# Inert Doublet Model signatures at Future $e^+e^-$ Colliders

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**Workshop on Connecting Insights in Fundamental Physics:
Standard Model and Beyond**

# Outline

1. Inert Doublet Model
2. Benchmark points
3. Leptonic analysis
    - Analysis strategy
    - Neutral scalar production
    - Charged scalar production
    - Prospects at higher energies
4. Semi-leptonic analysis
5. Conclusions

For more details:

- on benchmark points: JHEP 1812 (2018) 081, arXiv:1809.07712
- results for CLIC report: (leptonic signature) JHEP 1907 (2019) 053, arXiv:1811.06952

# Inert Doublet Model

One of the simplest extensions of the Standard Model (SM).

The scalar sector consists of two doublets:

- $\Phi_S$ is the SM–like Higgs doublet,
- $\Phi_D$ (inert doublet) has four additional scalars $H$, $A$, $H^\pm$.

$$\Phi_S = \begin{pmatrix} G^\pm \\ \frac{v+h+iG^0}{\sqrt{2}} \end{pmatrix} \qquad \Phi_D = \begin{pmatrix} H^\pm \\ \frac{H+iA}{\sqrt{2}} \end{pmatrix}$$

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$$\Phi_S = \left( \begin{array}{c} G^{\pm} \\ \frac{v+h+iG^0}{\sqrt{2}} \end{array} \right) \qquad \Phi_D = \left( \begin{array}{c} H^{\pm} \\ \frac{H+iA}{\sqrt{2}} \end{array} \right)$$

We assume a discrete $Z_2$ symmetry under which

- SM Higgs doublet $\Phi_S$ is *even*: $\Phi_S \to \Phi_S$ (also other SM→SM)
- inert doublet $\Phi_D$ is *odd*: $\Phi_D \to -\Phi_D$.

$\Rightarrow$ Yukawa-type interactions only for Higgs doublet ($\Phi_S$).
The inert doublet ($\Phi_D$) does not interact with the SM fermions!

$\Rightarrow$ The lightest inert particle is stable: a natural candidate for dark matter!
We assume the neutral scalar $H$ is the dark matter particle.

$$m_H < m_A,\ m_{H^{\pm}}$$

# Inert Doublet Model

After EWSB, the model contains a priori seven free parameters.

Two parameters can be fixed from the Standard Model ($v$, $m_h$).

We are left with five free parameters, which we take as:

⇒ three inert scalar masses: $m_H$, $m_A$, $m_{H^\pm}$
⇒ two couplings, eg. $\lambda_2$ and $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$

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Inert scalars couplings to $\gamma$, $W^\pm$ and $Z$ determined by SM parameters
⇒ well established predictions for production and decay rates!

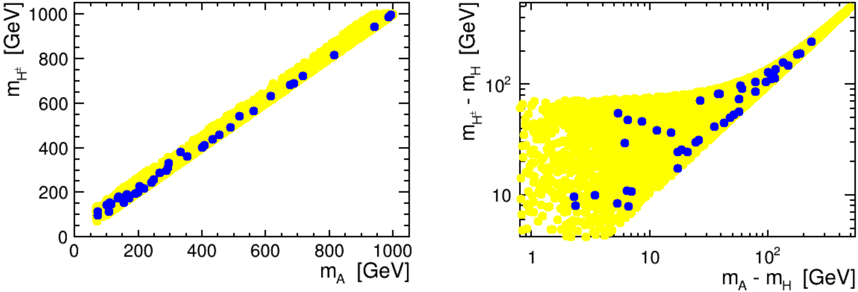
We scanned the IDM parameter space looking for scenarios consistent with current theoretical and experimental constraints, for masses up to 1 TeV.

For details and previous IDM parameter scan results see:

- Agnieszka Ilnicka, Maria Krawczyk, and Tania Robens, *Inert Doublet Model in light of LHC Run I and astrophysical data*, Phys. Rev. D93(5):055026, 2016, arXiv:1508.01671.
- Agnieszka Ilnicka, Tania Robens, and Tim Stefaniak, *Constraining Extended Scalar Sectors at the LHC and beyond*, Mod. Phys. Lett. A33(10n11):1830007, 2018, arXiv:1803.03594.

# IDM benchmark points

Out of about 15'000 points consistent with all considered constraints, we chose 41 benchmark points (including 20 "high mass") for detailed studies:



The selection was arbitrary, but we tried to

- cover wide range of scalar masses and the mass splittings
- get significant contribution to the relic density

For details see: JHEP 1812 (2018) 081, arXiv:1809.07712

For list of benchmark point parameters, see backup slides

# Analysis strategy

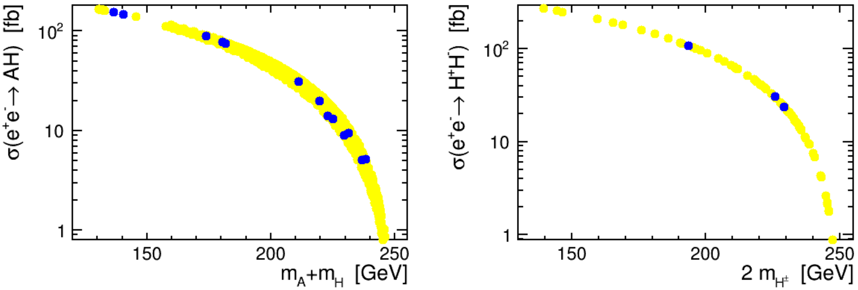
Production of IDM scalars at $e^+e^-$ colliders dominated by two processes:

$$e^+e^- \rightarrow A \;\; H \qquad\qquad e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 250 GeV:

13 benchmarks

3 benchmarks



Beam luminosity spectra not taken into account

# Analysis strategy

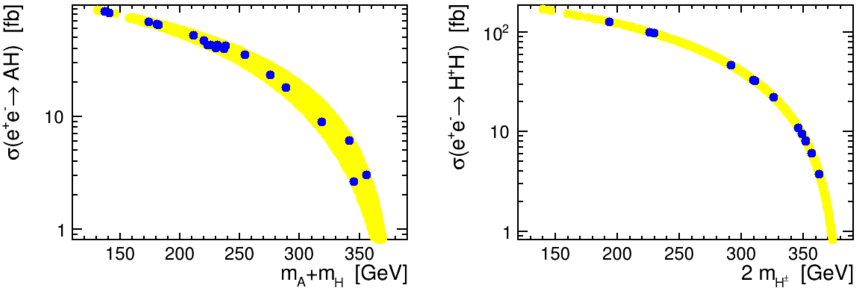
Production of IDM scalars at $e^+e^-$ colliders dominated by two processes:

$$e^+e^- \to A\ \ H \qquad\qquad e^+e^- \to H^+H^-$$

Leading-order cross sections for inert scalar production processes at 380 GeV:

20 benchmarks

15 benchmarks



Beam luminosity spectra not taken into account

# Analysis strategy

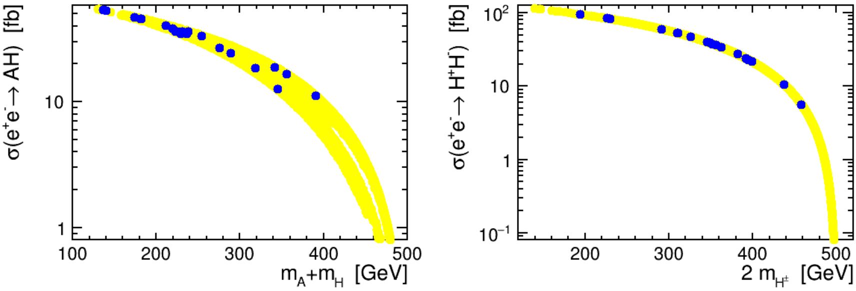
Production of IDM scalars at e$^+$e$^-$ colliders dominated by two processes:

$$e^+e^- \to A\ \ H \qquad\qquad e^+e^- \to H^+H^-$$

Leading-order cross sections for inert scalar production processes at 500 GeV:

21 benchmarks

21 benchmarks



Beam luminosity spectra not taken into account

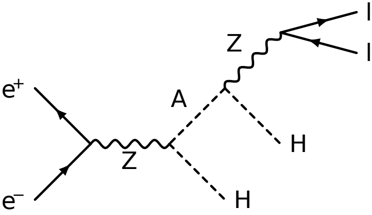
# Leptonic signatures

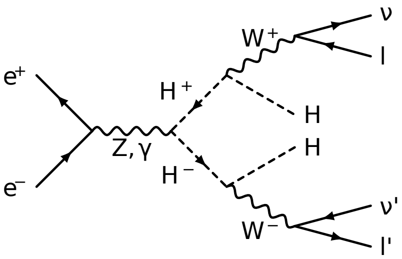
Same flavour lepton pair production can be considered a signature of the $AH$ production process followed by the $A$ decay:

$$e^+e^- \rightarrow HA \rightarrow HHZ^{(\star)} \rightarrow HH\mu^+\mu^-$$

while the production of the different flavour lepton pair is the expected signature for $H^+H^-$ production:

$$e^+e^- \rightarrow H^+H^- \rightarrow HHW^{+(*)}W^{-(*)} \rightarrow HH\ell^+\ell'^-\nu\bar{\nu}'$$





# Leptonic signatures

We consider two possible final state signatures:

- muon pair production, $\mu^+\mu^-$, for $AH$ production
- electron-muon pair production, $\mu^+e^-$ or $e^+\mu^-$, for $H^+H^-$ production

Both channels include contributions from $AH$ and $H^+H^-$ production!
In particular due to leptonic tau decays.

