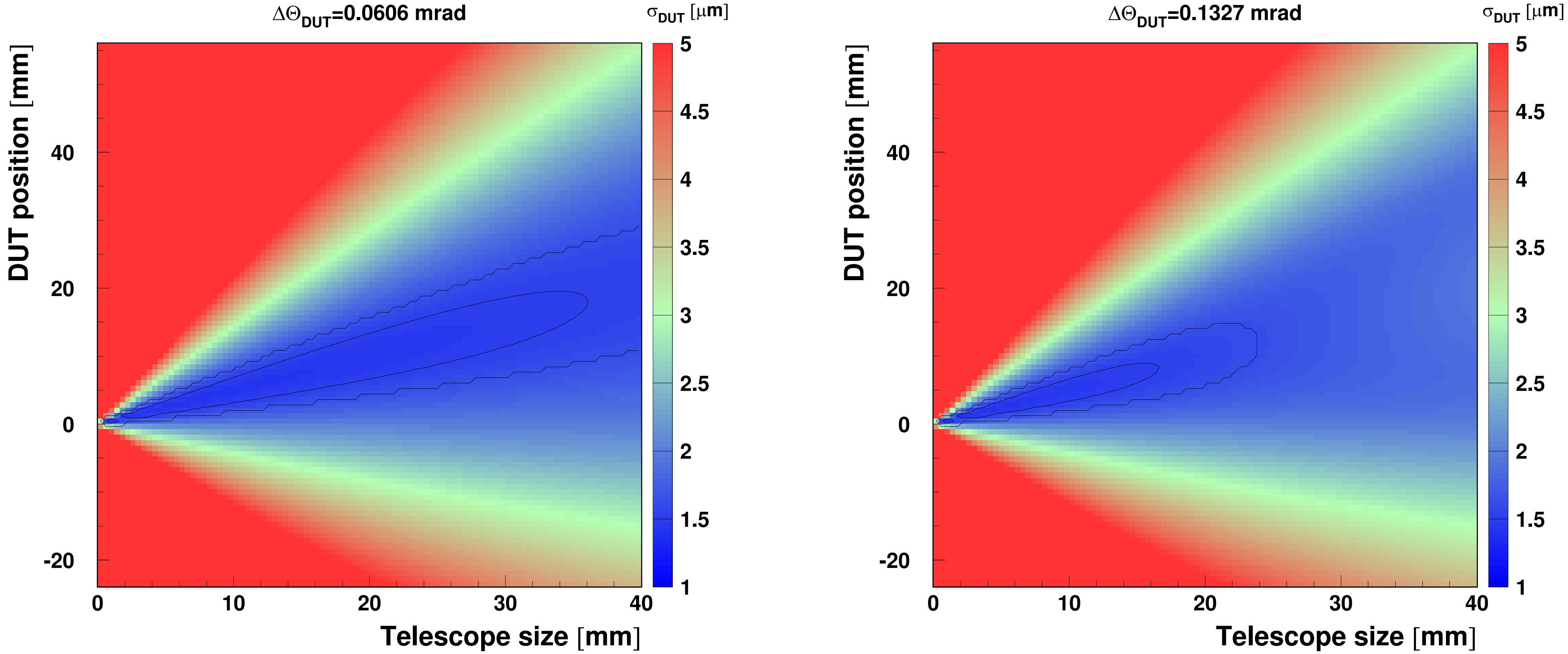


$\sigma_{pixel} = 2\ \mu m$, $\Delta\Theta_{pixel} = 0.0606$

Example

Simplest configuration

Error of position reconstruction at DUT, for "thin" and "thick" DUT.



Resolution is best when detectors are as close as possible...

Results

Optimal configurations

Expected position error at DUT can be calculated for any values of detector positions.

$\Rightarrow$ we can optimize the telescope setup

find distances between detector planes resulting in minimum position error at DUT

If no constraints are put on distances between planes,
smallest error is obtain when all distances go to zero:

$$\sigma_{DUT}^{min} = \lim_{\Delta x_i \to 0} \sigma_{DUT} = \frac{\sigma_{pixel}}{\sqrt{N-1}}$$

$\Rightarrow$ need to specify minimum and maximum allowed distance between planes
(d_{min} and d_{max})

