Telescope alignment in analytical approach

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Outline

- Introduction
- Analysis
- Simulation Results
- Alignment precision estimates
- Conclusions

Introduction

Analytical method for track fitting with multiple scattering has been developed
to study position measurement in the telescope and to suggest the best configuration.

Method has been verified using GEANT 4 simulation.

Qualitative improvement as compared to straight line fits,
whole sample of events can be used for analysis - no need for χ^2 cut

The optimum telescope setup is not uniquely defined, many possibilities
$\Rightarrow$ best configurations, depending on energy and telescope parameters, suggested.

Detailed results presented at the EUDET Annual meeting, see:
http://hep.fuw.edu.pl/u/zarnecki/talks/afz_eudet_ann06.pdf

This contribution:

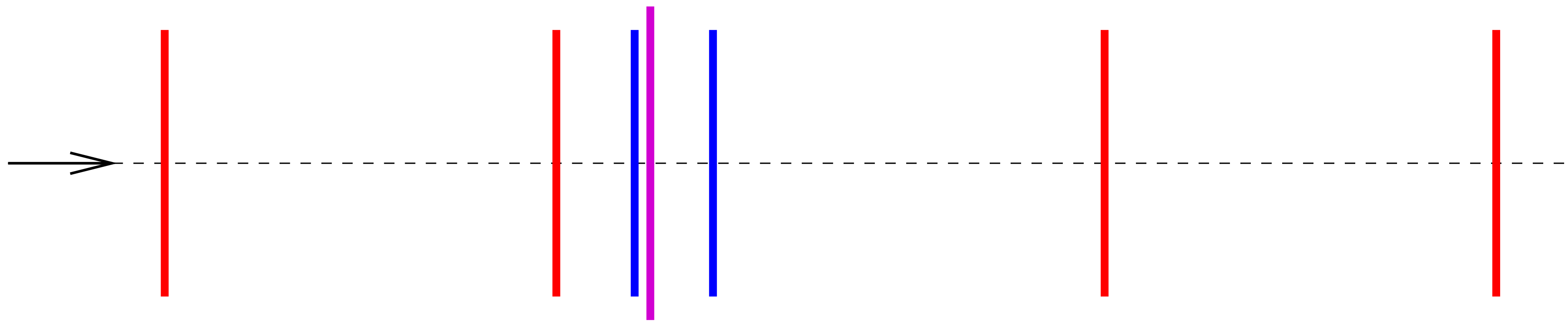
- simulation results include sensor alignment uncertainty
- estimates of sensor alignment precision from telescope data

Analysis

Simulation setup

GEANT 4 was used to simulate particle scattering in the telescope for the configuration optimum for the assumed telescope parameters:

- DUT with 500 μm thickness
- 2 high resolution sensor planes with 120 μm thickness, 1 μm position resolution
- 4 standard sensor planes with 120 μm thickness, 2 μm position resolution
- minimum distance between DUT and HR plane of 3 mm
- 6 GeV electron beam

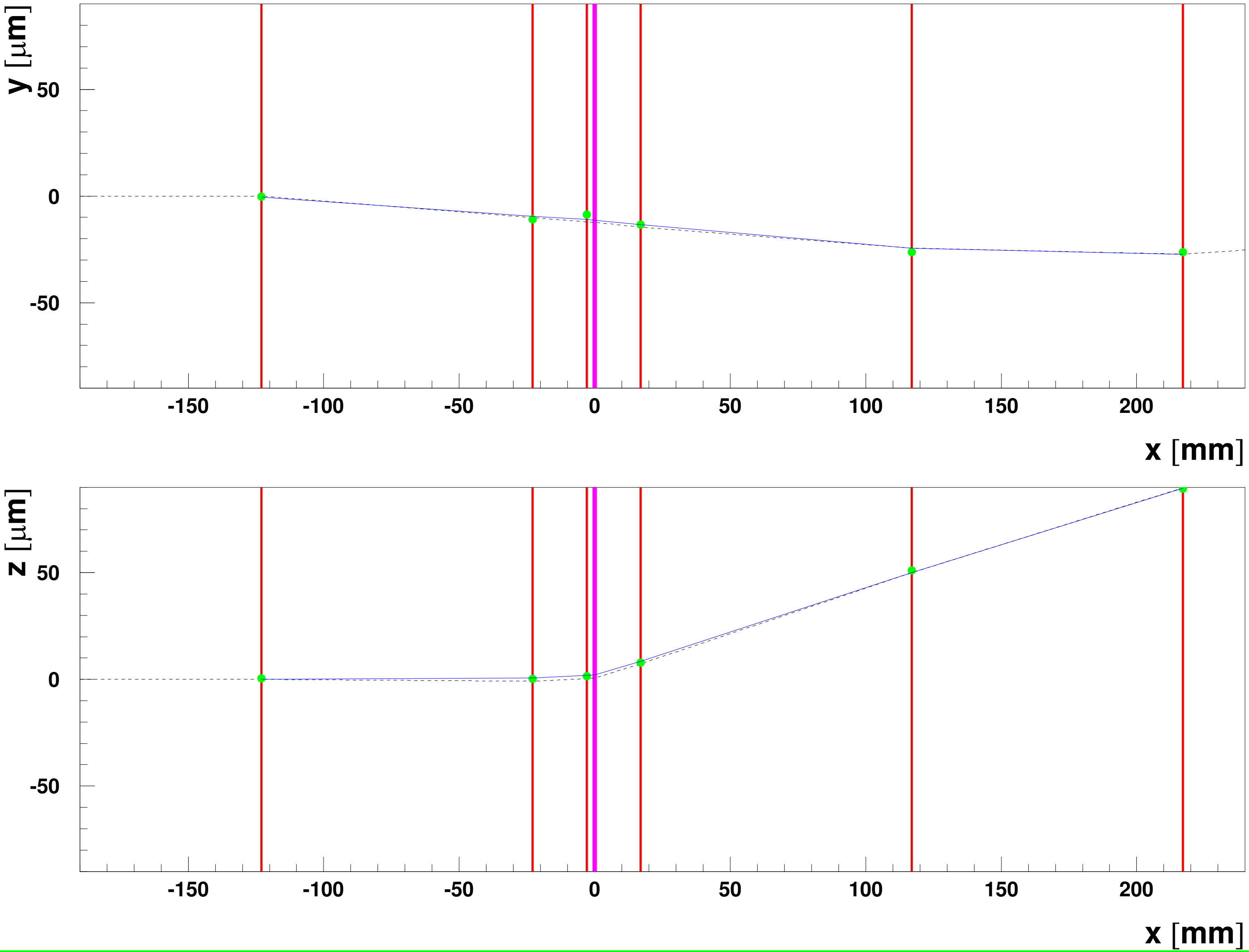


so called **WN–WW** configuration

Analysis

Simulation

Example of the GEANT 4 event



Color codes:

sensor planes

DUT

true particle path

measurements

fitted track

Analysis

Position at DUT

Reconstruction error distribution for the particle position at DUT

Fitted Gaussian distribution (red line):

$$\sigma_{DUT} = 0.840(\pm 0.006)\mu m$$

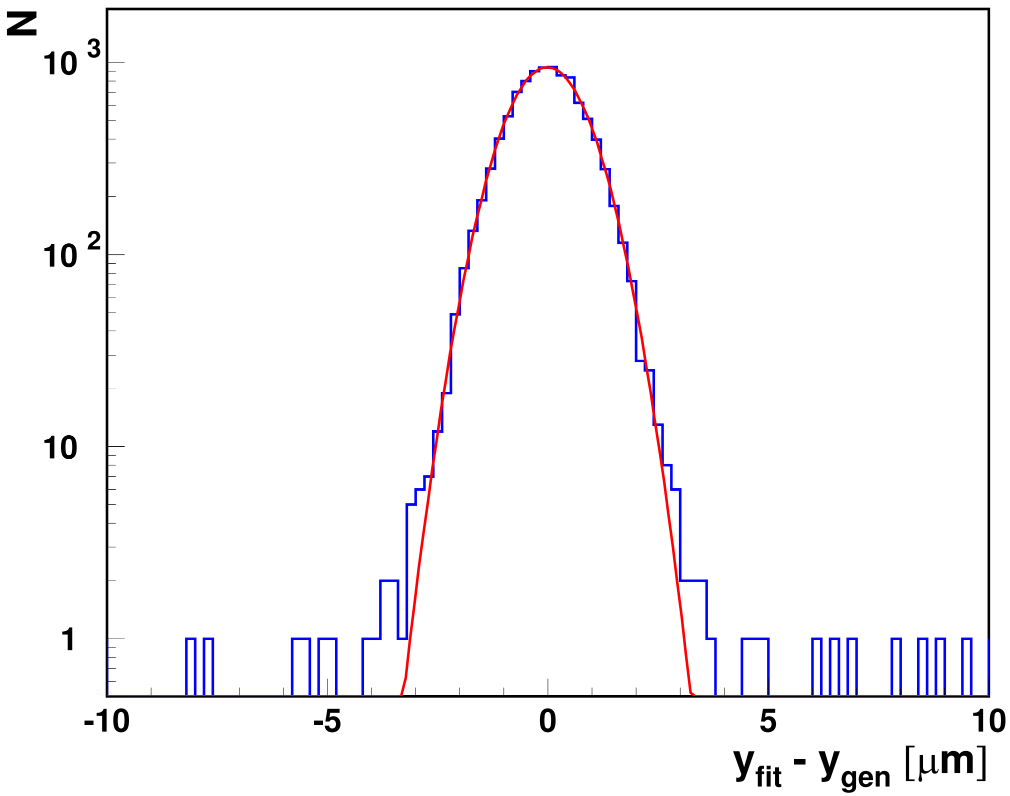
Removing 10% of events with worst χ^2:

$$\sigma_{DUT} = 0.805(\pm 0.007)\mu m$$

Expected resolution:

$$\sigma_{DUT} = 0.802\mu m$$

GEANT 4 events, 6 GeV electron beam



Analysis

Fit quality

We have 12 measurements
(6 planes $\times$ 2 position measurements)