Signal and background samples were generator with WHizard 2.2.8
based on the dedicated IDM model implementation in SARAH,
parameter files for benchmark scenarios were prepared using SPheno 4.0.3

Generator level cuts reflecting detector acceptance:

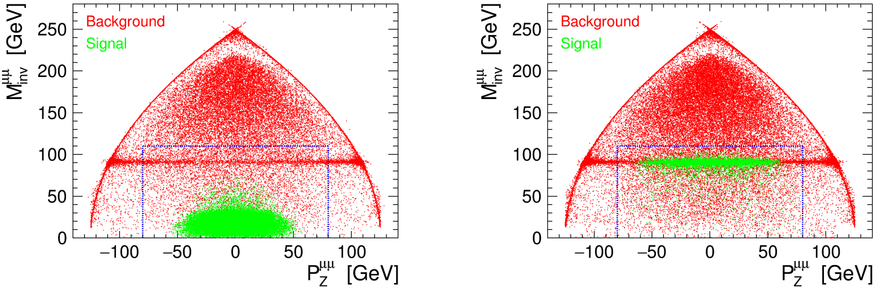
- require lepton energy $E_l > 5\,\text{GeV}$ and lepton angle $\Theta_l > 100\,\text{mrad}$
- no ISR photon with $E_\gamma > 10\,\text{GeV}$ and $\Theta_\gamma > 100\,\text{mrad}$

No detector resolution/efficiency taken into account
(but only electrons and muons in the final state)

# Neutral scalar production @ 250 GeV

Muon pair invariant mass, $\mathrm{M}_{inv}^{\mu\mu}$, as a function of the lepton pair long. momentum, $\mathrm{P}_{Z}^{\mu\mu}$, for IDM signal and SM background, at 250 GeV

BP1 BP9

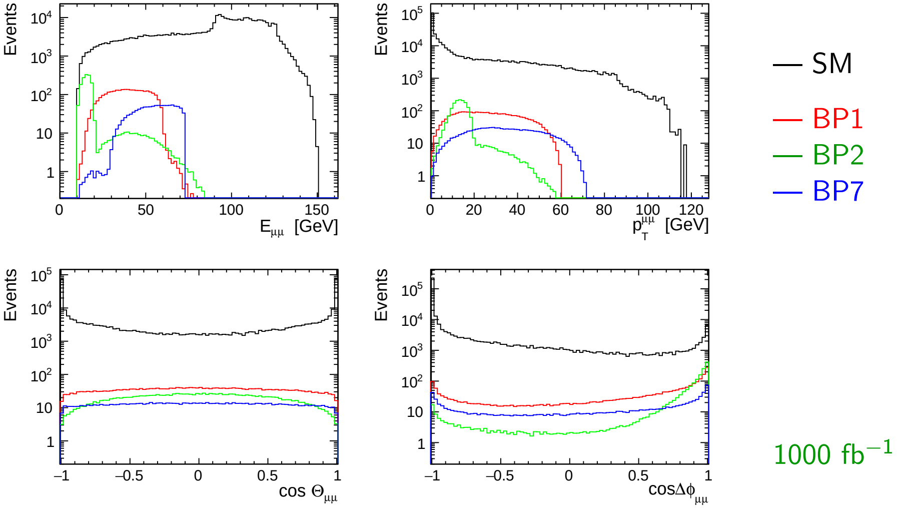


Background dominated by muon pair production $(e^+e^- \rightarrow \mu^+\mu^-)$ at nominal energy and radiative events $(e^+e^- \rightarrow \mu^+\mu^-\gamma)$

$\Rightarrow$ apply pre-selection cuts: $\mathrm{M}_{\mu\mu} < 0.33\sqrt{s}$ and $|\mathrm{P}_Z^{\mu\mu}| < 0.44\sqrt{s}$

# Neutral scalar production @ 250 GeV

Distributions of the kinematic variables describing the leptonic final state

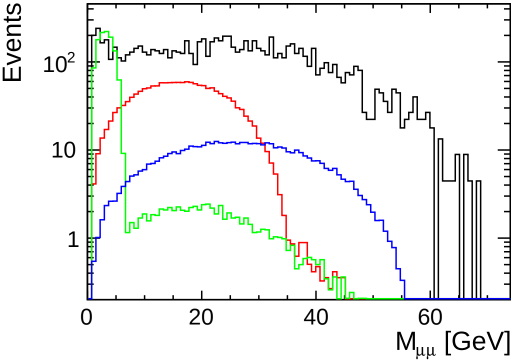


# Neutral scalar production @ 250 GeV

## Cut based approach

Lepton pair invariant mass distribution after selection cuts 1000 $\mathrm{fb}^{-1}$

- pair energy
  $E_{\mu\mu} < 75\,\mathrm{GeV}$
- transverse momentum
  $p_T^{\mu\mu} > 10\,\mathrm{GeV}$
- production angle
  $45^\circ < \Theta_{\mu\mu} < 135^\circ$
- azimuthal distance
  $|\Delta\varphi_{\mu\mu}| < \frac{\pi}{2}$



IDM signal would result in the visible excess in $M_{\mu\mu}$ distribution

15.9$\sigma$, 11.6$\sigma$ and 5.4$\sigma$, for BP1, BP2 and BP7

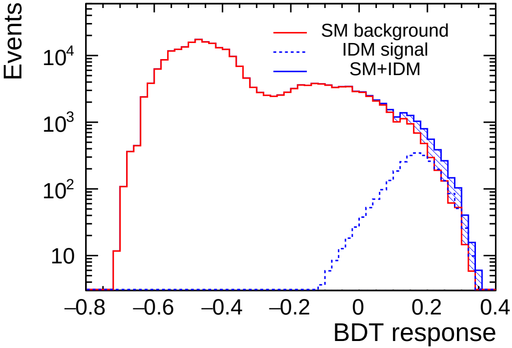
(without any cut on $M_{\mu\mu}$)

