

# Inert Doublet Model signatures at Future $e^+e^-$ Colliders

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- 1 Inert Doublet Model
- 2 Benchmark points
- 3 Leptonic analysis (generator level, general)
- 4 Semi-leptonic analysis (fast simulation for CLIC)
- 5 Conclusions

For more details:

- on benchmark points: JHEP 1812 (2018) 081, arXiv:1809.07712
- results for CLIC report: JHEP 1907 (2019) 053, arXiv:1811.06952 (leptonic signature)

One of the simplest extensions of the Standard Model (SM).  
The scalar sector consists of two doublets:

- $\Phi_S$  is the **SM-like Higgs** doublet,
- $\Phi_D$  (**inert doublet**) has four additional scalars  $H$ ,  $A$ ,  $H^\pm$ .

$$\Phi_S = \begin{pmatrix} G^\pm \\ \frac{v+h+iG^0}{\sqrt{2}} \end{pmatrix} \quad \Phi_D = \begin{pmatrix} H^\pm \\ \frac{H+iA}{\sqrt{2}} \end{pmatrix}$$

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We assume a discrete  $Z_2$  **symmetry** under which

- SM Higgs doublet  $\Phi_S$  is **even**:  $\Phi_S \rightarrow \Phi_S$  (also other SM  $\rightarrow$  SM)
- inert doublet  $\Phi_D$  is **odd**:  $\Phi_D \rightarrow -\Phi_D$ .

$\Rightarrow$  Yukawa-type interactions only for Higgs doublet ( $\Phi_S$ ).

The **inert doublet** ( $\Phi_D$ ) **does not interact with the SM fermions!**

$\Rightarrow$  The lightest inert particle is stable: a natural **candidate for dark matter!**

We assume the neutral scalar  $H$  is the dark matter particle.

$$m_H < m_A, m_{H^\pm}$$

After EWSB, the model contains a priori seven free parameters.

Two parameters can be fixed from the Standard Model ( $v$ ,  $m_h$ ).

We are left with **five free parameters**, which we take as:

⇒ three inert scalar masses:  $m_H$ ,  $m_A$ ,  $m_{H^\pm}$

⇒ two couplings, eg.  $\lambda_2$  and  $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$

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Inert scalars couplings to  $\gamma$ ,  $W^\pm$  and  $Z$  determined by SM parameters

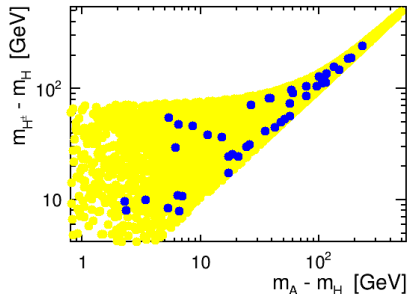
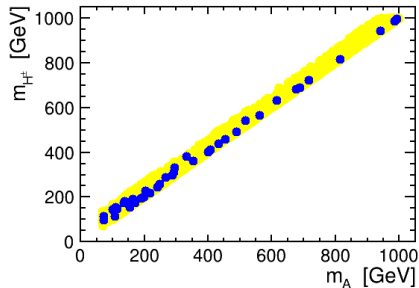
⇒ **well established predictions** for production and decay rates!

We scanned the IDM parameter space looking for scenarios consistent with current **theoretical** and **experimental constraints**, for masses up to 1 TeV.

For details and previous IDM parameter scan results see:

- Agnieszka Ilnicka, Maria Krawczyk, and Tania Robens, *Inert Doublet Model in light of LHC Run I and astrophysical data*, Phys. Rev. D93(5):055026, 2016, arXiv:1508.01671.
- Agnieszka Ilnicka, Tania Robens, and Tim Stefaniak, *Constraining Extended Scalar Sectors at the LHC and beyond*, Mod. Phys. Lett. A33(10n11):1830007, 2018, arXiv:1803.03594.

Out of about 15'000 points consistent with all considered constraints, we chose **41 benchmark points** (including 20 “high mass”) for detailed studies:



The selection was arbitrary, but we tried to

- cover wide range of scalar masses and the mass splittings
- get significant contribution to the relic density

For details see: [JHEP 1812 \(2018\) 081](#), [arXiv:1809.07712](#)

For list of benchmark point parameters, see backup slides

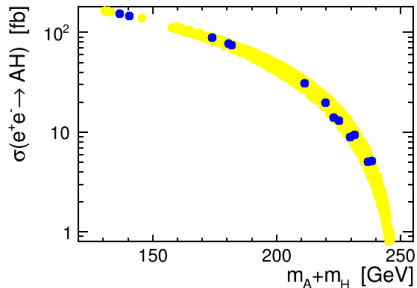
Production of IDM scalars at  $e^+e^-$  colliders dominated by two processes:

$$e^+e^- \rightarrow A H$$

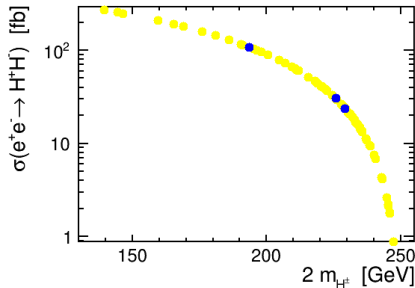
$$e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 250 GeV:

13 benchmarks



3 benchmarks



Beam luminosity spectra not taken into account



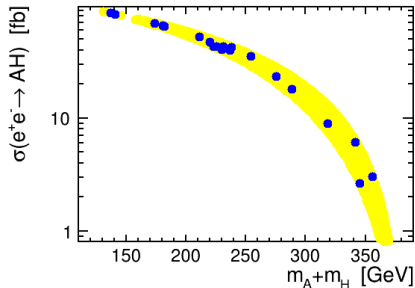
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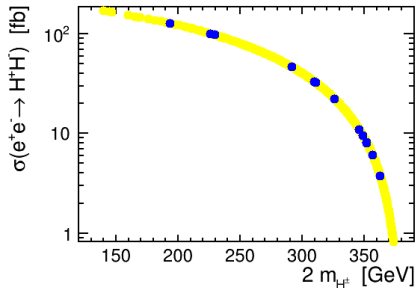
$$e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 380 GeV:

20 benchmarks



15 benchmarks



Beam luminosity spectra not taken into account

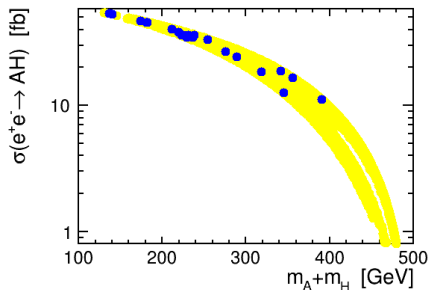
Production of IDM scalars at  $e^+e^-$  colliders dominated by two processes:

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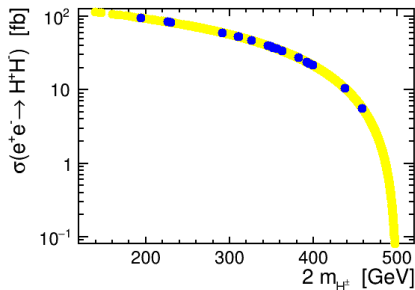
$$e^+e^- \rightarrow H^+H^-$$

Leading-order cross sections for inert scalar production processes at 500 GeV:

21 benchmarks



21 benchmarks



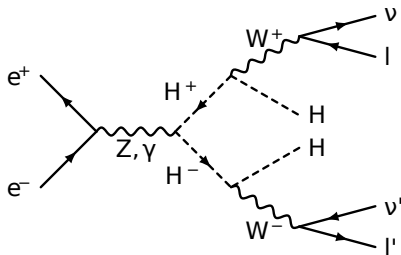
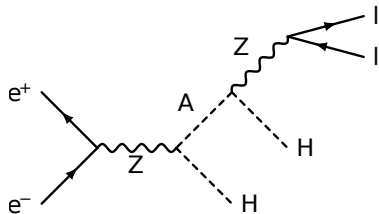
Beam luminosity spectra not taken into account

Same flavour lepton pair production can be considered a signature of the  $AH$  production process followed by the  $A$  decay:

$$e^+e^- \rightarrow HA \rightarrow HHZ^{(*)} \rightarrow HH\mu^+\mu^-$$

while the production of the different flavour lepton pair is the expected signature for  $H^+H^-$  production:

$$e^+e^- \rightarrow H^+H^- \rightarrow HHW^{(*)}W^{(*)} \rightarrow HH\ell^+\ell'^-\nu\bar{\nu}'$$



We consider two possible final state signatures:

- muon pair production,  $\mu^+\mu^-$ , for  $AH$  production
- electron-muon pair production,  $\mu^+e^-$  or  $e^+\mu^-$ , for  $H^+H^-$  production

Both channels include contributions from  $AH$  and  $H^+H^-$  production!  
In particular due to leptonic tau decays.

Signal and background samples were generated with WHizard 2.2.8  
based on the dedicated IDM model implementation in SARAH,  
parameter files for benchmark scenarios were prepared using SPheno 4.0.3

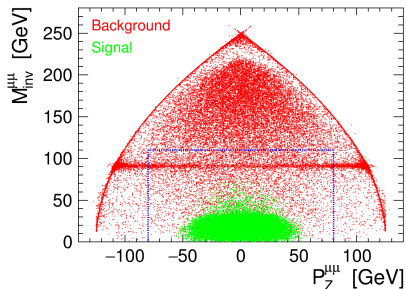
Generator level cuts reflecting detector acceptance:

- require lepton energy  $E_l > 5$  GeV and lepton angle  $\Theta_l > 100$  mrad
- no ISR photon with  $E_\gamma > 10$  GeV and  $\Theta_\gamma > 100$  mrad

