Consider a p.d.f. $f(x;\theta)$ where x represents the outcome of the experiment and θ is the unknown parameter for which we want to construct a confidence interval. The variable x could (and often does) represent an estimator for θ. Using $f(x;\theta)$ we can find for a pre-specified probability $1-\alpha$ and for every value of θ a set of values $x_1(\theta,\alpha)$ and $x_2(\theta,\alpha)$ such that

$$P(x_1 < x < x_2;\theta) = 1-\alpha = \int_{x_1}^{x_2} f(x;\theta)\,dx\ . \tag{32.39}$$

This is illustrated in Fig. 32.3: a horizontal line segment $[x_1(\theta,\alpha), x_2(\theta,\alpha)]$ is drawn for representative values of θ. The union of such intervals for all values of θ, designated in the figure as $D(\alpha)$, is known as the *confidence belt*. Typically the curves $x_1(\theta,\alpha)$ and $x_2(\theta,\alpha)$ are monotonic functions of θ, which we assume for this discussion.

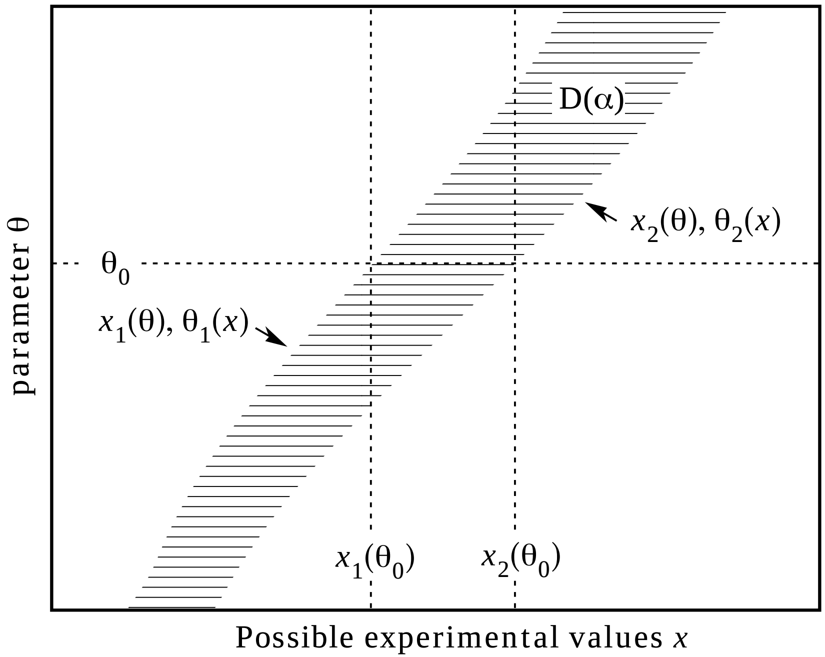


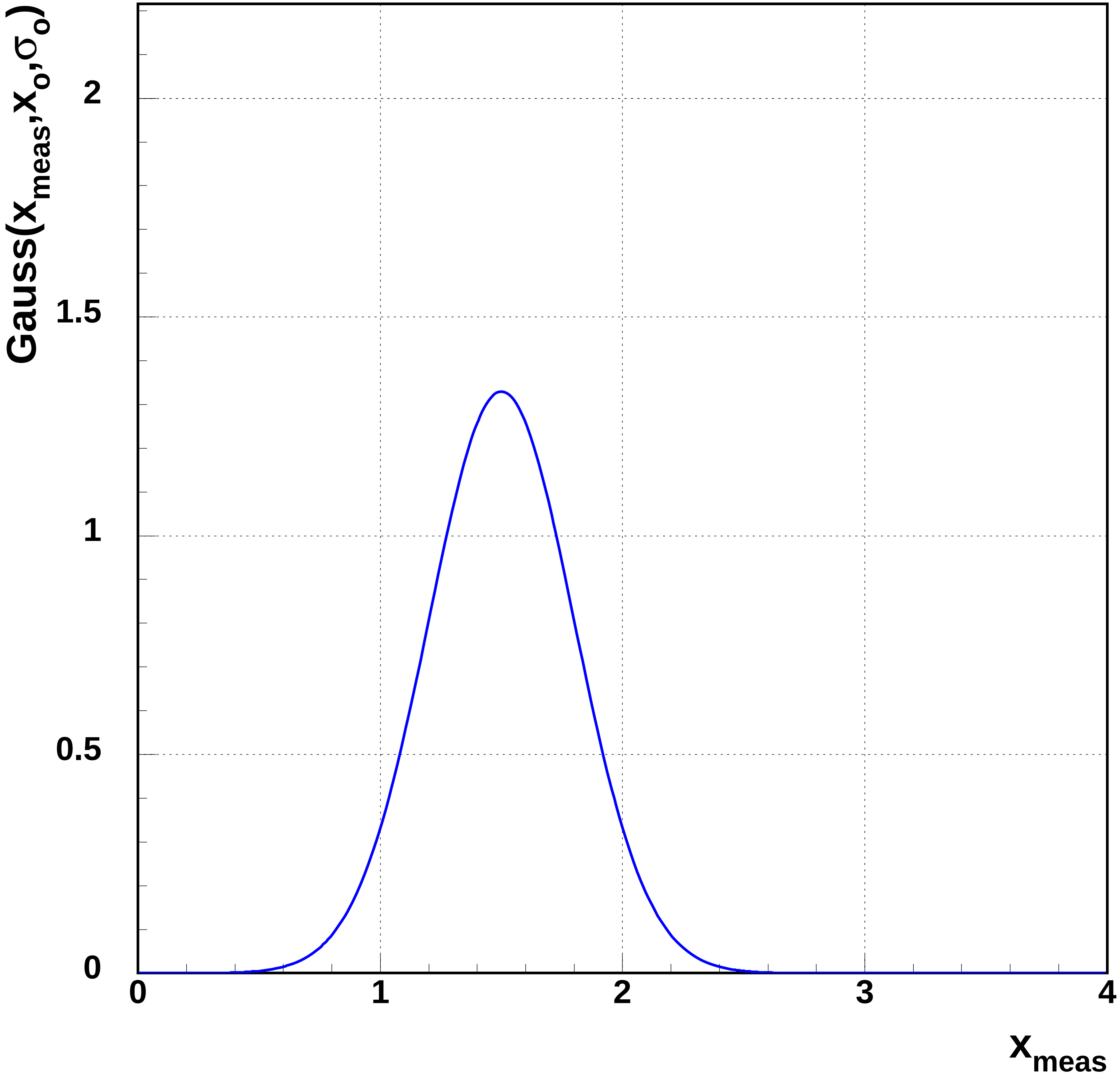
Figure 32.3: Construction of the confidence belt (see text).

32.3.1. *The Bayesian approach* :

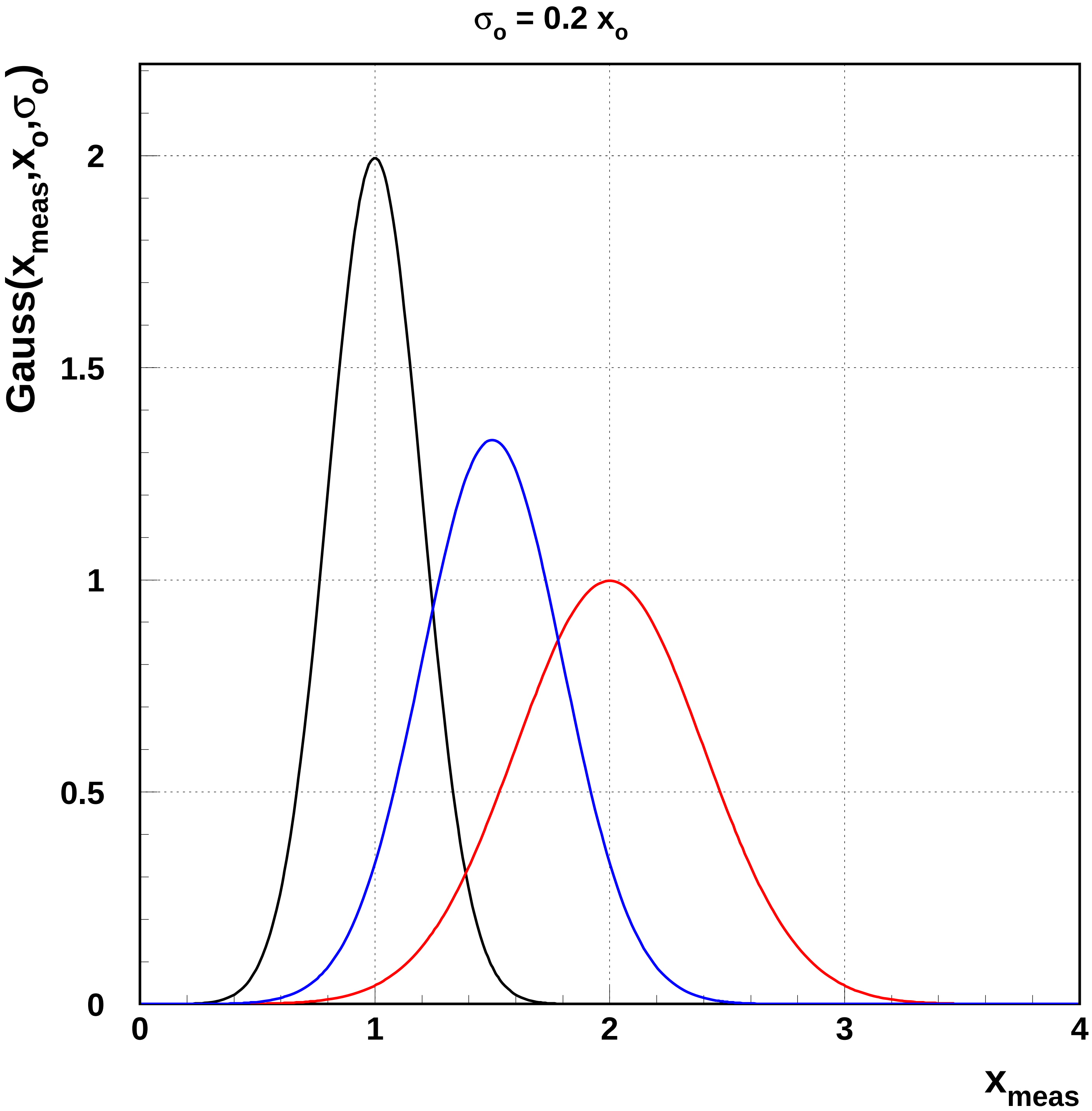
Suppose the outcome of the experiment is characterized by a vector of data $\boldsymbol{x}$, whose probability distribution depends on an unknown parameter (or parameters) $\boldsymbol{\theta}$ that we wish to determine. In Bayesian statistics, all knowledge about $\boldsymbol{\theta}$ is summarized by the posterior p.d.f. $p(\boldsymbol{\theta}|\boldsymbol{x})$, which gives the degree of belief for $\boldsymbol{\theta}$ to take on values in a certain region given the data $\boldsymbol{x}$. It is obtained by using Bayes' theorem,

$$p(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{L(\boldsymbol{x}|\boldsymbol{\theta})\pi(\boldsymbol{\theta})}{\int L(\boldsymbol{x}|\boldsymbol{\theta}')\pi(\boldsymbol{\theta}')\, d\boldsymbol{\theta}'} , \tag{32.30}$$

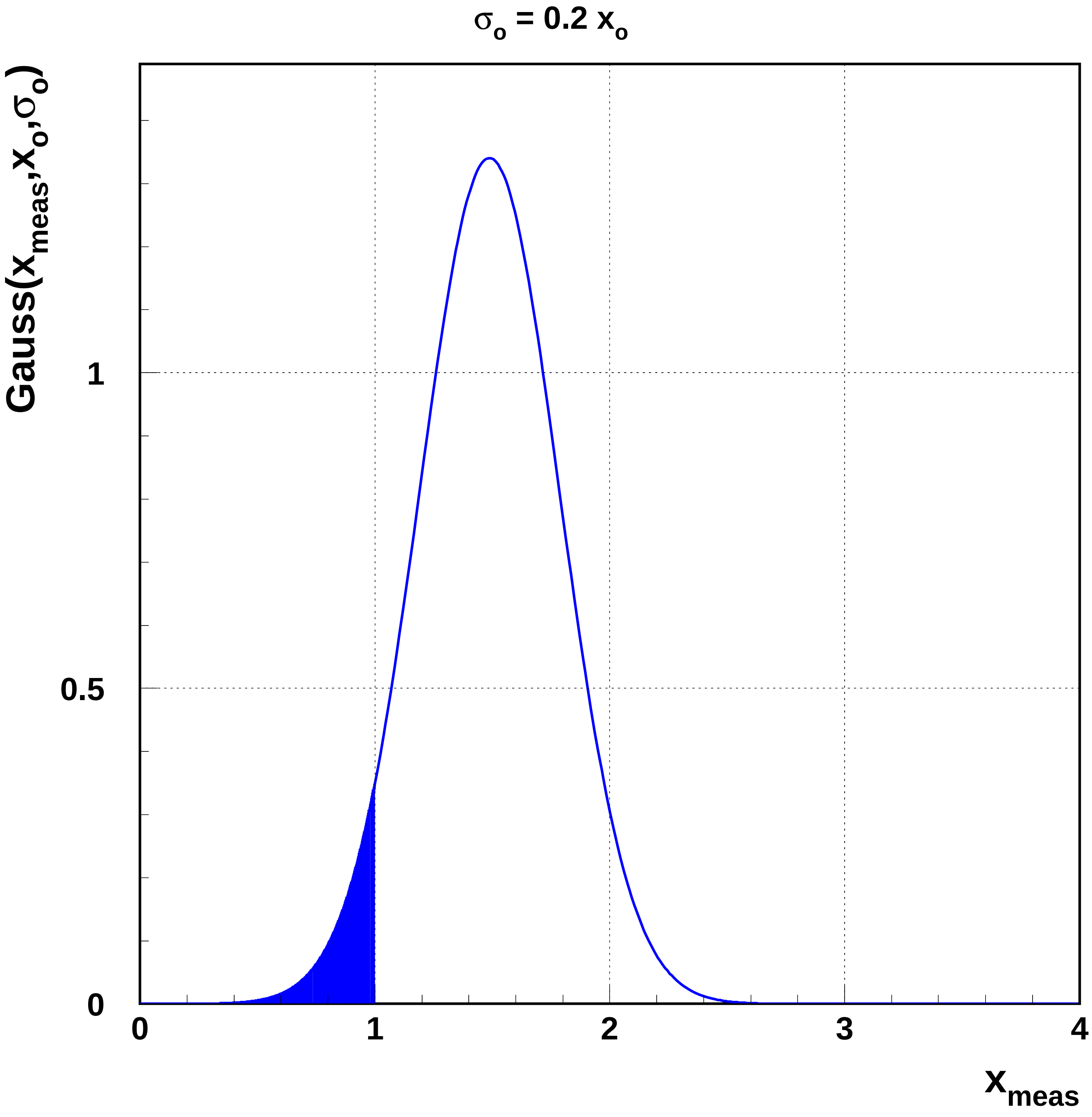
where $L(\boldsymbol{x}|\boldsymbol{\theta})$ is the likelihood function, *i.e.*, the joint p.d.f. for the data given a certain value of $\boldsymbol{\theta}$, evaluated with the data actually obtained in the experiment, and $\pi(\boldsymbol{\theta})$ is the prior p.d.f. for $\boldsymbol{\theta}$. Note that the denominator in (32.30) serves simply to normalize the posterior p.d.f. to unity.