# Neutral scalar production @ 250 GeV

## Multivariate analysis

BDT classifier with 8 input variables used for selection of signal events

Response distribution for $\mu\mu$ channel: BP1 scenario and SM background
unpolarised 1000 fb$^{-1}$ at $\sqrt{s} = 250$ GeV



$\Rightarrow$ signal significance of about 24 $\sigma$ for BDT$>$ 0.11

# Neutral scalar production

**Multivariate analysis** commmon approach

We train BDTs to separate the considered signal from the background
But we will not know in advance what to look for!
We will not know details of the model (scalar masses)

# Neutral scalar production

**Multivariate analysis** commmon approach

We train BDTs to separate the considered signal from the background
But we will not know in advance what to look for!
We will not know details of the model (scalar masses)

**Scenario-independent approach**

Divide the considered BP scenarios in two groups:

- scenarios with real $Z$ (or real $W$) production
- scenarios with virtual $Z$ (or virtual $W$) in intermediate state

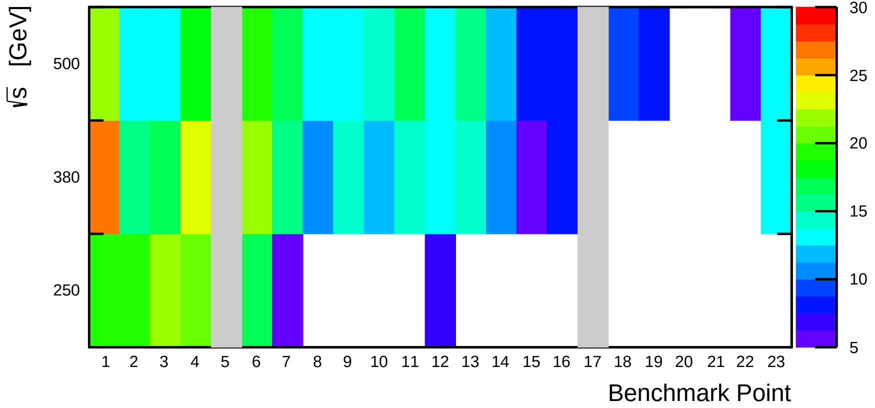
For each group: search for given BP (test sample)
while using all other scenarios to train BDT (training samples)

Corresponds to the assumption that two independent BDTs will be used in the analysis for the two cases...