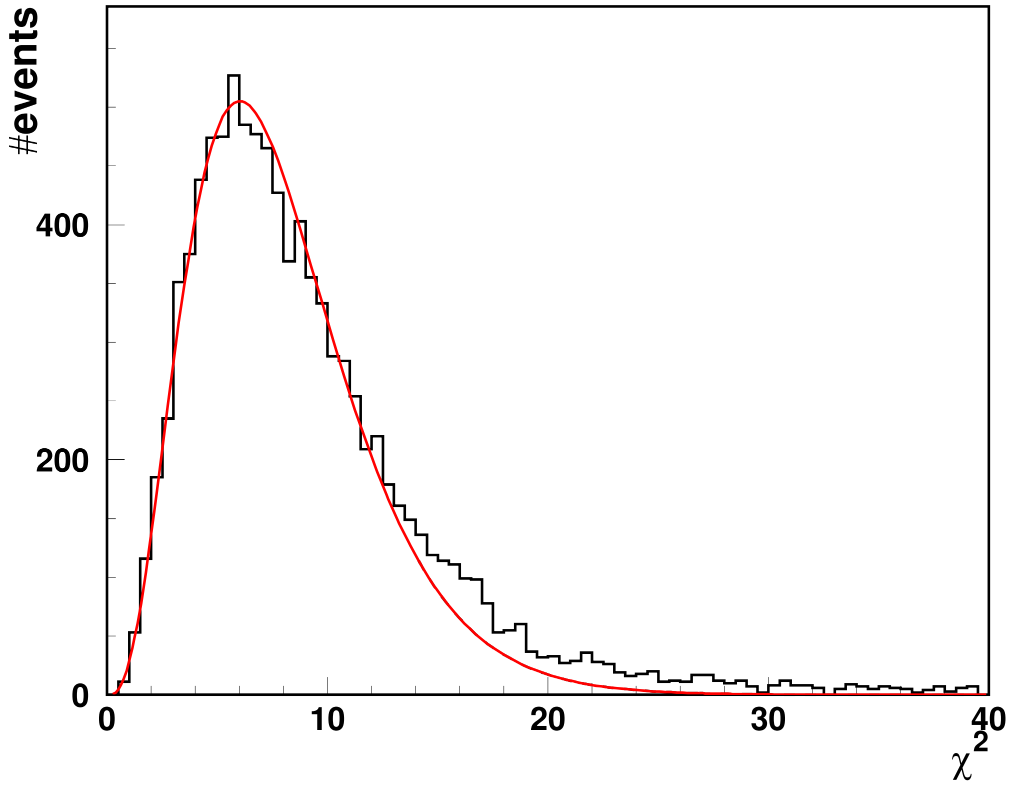
and fit 14 parameters
(2 position coordinates for 7 planes)

However, we also impose
10 constraints on scattering angles.

$\Rightarrow$ Number of degrees of freedom:

$$N_{df} \;=\; 12 + 10 - 14 = 8$$

χ^2 distribution for GEANT 4 events



Simulation Results

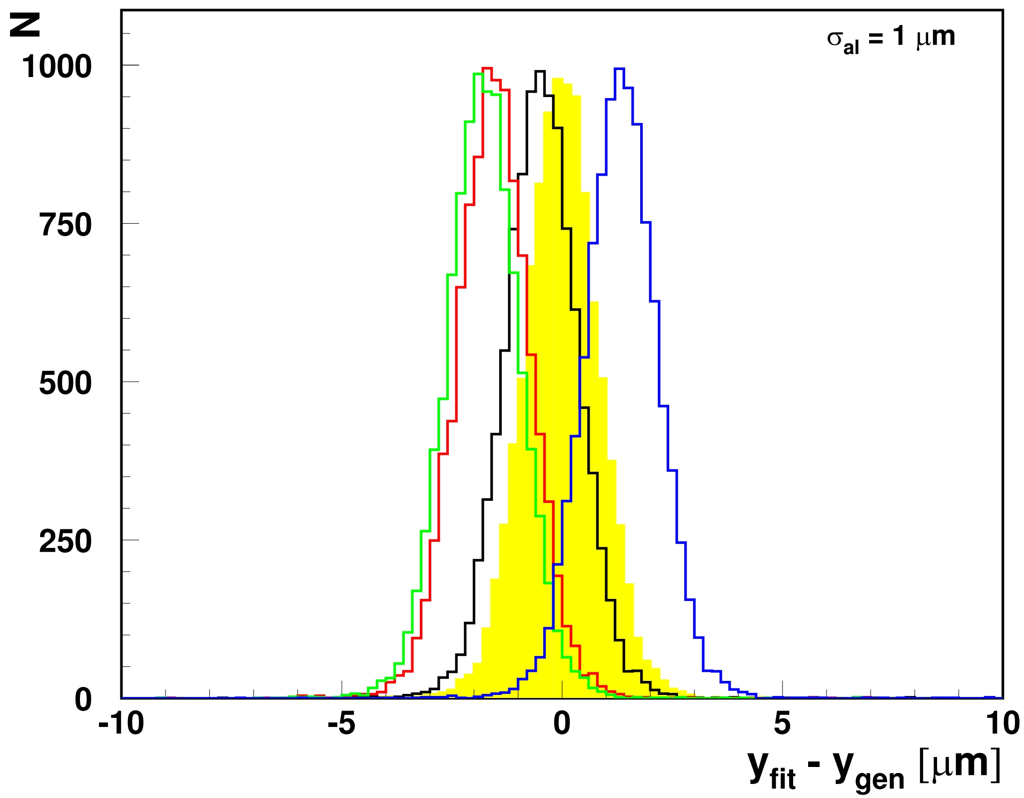
Misalignment

Reconstruction error distribution for the particle position at DUT

Four experiments with 1 μm alignment compared to perfect alignment (solid yellow)

Sensor position is randomly shifted for each simulated data set.

GEANT 4 events, 6 GeV electron beam



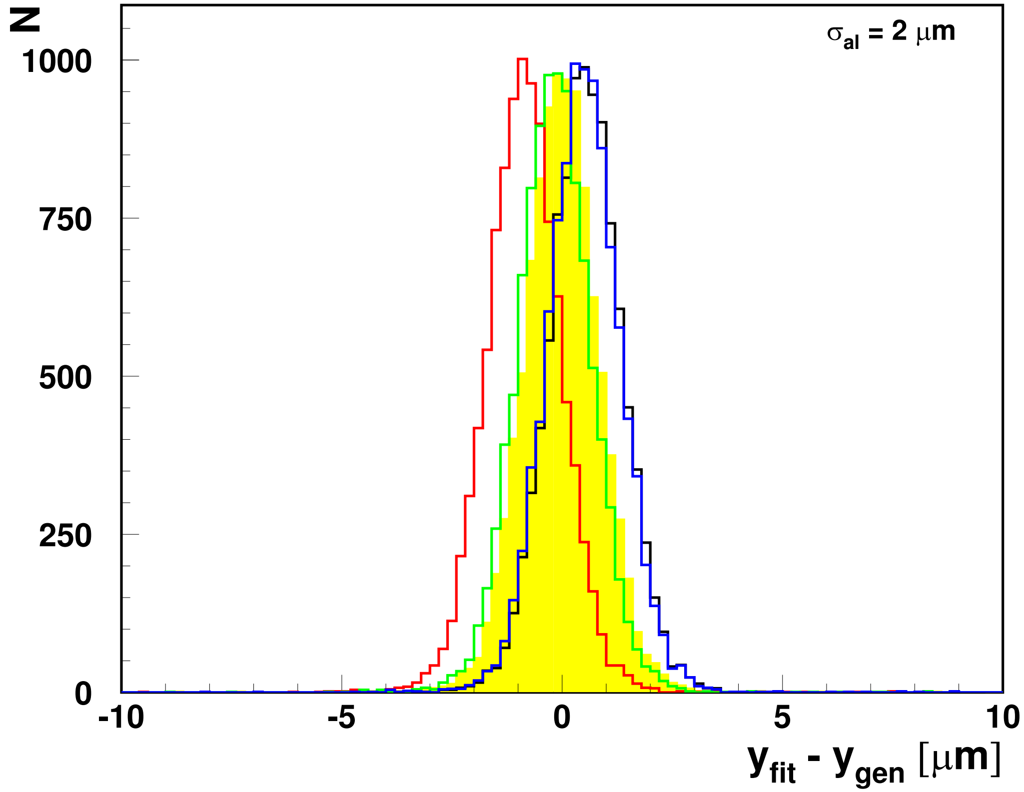
Simulation Results

Misalignment

Reconstruction error distribution for the particle position at DUT

Four experiments with 2 μm alignment compared to perfect alignment (solid yellow)