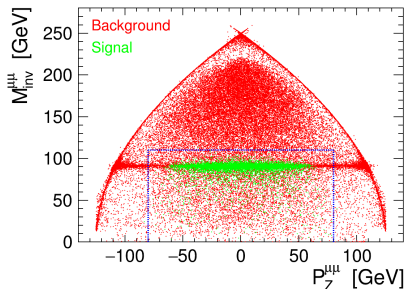
No detector resolution/efficiency taken into account  
(but only electrons and muons in the final state)

Muon pair invariant mass,  $M_{inv}^{\mu\mu}$ , as a function of the lepton pair long. momentum,  $P_Z^{\mu\mu}$ , for **IDM signal** and **SM background**, at 250 GeV

BP1



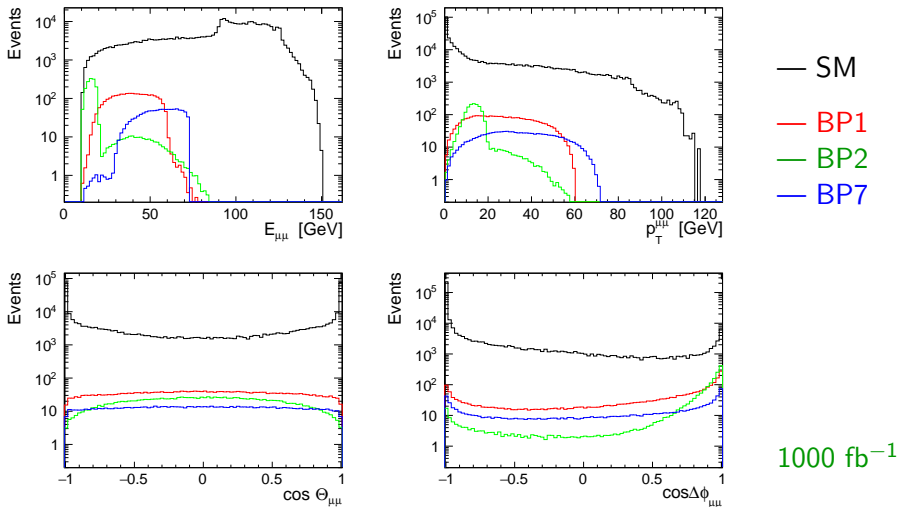
BP9



Background dominated by muon pair production ( $e^+e^- \rightarrow \mu^+\mu^-$ ) at nominal energy and radiative events ( $e^+e^- \rightarrow \mu^+\mu^-\gamma$ )

$\Rightarrow$  apply pre-selection cuts:  $M_{\mu\mu} < 0.33\sqrt{s}$  and  $|P_Z^{\mu\mu}| < 0.44\sqrt{s}$

## Distributions of the kinematic variables describing the leptonic final state

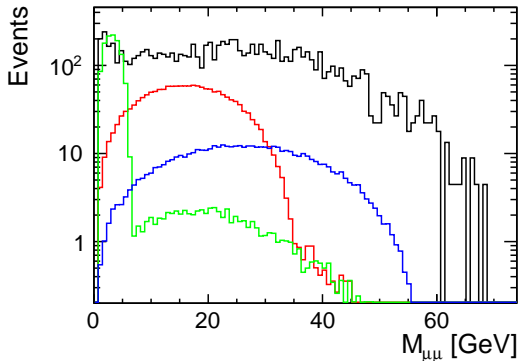


## Cut based approach

Lepton pair invariant mass distribution after selection cuts

1000 fb<sup>-1</sup>

- pair energy  
 $E_{\mu\mu} < 75$  GeV
- transverse momentum  
 $p_T^{\mu\mu} > 10$  GeV
- production angle  
 $45^\circ < \Theta_{\mu\mu} < 135^\circ$
- azimuthal distance  
 $|\Delta\varphi_{\mu\mu}| < \frac{\pi}{2}$



IDM signal would result in the visible excess in  $M_{\mu\mu}$  distribution

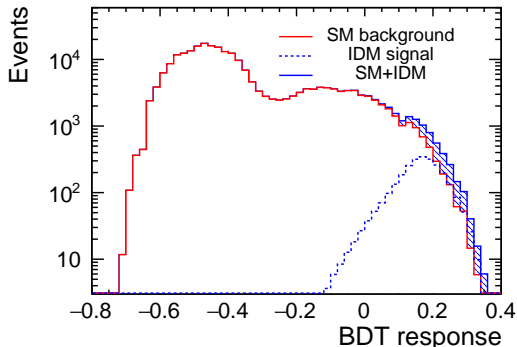
15.9 $\sigma$ , 11.6 $\sigma$  and 5.4 $\sigma$ , for BP1, BP2 and BP7

(without any cut on  $M_{\mu\mu}$ )

## Multivariate analysis

BDT classifier with 8 input variables used for selection of signal events

Response distribution for  $\mu\mu$  channel: BP1 scenario and SM background unpolarised  $1000 \text{ fb}^{-1}$  at  $\sqrt{s} = 250 \text{ GeV}$



⇒ signal significance of about  $24 \sigma$  for  $\text{BDT} > 0.11$



**Multivariate analysis**    common approach

We train BDTs to separate the **considered signal** from the background

**But we will not know in advance what to look for!**

We will not know details of the model (scalar masses)