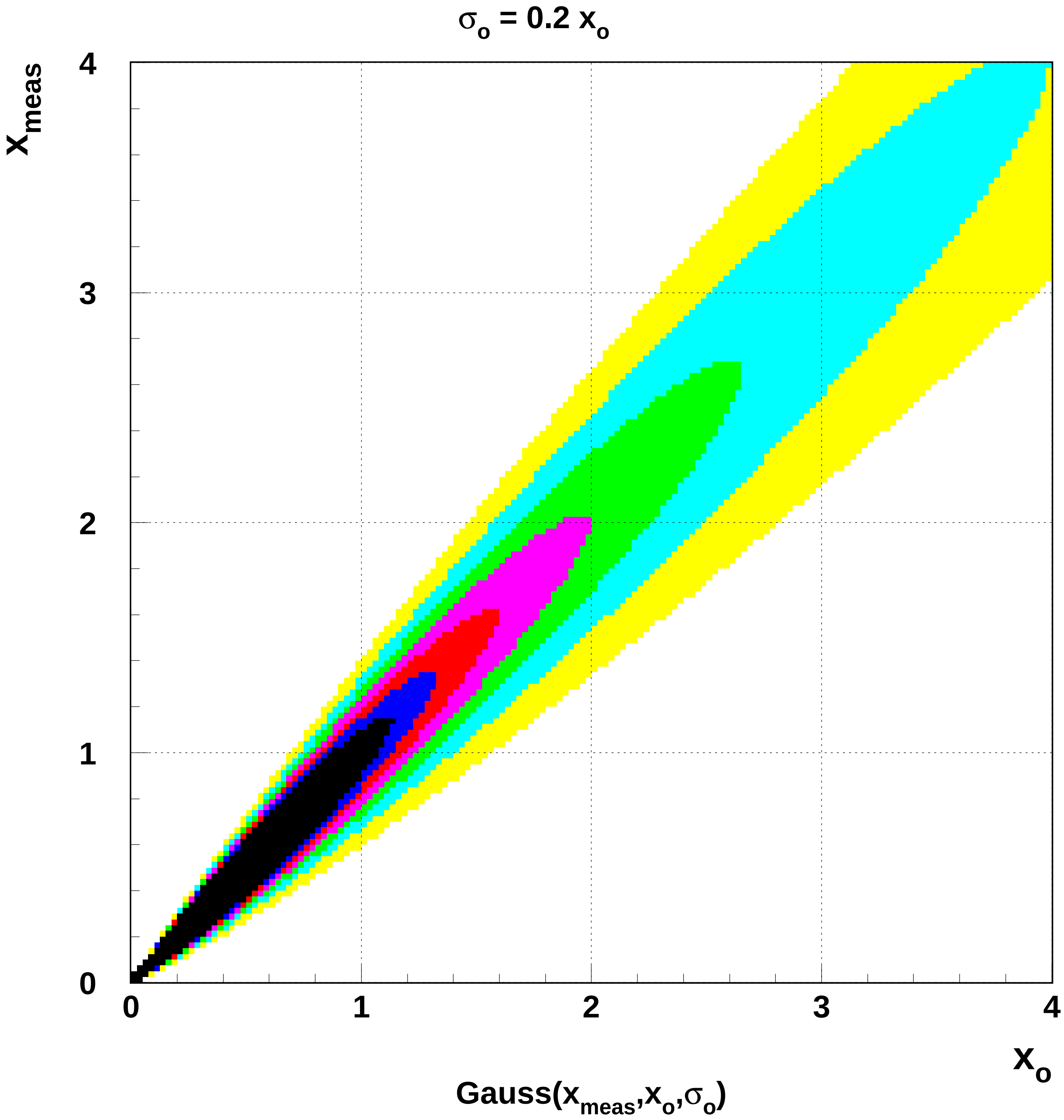
Gauss(x_{meas},x_o,σ_o)
2
1.5
1
0.5
0
0
1
2
3
4
x_{meas}



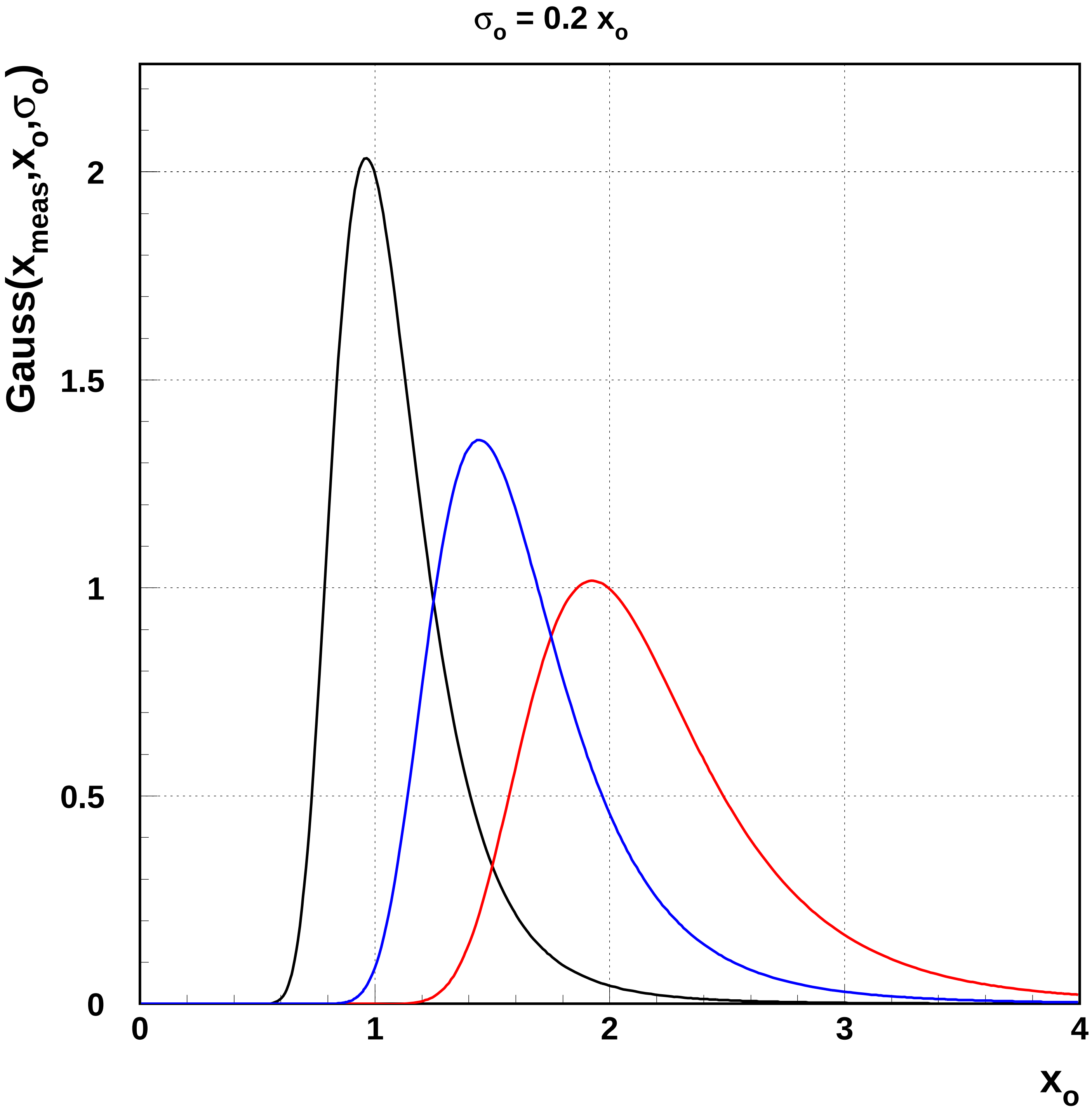
σo = 0.2 xo
Gauss(xmeas,xo,σo)
2
1.5
1
0.5
0
0
1
2
3
4
xmeas

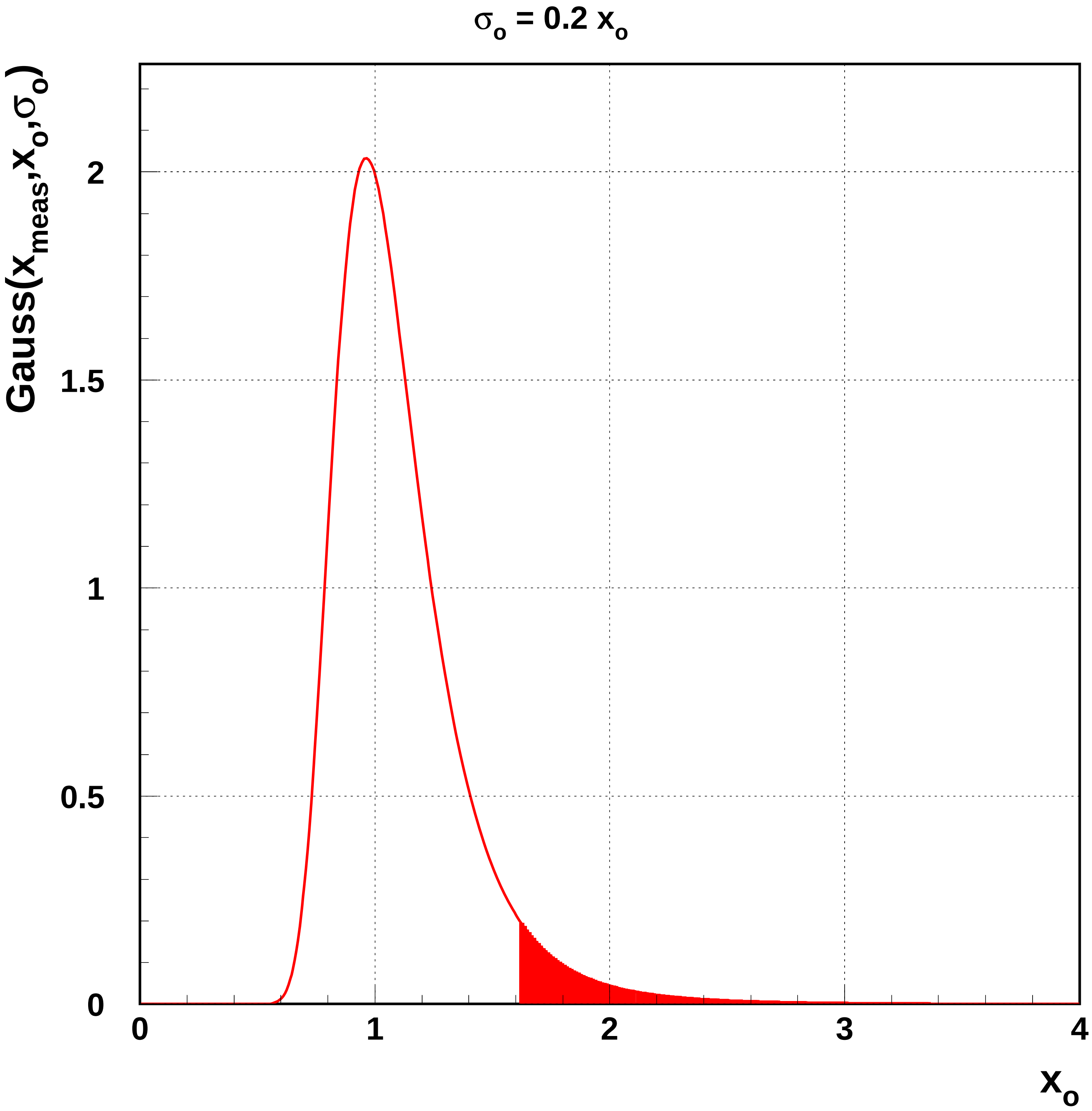


σo = 0.2 xo
Gauss(xmeas,xo,σo)
1
0.5
0
0
1
2
3
4
xmeas

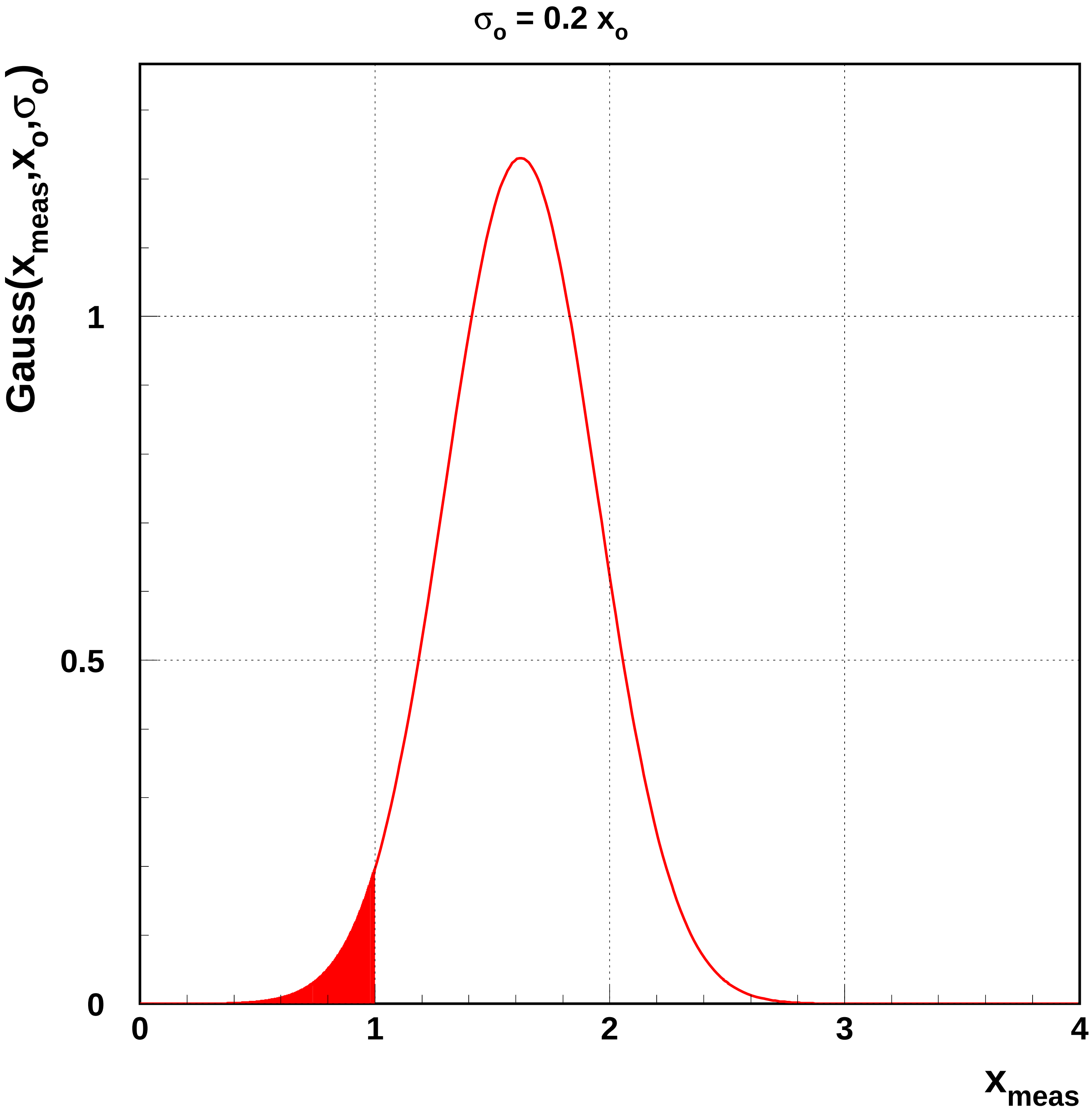


$\sigma_o = 0.2\ x_o$
x_{meas}
4
3
2
1
0
0
1
2
3
4
x_o
Gauss(x_{meas},x_o,σ_o)


σo = 0.2 xo
Gauss(xmeas,xo,σo)
2
1.5
1
0.5
0
0
1
2
3
4
xo



σo = 0.2 xo
Gauss(xmeas,xo,σo)
2
1.5
1
0.5
0
0
1
2
3
4
xo



σo = 0.2 xo
Gauss(xmeas,xo,σo)
1
0.5
0
0
1
2
3
4
xmeas

Search for contact interactions with ZEUS detector at HERA

Presentation of ZEUS Preliminary results for ICHEP'06

A.F.Żarnecki
Warsaw University

Outline:

- Introduction
- Models
- Data and analysis
- Systematic uncertainties
- Results

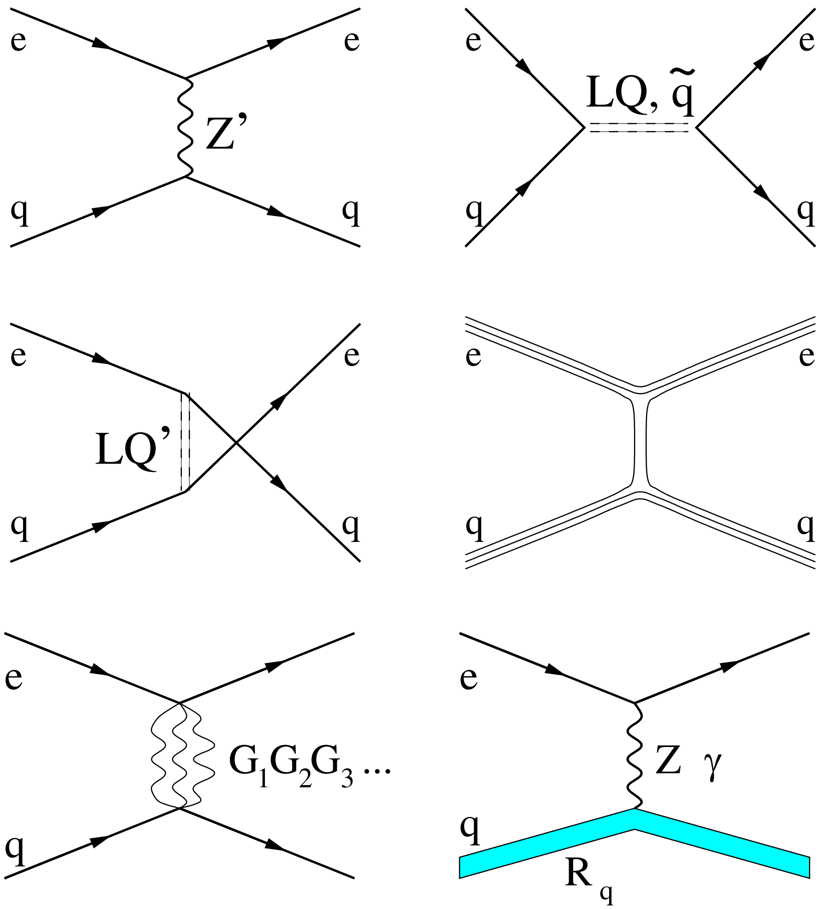
http://www-zeus.desy.de/~zarnecki/ZEUS_ONLY/ci_ed.html

ZEUS Meeting, June 28th, 2006

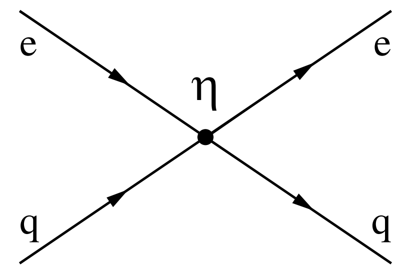
Models

Neutral Current eq Scattering

Possible "new physics" processes:



For $\sqrt{s}$ much smaller than "new" scale Λ



$eeqq$ contact interactions (CI)

Effective Lagrangian for vector $eeqq$ contact interactions:

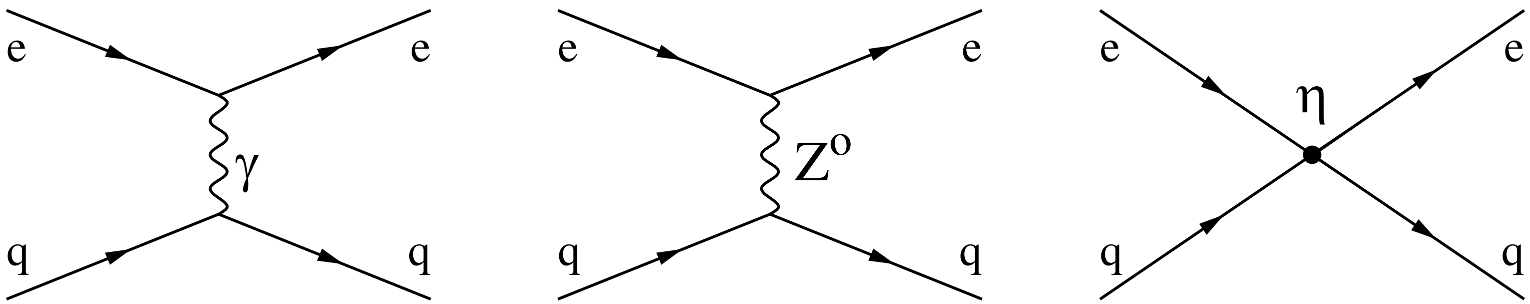
$$\mathcal{L}_{CI} = \sum_{\substack{\alpha,\beta=L,R \\ q}} \eta^{eq}_{\alpha\beta} \cdot (\bar{e}_\alpha \gamma^\mu e_\alpha)(\bar{q}_\beta \gamma_\mu q_\beta)$$

Scalar and tensor CI constrained beyond HERA sensitivity.

Models

Contact Interactions

Contact Interactions modify tree level $eq \rightarrow eq$ scattering amplitudes $M^{eq}_{\alpha\beta}$:



$$M^{eq}_{\alpha\beta}(Q^2) = \frac{e^2 e_q}{Q^2} - \frac{e^2}{sin^2\theta_W \cdot cos^2\theta_W} \cdot \frac{g^e_\alpha g^q_\beta}{Q^2 + m_Z^2} + \eta^{eq}_{\alpha\beta}$$

γ $\qquad Z^\circ$ $\qquad$?

$\eta^{eq}_{\alpha\beta}$ - 4 possible couplings for every flavor q

Different models assume different helicity structure of new interactions

Models

Cross-section formula

For NC e^-p DIS with polarized beam

$$\frac{d^2\sigma^{e^-p}}{dxdy} =$$

$$\frac{sx}{16\pi}\sum_q \quad q(x)\left\{\mathcal{P}_-|M^{eq}_{LL}|^2 + \mathcal{P}_+|M^{eq}_{RR}|^2 + (1-y)^2\left[\mathcal{P}_-|M^{eq}_{LR}|^2 + \mathcal{P}_+|M^{eq}_{RL}|^2\right]\right\}$$

$$+ \quad \bar{q}(x)\left\{\mathcal{P}_-|M^{eq}_{LR}|^2 + \mathcal{P}_+|M^{eq}_{RL}|^2 + (1-y)^2\left[\mathcal{P}_-|M^{eq}_{LL}|^2 + \mathcal{P}_+|M^{eq}_{RR}|^2\right]\right\}$$

$\Rightarrow$ most sensitive to η^{eq}_{LL} and η^{eq}_{RR} (q=u,d)

where: $\mathcal{P}_\pm = (1 \pm P)$

P is electron beam polarization

Models

Cross-section formula

For NC e^+p DIS with polarized beam

$$\frac{d^2\sigma^{e^+p}}{dxdy} =$$

$$\frac{sx}{16\pi}\sum_q \quad q(x)\left\{\mathcal{P}_+|M^{eq}_{LR}|^2 + \mathcal{P}_-|M^{eq}_{RL}|^2 + (1-y)^2\left[\mathcal{P}_+|M^{eq}_{LL}|^2 + \mathcal{P}_-|M^{eq}_{RR}|^2\right]\right\}$$

$$+ \quad \bar{q}(x)\left\{\mathcal{P}_+|M^{eq}_{LL}|^2 + \mathcal{P}_-|M^{eq}_{RR}|^2 + (1-y)^2\left[\mathcal{P}_+|M^{eq}_{LR}|^2 + \mathcal{P}_-|M^{eq}_{RL}|^2\right]\right\}$$

$\Rightarrow$ most sensitive to η^{eq}_{LR} and η^{eq}_{RL} (q=u,d)

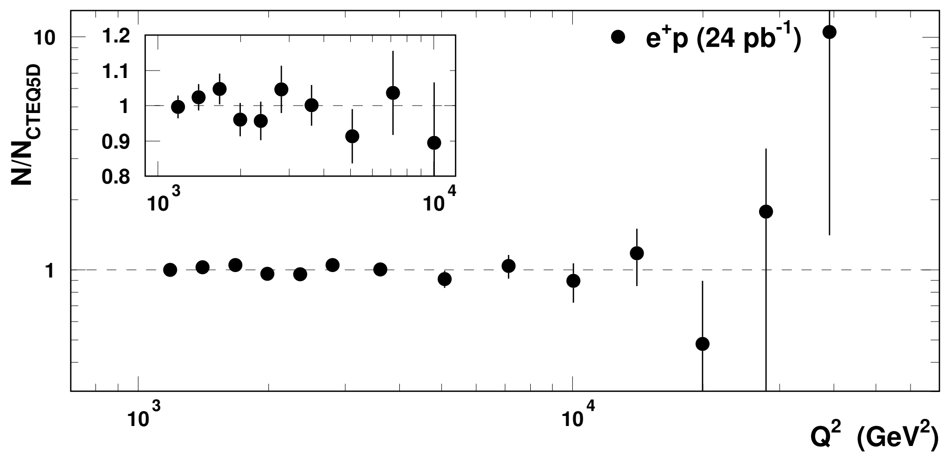
$\Rightarrow$ Combining e^+p and e^-p can significantly improve limits

Data and analysis

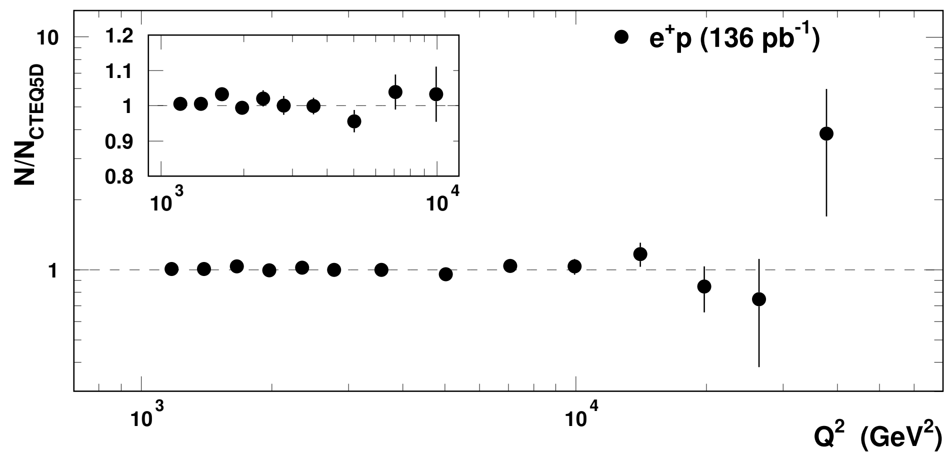
HERA-II data

All HERA data

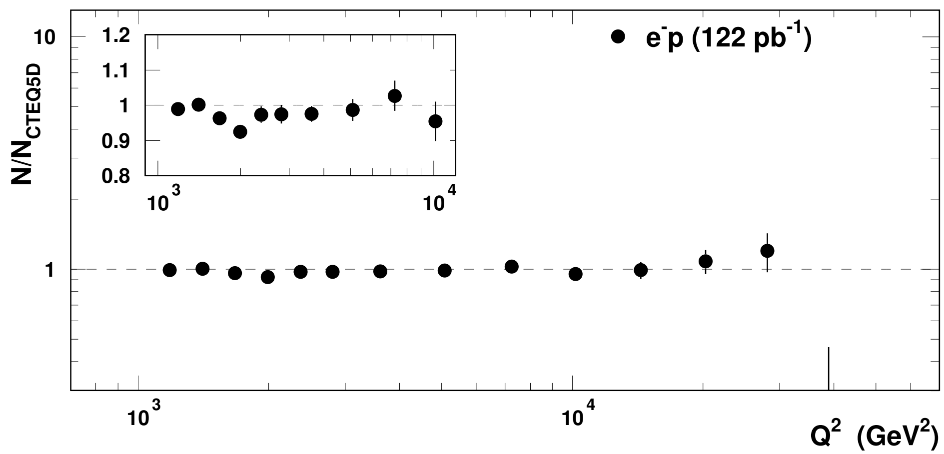
ZEUS 2003-2004



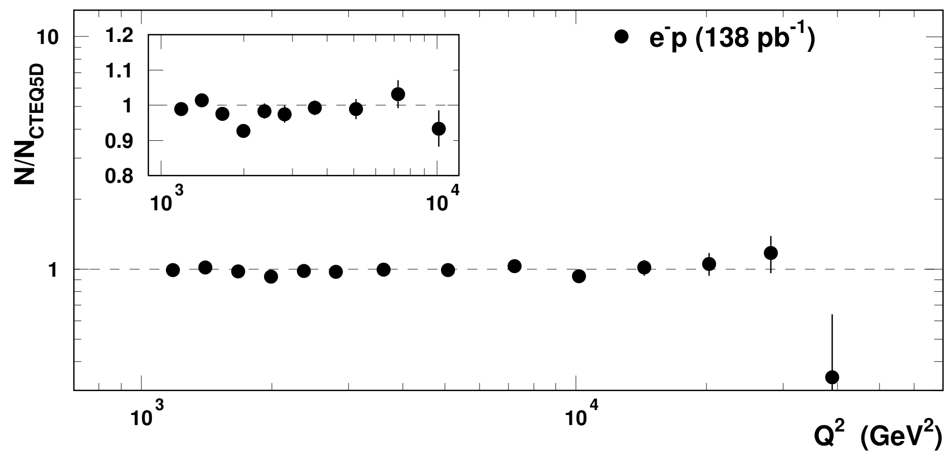
ZEUS 1994-2004



ZEUS 2004-2005



ZEUS 1998-2005



Models

General models

Also referred to as compositeness models

Couplings $\eta^{eq}_{\alpha\beta}$ are related to the "new physics" mass scale Λ by the formula:

$$\eta = \frac{\varepsilon \cdot g^2_{CI}}{\Lambda^2}$$

where g_{CI} is the coupling strength of new interactions and $\varepsilon = \pm 1$.

By convention we set $g^2_{CI} = 4\pi$.

Models conserving parity:

$$\eta^{eq}_{LL} + \eta^{eq}_{LR} - \eta^{eq}_{RL} - \eta^{eq}_{RR} = 0$$

Family universality assumed !

Models conserving parity:

Model	η^{ed}_{LL}	η^{ed}_{LR}	η^{ed}_{RL}	η^{ed}_{RR}	η^{eu}_{LL}	η^{eu}_{LR}	η^{eu}_{RL}	η^{eu}_{RR}
VV	$+\eta$	$+\eta$	$+\eta$	$+\eta$	$+\eta$	$+\eta$	$+\eta$	$+\eta$
AA	$+\eta$	$-\eta$	$-\eta$	$+\eta$	$+\eta$	$-\eta$	$-\eta$	$+\eta$
VA	$+\eta$	$-\eta$	$+\eta$	$-\eta$	$+\eta$	$-\eta$	$+\eta$	$-\eta$
X1	$+\eta$	$-\eta$			$+\eta$	$-\eta$		
X2	$+\eta$		$+\eta$		$+\eta$		$+\eta$	
X3	$+\eta$			$+\eta$	$+\eta$			$+\eta$
X4		$+\eta$	$+\eta$			$+\eta$	$+\eta$	
X5		$+\eta$		$+\eta$		$+\eta$		$+\eta$
X6			$+\eta$	$-\eta$			$+\eta$	$-\eta$
U1					$+\eta$	$-\eta$		
U2					$+\eta$		$+\eta$	
U3					$+\eta$			$+\eta$
U4						$+\eta$	$+\eta$	
U5						$+\eta$		$+\eta$
U6							$+\eta$	$-\eta$
Models violating parity:								
LL	$+\eta$				$+\eta$			
LR		$+\eta$				$+\eta$		
RL			$+\eta$				$+\eta$	
RR				$+\eta$				$+\eta$

Models

Large Extra Dimensions

Arkani-Hamed–Dimopoulos–Dvali Model

If gravity propagates in the $4+\delta$ dimensions, the effective mass scale M_S can be as low as 1 TeV.