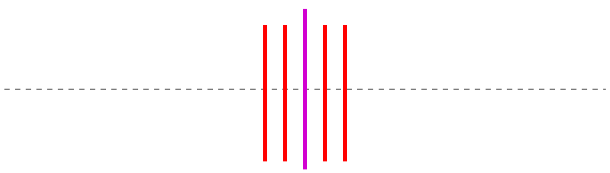
Results

Optimal configurations

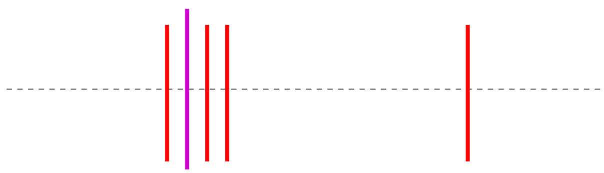
Depending on the assumed parameter values different configurations can turn out to be optimal i.e. minimizing position error at DUT (as found by MINUIT):

4 sensors + DUT

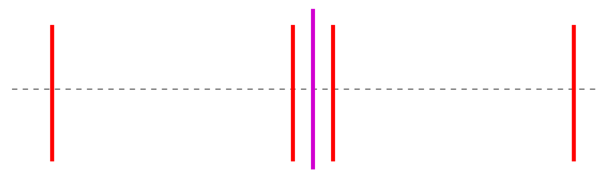
"Narrow" (N)



"Asymmetric" (A)

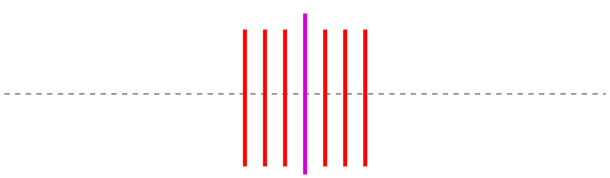


"Wide" (W)

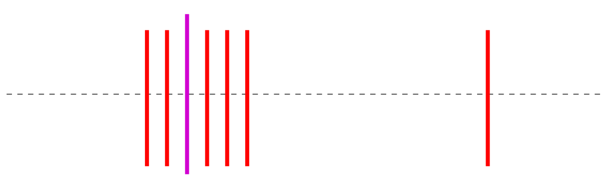


6 sensors + DUT

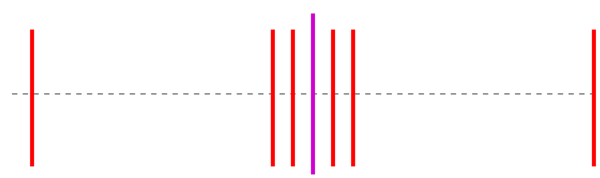
"Narrow"



"Asymmetric"



"Wide"



Results

Optimal configurations

Minimal error on position reconstruction at DUT, for 4 or 6 sensor planes, different detector thicknesses and beam energies.

Results of MINUIT minimization for $d_{min} = 10\ mm$ and $d_{max} = 100\ mm$

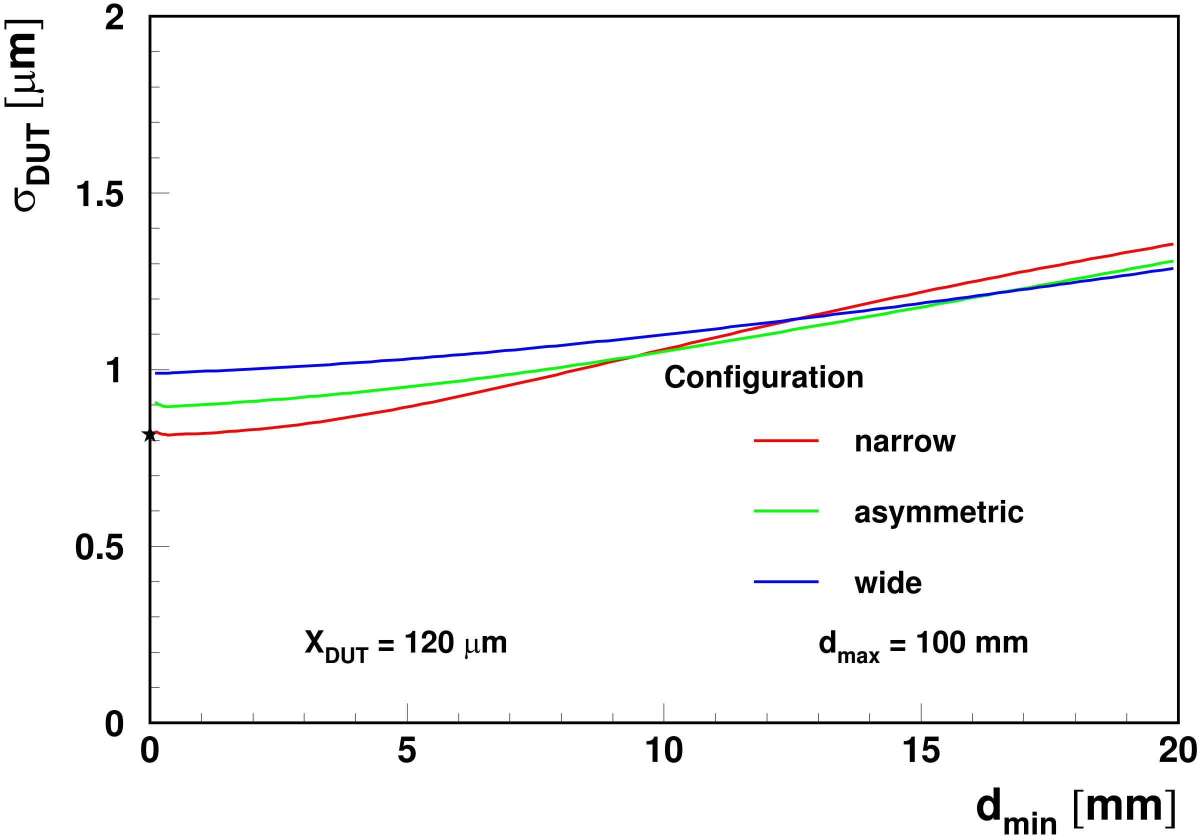
Beam	Sensor	DUT	minimal position error [μm]			
			4 planes		6 planes	
	[μm Si]	[μm Si]	σ_{DUT}	Conf.	σ_{DUT}	Conf.
6 GeV e^-	120	120	1.114	N	1.052	**A**
	120	500	1.362	**A**	1.222	**W**
100 GeV π^-	120	120	1.000	N	0.8178	N
	120	500	1.002	N	0.8208	N