GEANT 4 events, 6 GeV electron beam



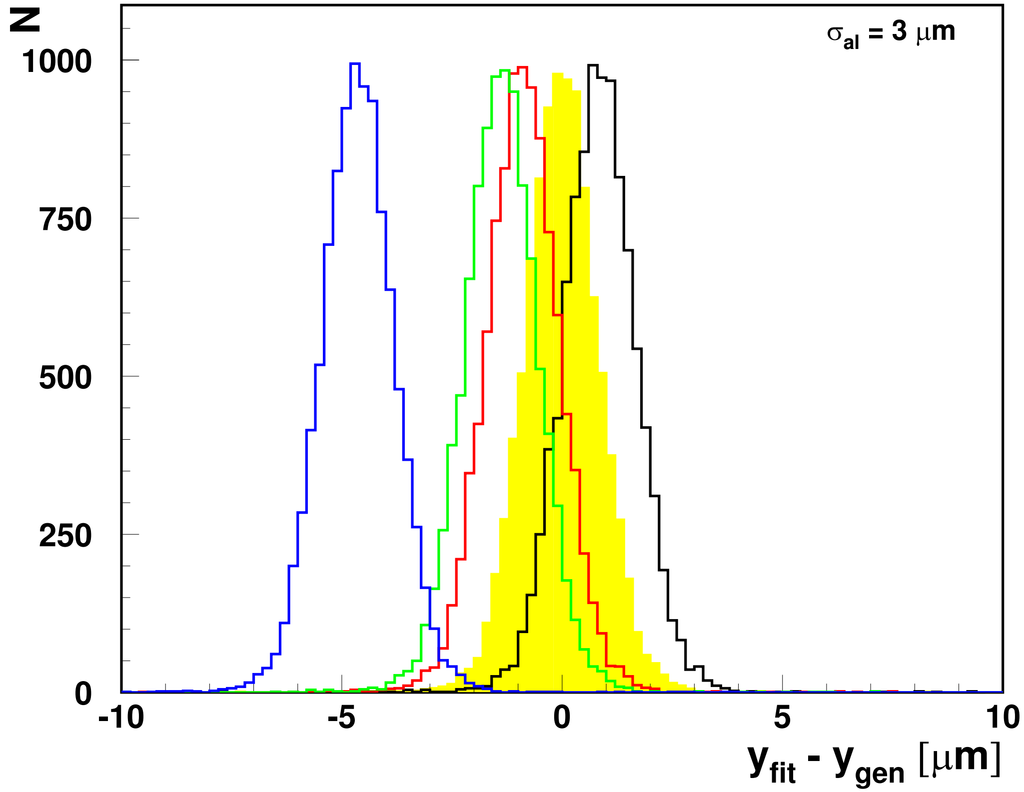
Simulation Results

Misalignment

Reconstruction error distribution for the particle position at DUT

Four experiments with 3 μm alignment compared to perfect alignment (solid yellow)

GEANT 4 events, 6 GeV electron beam



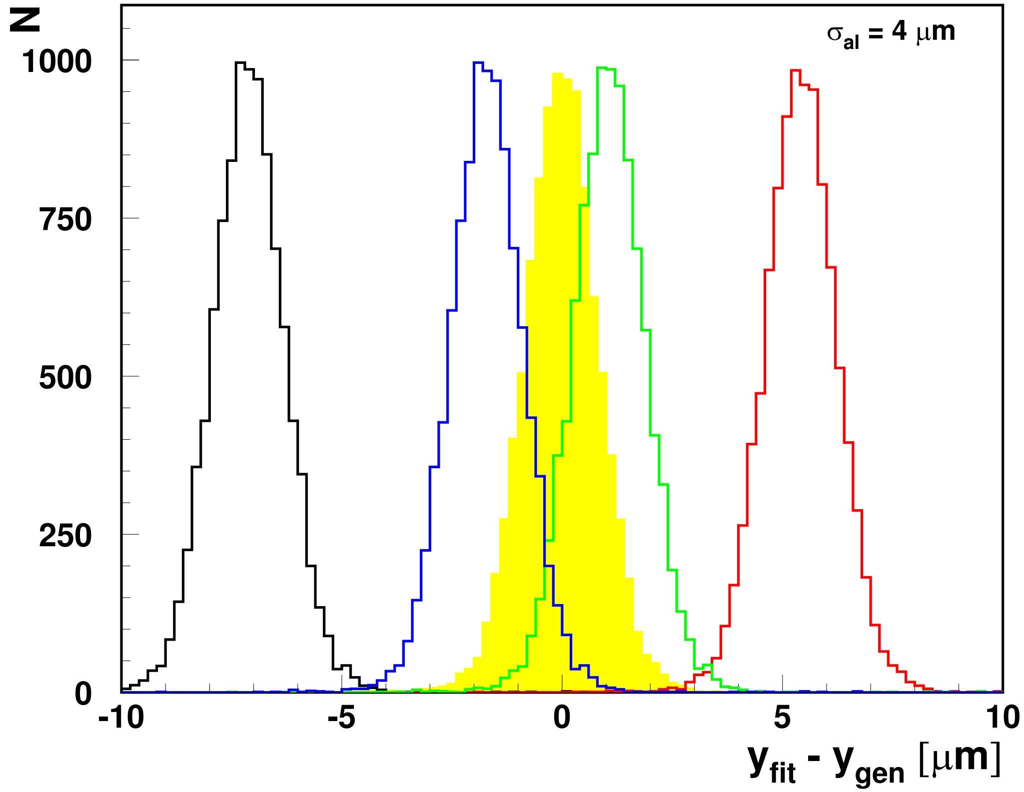
Simulation Results

Misalignment

Reconstruction error distribution for the particle position at DUT

Four experiments with 4 μm alignment compared to perfect alignment (solid yellow)

GEANT 4 events, 6 GeV electron beam



Simulation Results

Misalignment

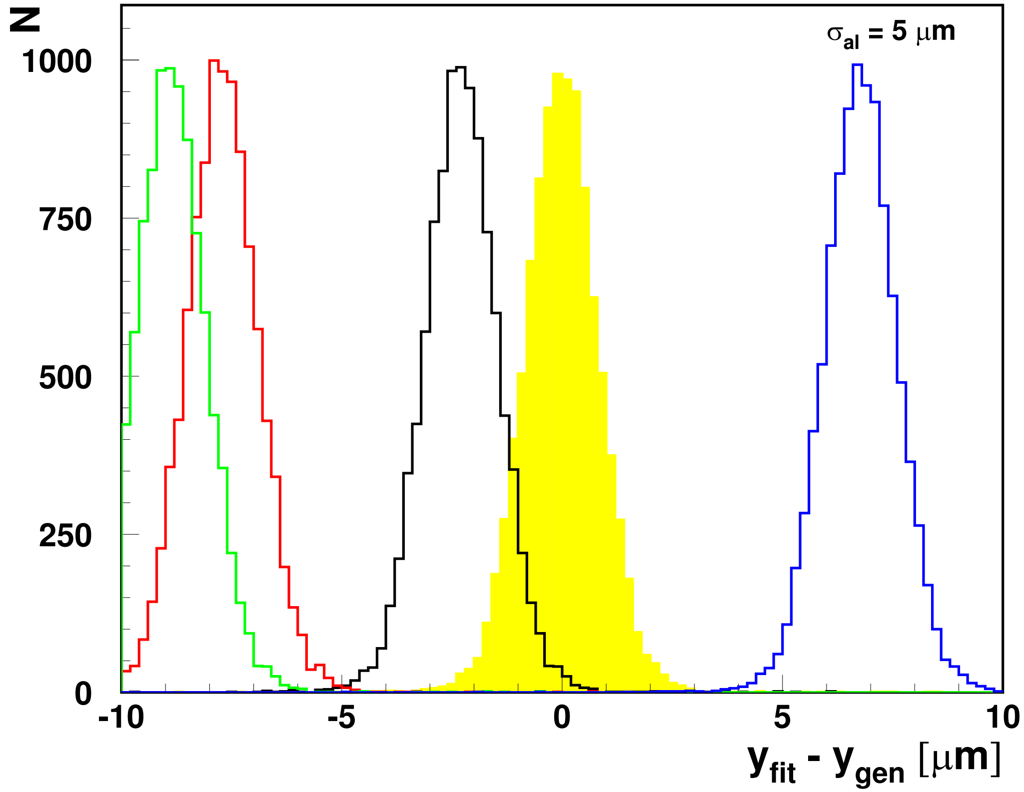
Reconstruction error distribution for the particle position at DUT

Four experiments with 5 μm alignment compared to perfect alignment (solid yellow)

Width of the distribution is unchanged !!!

Telescope misalignment is equivalent to DUT position shift.