# Neutral scalar production

## Significance of observation

Summary of results for the considered benchmark scenarios

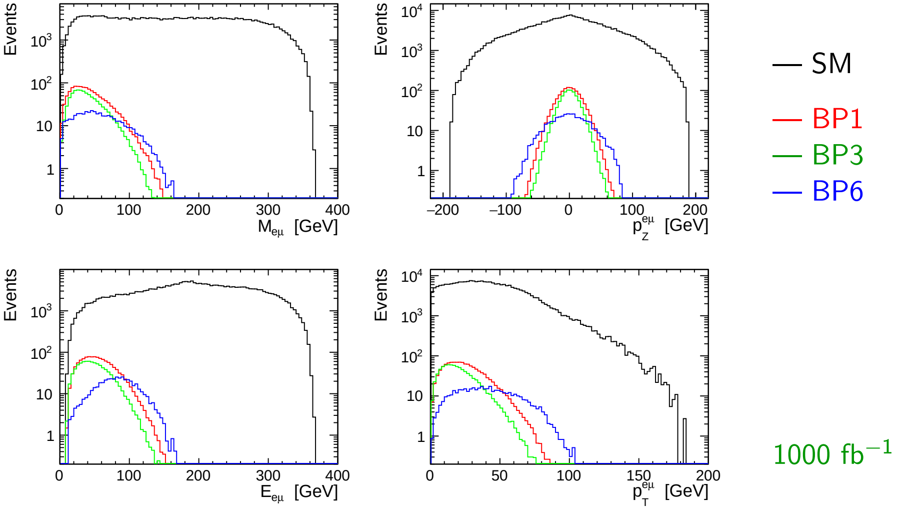


High significance of observation for scenarios accessible at given energy

Expected significance mainly related to the *AH* production cross section

# Charged scalar production @ 380 GeV

Distributions of the kinematic variables describing the leptonic final state



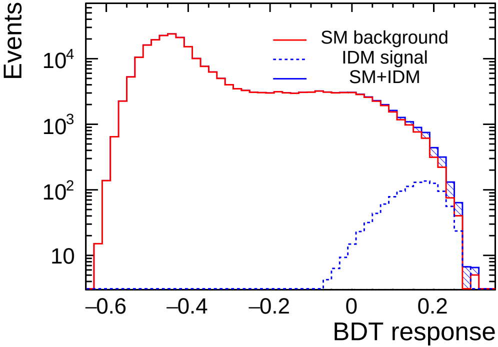
# Charged scalar production @ 380 GeV

**Multivariate analysis**

BDT classifier with 8 input variables used for selection of signal events

Response distribution for $e\mu$ channel: BP1 scenario and SM background

unpolarised 1000 fb$^{-1}$ at $\sqrt{s} = 380$ GeV

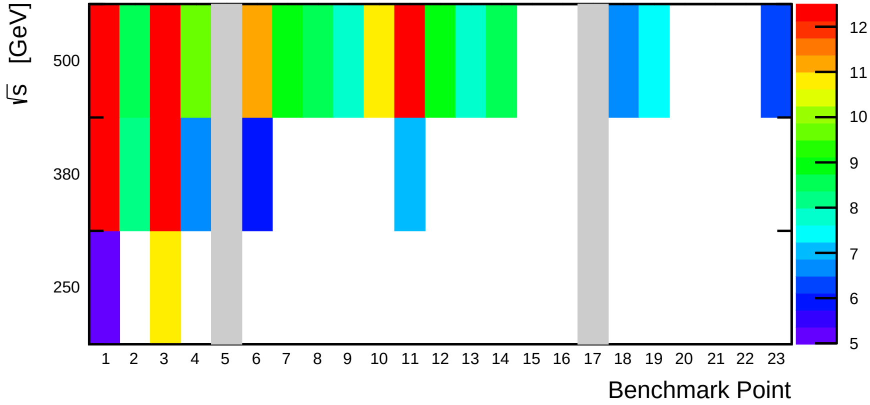


$\Rightarrow$ signal significance of about 17 $\sigma$ for BDT$> 0.12$

# Charged scalar production

**Significance of observation** scenario-independent approach

Summary of results for multivariate analysis of $e^{\pm}\mu^{\mp}$ final state



Fewer scenarios can be observed, clear need for 500 GeV

Significance reduced by about 10% by modified BDT training procedure

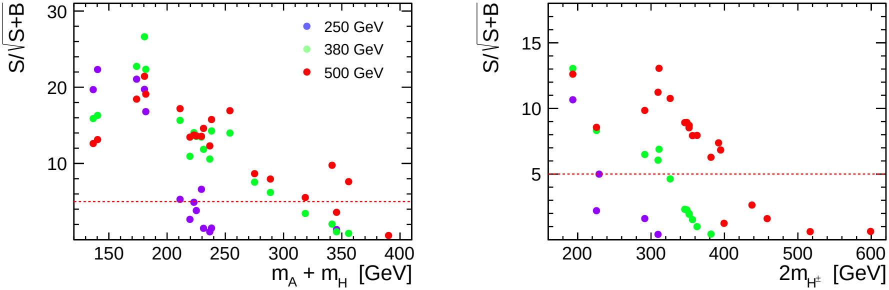
# Low energy summary

## Expected significance

Search for pair-production of IDM scalars, for different $\sqrt{s}$

$AH$ signature ($\mu^+\mu^-$)

$H^+H^-$ signature ($\mu^\pm e^\mp$)



Discovery reach mainly depends on the scalar masses!

- $m_A + m_H < 220, 300, 330$ GeV
- $m_{H^\pm} < 110, 160, 200$ GeV

for 1000 fb$^{-1}$ at $\sqrt{s} = 250, 380, 500$ GeV

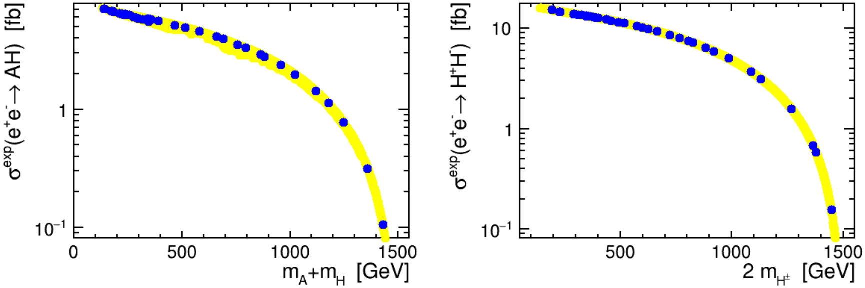
# Prospects at higher energies

Production of IDM scalars considered also for high energy stages of CLIC

JHEP 1907 (2019) 053, arXiv:1811.06952: results submitted to CLIC Physics Potential report

Leading-order cross sections for inert scalar production at 1.5 TeV:

$e^+e^- \to A\ H$ $e^+e^- \to H^+H^-$



Much smaller cross sections for light IDM scalar production ($\sim \frac{1}{s}$)!

Beam luminosity spectra not taken into account

# Prospects at higher energies

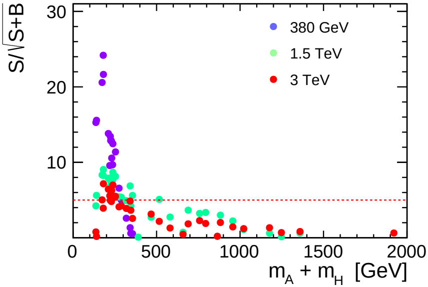
## IDM study for CLIC

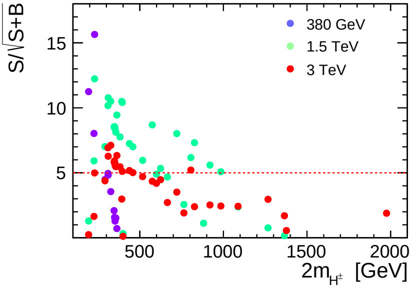
including luminosity spectra

Comparing CLIC running scenarios:

$1000\,\mathrm{fb}^{-1}$ at 380 GeV $2500\,\mathrm{fb}^{-1}$ at 1.5 TeV $5000\,\mathrm{fb}^{-1}$ at 3 TeV