Results

For 6 sensor planes

Error of position reconstruction at DUT, for DUT thickness of 120 μm

6 GeV e^{-} beam, 120 μm sensors

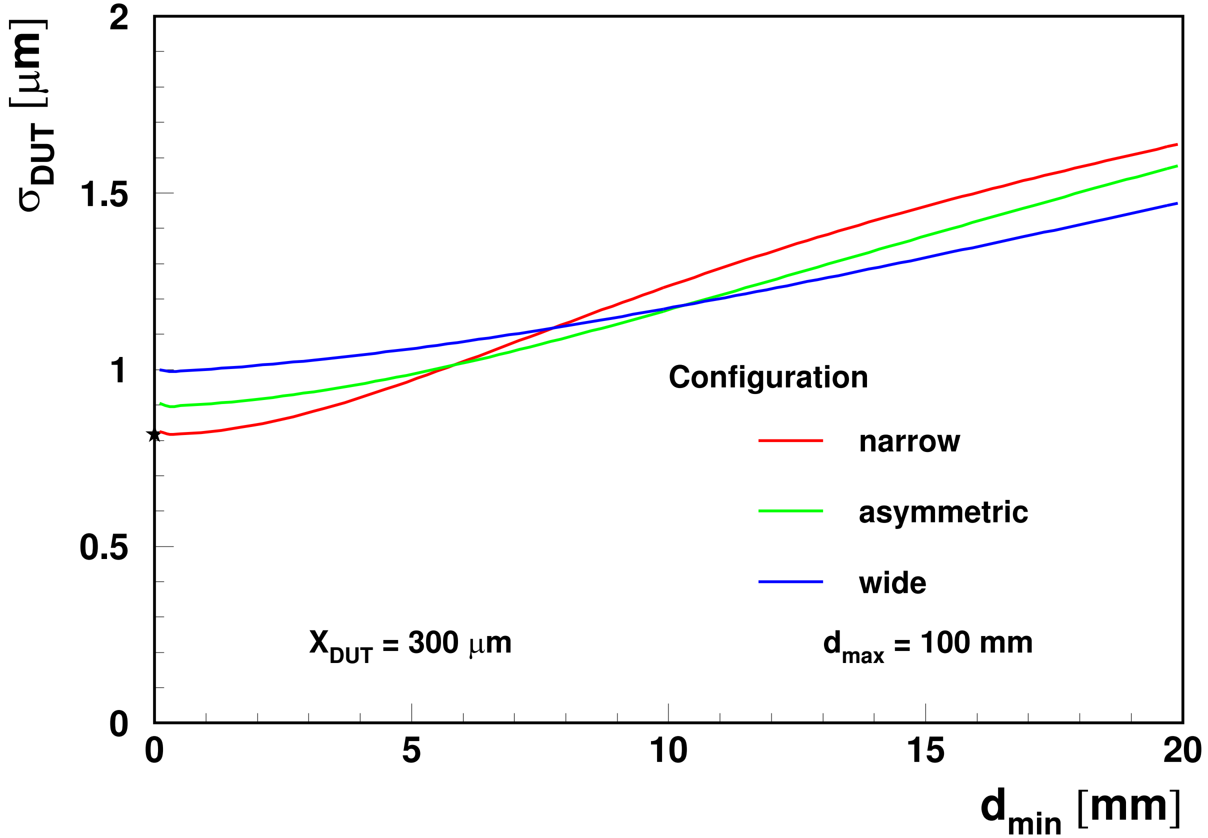


Results

For 6 sensor planes

Error of position reconstruction at DUT, for DUT thickness of 300 μm

6 GeV e^- beam, 120 μm sensors

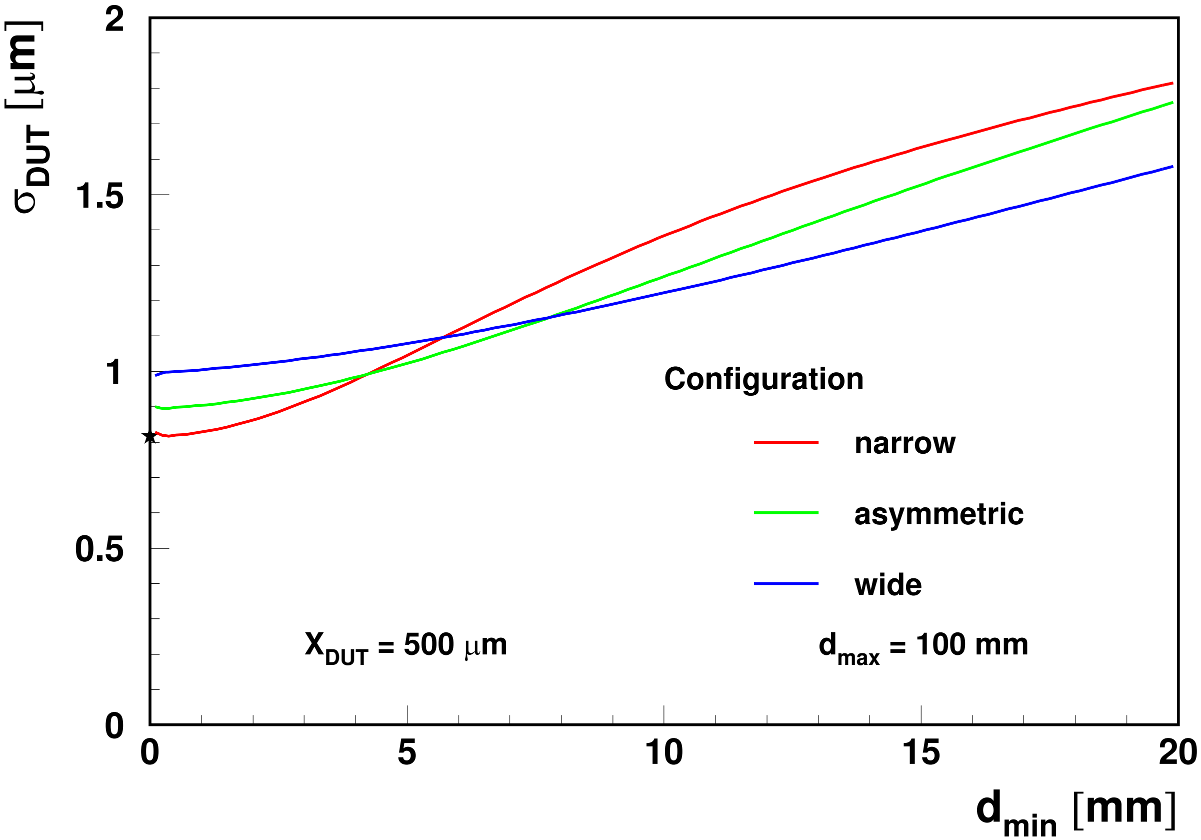


Results

For 6 sensor planes

Error of position reconstruction at DUT, for DUT thickness of 500 μm