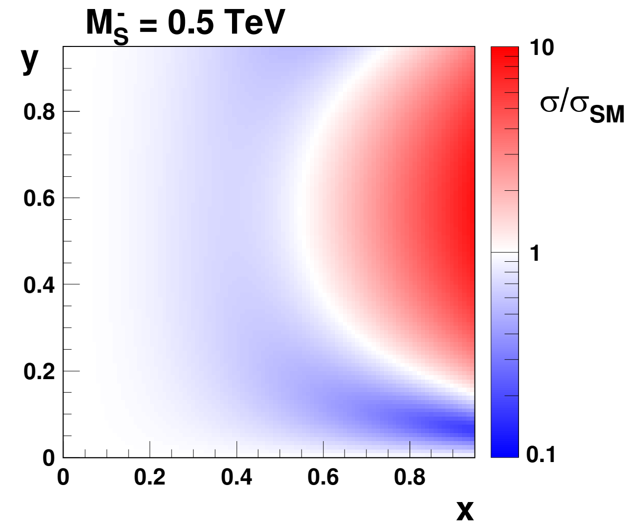
$\Rightarrow$ Gravitational interactions become comparable in strength to electroweak interactions.

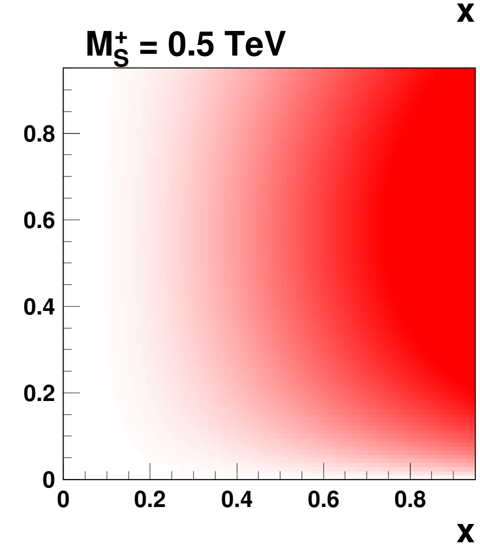
The contribution of graviton (Kaluza-Klein tower) exchange to the $e^{\pm}p$ NC DIS cross section can be described by an effective contact interaction type coupling:

$$\eta_G \;=\; \pm\lambda \cdot \frac{\mathcal{E}^2}{M_S^4}$$

where λ is the coupling strength and $\mathcal{E}$ is related to the energy scales of hard interaction. $(\sqrt{s}, Q^2)$

Cross-section deviations for e^-p:





Models

Leptoquarks Buchmüller-Rückl-Wyler (BRW) model

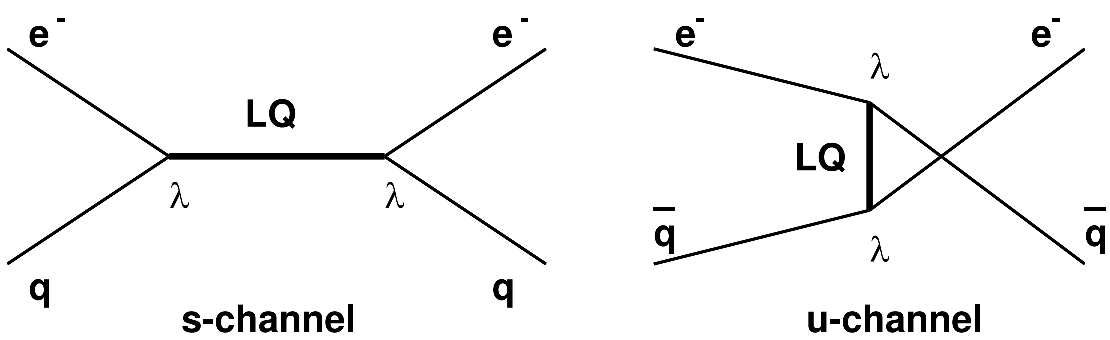
- $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariance
- lepton and baryon number conservation
- strong bounds from rare decays $\Rightarrow$ either left- or right-handed couplings
- family diagonal

$\Rightarrow$ 7 scalar and 7 vector leptoquarks

For high mass leptoquarks

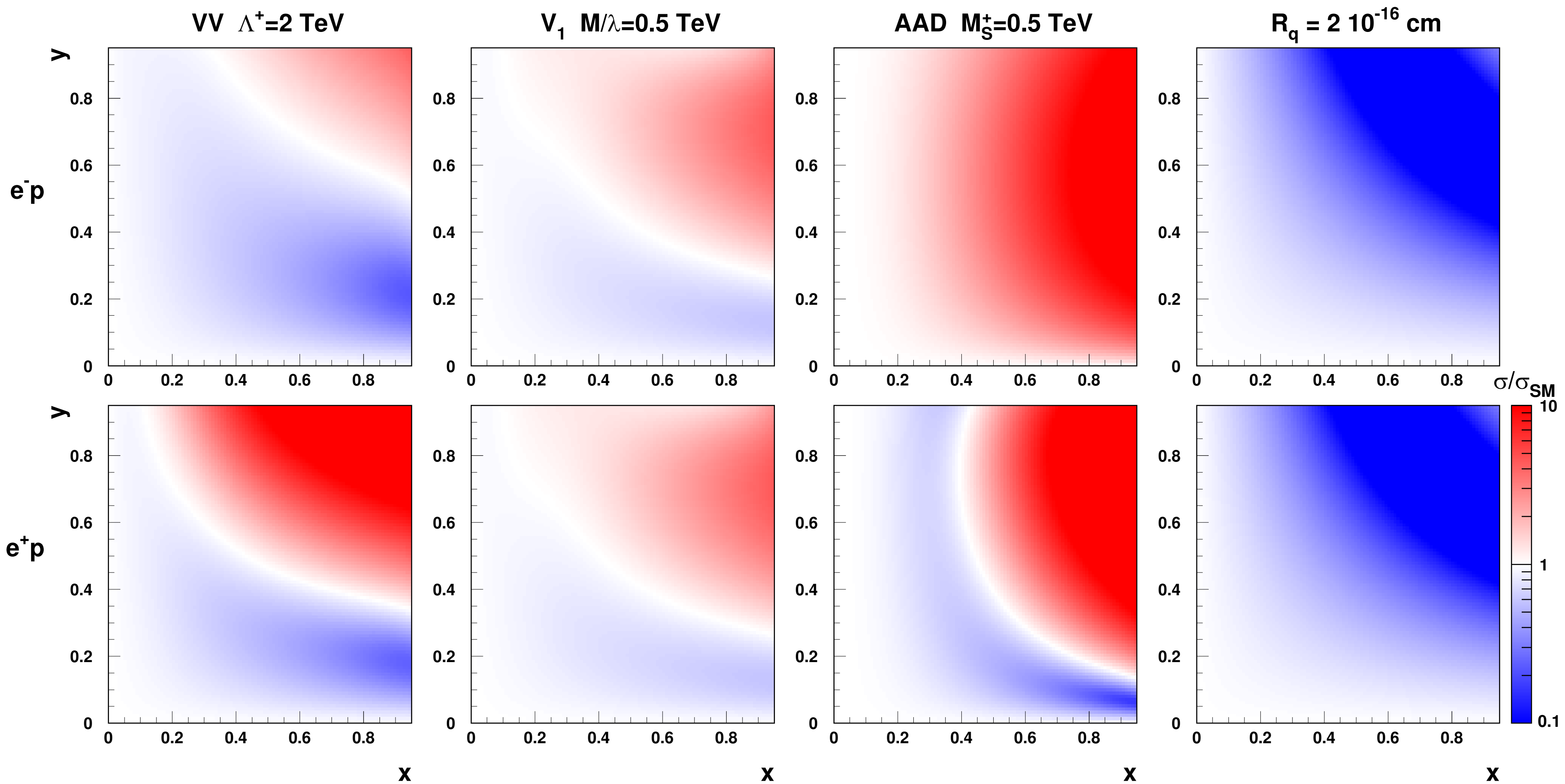
$$M_{LQ} \gg \sqrt{s}$$

both s- and u-channel important $\Rightarrow$



Models

Comparison of cross-section deviations expected in different models



In most cases, contact interaction contribution depends mainly on Q^2...

Models

Quark form factor

"classical" method to look for possible fermion (sub)structure.

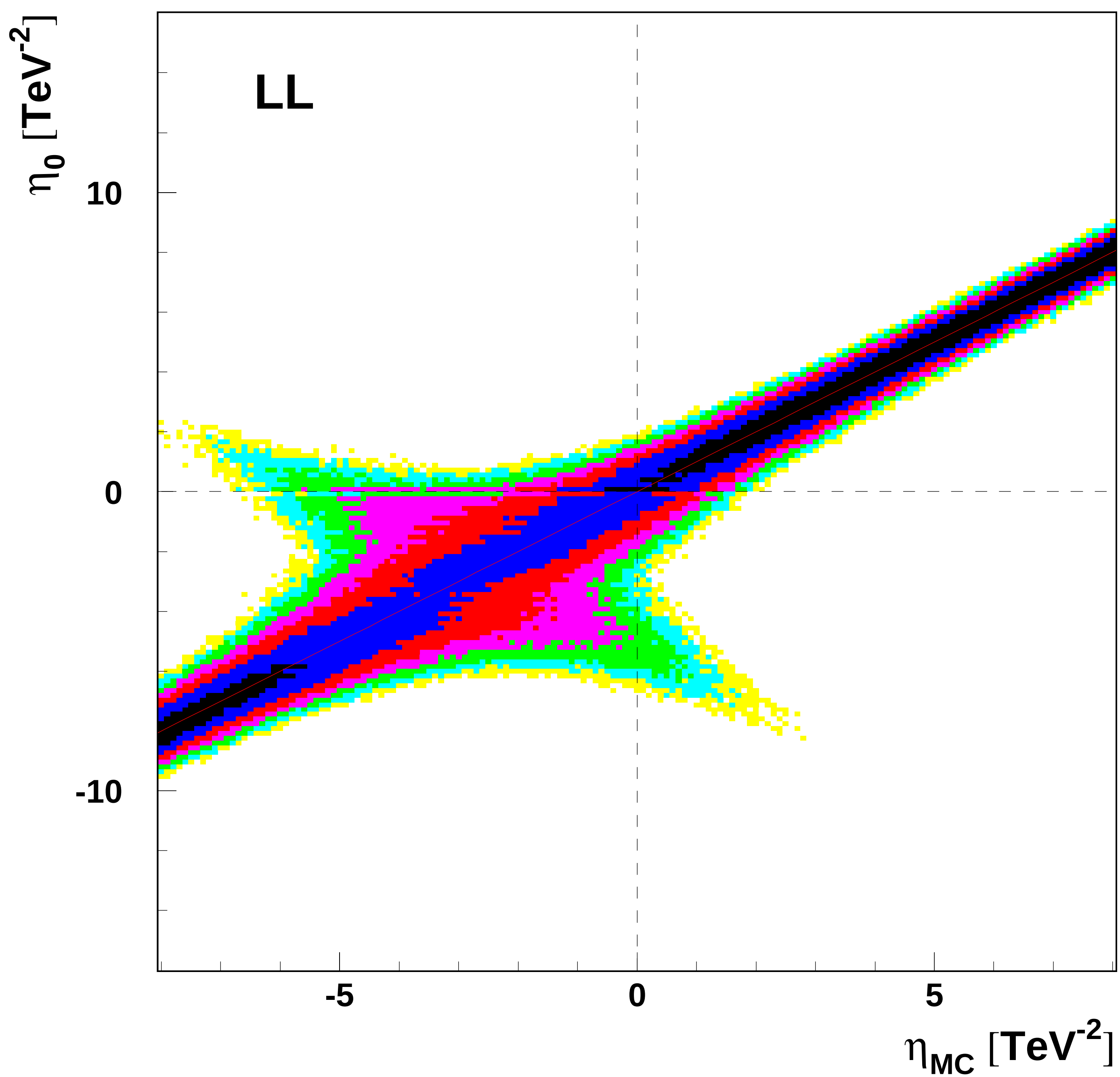
If a quark has finite size, the standard model cross-section is expected to decrease at high momentum transfer:

$$\frac{d\sigma}{dQ^2} = \frac{d\sigma^{SM}}{dQ^2} \cdot \left[1 - \frac{R_q^2}{6} Q^2\right]^2 \cdot \left[1 - \frac{R_e^2}{6} Q^2\right]^2$$

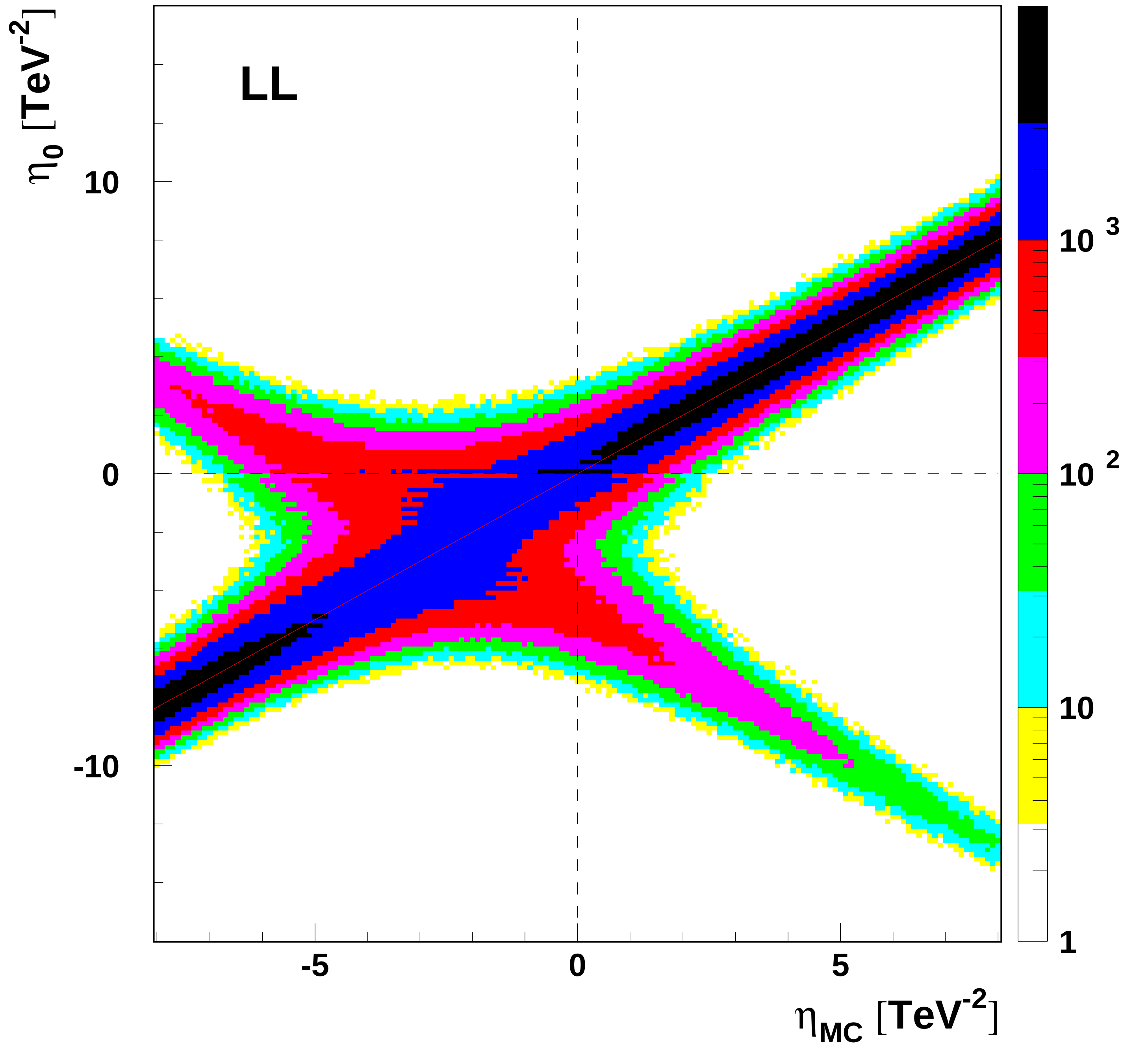
where R_q is the root mean-square radius of the electroweak charge distribution in the quark.