GEANT 4 events, 6 GeV electron beam



Analysis

Track fitting

Fitting a track, i.e. finding minimum of χ^2 is equivalent to solving the set of N equations:

$$\frac{\partial \chi^2}{\partial p_i} = 0, \qquad i = 1 \ldots N \qquad p_i \text{ - particle position in plane } i$$

This is transformed it to **matrix equation**:

$$\sum_j A_{ij}\, p_j = \varepsilon_i\, y_i \qquad y_i \text{ - measured position in plane } i$$

$$\text{where: } A_{ij} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial p_i\, \partial p_j}$$

Reconstructed position is given by linear combination of measured positions:

$$p_i = \sum_j \left(A^{-1}\right)_{ij} \varepsilon_j\, y_j$$

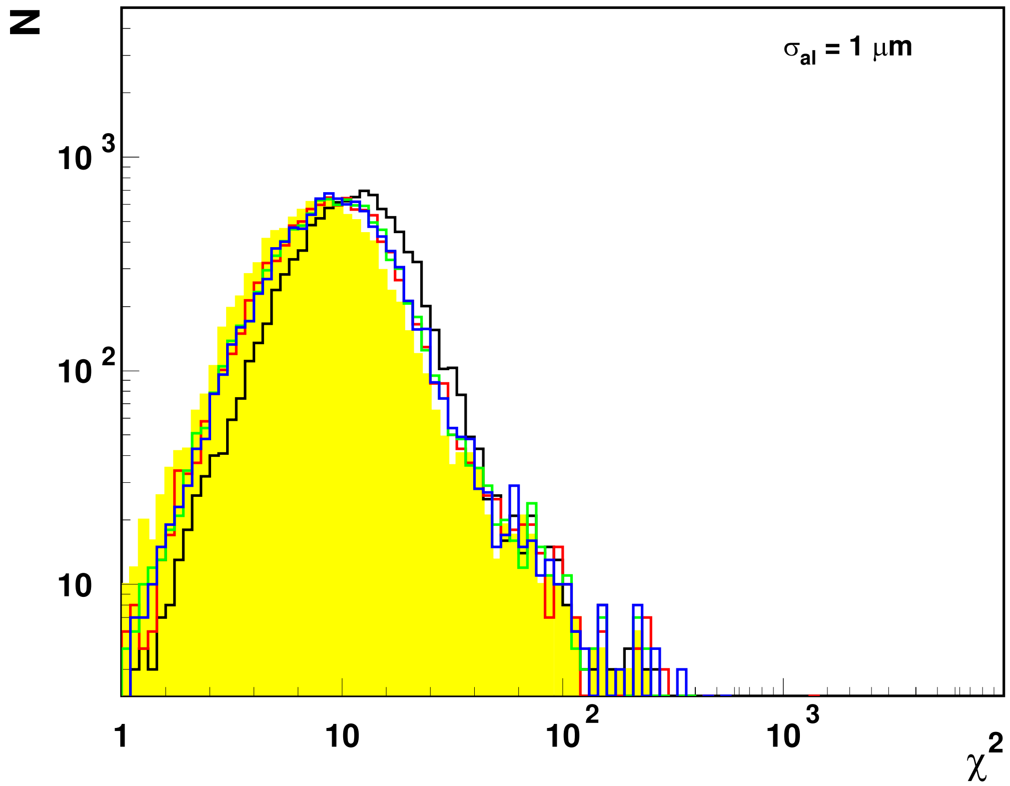
Misalignment is equivalent to constant offset in $y_i \Rightarrow$ results in systematic shift of p_i

Simulation Results

Fit quality

Four experiments with 1 μm alignment
compared to perfect alignment
(solid yellow)

χ^2 distribution for GEANT 4 events

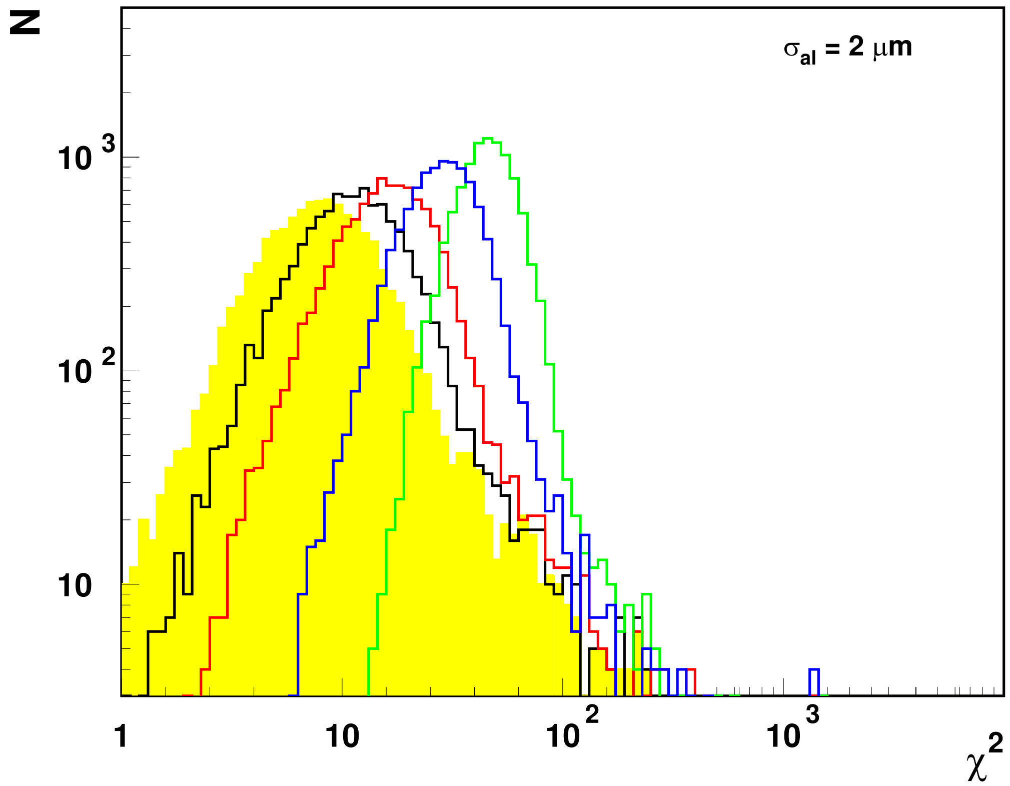


Simulation Results

Fit quality

Four experiments with 2 μm alignment
compared to perfect alignment
(solid yellow)

χ^2 distribution for GEANT 4 events

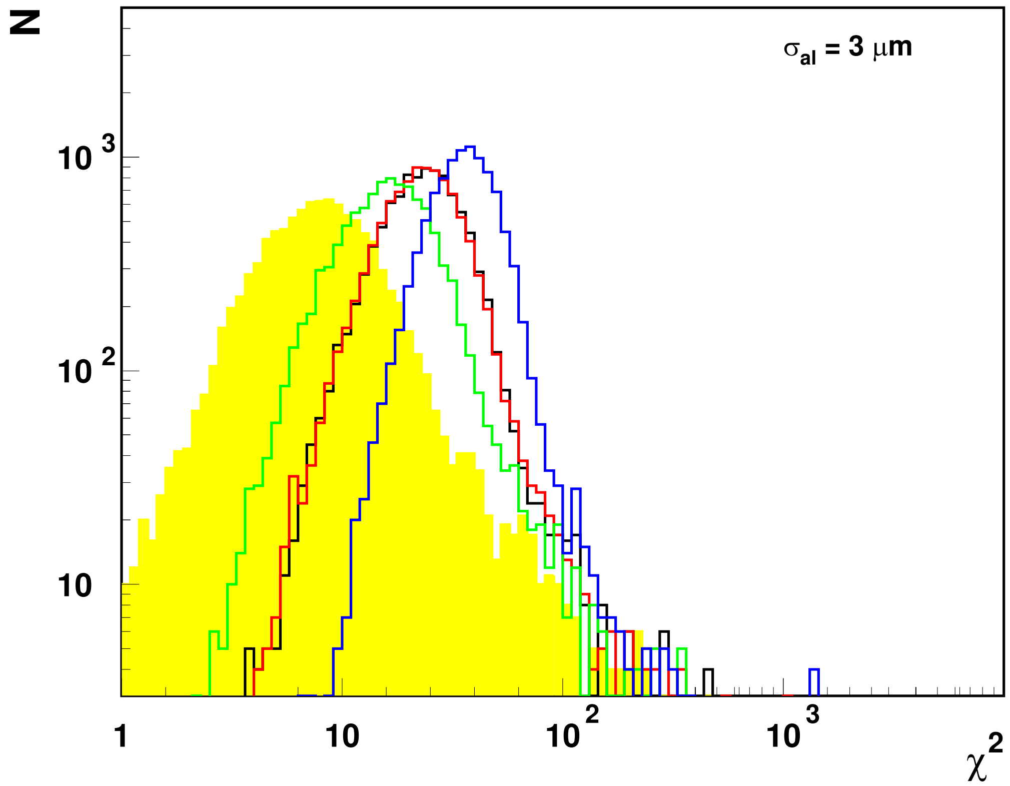


Simulation Results

Fit quality

Four experiments with 3 μm alignment
compared to perfect alignment
(solid yellow)