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We will not know details of the model (scalar masses)

## Scenario-independent approach

Divide the considered BP scenarios in **two groups**:

- scenarios with **real  $Z$**  (or **real  $W$** ) production
- scenarios with **virtual  $Z$**  (or **virtual  $W$** ) in intermediate state

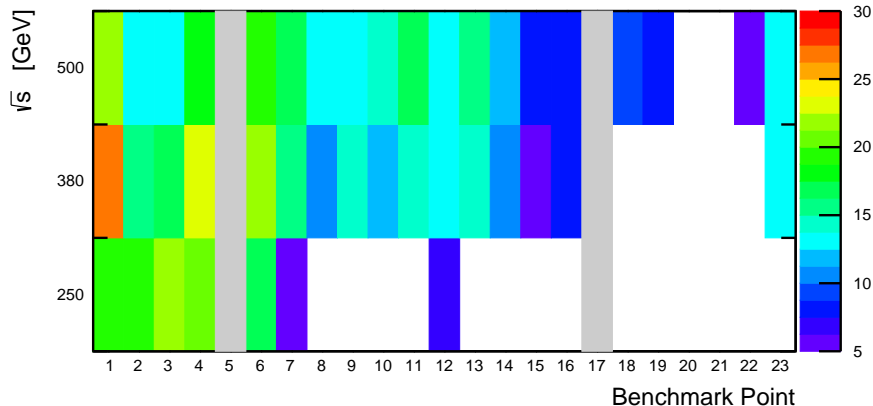
For each group: search for given BP (**test sample**)

while using **all other scenarios** to train BDT (**training samples**)

Corresponds to the assumption that two independent BDTs will be used in the analysis for the two cases...

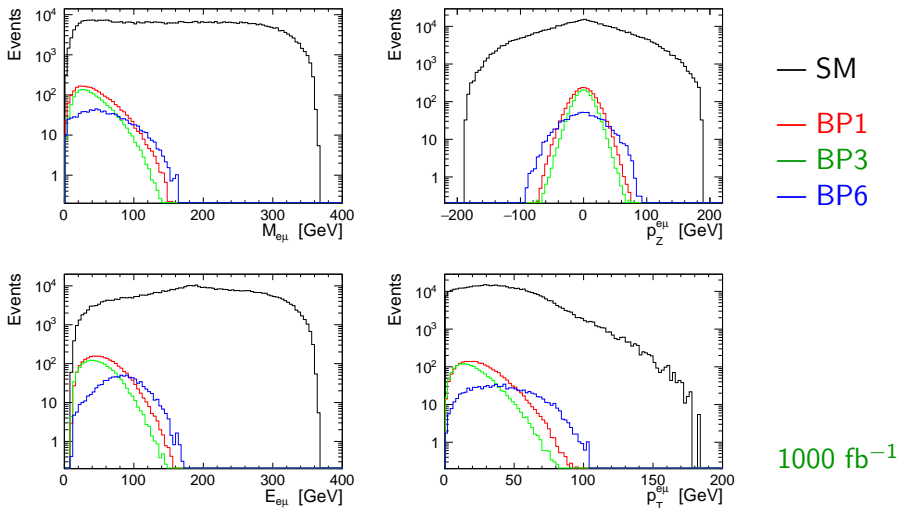
## Significance of observation

Summary of results for the considered benchmark scenarios



High significance of observation for scenarios accessible at given energy  
 Expected significance mainly related to the  $AH$  production cross section

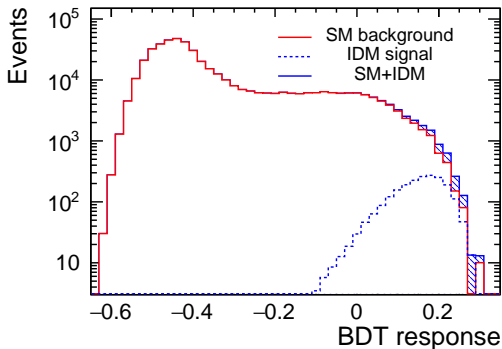
## Distributions of the kinematic variables describing the leptonic final state



## Multivariate analysis

BDT classifier with 8 input variables used for selection of signal events

Response distribution for  $e\mu$  channel: **BP1 scenario** and **SM background**  
unpolarised  $1000 \text{ fb}^{-1}$  at  $\sqrt{s} = 380 \text{ GeV}$