We do not consider the possibility of finite electron size...

same dependence expected for e^+p and e^-p !



LL
10
0
-10
-5
0
5
η0 [TeV-2]
ηMC [TeV-2]



LL
η0 [TeV-2]
ηMC [TeV-2]
10
0
-10
-5
0
5
1
10
10 2
10 3

Analysis

Probability function

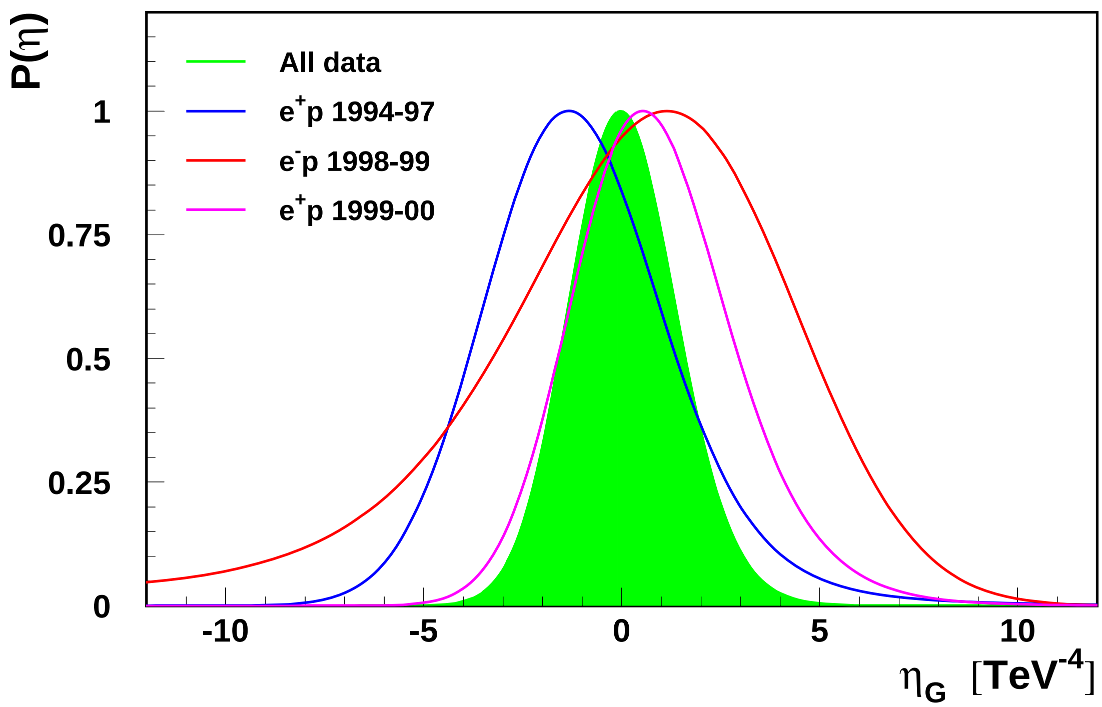
Observed numbers of events in Q^2 bins n_i are compared with the CI model expectations $\mu_i(\eta_G)$ using the probability function:

$$P(\eta_G) \sim \prod_i \frac{\mu_i(\eta_G)^{n_i} \cdot e^{-\mu_i(\eta_G)}}{n_i!}$$

where i runs over 14 Q^2 bins $\times$ 3 data taking periods.

$$\eta_G \equiv \pm \frac{\lambda}{M_S^4}$$

Resulting probability function for the nominal data:



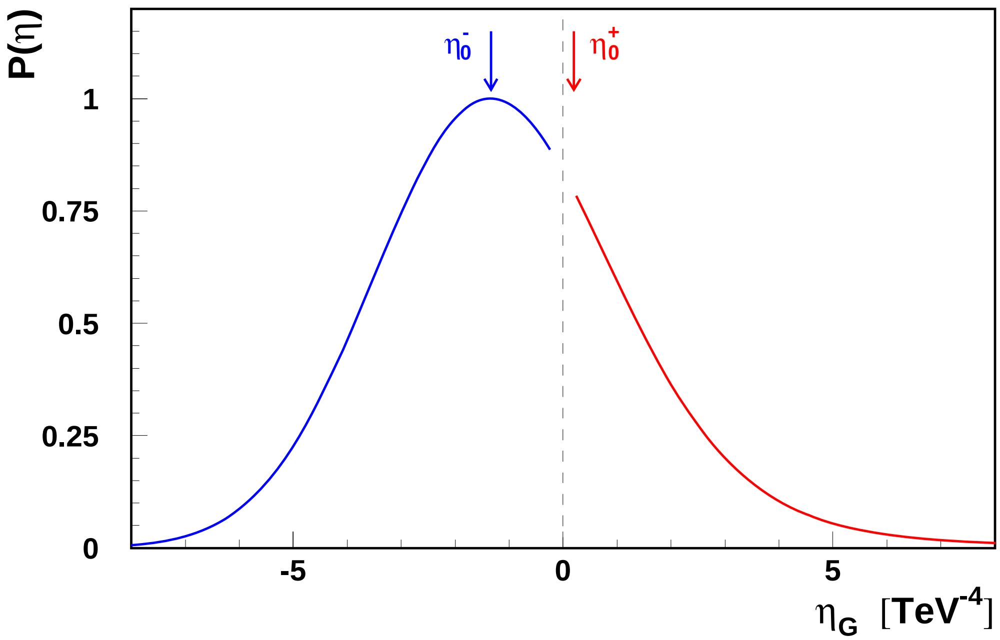
normalized to $\max_\eta P(\eta_G) = 1$

Analysis

Limit setting (1)

- Find coupling values giving best description of the data, separately for negative and positive couplings:

example



For ED model $P(\eta_G)$ has always only one maximum: either $\eta_\circ^+$ or $\eta_\circ^-$ is zero.

In general case (other CI models) two maxima can be found.

ED model, ZEUS 1994-2000 data:

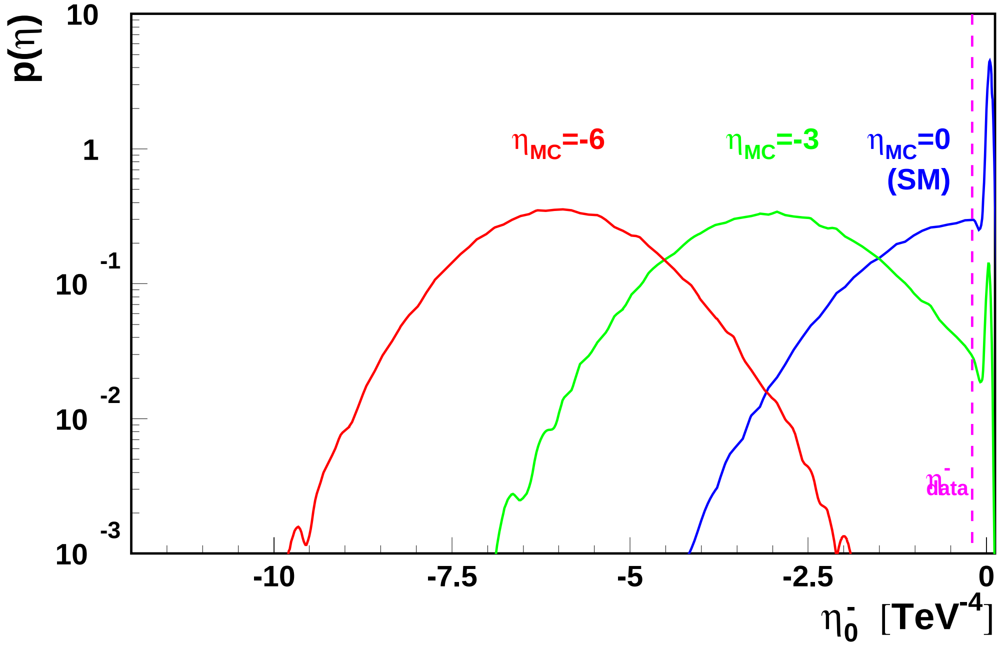
$$\eta_\circ^- = -0.02\ \text{TeV}^{-4}$$
$$\eta_\circ^+ = 0$$

very good agreement with the Standard Model

Analysis

Limit setting (2)

- Perform "MC experiments" (MCE) to find the expected distribution of $\eta_\circ^+$ and $\eta_\circ^-$ for Standard Model and for ED model with arbitrary coupling value η_{MC}



95% CL limit on η_G (for $\eta_G < 0$) is defined as η_{MC} value for which 95% of Monte Carlo experiments result in η_0^- value lower than the value η_{data}^- found for nominal data.

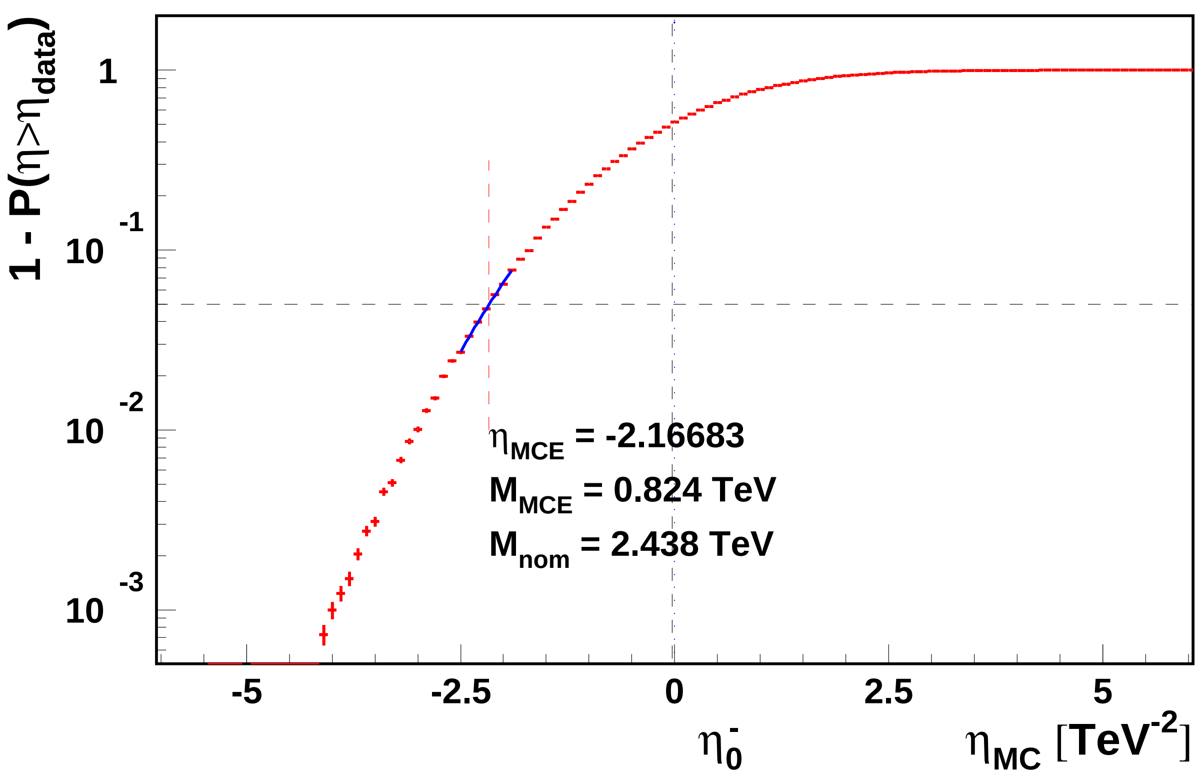
Analysis

Limit setting (3)

Method used to find 95% CL coupling limits with high (statistical) precision:

- calculate probability $P(|\eta_\circ^\pm| > |\eta_{data}|)$ for selected η_{MC} values (grid).
- interpolate between grid points using polynomial fit to $\ln(P)$.

nominal data, no systematics



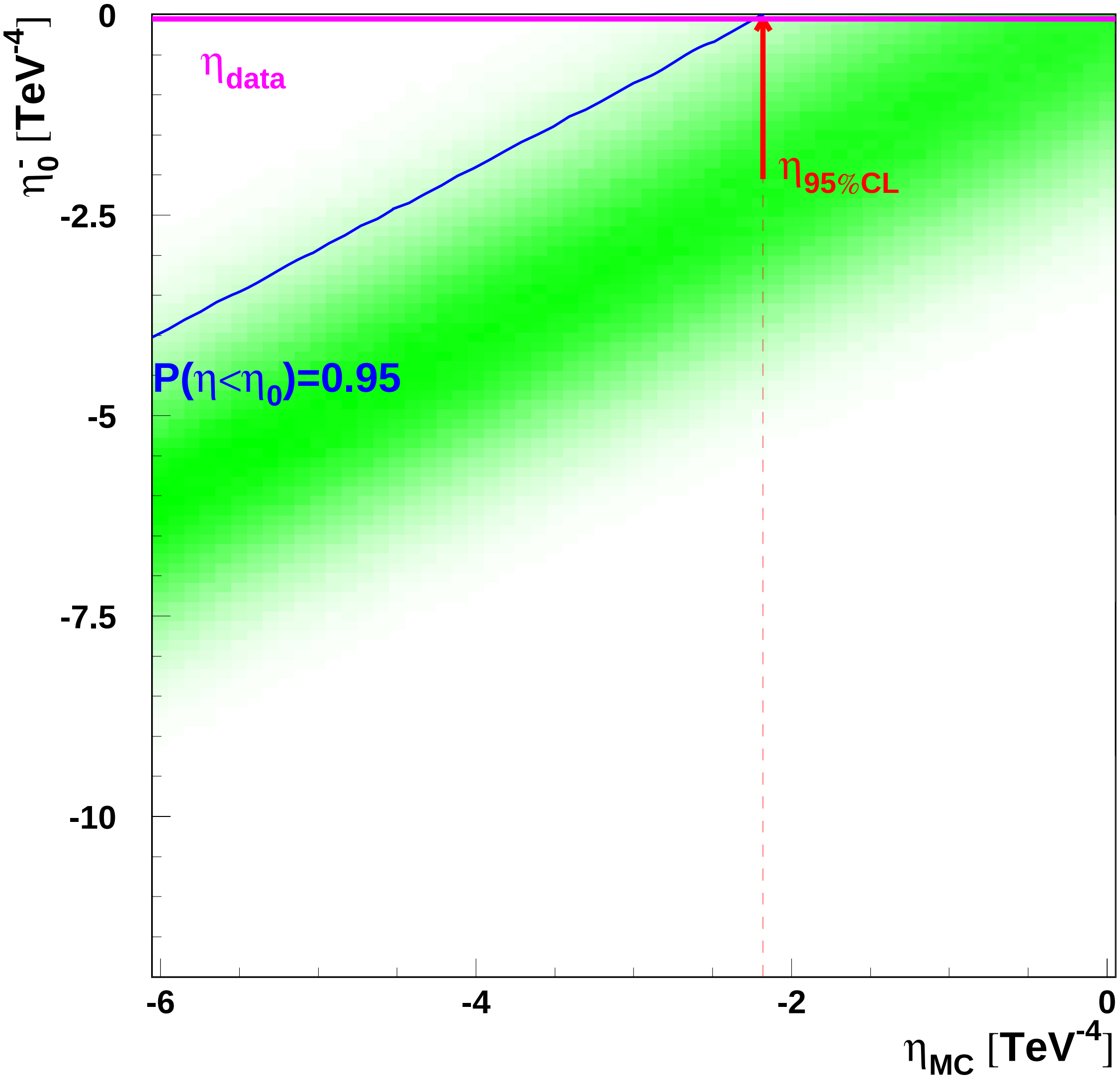
$\eta_G < -2.167$ TeV^{-4} on 95% CL

Analysis

Limit setting

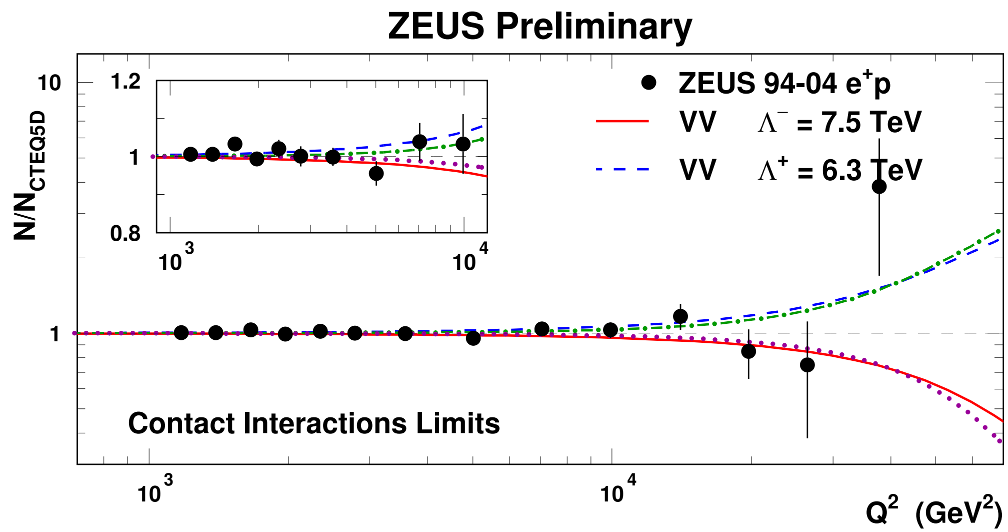
2-D probability distribution for η_0^- as a function of η_{MC}