χ^2 distribution for GEANT 4 events

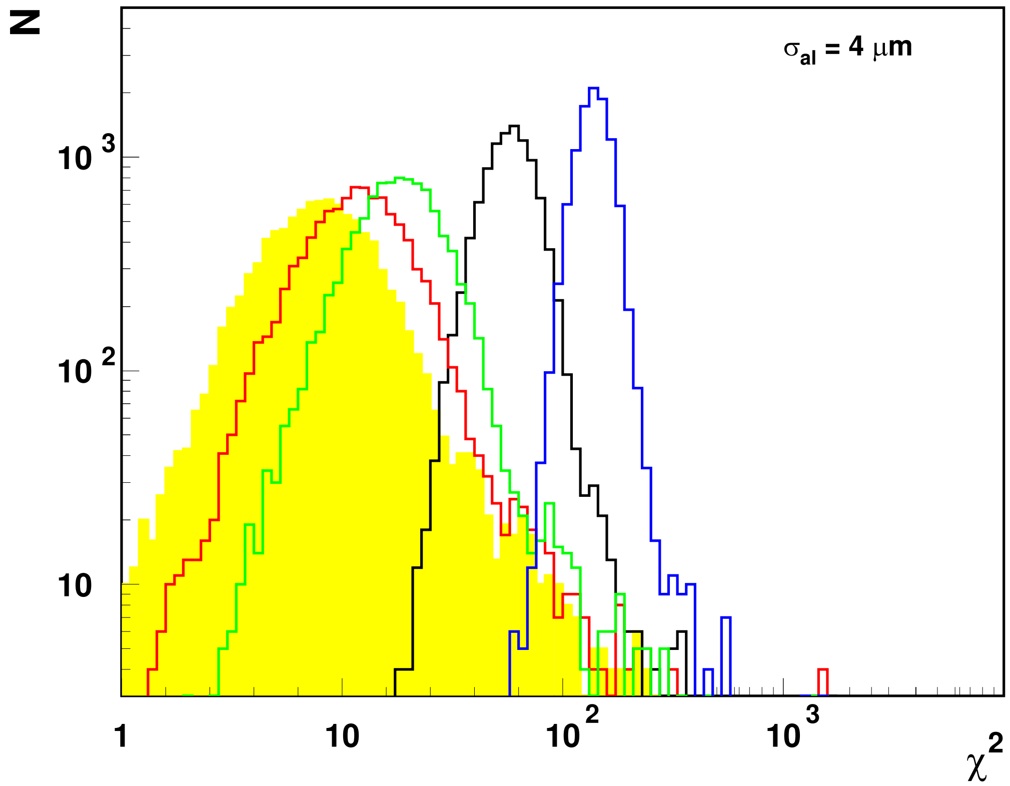


Simulation Results

Fit quality

Four experiments with 4 μm alignment
compared to perfect alignment
(solid yellow)

χ^2 distribution for GEANT 4 events

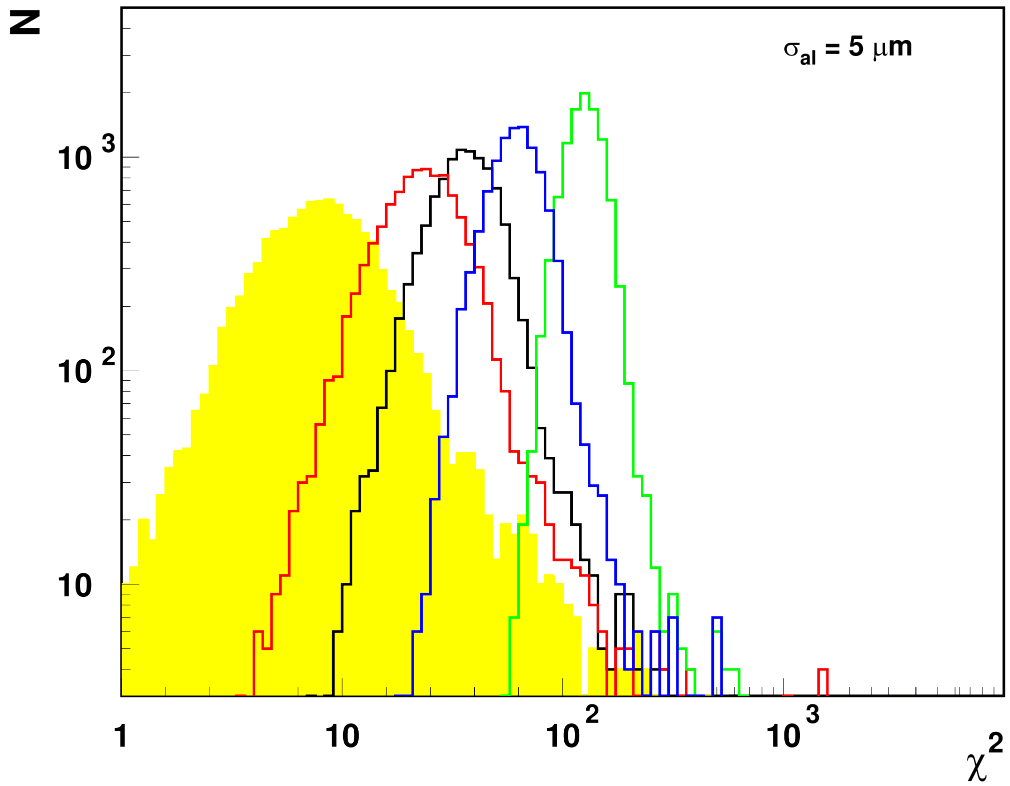


Simulation Results

Fit quality

Four experiments with 5 μm alignment
compared to perfect alignment
(solid yellow)

χ^2 distribution for GEANT 4 events

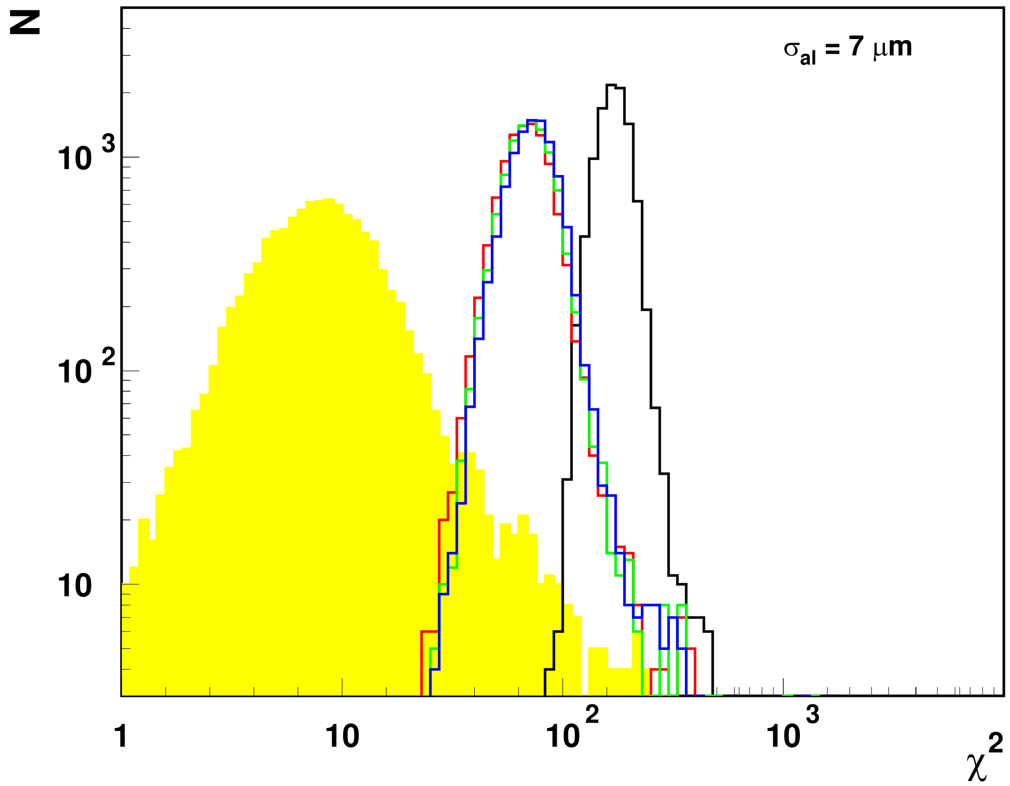


Simulation Results

Fit quality

Four experiments with $7\ \mu m$ alignment
compared to perfect alignment
(solid yellow)