$AH$ signature ($\mu^+\mu^-$) $H^+H^-$ signature ($\mu^\pm e^\mp$)





Only moderate increase in discovery reach for 1.5 TeV:

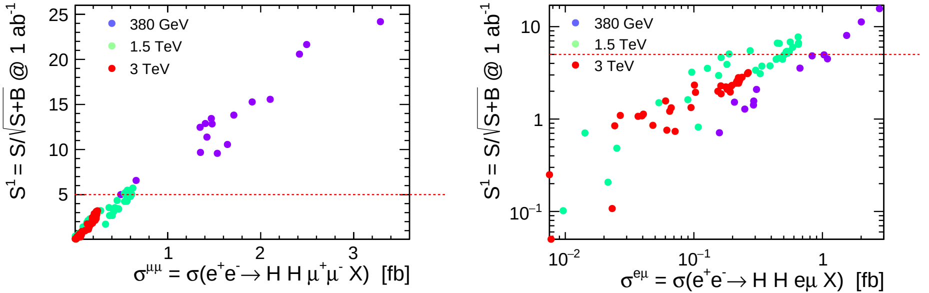
- neutral scalar production: $m_A + m_H < 450\,\mathrm{GeV}$ (290 GeV @ 380 GeV)
- charged scalar production: $m_{H^\pm} < 500\,\mathrm{GeV}$ (150 GeV @ 380 GeV)

## IDM study for CLIC

including luminosity spectra

Significance scaled to the same integrated luminosity of $1000\,\mathrm{fb}^{-1}$ as a function of the signal channel cross section

*AH* signature ($\mu^+\mu^-$) $H^+H^-$ signature ($\mu^\pm e^\mp$)



Expected significance mainly related to the signal channel cross section!

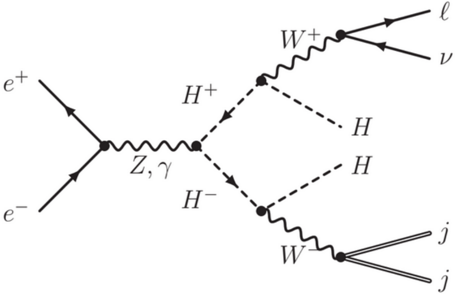
∼ 0.5 fb required for the discovery...

# Prospects at higher energies

## Semi-leptonic signature

Much higher significance can be expected for $H^+H^-$ production in the semi-leptonic final state (isolated lepton and two jets)

- energy and invariant mass recontruction for one of $W$ bosons
  $\Rightarrow$ better signal-background separation
- much larger branching fraction compared to $e\mu$: 2.25% $\Rightarrow$ 28.6%
  $\Rightarrow$ discovery reach should increase significantly



# Semi-leptonic signature

## Analysis framework

Event samples generated with Whizard 2.7.0

based on the dedicated IDM model implementation in SARAH,

parameter files prepared using SPheno 4.0.3 (as before)

fragmentation and hadronisation is simulated using PYTHIA 6.4

CLIC beam energy spectra taken into account

Consider running with -80% electron beam polarisation,

with $2\,\mathrm{ab}^{-1}$ collected at 1.5 TeV and $4\,\mathrm{ab}^{-1}$ collected at 3 TeV

Fast simulation of CLIC detector response with DELPHES

dedicated CLICdet model cards

beam related backgrounds taken into account

by additional jet energy-momentum smearing

DELPHES
fast simulation

# Semi-leptonic signature

**Signal signature** $e^+e^- \rightarrow H^+H^- \rightarrow H\,H\,W^+W^- \rightarrow H\,H\;qq'\;l\nu$

- two hadronic jets consistent with (real or virtual) W decay
- single lepton from leptonic W decay
- large missing (transverse) energy/momentum/mass
  (two invisible scalars H produced)

**Analysis flow**

Event reconstruction in DELPHES

- jets reconstructed with VLC algorithm (two exclusive jets)
- isolated leptons ($e^\pm$ and $\mu^\pm$) and photons identified
  require single leptons and no hard isolated photons (above 10 GeV)
- require no additional energy-flow in the detector (20 GeV cut)

# Semi-leptonic signature

## Backgrounds

Backgrounds which could result in the same final state simulated

Main contributions coming from $qql\nu$, $qqll$, $qql\nu l\nu$, $qql\nu\nu\nu$

Event pre-selection cuts applied on $M_{qq}$, $\Theta_{qq}$, $E_l$, $p^l_T$, $\Theta_l$

Preselection results for 3 TeV: two example BPs included

| channel | all exp. ev. | exp. ev. after preselec. | eff. |
|---|---|---|---|
| $H^+H^-$ (BP23) | 22716 | 8872 | 39.1% |
| $H^+H^-$ (HP15) | 11963 | 6063 | 50.7% |
| tot. backg. | 74877722 | 625494 | 0.84% |
| $qq\ell\ell$ | 12877040 | 78382 | 0.61% |
| $qq\ell\nu$ | 35326320 | 399470 | 1.13% |
| $qq\ell\nu\ell\nu$ | 317914 | 30742 | 9.67% |
| $qq\ell\nu\nu\nu$ | 360848 | 63581 | 17.62% |
| signal/backg. (BP23) | 0.0003 | 0.014 | |
| signal/backg. (HP15) | 0.00016 | 0.0097 | |