6 GeV e^- beam, 120 μm sensors



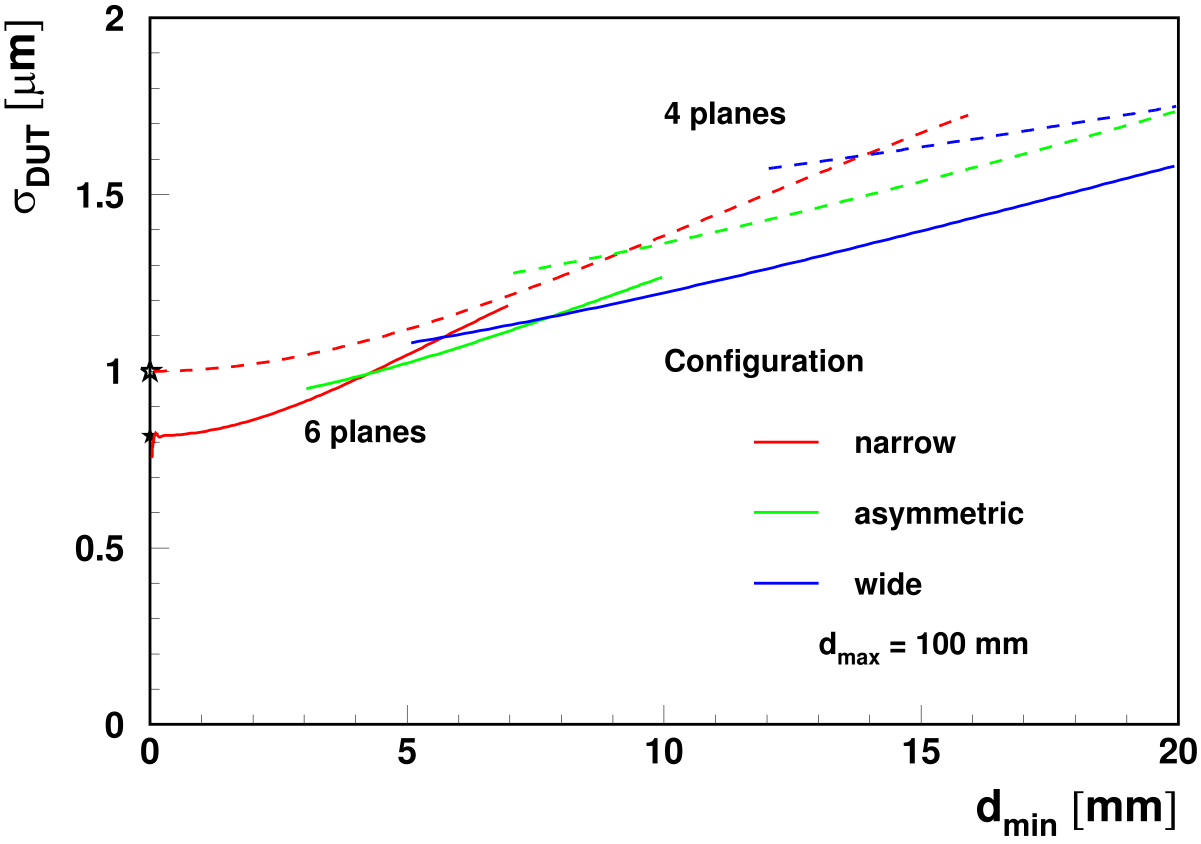
"Wide" configuration gives best position resolution for thick DUT and large d_{min}

Results

6 vs 4 sensor planes

Error of position reconstruction at DUT, for DUT thickness of 500 μm

6 GeV e^- beam, 120 μm sensors



When multiple scattering is properly taken into account, 6 sensor planes always give better position resolution than 4 planes.