χ^2 distribution for GEANT 4 events

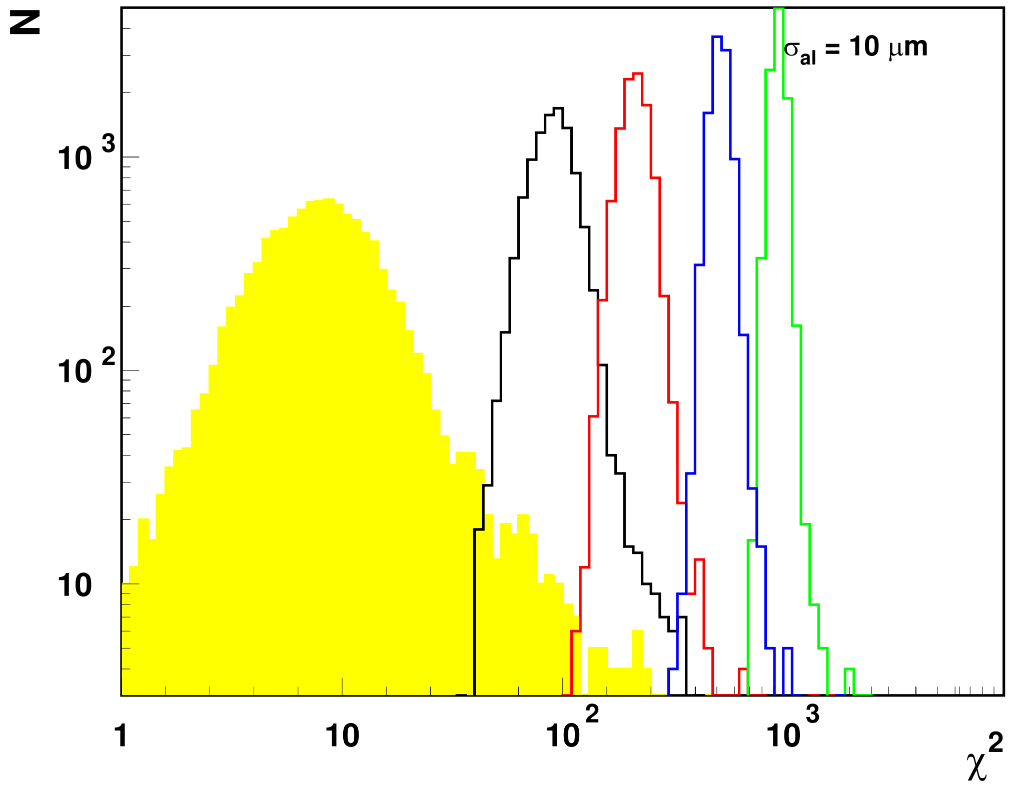


Simulation Results

Fit quality

χ^2 distribution for GEANT 4 events

Four experiments with 10 μm alignment
compared to perfect alignment
(solid yellow)



Simulation Results

Fit quality

Misalignment $\Rightarrow$ large χ^2 values, not related to the actual measurement.

χ^2 cut can no longer be used to remove poorly reconstructed tracks

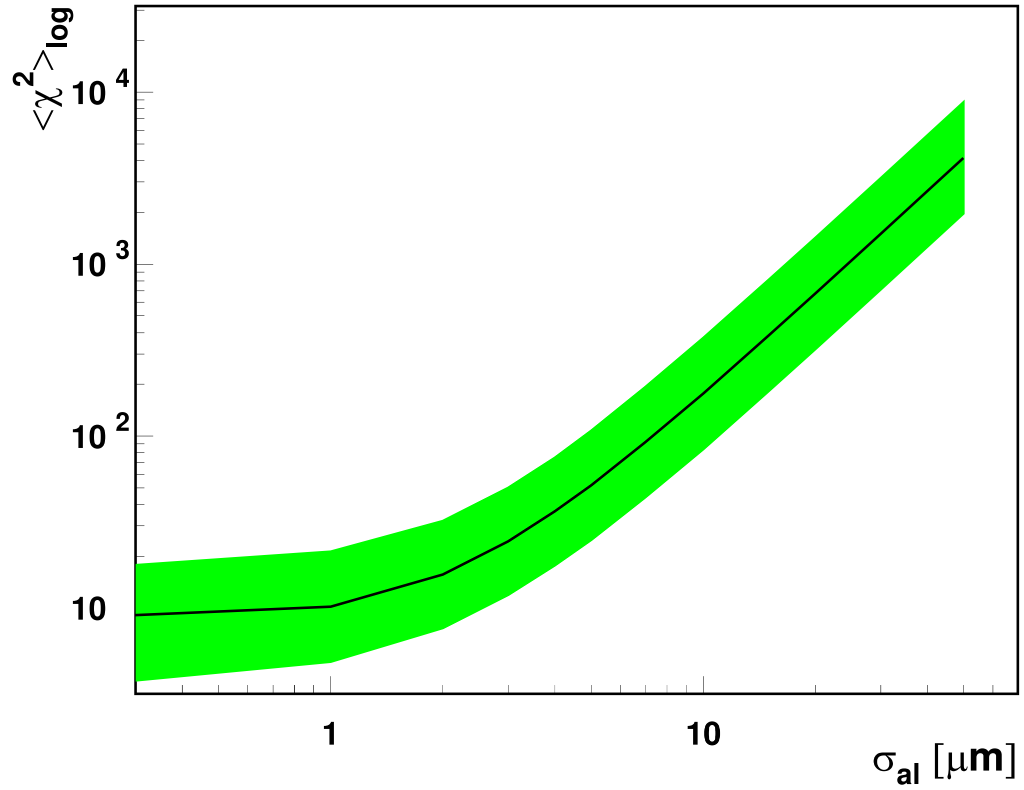
resolution can deteriorate slightly

If multiple hits are reconstructed in telescope layers, it is much more difficult to match hits to the track.

$\Rightarrow$ we should reduce alignment error to

$$\sigma_{al} \sim \sigma_{tel}$$

Mean and spread of $\log_{10} \chi^2$ from GEANT 4



Alignment

Consider only sensor displacement in transverse direction.

Effects of longitudinal shift should be much smaller.

No rotations.

Simple approaches

- Align to beam profile (each plane separately)
  "absolute" alignment, but poor precision $(10 - 100\mu m$?)
- Align to track extrapolated from the first plane
  only if beam angular spread negligible, limited by multiple scatterings

$\Rightarrow$ Alignment to first and last plane
"relative", but no other possibility if no precise constrain from the beam

Full fit of telescope alignment parameters to all measurements
should result in precision below $1\mu m$, but can be slow
hard to implement in analytical approach

Alignment

Possible approach

Interpolate track between first and last plane using line fit.

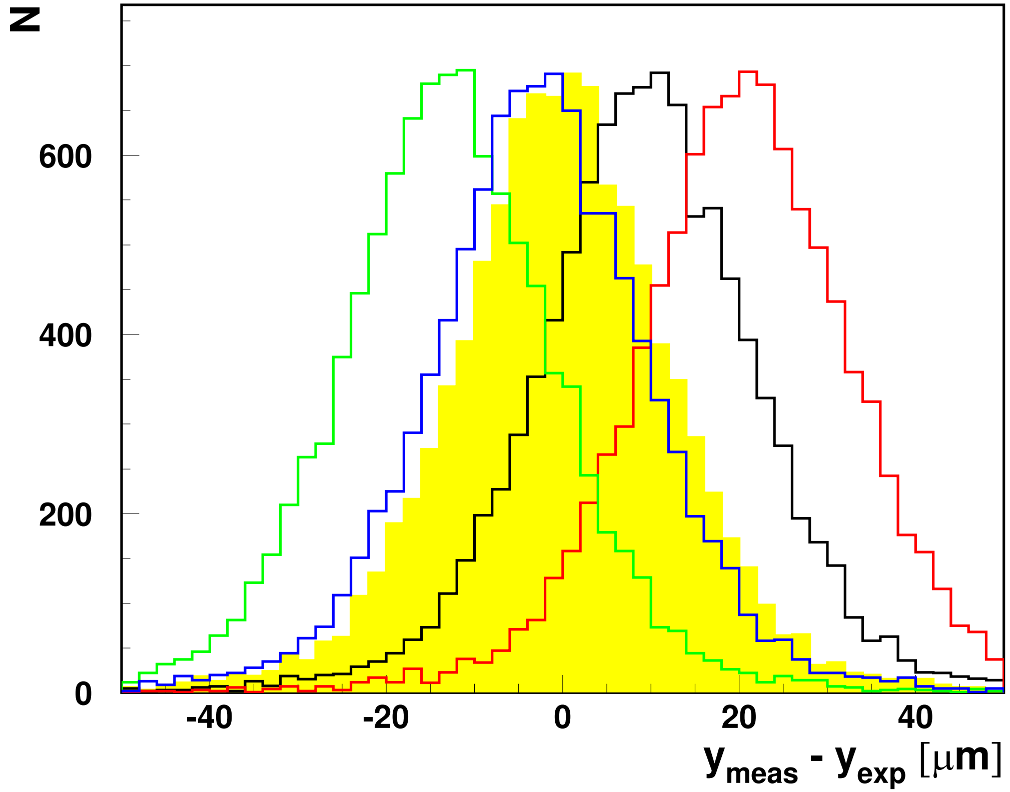
Multiple experiments
with 10 μm alignment uncertainty

Difference between position measured in 2nd telescope plane and the position expected from first and last plane ⇒

Position in the plane can be estimated with $\sim 11\mu m$ precision.