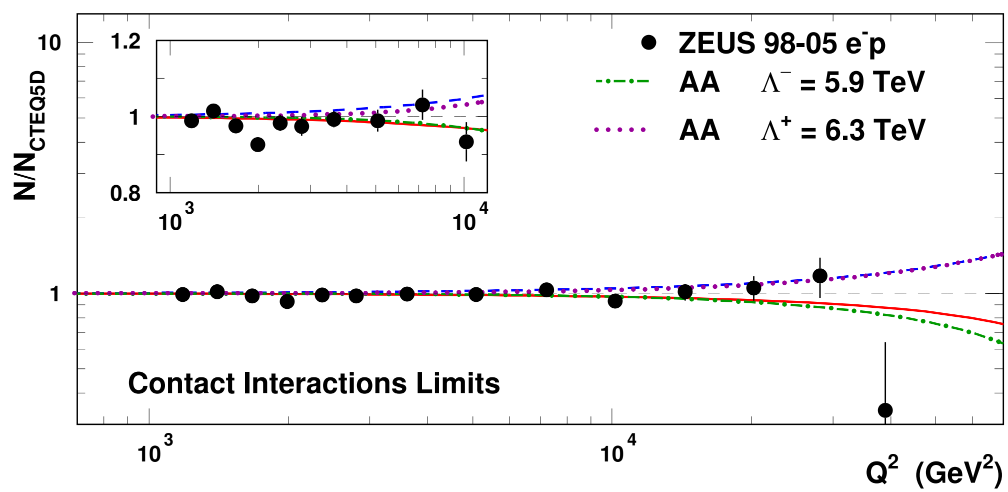
$$P(\eta_\circ^-;\eta_{MC})$$

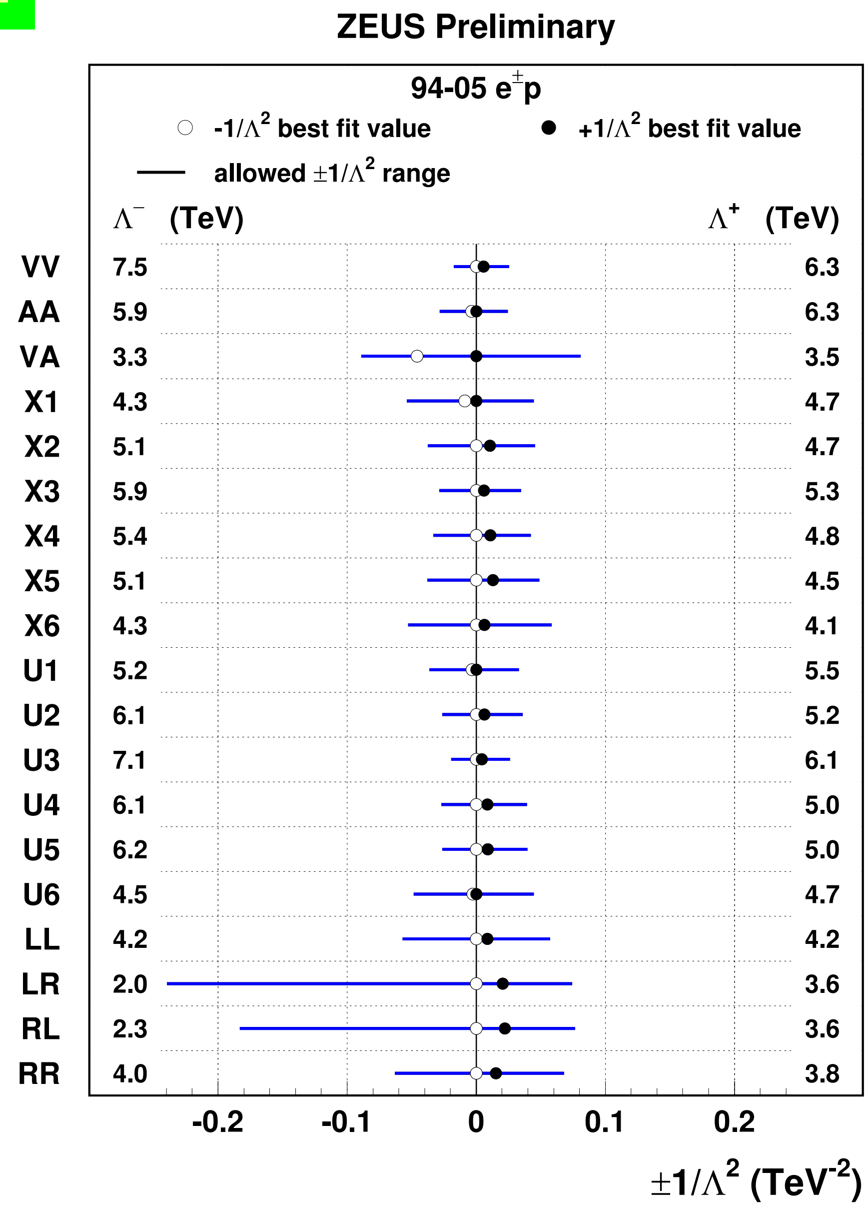


Results

Public plots: CI

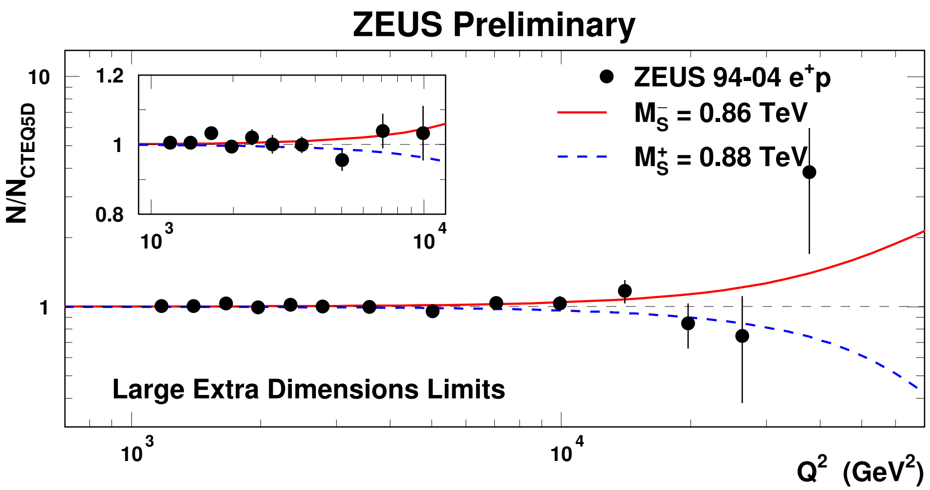


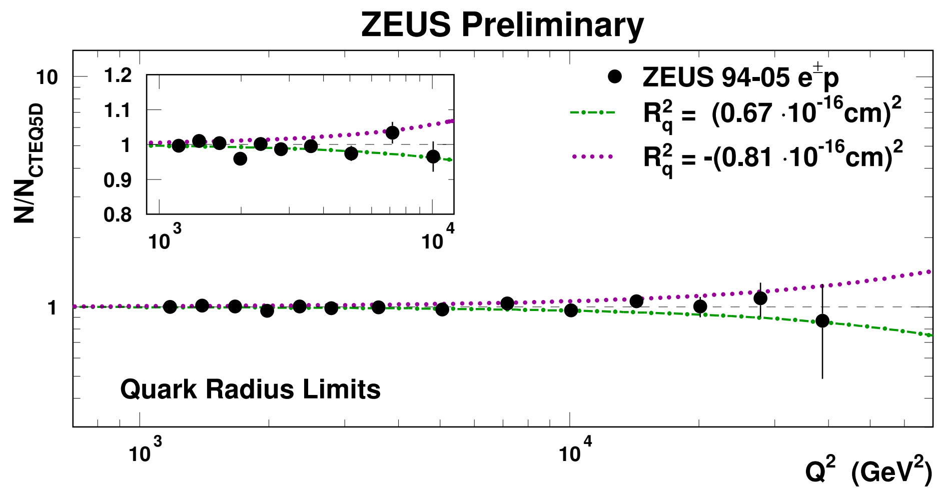


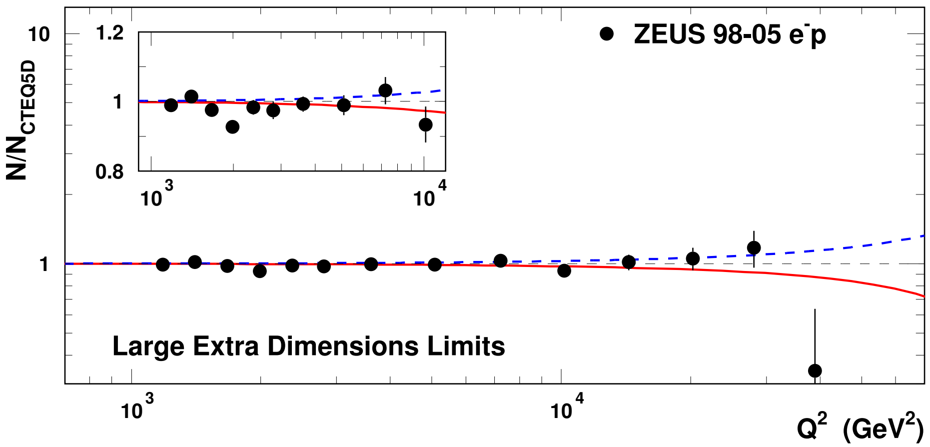


Results

Public plots: LED and R_q

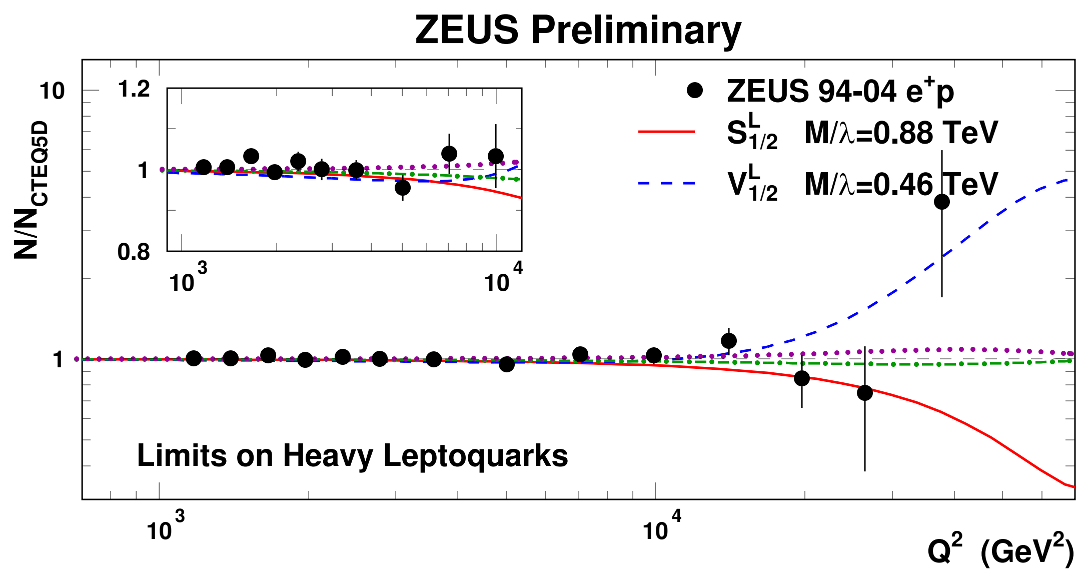


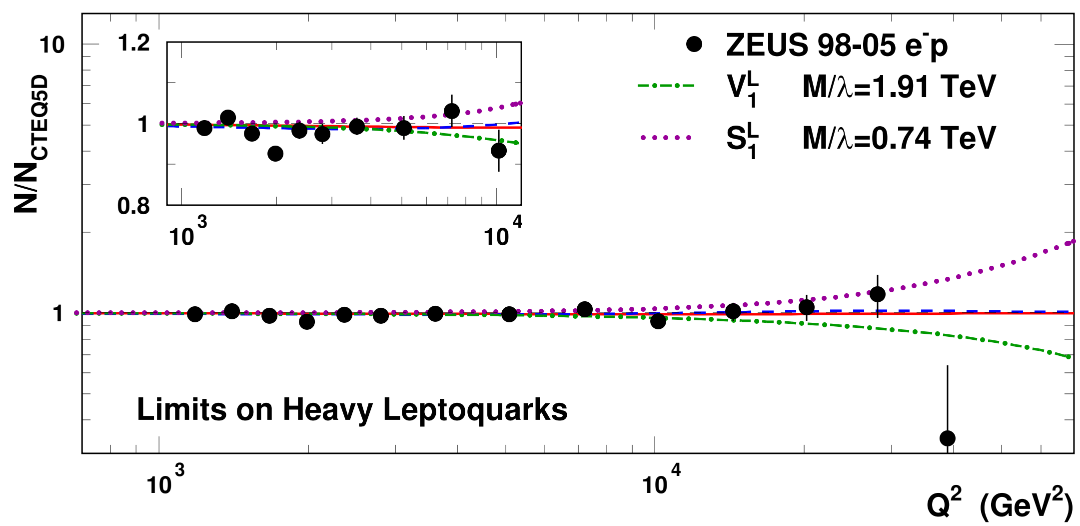




Results

Public plots: LQ



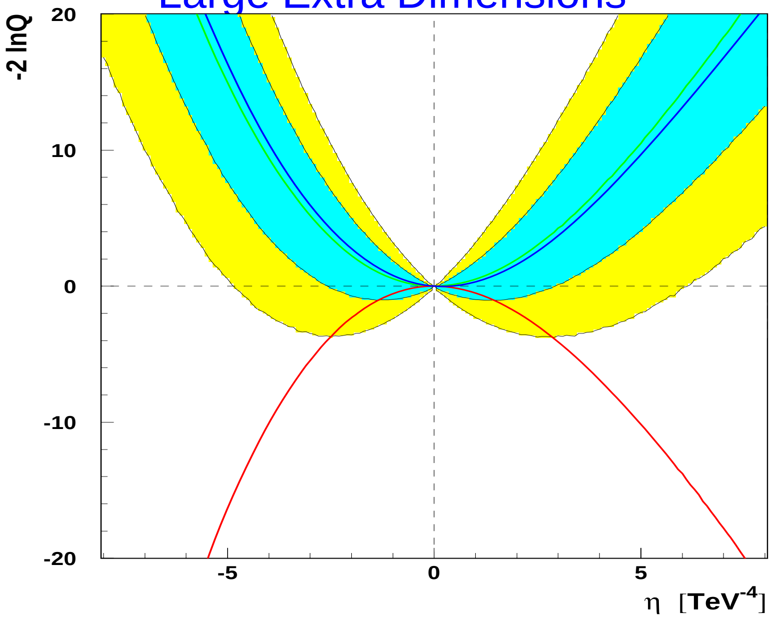


Comparison with SM

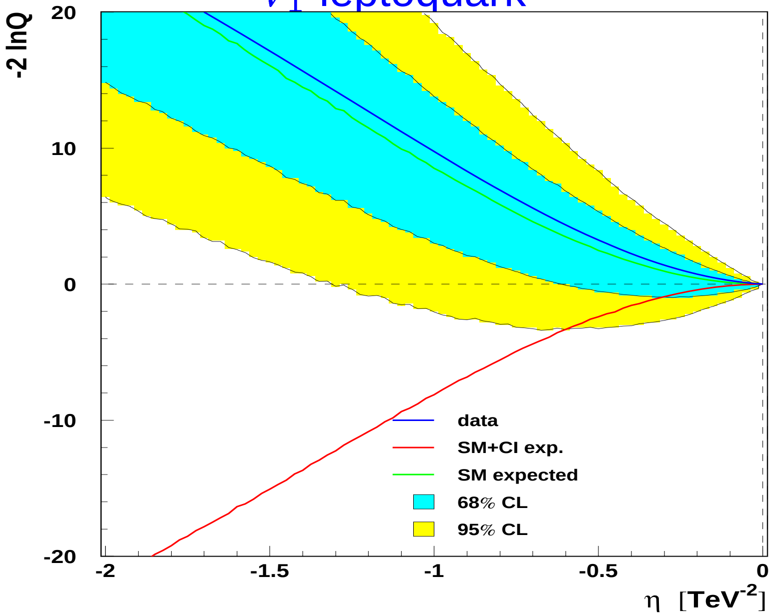
Use “LEP method” to show the agreement with SM