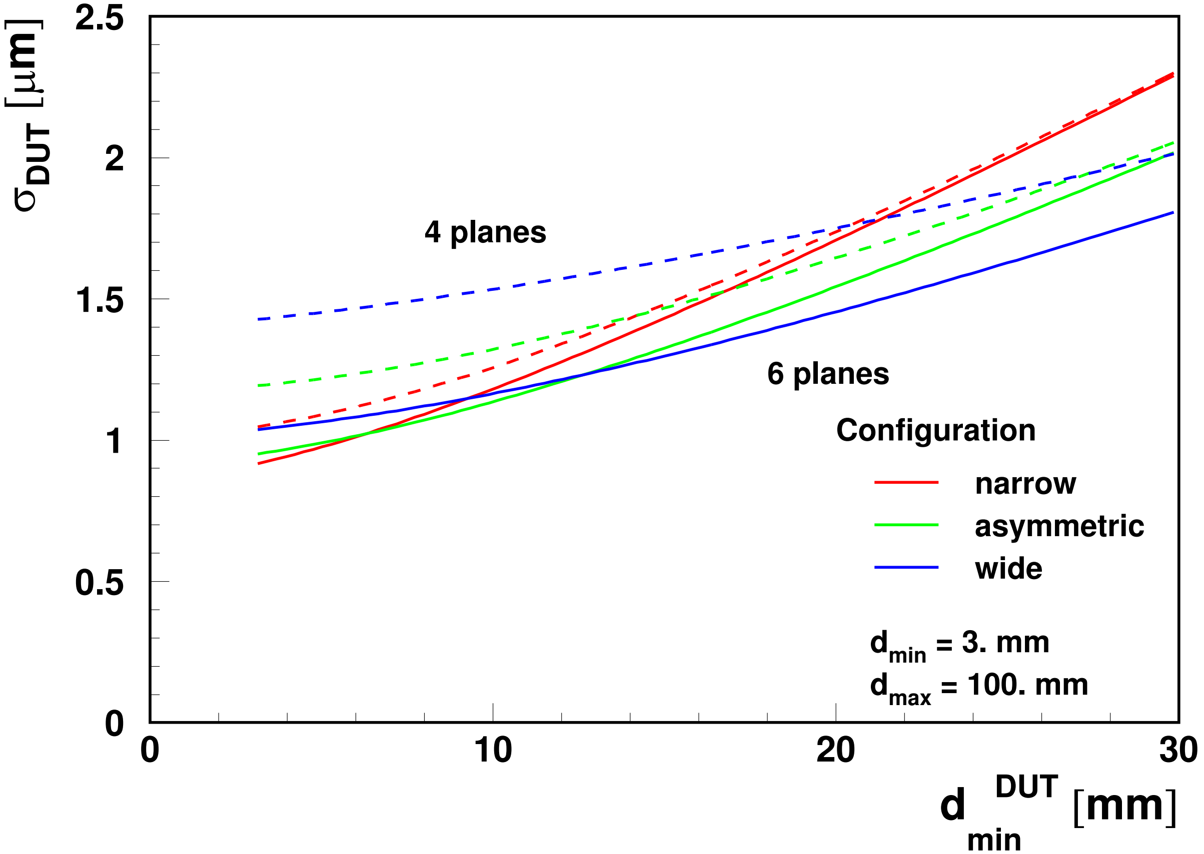
Results

More constraints...

Assume that pixel sensors can be placed closer to each other than to DUT.