(simulation agree with calculations)

GEANT 4 events, 6 GeV electrons



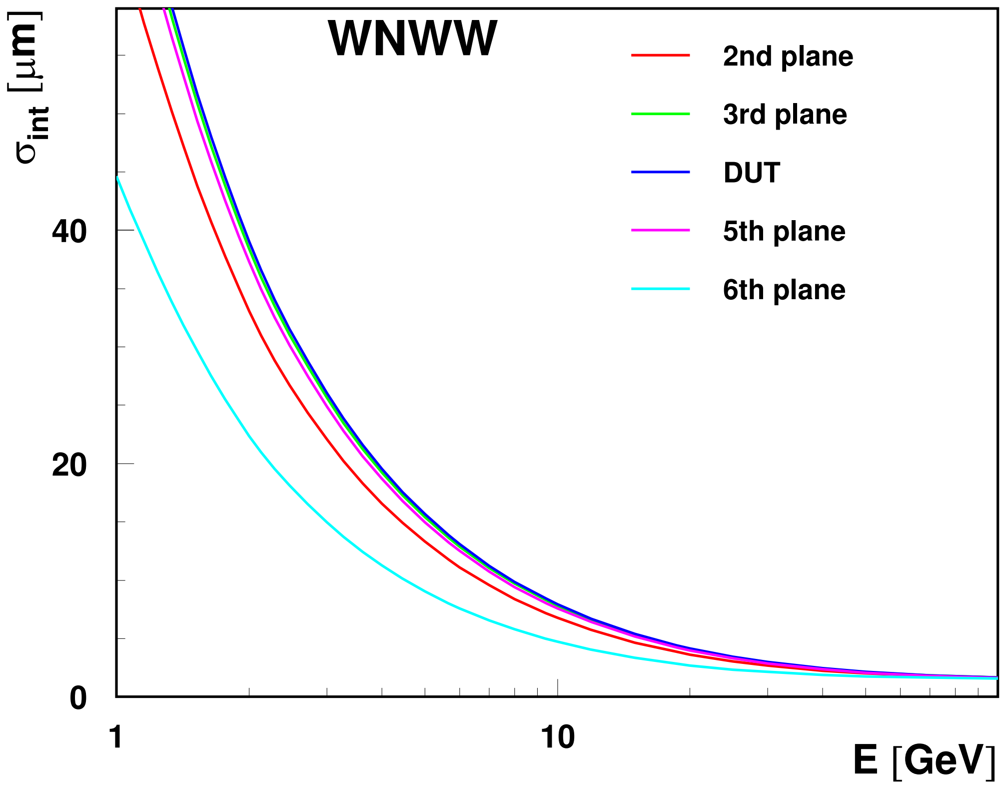
Alignment

Possible approach

Line fit to measurements in first and last telescope plane only ($\sigma = 2\mu m$).

Precision of particle position determination in other planes ⇒

$\Delta_{DUT} = 500\mu m$



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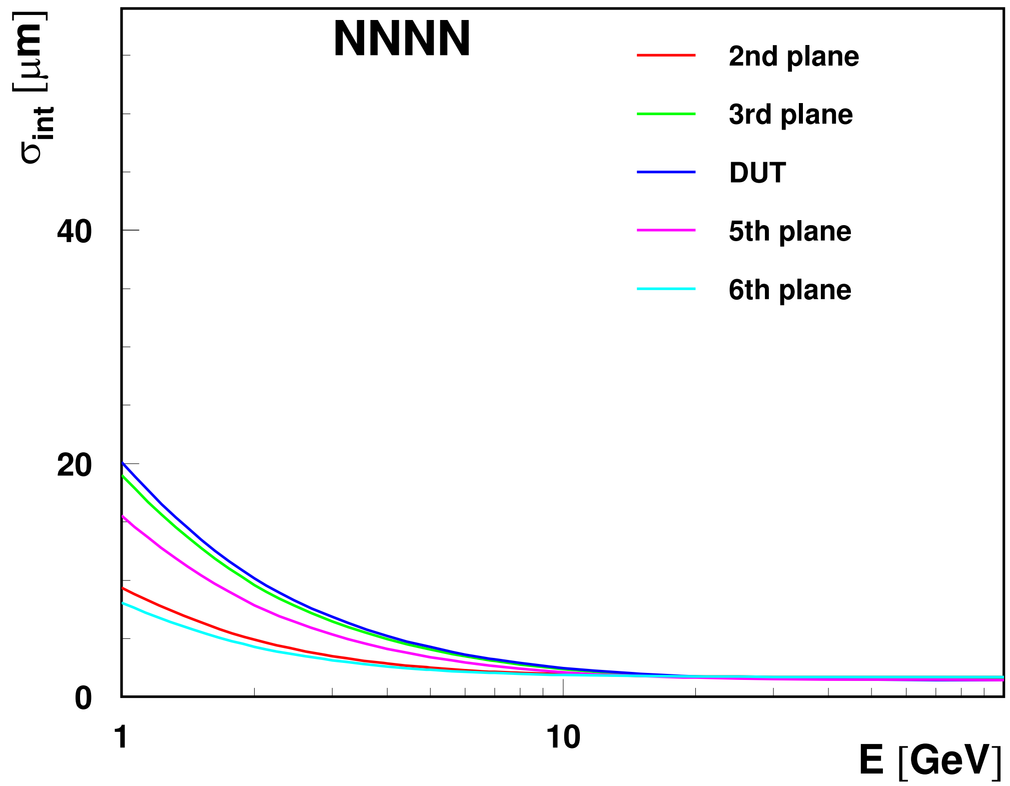


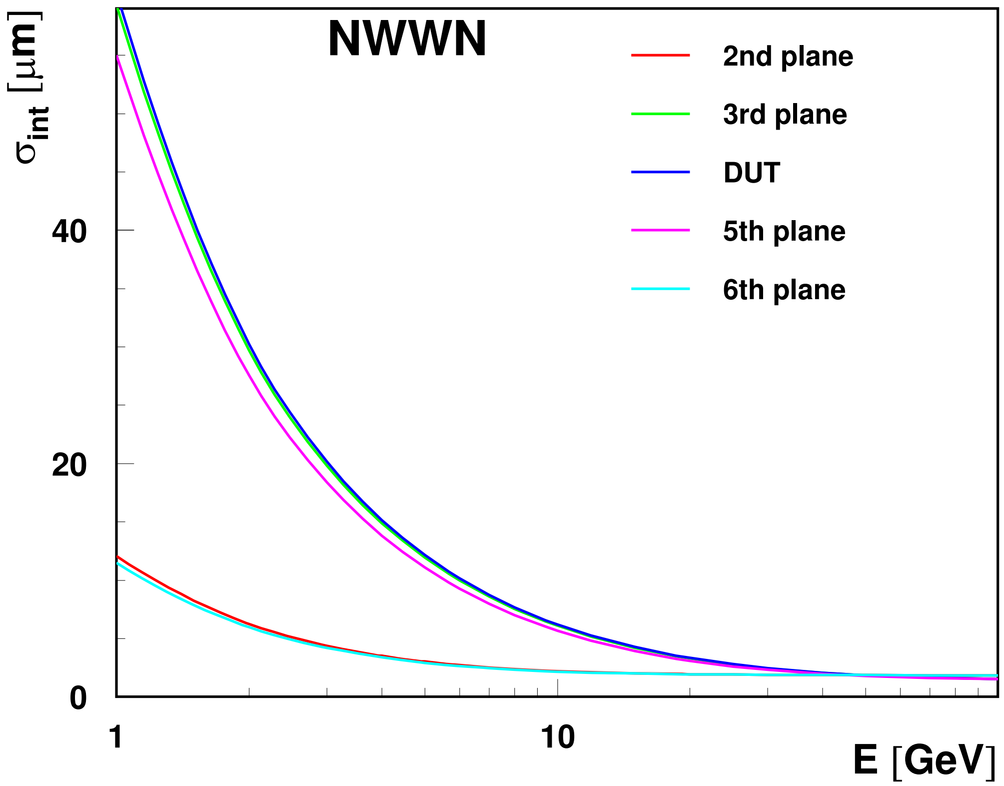
Alignment

Possible approach

$\Delta_{DUT} = 500\mu m$

Line fit to measurements in first and last telescope plane only $(\sigma = 2\mu m)$.

Precision of particle position determination in other planes $\Rightarrow$



With $\sim$ 1000 reconstructed tracks alignment to $\sim 1\mu m$ possible $(\frac{\sigma_{int}}{\sqrt{N}})$

Should be sufficient for simple on-line alignment check

Conclusions

Analytical track fitting is little sensitive to telescope misalignment.

Systematic shift in position, but position resolution at DUT unchanged.

Possible misalignment affects only the track quality estimate.

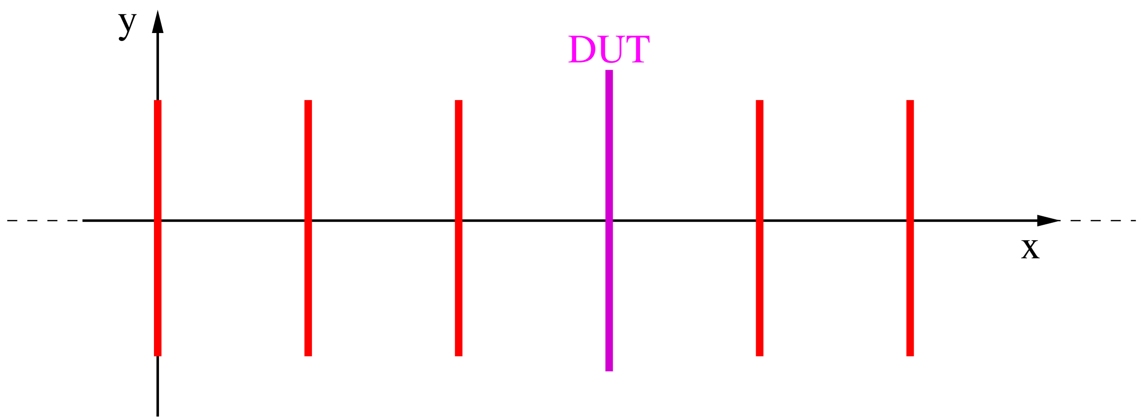
For proper selection of good tracks alignment to few μm needed.

Simple procedure of relative plane alignment,
based on a linear interpolation between first and last telescope plane,
can fulfill this requirement.

Influence of small sensor rotations (around beam axis) still to be studied...