$$-2 \ln Q \;=\; \chi^2(\eta) - \chi^2_{SM}$$

Large Extra Dimensions



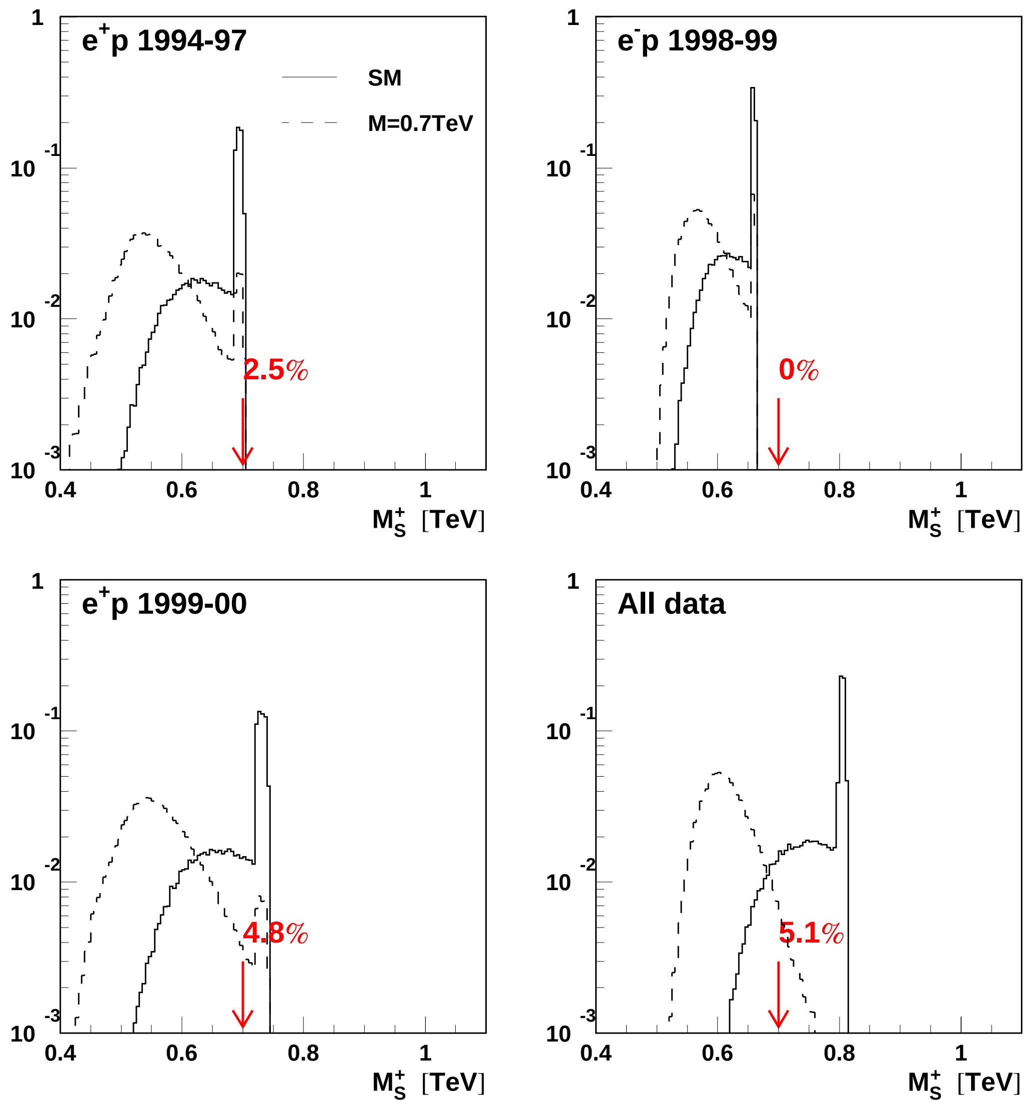
V_1 leptoquark



Comparison with expectations

Large Extra Dimensions

Limits expected for M_S^+ = 0.7 TeV
default ZEUS method



We get limits better than true M_S^+ in 5% of events
$\Rightarrow$ We do calculate 95% CL limit !

Search for Compositeness, Leptoquarks and Large Extra Dimensions in eq Contact Interactions at HERA

H1 Collaboration

Abstract

The reaction $e^+p \to e^+X$ is studied with the H1 detector at HERA. The data cover momentum transfers Q^2 between $200\ \mathrm{GeV}^2$ and $30,000\ \mathrm{GeV}^2$ and correspond to an integrated luminosity of $35.6\ \mathrm{pb}^{-1}$. The differential cross section $\mathrm{d}\sigma/\mathrm{d}Q^2$ is compared to the Standard Model expectation for neutral current scattering and analysed to search for $(\bar{e}e)(\bar{q}q)$ contact interactions. No evidence for new phenomena is observed. The results are used to set limits on scales within models of electron–quark compositeness, quark form factors and the exchange of virtual heavy leptoquarks. A search for gravitational effects mediated through the exchange of virtual gravitons which propagate into large extra dimensions is presented.

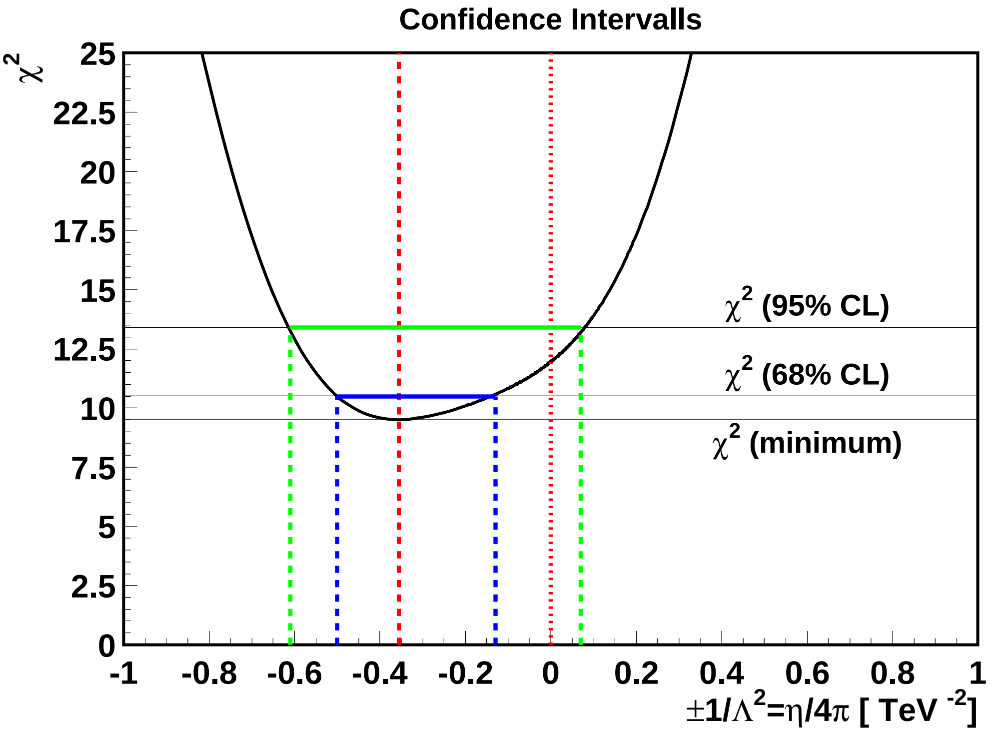
In order to derive quantitative tests of the Standard Model and to search for new physics hypotheses, a χ^2 analysis of the data is performed taking the dominant error sources and uncertainties into account. The χ^2 function is defined as

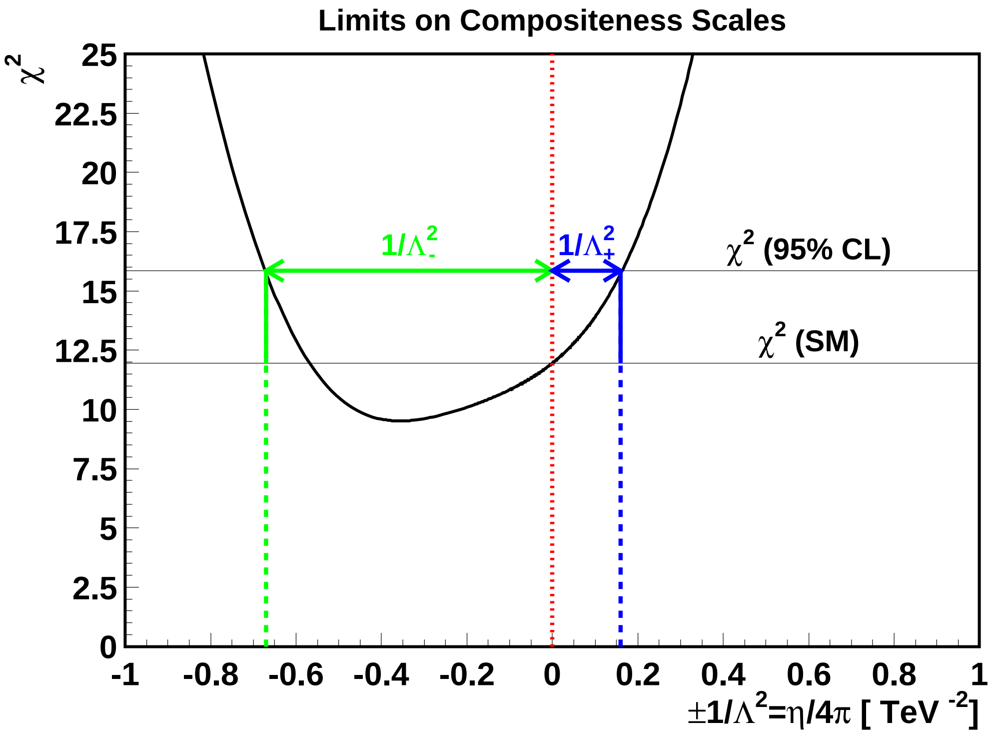
$$\chi^2 = \sum_i \left(\frac{\hat{\sigma}_i^{exp} f_n - \hat{\sigma}_i^{th} \left(1 - \sum_k \Delta_{ik}(\varepsilon_k)\right)}{\Delta \hat{\sigma}_i^{exp} f_n} \right)^2 + \left(\frac{f_n - 1}{\Delta f_n} \right)^2 + \sum_k \varepsilon_k^2 \, . \qquad (2)$$

Here $\hat{\sigma}_i$ denotes the experimental or theoretical cross section in the Q^2 bin i and f_n is the overall normalisation parameter with an uncertainty $\Delta f_n = 0.015$. The experimental error $\Delta \hat{\sigma}_i^{exp}$ includes statistical and uncorrelated systematic errors added in quadrature. The functions $\Delta_{ik}(\varepsilon_k)$ describe for the i^{th} bin effects due to correlated systematic errors associated to different sources k. They depend quadratically on the fit parameters ε_k, which may be interpreted as pulls, *i.e.* shifts caused by systematics normalised to their error estimates. There are three sources of correlated systematic errors taken into account: the experimental uncertainties of the positron energy scale and the scattering angle and the uncertainty of the strong coupling entering in the Standard Model prediction (see below).

A fit of the cross section $\mathrm{d}\sigma/\mathrm{d}Q^2$ to the Standard Model expectation using CTEQ5D parton densities yields $\chi^2/\mathrm{dof} = 12.3/16$ with a normalisation parameter $f_n = 1.004$. Limits of a model parameter are derived by varying the parameter until the χ^2 value changes by a certain amount with respect to the Standard Model fit, *e.g.* $\chi^2 - \chi^2_{SM} = 3.84$ for 95% confidence level (CL). Systematics due to different parton distributions are taken into account by always quoting the most conservative result of the various fits, *i.e.* the smallest value in case of a lower limit.