Error as a function of minimum distance to DUT, for DUT thickness of 500 μm

6 GeV e^- beam, 120 μm sensors, minimum distance between sensors $d_{min} = 3$ mm



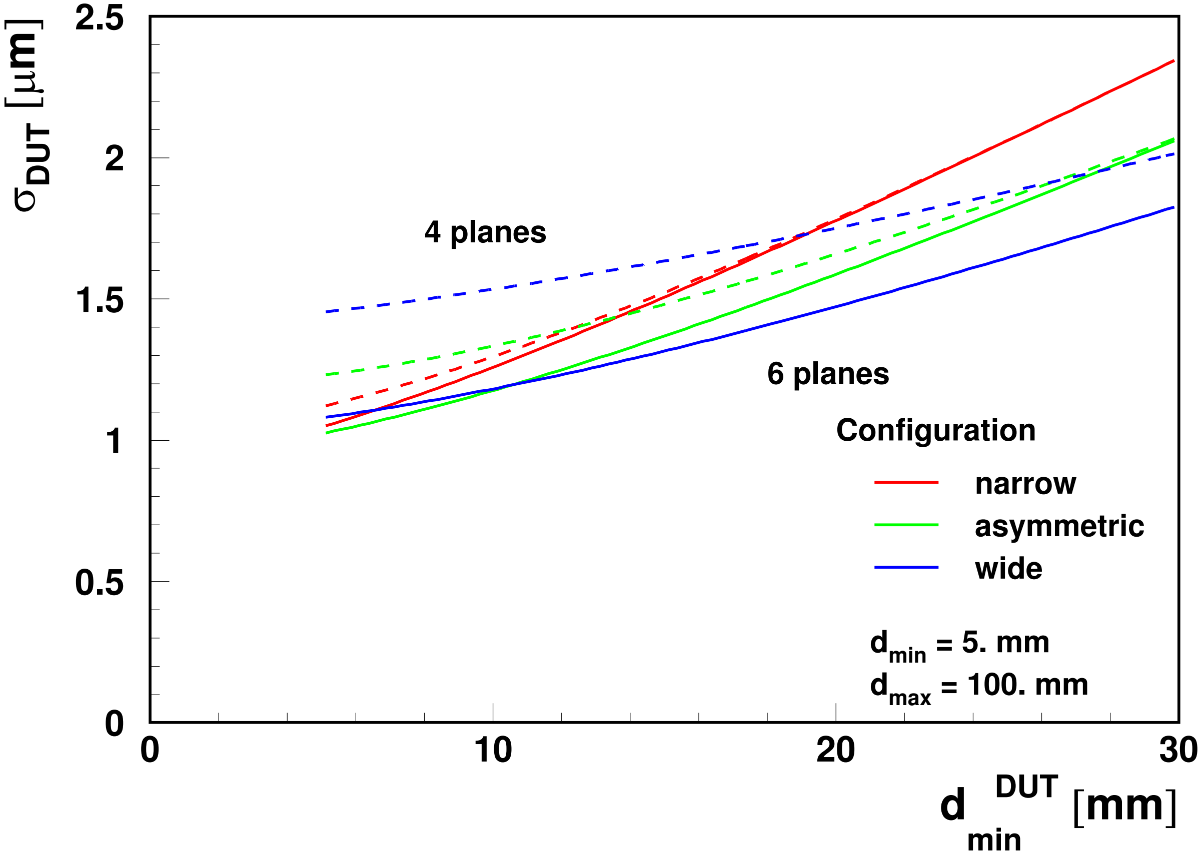
Results

More constraints...

Assume that pixel sensors can be placed closer to each other than to DUT.

Error as a function of minimum distance to DUT, for DUT thickness of 500 μm