# Semi-leptonic signature

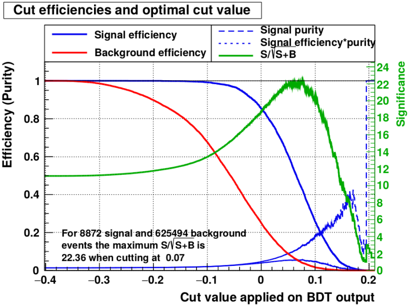
## Multivariate analysis

Final selection/significance estimate based on BDT with 11 input variables

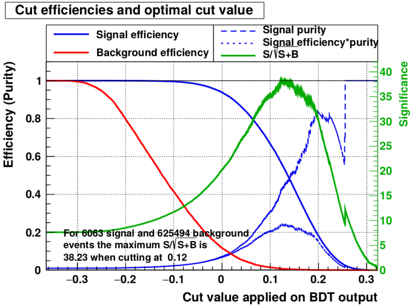
Two BDTs trained: for scenarios with virtual and real W production.

We do not optimise the selection for each particular scenario!

BP23 $\Delta m = 128$ GeV



HP15 $\Delta m = 45$ GeV



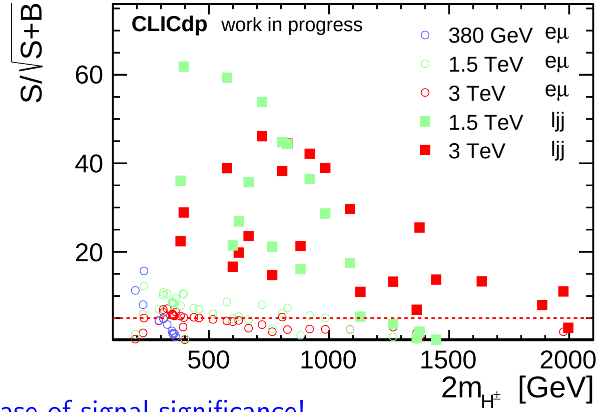
$$\Delta m = m_{H^\pm} - m_H$$

# Semi-leptonic signature

## Results

Summary of results obtained for the semi-leptonic channel

compared with leptonic channel results presented earlier



Huge increase of signal significance!

Discovery reach extended up to $m_{H^\pm} \sim 1$ TeV

# Conclusions

Inert Doublet Model is one of the simplest SM extensions providing natural candidate for dark matter

Light IDM scenarios (masses in 0(100 GeV) range) are still not excluded

Low mass IDM scenarios can be observed with high significance in the di-lepton channels already with 250 GeV $e^+e^-$ collider

Discovery reach increases for higher $\sqrt{s}$.
Significant improvement when looking at semi-leptonic final state!

Fast simulation results still to be confirmed with full simulation for selected BPs

Thank you!

# Additional references

## Inert Doublet Model

- A. Ilnicka, M. Krawczyk, T. Robens, *Inert Doublet Model in light of LHC Run I and astrophysical data*, Phys. Rev. D93:055026, 2016, 1508.01671.
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## Software

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# Backup slide

## IDM benchmark points

Constraints on inert scalar masses and couplings

- Theoretical
  - vacuum stability at tree level
  - perturbative unitarity
  - global minimum of the potential

- Experimental
  - (SM-like) Higgs boson mass and signal strenghts from LHC
  - Total widths of $W$ and $Z$ boson
  - Agreement with electroweak precision observables
  - Exclusion from SUSY searches at LEP and LHC experiments
    we use whatever is available, but not all recasts are done yet
  - Lower limit on $H^{\pm}$ width from long-lived charged particle searches
  - Direct bound by the dark matter nucleon scattering (LUX, XENON1T)
  - Planck upper limit on relic density

# Backup slide

## Low mass IDM benchmark points

| No. | $M_H$ | $M_A$ | $M_{H\pm}$ | $\lambda_2$ | $\lambda_{345}$ | $\Omega_c h^2$ |
|---|---|---|---|---|---|---|
| BP1 | 72.77 | 107.8 | 114.6 | 1.445 | -0.004407 | 0.1201 |
| BP2 | 65 | 71.53 | 112.8 | 0.7791 | 0.0004 | 0.07081 |
| BP3 | 67.07 | 73.22 | 96.73 | 0 | 0.00738 | 0.06162 |
| BP4 | 73.68 | 100.1 | 145.7 | 2.086 | -0.004407 | 0.08925 |
| BP6 | 72.14 | 109.5 | 154.8 | 0.01257 | -0.00234 | 0.1171 |
| BP7 | 76.55 | 134.6 | 174.4 | 1.948 | 0.0044 | 0.0314 |
| BP8 | 70.91 | 148.7 | 175.9 | 0.4398 | 0.0051 | 0.124 |
| BP9 | 56.78 | 166.2 | 178.2 | 0.5027 | 0.00338 | 0.08127 |
| BP10 | 76.69 | 154.6 | 163 | 3.921 | 0.0096 | 0.02814 |
| BP11 | 98.88 | 155 | 155.4 | 1.181 | -0.0628 | 0.002737 |
| BP12 | 58.31 | 171.1 | 173 | 0.5404 | 0.00762 | 0.00641 |
| BP13 | 99.65 | 138.5 | 181.3 | 2.463 | 0.0532 | 0.001255 |
| BP14 | 71.03 | 165.6 | 176 | 0.3393 | 0.00596 | 0.1184 |
| BP15 | 71.03 | 217.7 | 218.7 | 0.7665 | 0.00214 | 0.1222 |
| BP16 | 71.33 | 203.8 | 229.1 | 1.03 | -0.00122 | 0.1221 |
| BP18 | 147 | 194.6 | 197.4 | 0.387 | -0.018 | 0.001772 |
| BP19 | 165.8 | 190.1 | 196 | 2.768 | -0.004 | 0.002841 |
| BP20 | 191.8 | 198.4 | 199.7 | 1.508 | 0.008 | 0.008494 |
| BP21 | 57.48 | 288 | 299.5 | 0.9299 | 0.00192 | 0.1195 |
| BP22 | 71.42 | 247.2 | 258.4 | 1.043 | -0.00406 | 0.1243 |
| BP23 | 62.69 | 162.4 | 190.8 | 2.639 | 0.0056 | 0.06404 |