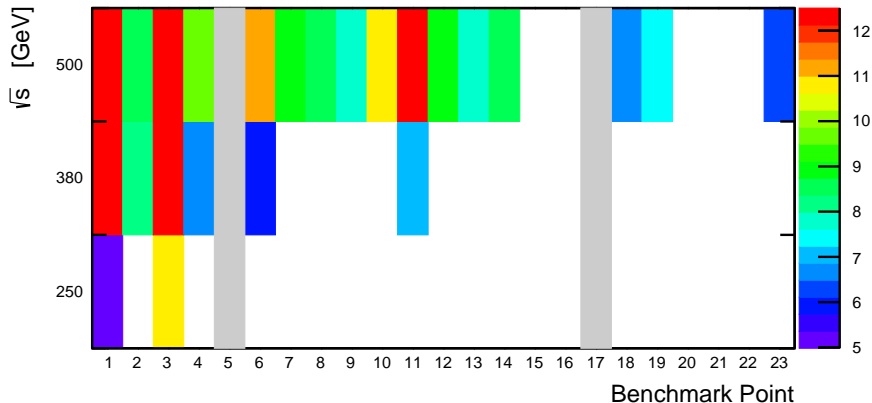


⇒ signal significance of about  $17 \sigma$  for  $\text{BDT} > 0.12$

## Significance of observation

scenario-independent approach

Summary of results for multivariate analysis of  $e^\pm\mu^\mp$  final state



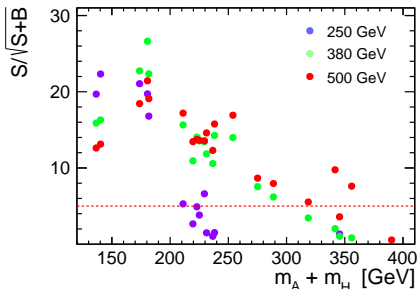
Fewer scenarios can be observed, clear need for 500 GeV

Significance reduced by about 10% by scenario-independent BDT training

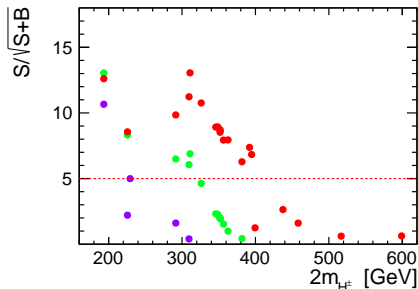
## Expected significance

Search for pair-production of IDM scalars, for different  $\sqrt{s}$

$AH$  signature ( $\mu^+\mu^-$ )



$H^+H^-$  signature ( $\mu^\pm e^\mp$ )



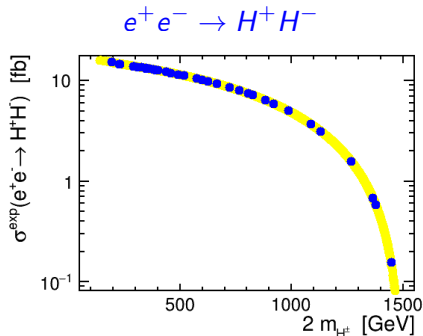
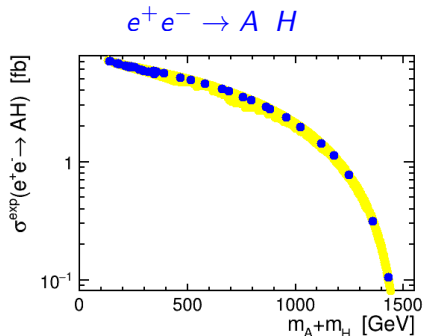
Discovery reach mainly depends on the scalar masses!

- $m_A + m_H < 220, 300, 330$  GeV
- $m_{H^\pm} < 110, 160, 200$  GeV

for  $1000 \text{ fb}^{-1}$  at  $\sqrt{s} = 250, 380, 500$  GeV

Production of IDM scalars considered also for high energy stages of CLIC  
 JHEP 1907 (2019) 053, arXiv:1811.06952: results submitted to CLIC Physics Potential report

Leading-order cross sections for inert scalar production at 1.5 TeV:



Much smaller cross sections for light IDM scalar production ( $\sim \frac{1}{s}$ )!

Beam luminosity spectra not taken into account

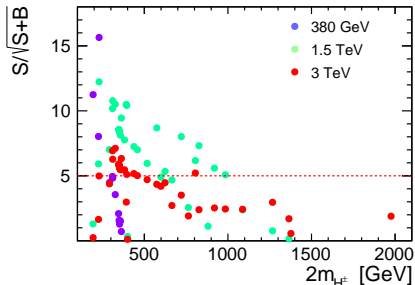
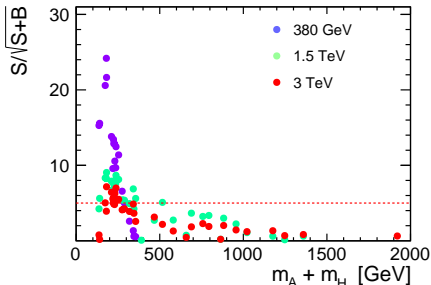


## IDM study for CLIC

including luminosity spectra

Comparing CLIC running scenarios:

$1000 \text{ fb}^{-1}$  at 380 GeV     $2500 \text{ fb}^{-1}$  at 1.5 TeV     $5000 \text{ fb}^{-1}$  at 3 TeV  
 AH signature ( $\mu^+\mu^-$ )                       $H^+H^-$  signature ( $\mu^\pm e^\mp$ )



Only moderate increase in discovery reach for 1.5 TeV:

- neutral scalar production:  $m_A + m_H < 450 \text{ GeV}$  (290 GeV @ 380 GeV)
- charged scalar production:  $m_{H^\pm} < 500 \text{ GeV}$  (150 GeV @ 380 GeV)

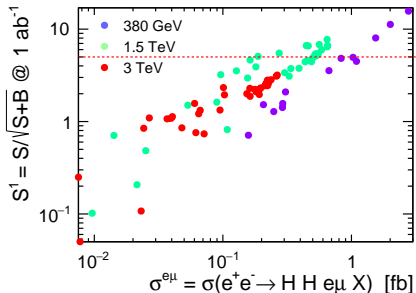
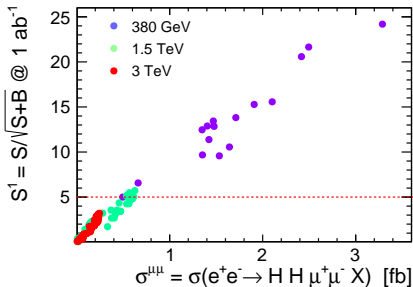
## IDM study for CLIC

including luminosity spectra

Significance scaled to the same integrated luminosity of  $1000 \text{ fb}^{-1}$  as a function of the signal channel cross section

$AH$  signature ( $\mu^+ \mu^-$ )

$H^+ H^-$  signature ( $\mu^\pm e^\mp$ )

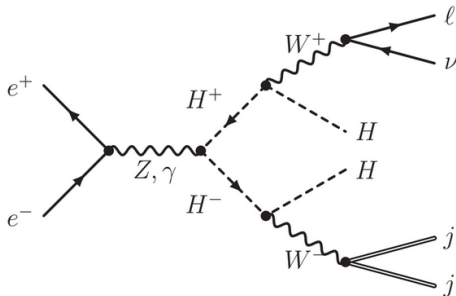


Expected significance mainly related to the signal channel cross section!  
 $\sim 0.5 \text{ fb}$  required for the discovery...

## Semi-leptonic signature

Much higher significance can be expected for  $H^+H^-$  production in the semi-leptonic final state (isolated lepton and two jets)

- energy and invariant mass reconstruction for one of  $W$  bosons  
⇒ better signal-background separation
- much larger branching fraction compared to  $e\mu$ : 2.25% ⇒ 28.6%  
⇒ discovery reach should increase significantly



## Analysis framework

Event samples generated with Whizard 2.7.0

based on the dedicated IDM model implementation in SARAH,  
parameter files prepared using SPheno 4.0.3 (as before)

fragmentation and hadronisation is simulated using PYTHIA 6.4

CLIC beam energy spectra taken into account

Consider running with -80% electron beam polarisation,  
with  $2 \text{ ab}^{-1}$  collected at 1.5 TeV and  $4 \text{ ab}^{-1}$  collected at 3 TeV

Fast simulation of CLIC detector response with DELPHES

dedicated CLICdet model cards

beam related backgrounds taken into account  
by additional jet energy-momentum smearing



**DELPHES**  
fast simulation

**Signal signature**  $e^+e^- \rightarrow H^+H^- \rightarrow HH W^+W^- \rightarrow HH qq' l\nu$

- two hadronic jets consistent with (real or virtual) W decay
- single lepton from leptonic W decay
- large missing (transverse) energy/momentum/mass  
(two invisible scalars H produced)

## Analysis flow

Event reconstruction in DELPHES

- jets reconstructed with VLC algorithm (two exclusive jets)
- isolated leptons ( $e^\pm$  and  $\mu^\pm$ ) and photons identified  
require single leptons and no hard isolated photons (above 10 GeV)
- require no additional energy-flow in the detector (20 GeV cut)

## Backgrounds

Backgrounds which could result in the same final state simulated

Main contributions coming from  $qq\nu$ ,  $qql$ ,  $qql\nu\nu$ ,  $qql\nu\nu\nu$

Event pre-selection cuts applied on  $M_{qq}$ ,  $\Theta_{qq}$ ,  $E_l$ ,  $p_T^l$ ,  $\Theta_l$

Preselection results for 3 TeV:

two example BPs included

channel	all exp. ev.	exp. ev. after preselec.	eff.
$H^+H^-$ (BP23)	22716	8872	39.1%
$H^+H^-$ (HP15)	11963	6063	50.7%
tot. backg.	74877722	625494	0.84%
$qqll$	12877040	78382	0.61%
$qql\nu$	35326320	399470	1.13%
$qql\nu\nu$	317914	30742	9.67%
$qql\nu\nu\nu$	360848	63581	17.62%
signal/backg. (BP23)	0.0003	0.014	
signal/backg. (HP15)	0.00016	0.0097	

## Multivariate analysis

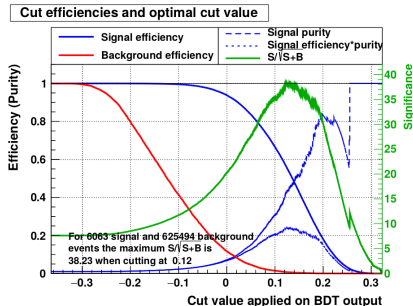
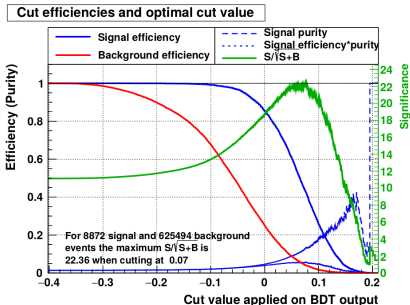
Final selection/significance estimate based on BDT with 11 input variables

Two BDTs trained: for scenarios with virtual and real W production.