6 GeV e^- beam, 120 μm sensors, minimum distance between sensors $d_{min} = 5$ mm



For 6 telescope planes "wide" configuration preferred, if $d_{min}^{DUT} > 10$ mm

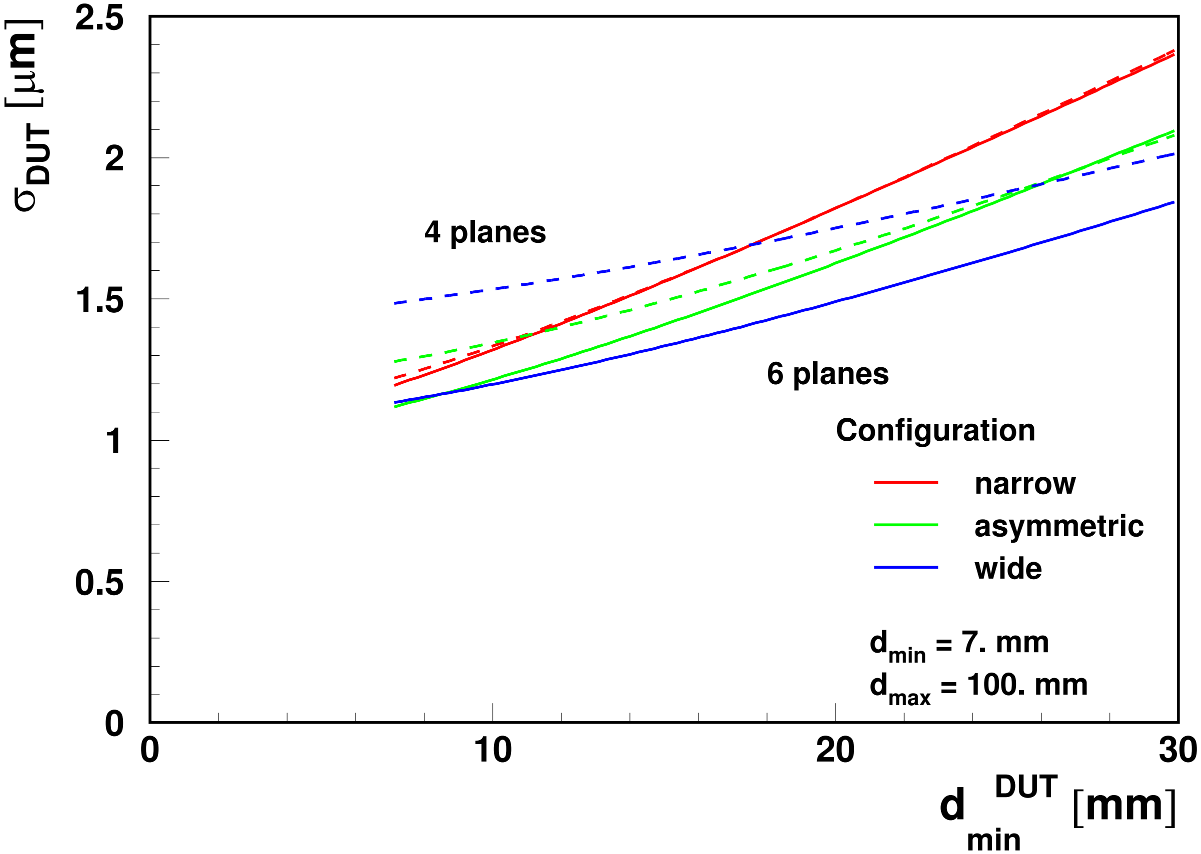
Results

More constraints...

Assume that pixel sensors can be placed closer to each other than to DUT.