Extraction of Limits



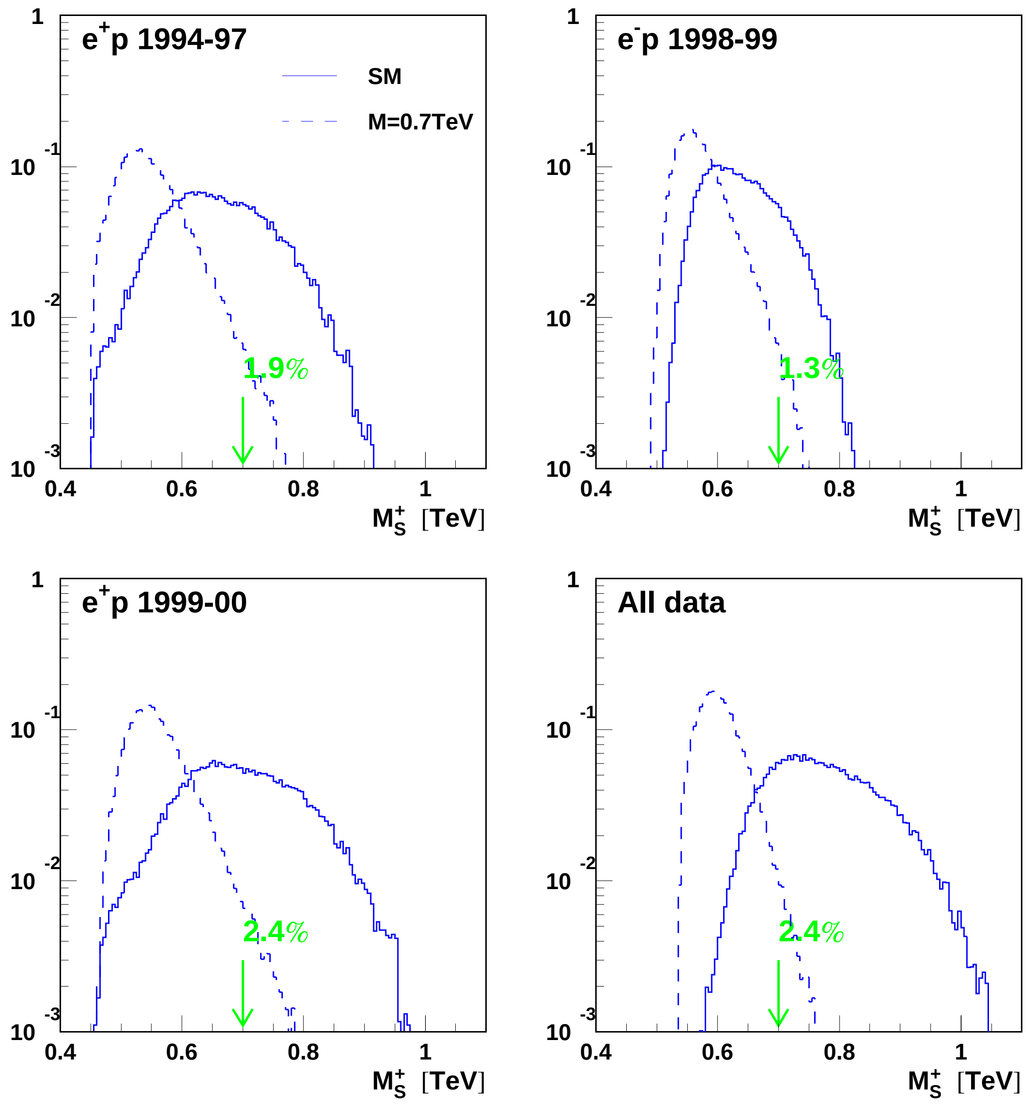

Limits on Compositeness Scales
χ2
25
22.5
20
17.5
15
12.5
10
7.5
5
2.5
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
±1/Λ2=η/4π [TeV -2]
1/Λ-2
1/Λ+2
χ2 (95% CL)
χ2 (SM)

Comparison with expectations

Large Extra Dimensions

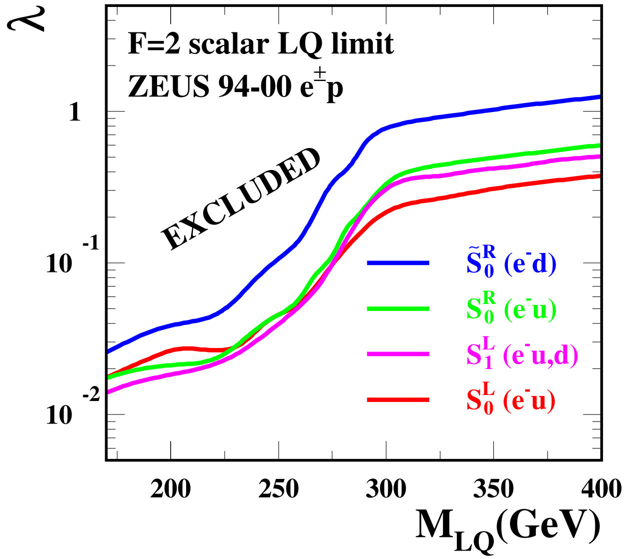
Limits expected for M_S^+ = 0.7 TeV

H1 method



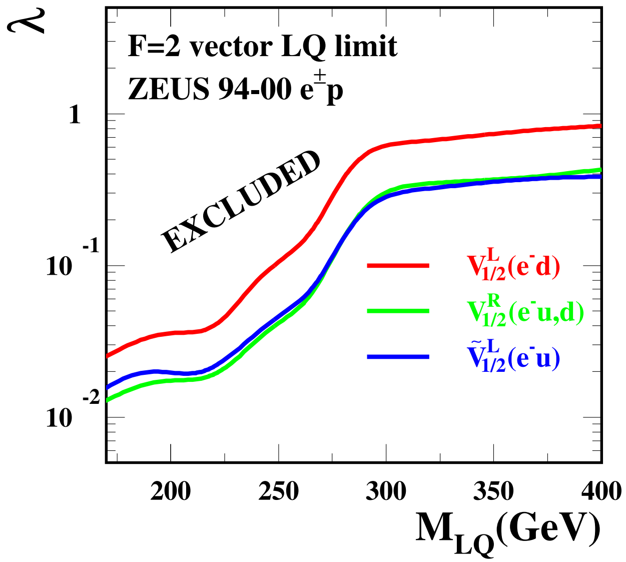
⇒ H1 method results rather in 97.5% CL limits !?

Limits on F=2 BRW LQ from ZEUS (HERA I)



F=2 **scalar** BRW LQ model

⇒Combine eq & νq channels



F=2 **vector** BRW LQ model

⇒ eq channel only

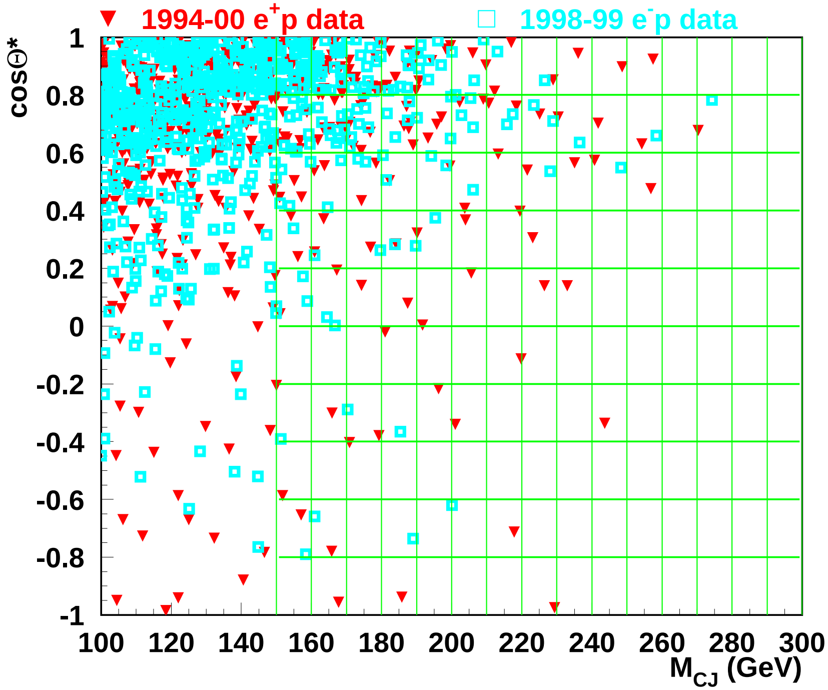


Figure 5.6: Distribution of selected NC DIS type events in the M_{ejs} - $\cos\theta^{\star}_{ejs}$ plane, for the $e^{-}p$ and the $e^{+}p$ data. The grid indicates bins used in the likelihood analysis.

L_i is the function of N_i and μ_i, thus also M_{LQ} and λ_{LQ}. The two dimensional likelihood L is the product of Poisson probabilities over all considered $\cos\theta^{*}$–M_{ljs} bins:

$$L(M_{LQ},\lambda_{LQ}) = \prod_i L_i = \prod_i e^{(-\mu_i)} \frac{{\mu_i}^{N_i}}{N_i!}. \tag{5.6}$$

In this analysis we adopted the Bayesian approach and the upper limit on the coupling strength as a function of M_{LQ}, $\lambda_{limit}(M_{LQ})$, was obtained by solving the equation[1]

$$\int_0^{\lambda^2_{\rm limit}} d\lambda^2 L(M_{LQ},\lambda) = 0.95 \int_0^{\infty} d\lambda^2 L(M_{LQ},\lambda). \tag{5.7}$$

The confidence level of the limit calculated with this method is not exactly equal, but is expected to be close to 95%. This assumption was verified using the so called Monte Carlo Experiments method. More details can be found in Appendix E.

The procedure for determining C.L. is the following:

1. Calculate global Likelihood for our nominal data, assuming SM expectations (i.e. λ=0):

$$\mathcal{L}_{data} = \mathcal{L}(data|SM).$$

2. Calculate the global likelihood for the nominal data taking $SM + LQ$ model expectations with $\lambda = \lambda_{lim}$:

$$\mathcal{L}'_{data} = \mathcal{L}(data|SM + LQ).$$

3. Calculate likelihood ratio Q for our data:

$$Q_{data} = \frac{\mathcal{L}'_{data}}{\mathcal{L}_{data}} = \frac{\mathcal{L}(data|SM + LQ)}{\mathcal{L}(data|SM)}.$$

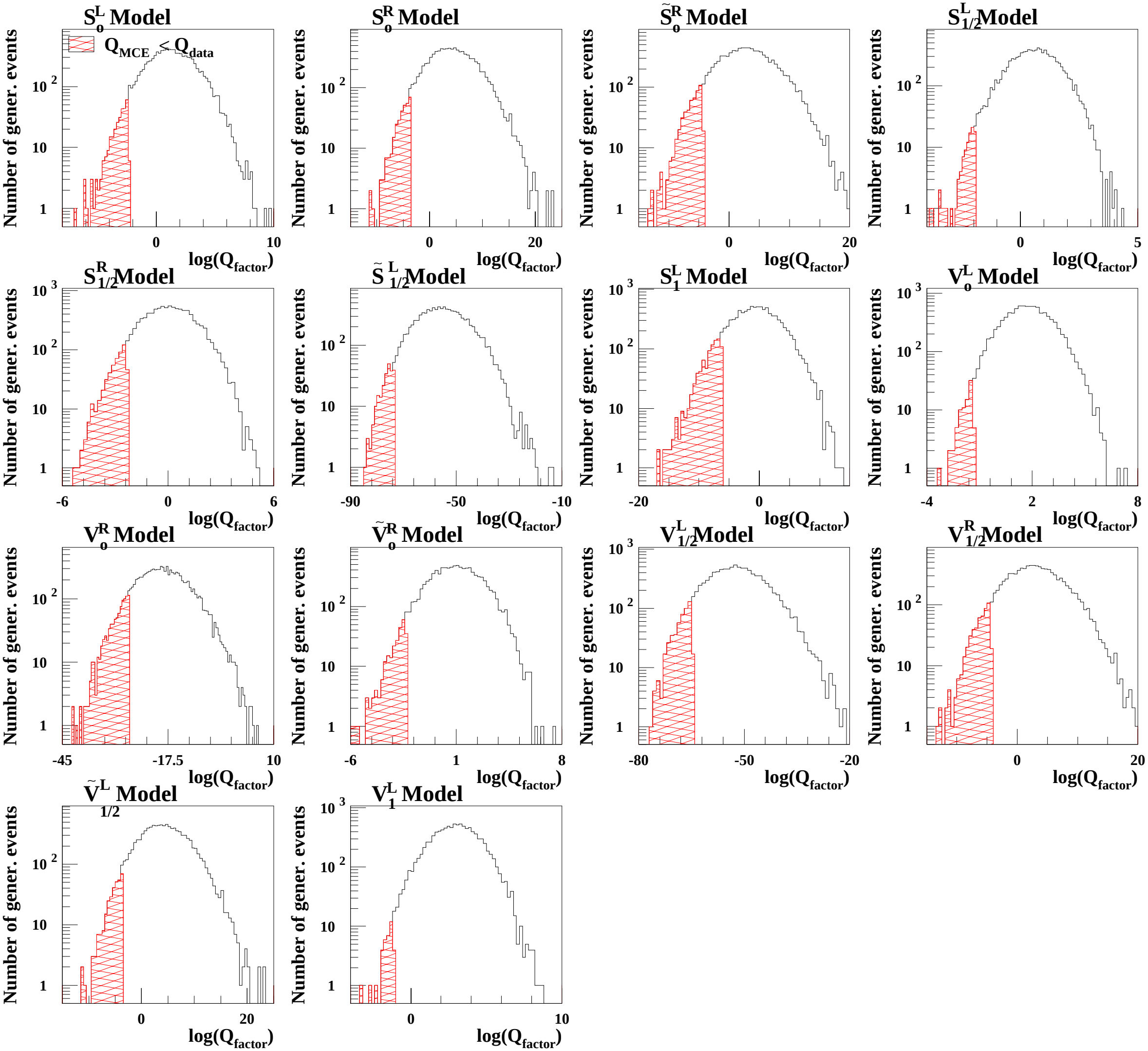
4. Run MC experiments, generating numbers of observed events in each bin according to $SM + LQ$ expectation for $\lambda = \lambda_{lim}$ (from Poisson distribution). For each MC experiment calculate likelihood ratio:

$$Q_{MCE} = \frac{\mathcal{L}'_{MCE}}{\mathcal{L}_{MCE}} = \frac{\mathcal{L}(MCE|SM + LQ)}{\mathcal{L}(MCE|SM)}.$$

5. After running many MC experiments, the C.L. of the limit can be defined as:

$$C.L. = \frac{N(Q_{MC} > Q_{data})}{N_{MC}},$$

i.e. the fraction of experiments for which the Q_{MCE} value was grater than that calculated for real data Q_{data}, so the agreement with SM was worse than in real data (or agreement with $SM + LQ$ better).



S_o^L Model
$Q_{MCE} < Q_{data}$
S_o^R Model
$\tilde{S}_o^R$ Model
$S_{1/2}^L$ Model
$S_{1/2}^R$ Model
$\tilde{S}_{1/2}^L$ Model
S_1^L Model
V_o^L Model
V_o^R Model
$\tilde{V}_o^R$ Model
$V_{1/2}^L$ Model
$V_{1/2}^R$ Model
$\tilde{V}_{1/2}^L$ Model
V_1^L Model
Number of gener. events
$\log(Q_{factor})$

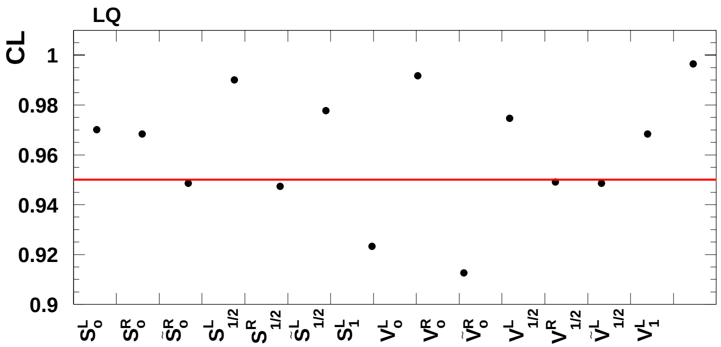


Figure E.4: Confidence Levels for the Yukawa coupling limits in different leptoquark models considered in this analysis.

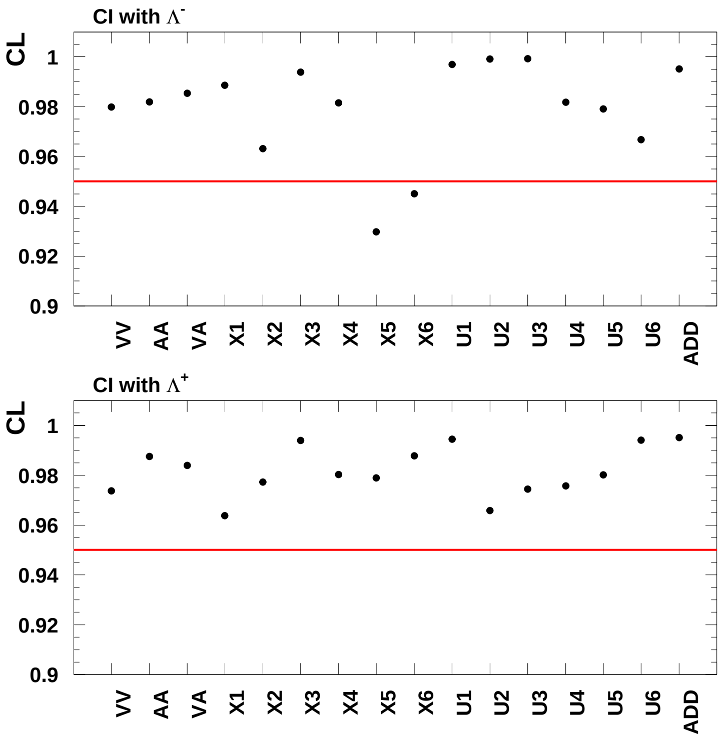


Figure E.5: Confidence Levels for mass scale limits Λ^- and Λ^+, for different contact interaction models considered in this analysis.