We do not optimise the selection for each particular scenario!

BP23  $\Delta m = 128 \text{ GeV}$

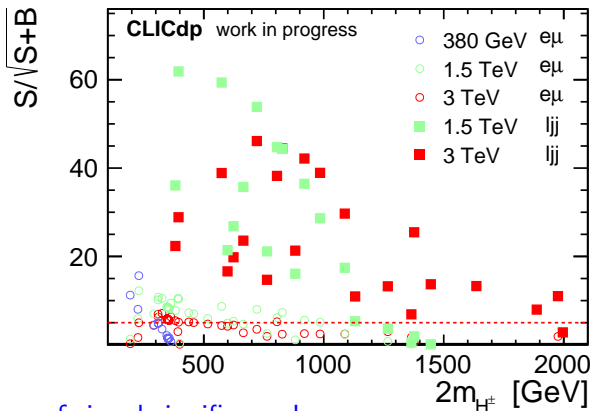
HP15  $\Delta m = 45 \text{ GeV}$



$$\Delta m = m_{H^\pm} - m_H$$

## Results

Summary of results obtained for the semi-leptonic channel compared with leptonic channel results presented earlier



Huge increase of signal significance!

Discovery reach extended up to  $m_{H^\pm} \sim 1$  TeV



**Inert Doublet Model** is one of the simplest SM extensions providing natural candidate for **dark matter**

**Light IDM scenarios** (masses in 0(100 GeV) range) are still not excluded

**Low mass IDM** scenarios can be observed with **high significance** in the di-lepton channels **already with 250 GeV**  $e^+e^-$  collider

**Discovery reach** increases for higher  $\sqrt{s}$ .

**Significant improvement** when looking at **semi-leptonic final state!**

Full simulation study starting, to confirm fast simulation results for **selected BPs**

Thank you!

## Inert Doublet Model

- A. Ilnicka, M. Krawczyk, T. Robens, *Inert Doublet Model in light of LHC Run I and astrophysical data*, Phys. Rev. D93:055026, 2016, 1508.01671.
- Nilendra G. Deshpande and Ernest Ma, *Pattern of Symmetry Breaking with Two Higgs Doublets*, Phys. Rev. D18:2574, 1978.
- Laura Lopez Honorez and Carlos E. Yaguna, *The inert doublet model of dark matter revisited*, JHEP 09:046, 2010, 1003.3125.
- Ethan Dolle, Xinyu Miao, Shufang Su, and Brooks Thomas, *Dilepton Signals in the Inert Doublet Model*, Phys. Rev. D81:035003, 2010, 0909.3094.
- A. Goudelis, B. Herrmann, and O. Stål, *Dark matter in the Inert Doublet Model after the discovery of a Higgs-like boson at the LHC*, JHEP 09:106, 2013, 1303.3010.

## Software

- Wolfgang Kilian, Thorsten Ohl, and Jurgen Reuter, *WHIZARD: Simulating Multi-Particle Processes at LHC and ILC*, Eur. Phys. J. C71:1742, 2011, [arXiv:0708.4233](https://arxiv.org/abs/0708.4233).
- Florian Staub, *Exploring new models in all detail with SARAH*, Adv. High Energy Phys. 2015:840780, 2015, [arXiv:1503.04200](https://arxiv.org/abs/1503.04200).
- Werner Porod, *SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at  $e^+e^-$  colliders*, Comput. Phys. Commun. 153:275–315, 2003, hep-ph/0301101.
- Andreas Hoecker, Peter Speckmayer, Joerg Stelzer, Jan Therhaag, Eckhard von Toerne, and Helge Voss, *TMVA: Toolkit for Multivariate Data Analysis*, PoS ACAT:040, 2007, physics/0703039.