Backup slide

Track fitting



Geometry can be specified by giving:

- N - number of detector planes (including DUT)
- x_i - position of each plane $(i = 1 \ldots N)$
- σ_i - position resolution in each plane $(i \neq i_{DUT})$
- $\Delta\theta_i$ - average scattering angle in each plane

Average scattering angle depends on the plane thickness Δ_i and the particle energy, and is calculated using Highland formula (Gaussian approximation).

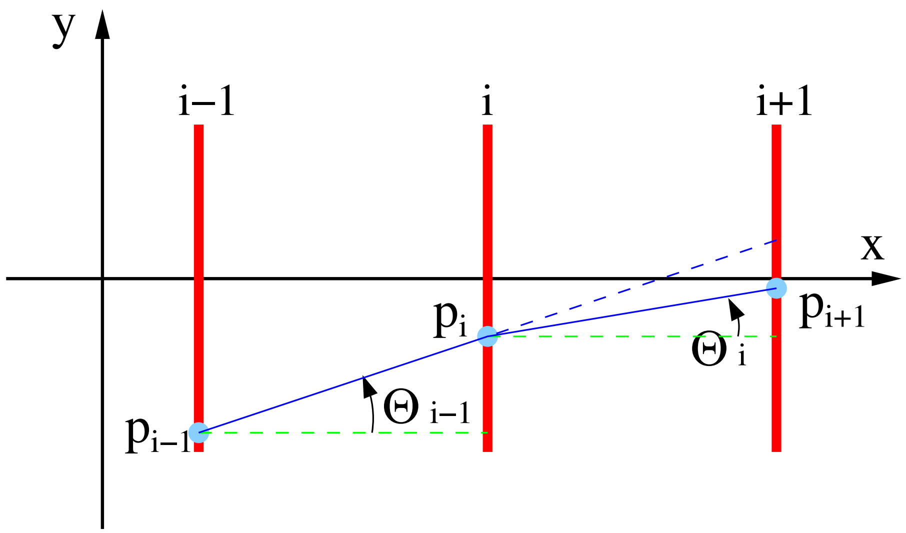
Backup slide

Track fitting (in one plane)

We want to determine track positions in each plane (including DUT), i.e. N parameters (p_i, $i = 1 \ldots N$), from $N-1$ measured positions in telescope planes (y_i, $i \neq i_{DUT}$).

However, we can use constraints on multiple scattering!

Contribution of plane i to χ^2 of the fit



position measurement — multiple scattering

$$\Delta\chi^2_i = \left(\frac{y_i - p_i}{\sigma_i}\right)^2 + \left(\frac{\Theta_i - \Theta_{i-1}}{\Delta\Theta_i}\right)^2$$

where: $$\Theta_i = \frac{p_{i+1} - p_i}{x_{i+1} - x_i}$$

Both terms present for planes $i \neq 1, i_{DUT}, N$,
first term missing for DUT, second for first and last plane

χ^2 minimum can be found by solving the matrix equation.

As a by-product we get also an expected error on the position reconstructed at DUT.

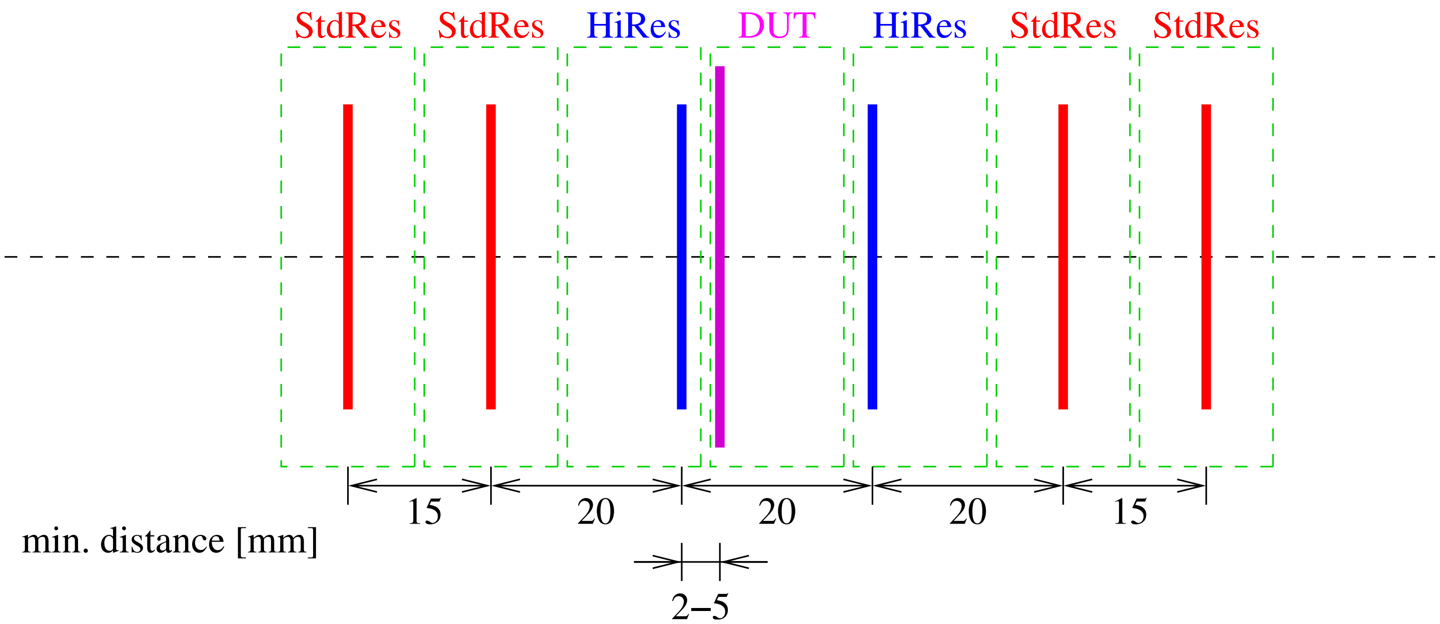
Backup slide

Realistic telescope geometry

thanks to W.Dulinski

The minimum distance between DUT and **one** of the telescope planes, d_{min}, is 5 mm (easy, realistic) or even 2 mm (hard, optimistic).

However, other distances can not be smaller than 15 or 20 mm:



In addition to standard sensor planes with 2 μm resolution we can consider adding one or two high resolution planes ($\sigma_{HR} \sim 1\mu m$) in front of and behind DUT

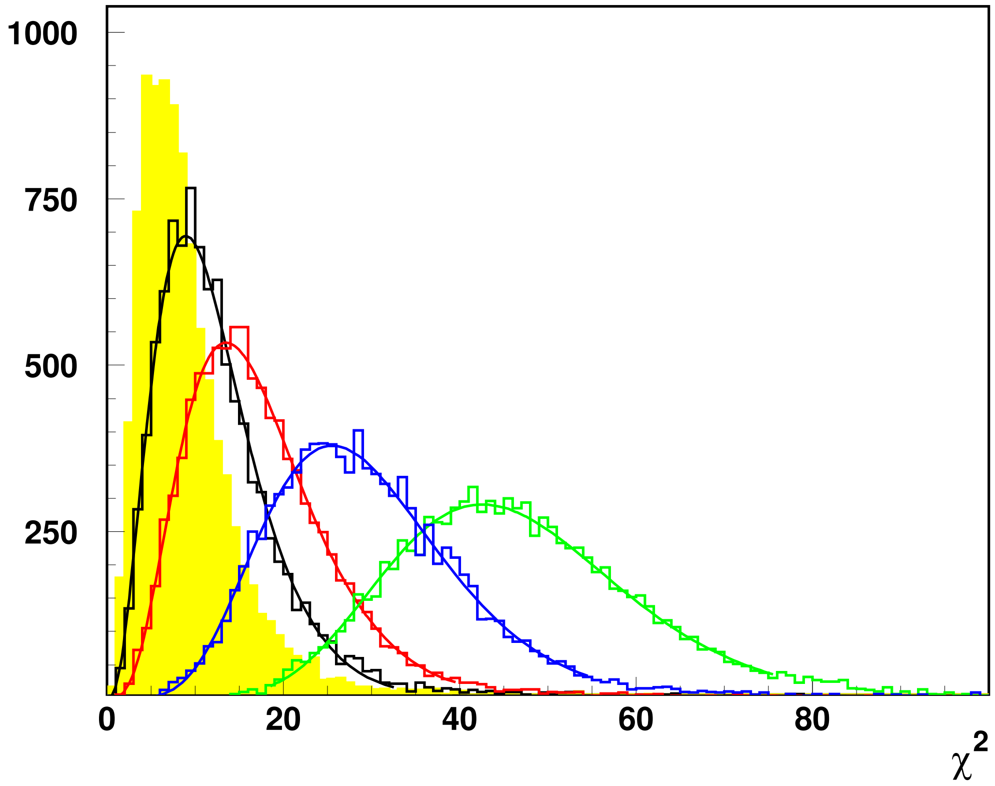
Backup slide

Fit quality

Multiple experiments
with 2 μm alignment uncertainty

χ^2 distribution can be described by the gamma distribution

GEANT 4 simulation



Backup slide

Fit quality

Multiple experiments
with 2 μm alignment uncertainty

χ^2 distribution can be described by the gamma distribution

Distributions significantly wider than expected for χ^2 distribution
(with increased number of degrees of freedom)

GEANT 4 simulation