Error as a function of minimum distance to DUT, for DUT thickness of 500 μm

6 GeV e^- beam, 120 μm sensors, minimum distance between sensors $d_{min} = 7$ mm



For 6 telescope planes "wide" configuration preferred, if $d_{min}^{DUT} > 10$ mm

For 4 telescope planes "asymmetric" configuration is best for $d_{min}^{DUT} \sim 10 - 25$ mm

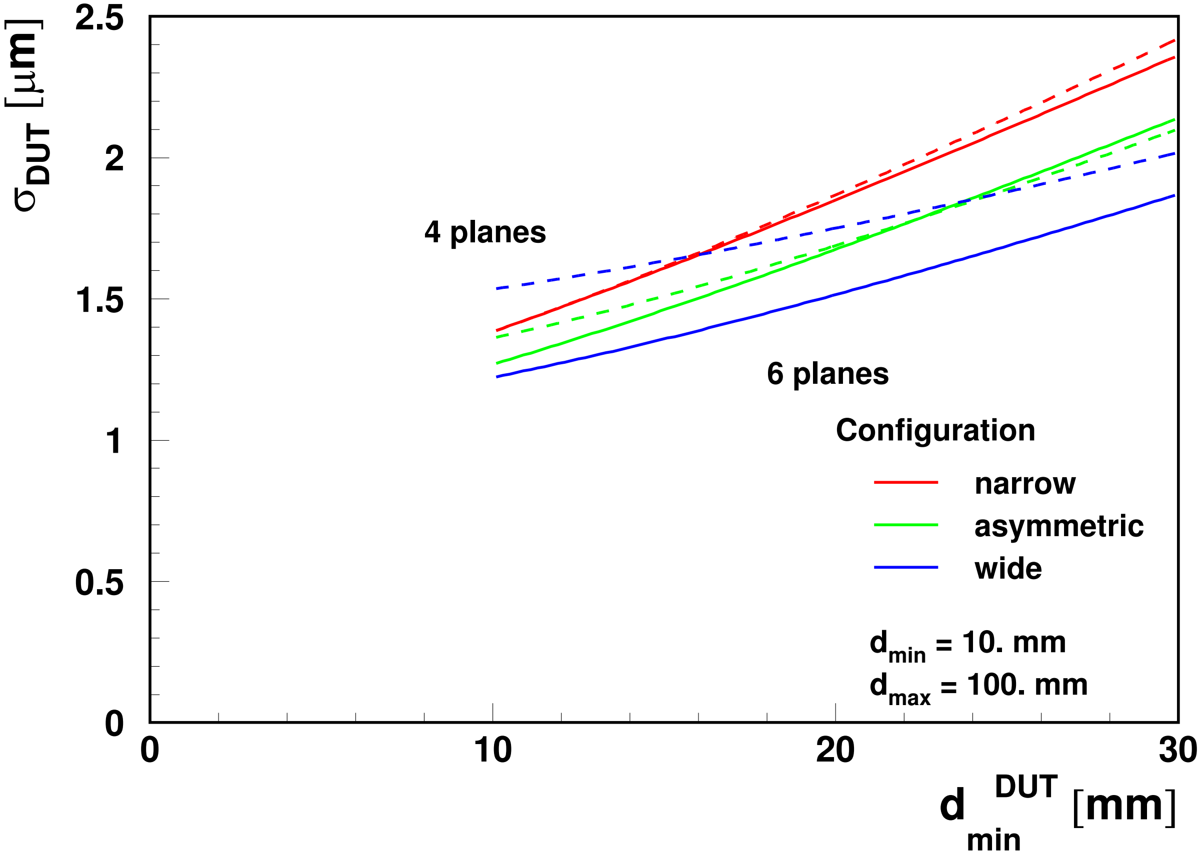
Results

More constraints...

Assume that pixel sensors can be placed closer to each other than to DUT.

Error as a function of minimum distance to DUT, for DUT thickness of 500 μm

6 GeV e^- beam, 120 μm sensors, minimum distance between sensors $d_{min} = 10$ mm



For 6 telescope planes "wide" configuration preferred, if $d_{min}^{DUT} > 10$ mm

For 4 telescope planes "asymmetric" configuration is best for $d_{min}^{DUT} \sim 10 - 25$ mm

Conclusions

In the idealized case, simple analytical method can be used
to describe the performance of the telescope and estimate errors.

For low energy beams taking into account multiple scattering is essential.

The optimum telescope setup depends on the assumed parameters.
If multiple scattering is small, “narrow” configuration is preferred.

The achieved error on the particle position at DUT depends
strongly on the minimum distance between the detector planes.

It is essential to place sensor planes as close to DUT as possible.

6 sensor planes always give better position resolution than 4 planes

Analytical results can be used to guide future simulation studies.