Note that BP5 and BP17 were excluded by the updated XENON1T limits, arXiv:1805.12562

# Backup slide

## High mass IDM benchmark points

| No. | $M_H$ | $M_A$ | $M_{H\pm}$ | $\lambda_2$ | $\lambda_{345}$ | $\Omega_c h^2$ |
|---|---|---|---|---|---|---|
| HP1 | 176 | 291.4 | 312 | 1.49 | -0.1035 | 0.0007216 |
| HP2 | 557 | 562.3 | 565.4 | 4.045 | -0.1385 | 0.07209 |
| HP3 | 560 | 616.3 | 633.5 | 3.38 | -0.0895 | 0.001129 |
| HP4 | 571 | 676.5 | 682.5 | 1.98 | -0.471 | 0.0005635 |
| HP5 | 671 | 688.1 | 688.4 | 1.377 | -0.1455 | 0.02447 |
| HP6 | 713 | 716.4 | 723 | 2.88 | 0.2885 | 0.03515 |
| HP7 | 807 | 813.4 | 818 | 3.667 | 0.299 | 0.03239 |
| HP8 | 933 | 940 | 943.8 | 2.974 | -0.2435 | 0.09639 |
| HP9 | 935 | 986.2 | 988 | 2.484 | -0.5795 | 0.002796 |
| HP10 | 990 | 992.4 | 998.1 | 3.334 | -0.051 | 0.1248 |
| HP11 | 250.5 | 265.5 | 287.2 | 3.908 | -0.1501 | 0.00535 |
| HP12 | 286.1 | 294.6 | 332.5 | 3.292 | 0.1121 | 0.00277 |
| HP13 | 336 | 353.3 | 360.6 | 2.488 | -0.1064 | 0.00937 |
| HP14 | 326.6 | 331.9 | 381.8 | 0.02513 | -0.06267 | 0.00356 |
| HP15 | 357.6 | 400 | 402.6 | 2.061 | -0.2375 | 0.00346 |
| HP16 | 387.8 | 406.1 | 413.5 | 0.8168 | -0.2083 | 0.0116 |
| HP17 | 430.9 | 433.2 | 440.6 | 3.003 | 0.08299 | 0.0327 |
| HP18 | 428.2 | 454 | 459.7 | 3.87 | -0.2812 | 0.00858 |
| HP19 | 467.9 | 488.6 | 492.3 | 4.122 | -0.252 | 0.0139 |
| HP20 | 505.2 | 516.6 | 543.8 | 2.538 | -0.354 | 0.00887 |

# Backup slide

## Signal processes for $\mu^+\mu^-$ final state

$$\begin{aligned} e^+e^- &\to \mu^+\mu^- \, HH, \\ &\to \mu^+\mu^-\nu_\mu\bar{\nu}_\mu \, HH, \\ &\to \tau^+\mu^-\nu_\tau\bar{\nu}_\mu \, HH, \;\; \mu^+\tau^-\nu_\mu\bar{\nu}_\tau \, HH, \\ &\to \tau^+\tau^- \, HH, \;\; \tau^+\tau^-\nu_\tau\bar{\nu}_\tau \, HH. \\ & \quad \text{with} \tau^\pm \to \mu^\pm\nu\nu \end{aligned}$$

## Signal processes for $e^\pm\mu^\mp$ final state

$$\begin{aligned} e^+e^- &\to \mu^+\nu_\mu \, e^-\bar{\nu}_e \, HH, \;\; e^+\nu_e \, \mu^-\bar{\nu}_\mu \, HH, \\ &\to \mu^+\nu_\mu \, \tau^-\bar{\nu}_\tau \, HH, \;\; \tau^+\nu_\tau \, \mu^-\bar{\nu}_\mu \, HH, \\ &\to e^+\nu_e \, \tau^-\bar{\nu}_\tau \, HH, \;\; \tau^+\nu_\tau \, e^-\bar{\nu}_e \, HH, \\ &\to \tau^+ \, \tau^- \, HH, \;\; \tau^+\nu_\tau \, \tau^-\bar{\nu}_\tau \, HH, \end{aligned}$$