## IDM benchmark points

### Constraints on inert scalar masses and couplings

- Theoretical

- vacuum stability at tree level
- perturbative unitarity
- global minimum of the potential

- Experimental

- (SM-like) Higgs boson mass and signal strengths from LHC
- Total widths of  $W$  and  $Z$  boson
- Agreement with electroweak precision observables
- Exclusion from SUSY searches at LEP and LHC experiments  
we use whatever is available, but not all recasts are done yet
- Lower limit on  $H^\pm$  width from long-lived charged particle searches
- Direct bound by the dark matter nucleon scattering (LUX, XENON1T)
- Planck upper limit on relic density

## Low mass IDM benchmark points

No.	$M_H$	$M_A$	$M_{H^\pm}$	$\lambda_2$	$\lambda_{345}$	$\Omega_c h^2$
BP1	72.77	107.8	114.6	1.445	-0.004407	0.1201
BP2	65	71.53	112.8	0.7791	0.0004	0.07081
BP3	67.07	73.22	96.73	0	0.00738	0.06162
BP4	73.68	100.1	145.7	2.086	-0.004407	0.08925
BP6	72.14	109.5	154.8	0.01257	-0.00234	0.1171
BP7	76.55	134.6	174.4	1.948	0.0044	0.0314
BP8	70.91	148.7	175.9	0.4398	0.0051	0.124
BP9	56.78	166.2	178.2	0.5027	0.00338	0.08127
BP10	76.69	154.6	163	3.921	0.0096	0.02814
BP11	98.88	155	155.4	1.181	-0.0628	0.002737
BP12	58.31	171.1	173	0.5404	0.00762	0.00641
BP13	99.65	138.5	181.3	2.463	0.0532	0.001255
BP14	71.03	165.6	176	0.3393	0.00596	0.1184
BP15	71.03	217.7	218.7	0.7665	0.00214	0.1222
BP16	71.33	203.8	229.1	1.03	-0.00122	0.1221
BP18	147	194.6	197.4	0.387	-0.018	0.001772
BP19	165.8	190.1	196	2.768	-0.004	0.002841
BP20	191.8	198.4	199.7	1.508	0.008	0.008494
BP21	57.48	288	299.5	0.9299	0.00192	0.1195
BP22	71.42	247.2	258.4	1.043	-0.00406	0.1243
BP23	62.69	162.4	190.8	2.639	0.0056	0.06404

Note that BP5 and BP17 were excluded by the updated XENON1T limits, [arXiv:1805.12562](https://arxiv.org/abs/1805.12562)

## High mass IDM benchmark points

No.	$M_H$	$M_A$	$M_{H^\pm}$	$\lambda_2$	$\lambda_{345}$	$\Omega_c h^2$
HP1	176	291.4	312	1.49	-0.1035	0.0007216
HP2	557	562.3	565.4	4.045	-0.1385	0.07209
HP3	560	616.3	633.5	3.38	-0.0895	0.001129
HP4	571	676.5	682.5	1.98	-0.471	0.0005635
HP5	671	688.1	688.4	1.377	-0.1455	0.02447
HP6	713	716.4	723	2.88	0.2885	0.03515
HP7	807	813.4	818	3.667	0.299	0.03239
HP8	933	940	943.8	2.974	-0.2435	0.09639
HP9	935	986.2	988	2.484	-0.5795	0.002796
HP10	990	992.4	998.1	3.334	-0.051	0.1248
HP11	250.5	265.5	287.2	3.908	-0.1501	0.00535
HP12	286.1	294.6	332.5	3.292	0.1121	0.00277
HP13	336	353.3	360.6	2.488	-0.1064	0.00937
HP14	326.6	331.9	381.8	0.02513	-0.06267	0.00356
HP15	357.6	400	402.6	2.061	-0.2375	0.00346
HP16	387.8	406.1	413.5	0.8168	-0.2083	0.0116
HP17	430.9	433.2	440.6	3.003	0.08299	0.0327
HP18	428.2	454	459.7	3.87	-0.2812	0.00858
HP19	467.9	488.6	492.3	4.122	-0.252	0.0139
HP20	505.2	516.6	543.8	2.538	-0.354	0.00887

## Signal processes for $\mu^+\mu^-$ final state

$$\begin{aligned}
 e^+e^- &\rightarrow \mu^+\mu^- HH, \\
 &\rightarrow \mu^+\mu^-\nu_\mu\bar{\nu}_\mu HH, \\
 &\rightarrow \tau^+\mu^-\nu_\tau\bar{\nu}_\mu HH, \quad \mu^+\tau^-\nu_\mu\bar{\nu}_\tau HH, \\
 &\rightarrow \tau^+\tau^- HH, \quad \tau^+\tau^-\nu_\tau\bar{\nu}_\tau HH. \\
 &\text{with } \tau^\pm \rightarrow \mu^\pm\nu\nu
 \end{aligned}$$

## Signal processes for $e^\pm\mu^\mp$ final state

$$\begin{aligned}
 e^+e^- &\rightarrow \mu^+\nu_\mu e^-\bar{\nu}_e HH, \quad e^+\nu_e \mu^-\bar{\nu}_\mu HH, \\
 &\rightarrow \mu^+\nu_\mu \tau^-\bar{\nu}_\tau HH, \quad \tau^+\nu_\tau \mu^-\bar{\nu}_\mu HH, \\
 &\rightarrow e^+\nu_e \tau^-\bar{\nu}_\tau HH, \quad \tau^+\nu_\tau e^-\bar{\nu}_e HH, \\
 &\rightarrow \tau^+\tau^- HH, \quad \tau^+\nu_\tau \tau^-\bar{\nu}_\tau HH,
 \end{aligned}$$